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Gravitational Effects on a Hydrogen Atom: Length Contraction and Time Redefinition

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Abstract: This paper investigates the behavior of a hydrogen atom in Schwarzschild spacetime using Quantum Field Theory in Curved Spacetime (QFT-CS), employing a novel global Cartesian-like coordinate system and the vierbein formalism. Our primary contribution is the discovery of a gravitational length contraction effect encoded in the modification of electron's probability density, quantified as $B_0^2 = e_z^3$, which emerges from solving the Dirac equation and reveals a spatial compression subtle near Earth ($\Delta B_0^2 \sim 10^{-26}$ across the bohr radius) but significant in stronger fields. We further reinterpret gravitational redshift and time dilation as local quantum phenomena driven by atomic energy-level modulation via e_0^t, challenging their classical depictions and proposing time as a process-specific parameter rather than a universal dimension. These findings advance the quantum-gravitational synthesis, offering new insights into spacetime's interaction with quantum systems and suggesting testable signatures in extreme gravitational environments.

Keywords: Quantum Field Theory in Curved Spacetime; time dilation; Schwarzschild space-time

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1. Introduction

The interplay between quantum mechanics and general relativity remains a fundamental challenge in modern physics, particularly in understanding the behavior of quantum systems within curved spacetime. Quantum Field Theory in Curved Spacetime (QFT-CS) provides a crucial framework for addressing this issue, revealing gravitational effects on quantum fields that extend beyond classical geodesic motion. Early studies, such as those by DeWitt and Brehme [1], identified deviations from geodesic motion due to quantum wave effects, while subsequent research has examined energy-level shifts in atoms caused by spacetime curvature [2–6].

Considerable progress has been made in analyzing one-electron atoms, particularly hydrogen, within the Schwarzschild metric, which describes the spacetime around a static, spherically symmetric mass. Parker [2] and Pimentel [3] pioneered calculations of gravitationally induced perturbations to the hydrogen spectrum, while Zhao et al. [4] and Gill et al. [5] extended these results to stationary atoms and strong gravitational fields, respectively. These studies typically employ local coordinate systems, such as Riemann Normal Coordinates (RNC) or Fermi Normal Coordinates (FNC) [7,8], which approximate spacetime as locally flat at a single point. While effective in capturing local curvature effects, these coordinate systems are inherently limited in that they cannot directly compare atomic energy levels at spatially separated points in a global gravitational field. Furthermore, these approaches do not explicitly manifest time dilation within the metric itself, as the temporal component is normalized locally. In this work, we adopt a global perspective by reformulating the Schwarzschild metric in Cartesian-like coordinates, defined by $r = \sqrt{x^2 + y^2 + z^2}$, and applying the vierbein formalism to analyze a hydrogen atom in a weak gravitational field, such as near Earth's surface. This approach, detailed in Section 2, enables us to express the Dirac equation in a manner that retains the global structure of the metric. Unlike prior studies, our formulation reveals that gravitationally induced energy shifts are predominantly governed by the temporal vierbein component e_t^0, which reflects the gravitational potential, rather than by local curvature terms alone. Moreover, we identify a

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novel effect: a gravitational contraction of the electron's probability density, modulated by the spatial vierbein component e_z^3. This effect, quantified as $B_0^2 = e_z^3$, leads to a gravitational compression of the wave function, a phenomenon not readily apparent in normal coordinate approaches.

Gravitational redshift and time dilation are traditionally interpreted in general relativity (GR) as consequences of spacetime curvature affecting the passage of time and the frequency of light. Our approach, however, offers a distinct interpretation: we argue that both phenomena arise primarily from the gravitational modulation of atomic energy levels rather than from an intrinsic warping of spacetime itself. In this framework, the observed frequency shifts and variations in clock rates emerge as consequences of local quantum field interactions influenced by gravitational potentials. This perspective diverges from the conventional view, which attributes these effects solely to the geometric deformation of spacetime. While the emergent nature of time has been widely discussed in quantum gravity [9–19], our contribution lies in directly linking this emergence to observable effects such as time dilation and gravitational redshift, grounded in measurable quantum field interactions. By examining how gravitational potentials influence atomic transitions—and consequently, the functioning of timekeeping devices—we propose a novel perspective that integrates quantum field dynamics with gravitational effects, challenging conventional interpretations based purely on spacetime curvature.

Beyond technical refinements, our results offer new insights into the quantum-gravitational interplay. In Section 3, we reinterpret gravitational redshift and time dilation as local quantum phenomena driven by the modulation of atomic energy levels through e_t^0, rather than as intrinsic changes in photon frequency or a universal temporal flow. This perspective challenges the classical notion of time as a uniform and absolute dimension, suggesting instead that it is a process-dependent parameter shaped by gravitational interactions with quantum systems. By contrasting our global-coordinate formulation with previous local-coordinate approaches, we not only confirm established energy shifts but also introduce new theoretical insights with implications for precision measurements and the conceptual foundations of spacetime.

Our theoretical framework employs local Minkowski coordinates (LMCs) and the tetrad formalism to analyze curved spacetime. We use Greek indices (μ, ν) for global coordinates, which describe the spacetime geometry in general relativity, and Latin indices $(a, b = 0, 1, 2, 3)$ for LMCs, representing the flat tangent space at each point. Here, $a, b = 0$ denote the temporal component, while $a, b = 1, 2, 3$ correspond to spatial components. For three-dimensional spatial indices, we use $i, j = 1, 2, 3$. The Minkowski metric is given by $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$, a convention maintained throughout this work.

2. The Hydrogen Atom in Weak Schwarzschild Space-Time

This section examines the hydrogen atom in the weak Schwarzschild spacetime, a foundational solution describing static, spherically symmetric gravitational fields. By solving the Dirac equation using tetrads (vierbeins) tailored to this spacetime, we uncover how spacetime curvature modulates the electron's probability density. In the weak-field regime—such as near Earth's surface—the analysis reveals a gravitational compression of the electron's wavefunction, encoded in the amplitude factor $B_0(z) \propto \sqrt{e_z^3}$, where e_z^3 is a vierbein component tied to the local gravitational potential. This compression, distinct from kinematic relativistic effects, emerges from the interplay between the Coulomb interaction and spacetime curvature, offering insights into quantum phenomena in environments ranging from terrestrial labs to neutron stars. The results highlight how even weak gravitational fields subtly reshape quantum probability structures, with implications for precision atomic spectroscopy and tests of quantum gravity in astrophysical regimes.

2.1. Metric and Vierbein in Cartesian Coordinates

The Schwarzschild spacetime, which characterizes the geometry surrounding a spherically symmetric, non-rotating mass, is conventionally expressed in spherical coordinates. The line element in this system is given by

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2, \tag{1}$$

where $r_s = \frac{2GM}{c^2}$ denotes the Schwarzschild radius, with G as the gravitational constant, M the mass of the central body, and c the speed of light.

While spherical coordinates are well-suited for describing the inherent symmetry of the Schwarzschild solution, certain applications—such as the study of a hydrogen atom in a gravitational field—benefit from a transformation to the Cartesian coordinates (t, x, y, z), where $r = \sqrt{x^2 + y^2 + z^2}$, $\cos\theta = z/r$, and $\tan\varphi = y/x$. In this coordinate system, the metric tensor becomes

$$g_{\mu\nu}(t,x,y,z) = \begin{pmatrix} -\frac{r-r_s}{r} & 0 & 0 & 0 \\ 0 & 1 + \frac{r_s x^2}{(r-r_s)r^2} & \frac{r_s xy}{(r-r_s)r^2} & \frac{r_s xz}{(r-r_s)r^2} \\ 0 & \frac{r_s xy}{(r-r_s)r^2} & 1 + \frac{r_s y^2}{(r-r_s)r^2} & \frac{r_s yz}{(r-r_s)r^2} \\ 0 & \frac{r_s xz}{(r-r_s)r^2} & \frac{r_s yz}{(r-r_s)r^2} & 1 + \frac{r_s z^2}{(r-r_s)r^2} \end{pmatrix}, \tag{2}$$

with the notable property that the determinant satisfies $\sqrt{-g} = 1$, simplifying volume element computations. The vierbein (tetrad) e^a_μ, which is defined by the relation $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$, can be expressed as

$$e^a_\mu = \begin{pmatrix} \frac{1}{f_{r_s}+1} & 0 & 0 & 0 \\ 0 & 1 + \frac{x^2}{r^2} f_{r_s} & \frac{xy}{r^2} f_{r_s} & \frac{xz}{r^2} f_{r_s} \\ 0 & \frac{xy}{r^2} f_{r_s} & 1 + \frac{y^2}{r^2} f_{r_s} & \frac{yz}{r^2} f_{r_s} \\ 0 & \frac{xz}{r^2} f_{r_s} & \frac{yz}{r^2} f_{r_s} & 1 + \frac{z^2}{r^2} f_{r_s} \end{pmatrix}, \tag{3}$$

where $f_{r_s} \equiv \sqrt{\frac{r}{r-r_s}} - 1$. The inverse vierbein e^μ_a, satisfying $e^\mu_a e^b_\mu = \delta^b_a$, is given by

$$e^\mu_a = \begin{pmatrix} f_{r_s} + 1 & 0 & 0 & 0 \\ 0 & 1 - \frac{x^2}{r^2}\frac{f_{r_s}}{f_{r_s}+1} & -\frac{xy}{r^2}\frac{f_{r_s}}{f_{r_s}+1} & -\frac{xz}{r^2}\frac{f_{r_s}}{f_{r_s}+1} \\ 0 & -\frac{xy}{r^2}\frac{f_{r_s}}{f_{r_s}+1} & 1 - \frac{y^2}{r^2}\frac{f_{r_s}}{f_{r_s}+1} & -\frac{yz}{r^2}\frac{f_{r_s}}{f_{r_s}+1} \\ 0 & -\frac{xz}{r^2}\frac{f_{r_s}}{f_{r_s}+1} & -\frac{yz}{r^2}\frac{f_{r_s}}{f_{r_s}+1} & 1 - \frac{z^2}{r^2}\frac{f_{r_s}}{f_{r_s}+1} \end{pmatrix}. \tag{4}$$

The choice of Cartesian coordinates over spherical coordinates is motivated by the specific physical context under consideration. While spherical coordinates naturally reflect the symmetry of the Schwarzschild spacetime, they are less convenient when dealing with systems exhibiting directional dependence, such as a hydrogen atom with its nucleus fixed along the z-axis near Earth's surface. Cartesian coordinates offer several advantages:

- **Directional Alignment:** The hydrogen atom's nucleus, positioned along the z-axis, introduces a preferred direction that breaks spherical symmetry. Cartesian coordinates align naturally with this configuration, simplifying the representation of spatial relationships.
- **Weak-Field Perturbations:** Near Earth's surface, the gravitational field is weak ($r_s/r \ll 1$), and deviations from flat spacetime are small. In Cartesian coordinates, these deviations manifest as perturbative corrections to the Minkowski metric, facilitating analytical approximations.

Thus, the transformation to Cartesian coordinates is a pragmatic choice tailored to the problem's physical constraints and computational requirements.

2.2. Dirac Equation of a Hydrogen Atom in Weak Gravitational Field

The Dirac equation for a hydrogen atom in curved space-time is given by

$$\left[-i\hbar c e^\mu_i \alpha^i \nabla_\mu + \beta m_e c^2 + V_{int}(\vec{x})\right]\Psi(t,\vec{x}) = i\hbar e^t_0 \nabla_t \Psi(t,\vec{x}), \tag{5}$$

where e_i^μ is the vielbein. The Dirac matrices are defined as

$$\beta = \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \alpha^i = \gamma^0\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix},$$

where σ^i are the Pauli matrices. The covariant derivative $\nabla_\mu = \partial_\mu + \Gamma_\mu$ includes the spin connection $\Gamma_\mu \equiv -\frac{1}{8}\omega_\mu^{ab}[\gamma_a, \gamma_b]$, with ω_μ^{ab} given by [20]

$$\omega_\mu^{ab} = e_\nu^a e^{\sigma b}\Gamma^\nu_{\mu\sigma} - e^{\nu b}\partial_\mu e_\nu^a.$$

Consider a hydrogen atom situated near Earth's surface, with its nucleus fixed along the z-axis. The electron's spatial extent is approximately the Bohr radius, $a_0 = 5.29 \times 10^{-11}$ m, while the distance from Earth's center is on the order of the Earth's radius, $z \approx 6 \times 10^6$ m. Consequently, the ratios x/z and y/z are approximately 10^{-17}, allowing the radial distance to be approximated as $r \approx z$. This approximation renders the off-diagonal metric components—proportional to x/z and y/z in Eq. (2)—negligible, on the order of 10^{-17}. The approximations employed are physically motivated and mathematically consistent:

- **Christoffel Symbol Reduction:** With off-diagonal terms suppressed, only a subset of Christoffel symbols remains significant, streamlining subsequent calculations.
- **Weak-Field Limit:** The condition $z \gg r_s$ justifies series expansions of metric and vierbein components, retaining only leading-order gravitational corrections.

These simplifications enable an efficient analysis of gravitational effects on the hydrogen atom, balancing accuracy with computational tractability.

Under these conditions, the non-vanishing Christoffel symbols simplify to

$$\Gamma^t_{tz} = \Gamma^t_{zt} = \frac{1}{2}g^{tt}\partial_z g_{tt}, \quad \Gamma^z_{tt} = -\frac{1}{2}g^{zz}\partial_z g_{tt},$$
$$\Gamma^z_{xx} = g^{zz}\partial_x g_{xz}, \quad \Gamma^z_{yy} = g^{zz}\partial_y g_{yz}, \quad \Gamma^z_{zz} = \frac{1}{2}g^{zz}\partial_z g_{zz}.$$

The partial derivatives of the vierbein components are computed as

$$\partial_z e_t^0 = \frac{r_s\sqrt{z}}{2z^2\sqrt{z-r_s}}, \quad \partial_z e_z^3 = -\frac{r_s\sqrt{z-r_s}}{2(z-r_s)^2\sqrt{z}},$$
$$\partial_x e_x^3 = \partial_x e_z^1 = \partial_y e_y^3 = \partial_y e_z^2 = \frac{\sqrt{z/(z-r_s)} - 1}{z}.$$

Consequently, the spin connection components reduce to

$$\omega_t^{03} = -\omega_t^{30} = \frac{r_s}{2z^2},$$
$$\omega_x^{13} = \omega_y^{23} = -\omega_x^{31} = -\omega_y^{32} = \frac{\sqrt{(z-r_s)/z} - 1}{z} \approx -\frac{r_s}{2z^2},$$

where the approximation $\sqrt{(z-r_s)/z} \approx 1 - \frac{r_s}{2z}$ holds in the weak-field limit ($z \gg r_s$).

In the context of the Dirac equation in curved spacetime, the interaction term involving the gamma matrices and Christoffel symbols simplifies to

$$\gamma^0 e_0^t \Gamma_t + \gamma^x\Gamma_x + \gamma^y\Gamma_y \approx -\frac{r_s}{4z^2}\begin{pmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{pmatrix}, \tag{6}$$

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where $e_0^t \approx e_1^x \approx e_2^y \approx 1$, with second-order terms in $(r_s/z)^2$ neglected due to their small magnitude ($\sim 10^{-18}$ near Earth's surface, where $r_s \approx 8.87$ mm for Earth).

The Coulomb interaction in this weak Schwarzschild metric is given by [see appendix]

$$V_{int} = q e_0^t A_t = \frac{-q^2}{4\pi\epsilon_0 \sqrt{x^2 + y^2 + f^2(z)}} + \epsilon(V_{int}), \tag{7}$$

where $\epsilon(V_{int})$ represents the small correction term induced by the perturbation of the metric inside of the hydrogen atom.

2.3. Probability Density Modulation of the Hydrogen Atom

The Dirac equation, when expressed in matrix form in a curved spacetime, takes the following structure:

$$\begin{pmatrix} i\hbar e_0^t \partial_t - V_{\text{int}} - mc^2 & i\hbar c\left(e_j^\mu \sigma^j \partial_\mu - \frac{r_s}{4z^2}\sigma^3\right) \\ i\hbar c\left(e_j^\mu \sigma^j \partial_\mu - \frac{r_s}{4z^2}\sigma^3\right) & i\hbar e_0^t \partial_t - V_{\text{int}} + mc^2 \end{pmatrix} \begin{pmatrix} \psi_1^s \\ \psi_2^s \end{pmatrix} = 0, \tag{8}$$

where m is the particle mass, s indicates the spin index. The general solution to this equation can be written as:

$$\begin{pmatrix} \psi_1^s \\ \psi_2^s \end{pmatrix} = e^{-\frac{i}{\hbar} e_t^0 E_0 t} \left[B_0(z) \begin{pmatrix} \phi_1^s(x, y, f(z)) \\ \phi_2^s(x, y, f(z)) \end{pmatrix} + \begin{pmatrix} B_1^s(\vec{r}) \\ B_2^s(\vec{r}) \end{pmatrix} \right], \tag{9}$$

where $B_0(z)$ is a scalar modulation factor dependent on the radial coordinate z, ϕ_1^s and ϕ_2^s are spinor components, and $\begin{pmatrix} B_1^s(\vec{r}) \\ B_2^s(\vec{r}) \end{pmatrix}$ represents small perturbative corrections.

In the zeroth-order approximation, neglecting correction terms, we obtain:

$$B_0(z) \begin{pmatrix} E_0 - V_{\text{int}} - mc^2 & i\hbar c\, e_i^\mu \sigma^i \partial_\mu \\ i\hbar c\, e_i^\mu \sigma^i \partial_\mu & E_0 - V_{\text{int}} + mc^2 \end{pmatrix} \begin{pmatrix} \phi_1^s(x, y, f(z)) \\ \phi_2^s(x, y, f(z)) \end{pmatrix} = 0. \tag{10}$$

Here, E_0 represents the energy levels of the atom at spatial infinity ($z \to \infty$). The correction terms, when considered, satisfy a coupled equation:

$$\begin{gathered} \begin{pmatrix} \epsilon(V_{int}) & i\hbar c\left(-\frac{r_s}{4z^2}\sigma^3\right) \\ i\hbar c\left(-\frac{r_s}{4z^2}\sigma^3\right) & \epsilon(V_{int}) \end{pmatrix} \begin{pmatrix} B_0(z)\phi_1^s(x, y, f(z)) \\ B_0(z)\phi_2^s(x, y, f(z)) \end{pmatrix} + \\ \begin{pmatrix} i\hbar e_0^t \partial_t - V_{\text{int}} - mc^2 & i\hbar c(e_j^\mu \sigma^j \partial_\mu - \frac{r_s}{4z^2}\sigma^3) \\ i\hbar c(e_j^\mu \sigma^j \partial_\mu - \frac{r_s}{4z^2}\sigma^3) & i\hbar e_0^t \partial_t - V_{\text{int}} + mc^2 \end{pmatrix} \begin{pmatrix} B_1^s(\vec{r}) e^{-\frac{i}{\hbar} e_t^0 E_0 t} \\ B_2^s(\vec{r}) e^{-\frac{i}{\hbar} e_t^0 E_0 t} \end{pmatrix} = 0. \end{gathered} \tag{11}$$

In the zeroth-order approximation, the solution simplifies to:

$$\begin{pmatrix} \psi_1^s \\ \psi_2^s \end{pmatrix} = e^{-\frac{i}{\hbar} e_t^0 E_0 t} B_0(z) \begin{pmatrix} \phi_1^s(x, y, f(z)) \\ \phi_2^s(x, y, f(z)) \end{pmatrix}, \tag{12}$$

where the spinor components ϕ_1^s and ϕ_2^s exhibit a structure analogous to those in Minkowski spacetime. However, in the presence of a gravitational field, the coordinate z is replaced by a gravitationally modified coordinate $f(z)$, and the components are further modulated by a gravitational factor $B_0(z)$.

Notably, the energy levels of the atom within a gravitational field are given by $e_t^0 E_0$, making them explicitly dependent on location. This position-dependent energy shift has profound physical implications, directly leading to observable phenomena such as gravitational redshift and time dilation. A detailed exploration of this relationship will be presented in Section 3, where we will analyze the interplay between these effects and their broader consequences.

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To determine $B_0(z)$, we apply the continuity equation for the probability current:

$$\nabla_\mu(\bar{\psi}\gamma^\mu\psi) = \nabla_\mu\left(B_0^2\,\bar{\phi}\,\gamma^\mu\phi\right) = B_0^2\,\partial_x(\bar{\phi}\,\gamma^1\phi) + B_0^2\,\partial_y(\bar{\phi}\,\gamma^2\phi) + \partial_z\left(B_0^2\,\bar{\phi}\,e_3^z\,\gamma^3\phi\right) = 0. \tag{13}$$

Given $\partial_z\phi(x,y,f(z)) = e_z^3\,\partial_f\phi(x,y,f(z))$ and the flat-space continuity condition:

$$\partial_x(\bar{\phi}\,\gamma^1\phi) + \partial_y(\bar{\phi}\,\gamma^2\phi) + \partial_f(\bar{\phi}\,\gamma^3\phi) = 0, \tag{14}$$

we derive:

$$B_0^2 = e_z^3. \tag{15}$$

This result establishes a direct correspondence between the probability density amplitude $B_0(z)$ and the vierbein component e_z^3, which encodes the local gravitational field geometry. The amplitude $B_0(z)$ governs the spatial modulation of the wavefunction: in proximity to gravitational sources (small z), the probability density exhibits a relativistic compression, whereas at larger z, the normalization adapts to conserve total probability. This curvature-induced asymmetry subtly distorts the spatial profile of bound quantum states, such as those of the hydrogen atom.

As a concrete example, consider a hydrogen atom near Earth's surface. The difference between the probability density across the Bohr radius r_0 is quantified by the asymmetry parameter:

$$\Delta B_0^2 = \frac{r - r_0}{r - r_s - r_0} - \frac{r + r_0}{r - r_s + r_0} \approx \frac{2r_0 r_s}{r^2}, \tag{16}$$

where r denotes the radial distance from Earth's center to the proton. Substituting $r_0 \approx 5.3 \times 10^{-11}$ m (Bohr radius) and $r \approx 6.37 \times 10^6$ m (Earth's radius), we compute $\Delta B_0^2 \sim 10^{-26}$—a value far below the threshold of current experimental detectability. A dramatic contrast emerges near compact astrophysical objects ($r \sim r_s$), where the vierbein component e_z^3 varies steeply across atomic scales. Here, ΔB_0^2 is amplified by several orders of magnitude, potentially inducing measurable distortions in bound-state radii. Such effects could serve as novel signatures of quantum-gravitational coupling, accessible through precision spectroscopy of matter in extreme environments—for instance, in neutron star atmospheres or accretion disks around black holes. While terrestrial experiments remain beyond present technical capabilities, advancements in high-resolution astrophysical observations may enable empirical exploration of these curvature-driven quantum phenomena in the future.

We observe that the results B_0 obtained for a bound system differ fundamentally from those of a free particle, as discussed in [21]. For a free particle, the probability density decreases with increasing gravitational strength, as B_0^2 is inversely proportional to the particle's momentum. Since the momentum increases as the particle moves toward the gravitational center, probability current conservation is maintained. For a static electron bound by a Coulomb potential, the probability distribution distorted by gravity is encoded in the factor e_z^3 , which increases in stronger gravitational fields. This factor arises from the derivative of the function $f(z)$, which encapsulates the gravitationally induced distortion of the electron's probability distribution. In contrast, For a bound system, considering the entire system—such as a hydrogen atom including the proton—the system as a whole can be approximated as a free particle. Consequently, the probability density distribution B_0^2 of this combined system becomes approximately inversely proportional to its total momentum.

In summary, our analysis demonstrates that gravitational fields induce a measurable compression of the electron's probability density through the amplitude factor $B_0(z)$. This factor, tied to the vierbein component e_z^3, encapsulates the influence of spacetime curvature on quantum states and provides a clear link between the distance z from the gravitational source and the local probability density amplitude. Future investigations may further explore how this gravitational modulation affects other quantum phenomena, potentially offering new insights into quantum field theory in strong gravitational regimes.

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3. Gravitational Redshift, Time Dilation, and the Nature of Time

Traditional interpretations of gravitational redshift and time dilation, rooted in general relativity (GR), treat these phenomena as universal effects arising from spacetime geometry: all clocks, irrespective of their mechanism, are predicted to slow identically in gravitational fields due to the metric's temporal component, g_{tt} [22]. This view posits spacetime as a dynamic, physical continuum shaping physical processes. However, foundational critiques have challenged this assumption. Komar [23] questioned whether spacetime is a tangible entity or a mathematical construct encoding event relationships, while recent perspectives by Rovelli [24] suggest spacetime emerges from quantum interactions, with Marletto and Vedral [25] proposing quantum probes to test this hypothesis. Most recently, a unitary QFT-CS approach [26] treats time as a parameter emergent from quantum dynamics, not a spacetime dimension. Despite these advances, the prevailing paradigm retains time dilation as a universal property of a geometric spacetime fabric. In this section, we radically depart from this view, building on our global Schwarzschild coordinate analysis from Section 2. We propose that gravitational redshift and time dilation are local quantum phenomena, driven by the modulation of atomic energy levels via the temporal vierbein e_t^0, rather than intrinsic spacetime effects. This leads to a redefinition of time as a process-specific parameter shaped by system-dependent gravitational interactions, challenging the notion of a uniform temporal continuum and aligning with relational interpretations of spacetime.

Section 3 proceeds as follows: Subsection 3.1 reinterprets gravitational redshift through atomic energy shifts in QFT-CS. Subsection 3.2 analyzes time dilation using a twin paradox with dual clock types, exposing its process-specific nature. Subsection 3.3 redefines time as a parameter shaped by physical systems' dynamics, and Subsection 3.4 discusses implications for time travel and wormholes, precluded by the non-universality of time.

3.1. Gravitational Redshift and Atomic Clocks as Local Quantum Field Phenomena

In general relativity, gravitational redshift is traditionally interpreted as a shift in the frequency of a photon as it traverses a gravitational field, while time dilation is attributed to the slowing down of time in regions of strong gravity. However, the framework of QFT-CS offers an alternative perspective on these phenomena.

As previously established [21], the wave function of a single-mode photon propagating along a geodesic in Schwarzschild spacetime is given by $B_k e^{-i\int p_\mu dx^\mu}$. Importantly, the temporal component of the four-momentum, p_t, remains conserved due to the geodesic equation. As a result, the energy-dependent contribution to the photon's wave function takes the form $e^{-\frac{i}{\hbar}p_t t}$, where p_t remains constant throughout the photon's propagation.

In contrast, the energy-dependent component of an atomic wave function, representing either the emitter or absorber of the photon, is given by

$$e^{-\frac{i}{\hbar}e_t^0 E_0 t}, \tag{17}$$

where E_0 represents the atomic energy levels in Minkowski spacetime, corresponding to their values at spatial infinity in Schwarzschild spacetime. The factor e_t^0 accounts for the gravitational influence on energy levels, which vary depending on the atom's location in the gravitational field.

This perspective reveals that the gravitational frequency shift does not result from an intrinsic change in the photon's frequency during propagation. Instead, it arises due to the variation in atomic energy levels at different gravitational potentials. The photon maintains a constant frequency, but because the energy gap between atomic states changes with altitude, an observer measuring frequency relative to local atomic transitions perceives a redshift.

In a similar vein, we assert that time dilation, as observed in atomic clocks, arises from the gravitational modulation of atomic energy levels across varying field strengths. Clocks measure time by counting periodic processes: spring clocks tally mechanical vibrations, pendulum clocks track swings governed by gravitational acceleration, and atomic clocks count quartz crystal vibrations

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stabilized in resonance with atomic transition energies. For example, a cesium atomic clock defines one second by counting $N_{\rm at}$ vibrations of a quartz oscillator, locked to the hyperfine transition of cesium-133 ($F = 3 \leftrightarrow F = 4$, $\Delta E \approx 9.192\,\mathrm{GHz} \times h$) via a microwave field tuned to this frequency. The clock increments its displayed time by $T_{\rm dis} = 1/N_{\rm at}$ seconds per quartz cycle.

The underlying mechanism integrates quantum electrodynamics and feedback control: a quartz oscillator generates vibrations, which drive a microwave field to probe the cesium atoms' transition. When the microwave frequency matches $\nu = \Delta E/h$, where $\Delta E = e_t^0 \Delta E_0$ and $e_t^0 = 1/\sqrt{-g_{tt}}$ reflects the gravitational potential, a feedback loop adjusts the quartz frequency to maximize resonance, stabilizing its count. In stronger gravitational fields, e_t^0 decreases, reducing ΔE and thus the resonant frequency, slowing the quartz vibrations and manifesting as time dilation. This dependence on g_{tt} highlights time dilation as a local quantum phenomenon specific to atomic dynamics, distinct from mechanical clocks governed by macroscopic forces.

3.2. Twin Paradox with Dual Clock Types

It is imperative to recognize that clocks are merely instruments, and the time they display reflects only the count of specific physical events occurring within their mechanisms. Consequently, in time dilation experiments comparing two identical clocks, what is measured is not an absolute temporal flow but the respective numbers of these events—e.g., oscillations or transitions—recorded by each device. The transformation of this count under varying conditions, such as high-speed motion or gravitational gradients, hinges solely on the clock's underlying physics. For atomic clocks, this necessitates a theoretical analysis of how atomic energy levels, governed by the local metric component e_t^0, shift across different gravitational fields, yielding $\Delta E = e_t^0 \Delta E_0$. While mechanical clocks (e.g., pendulum and spring clocks) couple to gravitational acceleration $g \propto -\nabla g_{tt}$, involving metric derivatives. Thus, atomic and mechanical clocks probe distinct gravitational aspects—local potential versus its gradient—leading to divergent responses in non-uniform fields.

To further illustrate the implications of the QFT-CS framework, we consider a twin paradox scenario involving two types of clocks: atomic clocks and mechanical clocks (such as spring clocks). Alice and Bob are twins who carry identical pairs of clocks—an atomic clock and a mechanical clock—initially synchronized. Bob remains in orbit around a massive planet, while Alice lands on its surface, experiencing a stronger gravitational field. Upon re-reunification, the clocks display the following:

Atomic Clocks: Alice's atomic clock shows less elapsed time (t_A) compared to Bob's atomic clock (t_B), i.e., $t_A < t_B$. This is because the gravitational potential on the planet's surface is stronger, leading to $e_{t,A}^0 < e_{t,B}^0$.

Mechanical Clocks: Bob's mechanical clock may show less elapsed time compared to Alice's mechanical clock, depending on its design. This is because the effective gravitational acceleration g experienced by Bob in orbit is weaker than that experienced by Alice on the planet's surface. Mechanical clocks measure time based on mechanical oscillations, which are influenced by the local gravitational field. A weaker g in orbit could result in a slower oscillation frequency for Bob's mechanical clock, causing it to accumulate less time compared to Alice's.

This discrepancy—Alice younger by atomic measure, Bob by mechanical—highlights that time dilation is process-specific. Atomic clocks align with GR's predictions, while mechanical clocks, sensitive to ∇g_{tt}, deviate, underscoring their unsuitability for direct relativistic tests. Biological aging, involving chemical and mechanical processes, may similarly diverge from atomic clock predictions. For instance, metabolic rates might scale with g, complicating gravitational effects on physiology beyond QFT-CS's atomic focus.

The twin paradox scenario with atomic and mechanical clocks demonstrates that the concept of time dilation is not universally applicable across all timekeeping devices. Instead, it depends on the specific physical processes underlying each type of clock. This has important implications for experimental tests of relativity and the design of precision timekeeping devices.

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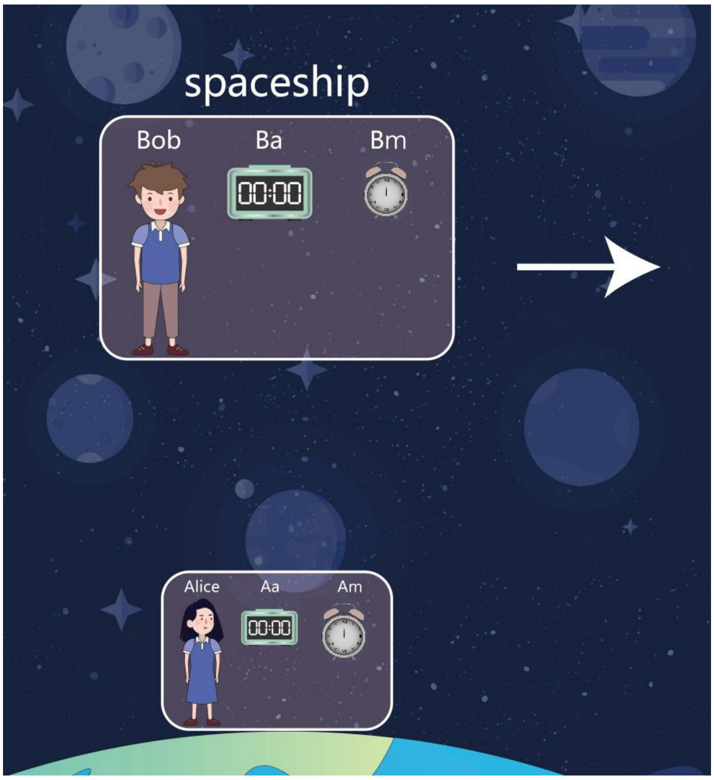

Figure 1. Dual Clock Types of Twin Paradox. Alice remains on the surface of a massive planet, experiencing a stronger gravitational field, while Bob orbits above. The atomic clocks (sensitive to e_t^0) show Alice's clock Aa ticking slower ($t_A < t_B$) than Bob's clock Ba, while the mechanical clocks (sensitive to ∇g_{tt}) may show Bob's clock Bm ticking slower than Alice's clock Am, depending on design. This highlights the process-specific nature of time dilation.

3.3. The Nature of Time

The QFT-CS framework unveils a critical insight: time is not a universal, intrinsic property of spacetime but a parameter shaped by the specific physical process it describes. In general relativity (GR), spacetime—embodied in the metric $g_{\mu\nu}$—serves as a mathematical scaffold to model the dynamics of physical systems, not as a tangible entity with inherent uniformity. This perspective reveals that phenomena such as atomic transitions, mechanical oscillations, or chemical reactions may exhibit distinct temporal rates under varying gravitational influences, challenging the notion of a singular, absolute time.

This view departs from the traditional conception of spacetime as a physical continuum independent of matter's governing laws, which assumes that coordinate transformations apply uniformly across all physical regimes. Yet, GR's operational role belies this assumption: the Einstein field equations describe how mass-energy shapes particle motion, with spacetime inferred from these interactions rather than directly measured. QFT-CS extends this relational stance, positing that spacetime emerges from quantum field dynamics, lacking an observable, standalone existence.

In QFT-CS, time's role as a mathematical parameter is process-specific, its rate determined by how gravity interacts with each system. Atomic clocks, for instance, track time via transition frequencies modulated by the local metric component e_t^0, while pendulum clocks rely on gravitational acceleration g. Chemical reactions or biological processes might blend these influences, producing unique temporal signatures. This divergence—evident in the differing responses of atomic and mechanical clocks to gravitational gradients—demonstrates that time is not a monolithic flow but a reflection of system-specific dynamics.

Consequently, the traditional view of spacetime as a robust, transformable entity collapses under scrutiny. QFT-CS suggests that time arises from the interplay of physical processes with gravity—e.g., e_t^0-driven atomic transitions or g-dependent oscillations—rather than an innate property of a physical

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continuum. Biological clocks, such as metabolic cycles, further underscore this complexity, potentially misaligning with mechanical or atomic measures. Time, therefore, lacks independent meaning without reference to a specific phenomenon; it is a derived construct, not a universal dimension, dismantling the assumption of a uniform temporal rate across all regimes.

3.4. Implications for Time Travel and Wormholes

The QFT-CS redefinition of time as a process-dependent parameter carries significant consequences for concepts like time travel and wormholes, which hinge on a uniform spacetime amenable to global manipulation. Conventional time travel scenarios—via closed timelike curves (CTCs) in solutions like Gödel's universe or Kerr spacetime [27]—presume that time is a single, traversable coordinate, uniformly shifting all physical systems. Similarly, traversable wormholes [28] assume spacetime can be engineered to connect distant points or times consistently across all phenomena. Our analysis, however, reveals these assumptions as untenable.

Time travel through CTCs envisions a synchronized temporal shift for all processes—atomic, mechanical, and biological. Yet, QFT-CS demonstrates that time's flow is tied to specific dynamics, such as $\Delta E = e_t^0 \Delta E_0$ for atomic transitions or g-driven oscillations for mechanical clocks. In a gravitational gradient, an atomic clock's rate diverges from a mechanical's, as explored earlier. A traveler entering a CTC might find their atomic clock registering fewer oscillations (an earlier "time"), while mechanical or biochemical processes (e.g., protein folding) advance due to differing gravitational couplings. This desynchronization renders time travel incoherent: without a process-independent time coordinate, "traveling back" loses physical grounding, as time is a fragmented, system-specific measure, not a universal entity.

Wormholes face a parallel critique. Proposed as spacetime shortcuts stabilized by exotic matter [28], they assume a cohesive geometry that all phenomena experience identically. QFT-CS, however, frames spacetime as an emergent construct from particle interactions, not a malleable medium. The metric $g_{\mu\nu}$ dictates local effects—e.g., e_t^0 for atomic clocks—but offers no basis for a globally manipulable fabric. A wormhole connecting regions of differing potential would yield varying energy levels and probability densities (e.g., B_k), disrupting uniform transit across diverse systems. Even if feasible, wormholes would not universally "connect times," as atomic, mechanical, and biological clocks would respond differently, echoing the twin paradox discrepancy.

Thus, QFT-CS precludes both time travel and wormholes by denying the universal time dilation they require. This aligns with quantum constraints on exotic matter [29] and Hawking's Chronology Protection Conjecture [27], but goes further: time's process-specific nature fundamentally undermines the coherence of such paradigms, reinforcing spacetime's relational, rather than substantive, character.

3.5. Conclusion

The QFT-CS framework unifies gravitational redshift and time dilation as local quantum phenomena, driven by the gravitational modulation of atomic energy levels through $e_t^0 = 1/\sqrt{-g_{tt}}$. Atomic clocks, rooted in quantum field dynamics, faithfully reflect GR predictions, with transition frequencies ($\Delta E = e_t^0 \Delta E_0$) tied to the local potential, while mechanical clocks—responsive to metric gradients (∇g_{tt})—diverge, clarifying gravitational timekeeping discrepancies. This process-specific insight extends to complex systems, raising questions for gravitational biology: how do molecular or biological rates, potentially influenced by g_{tt} or g, vary with gravity?

This unified description validates relativistic effects for quantum systems while exposing the limitations of mechanical probes, offering a lens to explore gravity's broader impact. Central to this framework is a redefinition of time: rather than a universal flow, time emerges as a parameter shaped by the dynamics of specific processes—atomic transitions, oscillations, or biochemical reactions—each uniquely modulated by gravity. Clocks count these phenomena, their rates reflecting gravitational interactions, not an intrinsic temporal continuum. This challenges spacetime's traditional status as a tangible entity, aligning with GR's view of it as a mathematical tool encoding physical relationships.

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The implications are far-reaching. By grounding time in measurable, system-specific dynamics, QFT-CS precludes universal spacetime transformations like time travel or wormhole traversal, which assume a cohesive fabric absent here. Spacetime emerges as relational, inferred from particle behavior, not an independent stage. This perspective deepens our understanding of quantum-gravitational interplay, inviting exploration of gravity's effects on quantum systems and biological processes, and advocating a nuanced, phenomenon-centric approach to temporal dynamics.

4. Discussion

This study leverages QFT-CS to explore a hydrogen atom in the Schwarzschild geometry, revealing key quantum-gravitational effects through a novel global coordinate approach. By transforming the metric into Cartesian-like coordinates and employing the vierbein formalism, we uncover a gravitational length contraction effect and reinterpret classical gravitational phenomena, offering fresh insights into their quantum underpinnings.

In Section 2, we demonstrate that the electron's probability density contracts under gravity, quantified as $B_0^2 = e_z^3$, the spatial vierbein component. Near Earth, this effect is subtle ($\Delta B_0^2 \sim 10^{-26}$ across the Bohr radius), but it amplifies near compact objects ($r \sim r_s$), potentially altering atomic structure measurably. This spatial modulation, driven by the global metric, complements energy-level shifts governed by the temporal vierbein e_t^0, providing a dual perspective on gravity's influence on quantum states.

Section 3 extends these findings to reinterpret gravitational redshift and time dilation as local quantum phenomena. Redshift arises from variations in atomic energy levels ($\Delta E = e_t^0 \Delta E_0$) across potentials, not photon frequency shifts, while time dilation reflects process-specific frequency adjustments in atomic clocks, distinct from mechanical systems. This leads to a broader conclusion: time is not a universal dimension but a parameter tied to specific gravitational interactions, challenging notions of a manipulable spacetime fabric.

Our contributions—the length contraction effect and the process-specific nature of time—highlight gravity's nuanced impact on quantum systems. The former suggests a detectable signature in strong fields, while the latter redefines temporal dynamics, opening avenues for exploring quantum effects in extreme environments and complex systems like biological processes. Together, they advance the synthesis of quantum mechanics and general relativity, emphasizing a phenomenon-centric approach to spacetime.

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Appendix: Coulomb Potential in Curved Spacetime

The Coulomb potential in curved spacetime arises from the covariant formulation of Maxwell's equations:

$$\nabla_\mu F^{\mu\nu} = \mu_0 J^\nu, \tag{A18}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ represents the electromagnetic field tensor, and J^ν denotes the four-current density. For a static point charge at the origin in Schwarzschild spacetime, the four-current takes a particularly simple form. In the coordinate basis, we have $J^\nu = (J^t, 0, 0, 0)$, where the temporal component incorporates both the metric structure and charge distribution:

$$J^t = e_0^t \rho = e_0^t \frac{q\delta^{(3)}(\vec{r})}{\sqrt{\gamma}}. \tag{A19}$$

Here, γ is the determinant of the induced spatial metric $\gamma_{\mu\nu} = e_\mu^i e_\nu^j \delta_{ij}$, with e_μ^i being the tetrad fields that relate the coordinate basis to an orthonormal frame.

The Maxwell equation for the time component in curved spacetime simplifies to:

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$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}F^{\mu t}) = \mu_0 J^t, \tag{A20}$$

where the field tensor components are given by $F^{\mu t} = g^{\alpha\mu}g^{t\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha)$. For the Schwarzschild metric in Cartesian-like coordinates where $\sqrt{-g} = 1$, this equation reduces to:

$$g^{tt}\nabla^2 A_t + \partial_\mu(g^{tt}g^{\alpha\mu})\partial_\alpha A_t = q\mu_0 e_0^t \frac{\delta^{(3)}(\vec{r})}{e_z^3}. \tag{A21}$$

where $\nabla^2 \equiv g^{xx}\partial_x^2 + g^{yy}\partial_y^2 + g^{zz}\partial_z^2$ as we have neglected small off-diagonal metric components, indicated in Eq. (2). For weak gravitational fields, such as those encountered near Earth's surface, the term $\partial_\mu(g^{tt}g^{\alpha\mu})$ can be treated as a perturbation. To zeroth order, the solution for the electrostatic potential takes the form:

$$A_t = \frac{q}{4\pi\epsilon_0 e_0^t\sqrt{x^2 + y^2 + f^2(z)}} + \epsilon(A_t), \tag{A22}$$

where the function $f(z)$ is:

$$f(z) = \int_{z_p}^{z} e_{z'}^3 dz'. \tag{A23}$$

Here, z_p denotes the proton's position relative to Earth's center, and $\epsilon(A_t)$ represents higher-order corrections arising from metric derivatives in the proton's vicinity. The tetrad component e_z^3 accounts for the global geometric effects on vertical distances. This solution demonstrates how general relativistic effects modify the classical Coulomb potential through both the explicit metric dependence and the tetrad fields that mediate between coordinate and physical measurements.

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