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Finite Informational Capacity and the Emergence of the Speed of Light

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Abstract

We introduce a projectional framework—termed TAP (Transcendental Projection)—in which the observable (3!+!1)-dimensional universe arises as the projected image of a higher-dimensional informational manifold under an operator $\mathcal{P}$. Finite temporal and spatial informational capacities on the projection surface enforce, through a constrained variational principle, a strictly hyperbolic effective field equation with characteristic velocity $c = \sqrt{\beta_{\text{eff}}/\alpha_{\text{eff}}}$. Within this structure, non-retentive (massless) modes necessarily propagate at c, while retentive modes are restricted to subluminal velocities. An associated energy–flux inequality provides a local information-theoretic bound, establishing the universal speed limit as an emergent capacity ratio rather than a postulate. We further situate this result in relation to relativity, quantum field theory, holography, and cosmology.

Keywords: speed of light; universal constant; projectional dynamics; Transcendental Projection (TAP); informational capacity; hyperbolic field equations; causality and Lorentz invariance; massless vs massive excitations; arrow of time; entropy emergence; holography; information geometry

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1. Introduction

The constancy of the speed of light, c, and the existence of massless excitations such as photons stand as cornerstones of modern physics. In special relativity, c appears as a structural constant of the Minkowski metric, delineating causal cones and enforcing Lorentz invariance [1,2]. In quantum field theory (QFT), the photon is identified as the massless gauge boson of the $U(1)$ symmetry, with its strictly luminal propagation guaranteed by gauge invariance [3]. In cosmology, the cosmic microwave background (CMB) is treated as a relic photon bath encoding the universe's thermal history, with photon dynamics central to its interpretation [4].

Yet, despite these successes, the existence of a universal speed limit is not derived from deeper first principles but rather imposed axiomatically. Relativity stipulates c as a postulate, while QFT encodes it into the causal structure of field commutators. This raises a foundational question: *why must there be a finite maximum velocity for information transfer, and why do certain excitations—specifically photons—propagate exactly at this bound?* Addressing this requires a framework in which the speed limit is not assumed, but instead emerges as a consequence of more primitive constraints.

Various approaches have attempted to address this gap. Information-theoretic perspectives, such as Wheeler's "It from Bit" program [5], emphasize the primacy of information over geometry but do not explain how a finite c arises. Holographic principles [6,7] relate spacetime degrees of freedom to lower-dimensional boundaries, suggesting a connection between information content and causal structure, but stop short of deriving a universal velocity bound. Likewise, loop quantum gravity, spin networks, and causal set models hint at discretized or capacity-limited structures, yet have not demonstrated the emergence of c as a derived constant.

In this work, we advance the *Transcendental Projection (TAP)* framework, in which the observable (3+1)-dimensional universe is understood as the projected image of a higher-dimensional informational manifold under a projection operator $\mathcal{P}$. The central hypothesis is that the projection surface—the effective spacetime on which physical processes unfold—possesses *finite informational capacity*, both in

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temporal update rate and spatial resolution. These finite capacities impose intrinsic constraints on how rapidly information can popagate across the surface. The TAP framework was originally introduced in our earlier work on time's arrow and entropy emergence *(doi 10.20944/preprints202508.0342.v1)* , and here it is extended to derive the universal speed limit as a capacity-induced structural bound.

We formalize this by introducing a constrained variational principle for the projected field $\phi(x, t)$, which yields a hyperbolic partial differential equation with characteristic velocity

$$c = \sqrt{\frac{\beta}{\alpha}},$$

where α and β encode temporal and spatial capacity weightings, respectively. In this formulation, the universal speed limit arises not as an external postulate but as the natural consequence of projectional geometry and finite informational capacity.

Within TAP, the photon appears naturally as a *boundary mode* with vanishing retention parameter $(\mu = 0)$, propagating precisely at the emergent capacity-imposed speed limit c. Excitations with nonzero retention $(\mu > 0)$, by contrast, accumulate information on the projection surface and therefore propagate subluminally, consistent with massive particles. This duality provides an integrated account of why massless particles move at c, while massive excitations are restricted to $v < c$.

The broader implication is that fundamental constants and structural features of physics—most notably the invariance of c—may be reinterpreted as emergent properties of projectional capacity rather than axiomatic givens. This perspective reframes puzzles about causality, Lorentz invariance, and the arrow of time, while opening new possibilities for unifying relativity, QFT, and cosmology under a common information-geometric principle.

[1] Einstein, *Relativity* (1905/1915)
[2] Feynman, *The Character of Physical Law* (1965)
[3] Peskin & Schroeder, *An Introduction to QFT*
[4] Dodelson, *Modern Cosmology*
[5] Wheeler, "It from Bit"
[6] 't Hooft, on holography
[7] Susskind, on holography

2. Projection Operator and Capacity Constraints

2.1. Setting and Notation

We model the observable universe as an effective (3+1)-dimensional interface $\Sigma \cong \mathbb{R}^3 \times \mathbb{R}$ (with spatial coordinate $x \in \mathbb{R}^3$ and time $t \in \mathbb{R}$) on which a projected field $\phi : \Sigma \to \mathbb{R}$ (or $\mathbb{C}$, or a vector bundle, as appropriate) is defined. The "ontic" or higher-dimensional informational state is denoted $\Phi : \mathcal{M}_H \to \mathbb{R}$, where $\mathcal{M}_H$ is a (possibly non-compact) manifold of dimension $D > 4$. A projection operator

$$\mathcal{P} : \ \mathcal{H}(\mathcal{M}H) \to \mathcal{F}(\Sigma)$$

maps Φ to ϕ via an integral transform with kernel K:

$$\phi(x,t) \ = \ (\mathcal{P}\Phi)(x,t) \ := \int_{\mathcal{M}_H} K(x,t;\xi)\,\Phi(\xi)\,d\xi, \qquad (x,t) \in \Sigma,\ \xi \in \mathcal{M}_H. \tag{2.1}$$

We assume Φ and K lie in appropriate function spaces (e.g., Sobolev spaces) such that ϕ is well-defined and finite. Throughout, we use ∂_t and ∇ for time and spatial derivatives on Σ, and $\Delta = \nabla \cdot \nabla$ for the spatial Laplacian.

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2.2. Projection Operator: Structural Axioms

We now state the structural hypotheses (A1–A5) that define the projection mechanism and the interface's informational limitations. These axioms are not mere conveniences: each feeds directly into the variational construction and, subsequently, the emergence of a finite characteristic speed.

A1 (Idempotent Projection). $\mathcal{P}$ is idempotent: $\mathcal{P}^2 = \mathcal{P}$. Operationally, once information has been projected onto Σ, re-applying $\mathcal{P}$ does not alter ϕ. This encodes the notion that "observation lives on the interface."

A2 (Local Causality / Finite Influence Domain). For any localized perturbation $\delta\Phi$ supported in a small neighborhood of $\xi_0 \in \mathcal{M}_H$ and introduced at interface time t_0, its induced response $\delta\phi$ on Σ has a finite influence domain: there exists a constant $c > 0$ (to be determined endogenously) such that

$$\delta\phi(x,t) = 0 \quad \text{whenever} \quad \|x - x_0\| > c\,(t - t_0) \;\; \text{and} \;\; t \geq t_0,$$

where x_0 is the image point(s) on Σ corresponding to ξ_0 via K. This is a causality postulate tailored to the projected dynamics; we will show later that the same c emerges as the characteristic speed of the governing PDE.

A3 (Finite Capacity Bounds). The interface has finite informational capacity in both temporal update and spatial resolution. We encode this with bounded "information density" $i(x,t)$ and "information flux" $j(x,t)$, which satisfy a continuity-type balance:

$$\partial_t i(x,t) \;+\; \nabla \cdot j(x,t) \;=\; s(x,t), \tag{2.2}$$

with capacity bounds

$$0 \;\leq\; i(x,t) \;\leq\; I_{\max}, \qquad \|j(x,t)\| \;\leq\; J_{\max} \quad \text{for all } (x,t) \in \Sigma.$$

Here s represents the net injection from Φ through the kernel K into Σ. The precise constitutive relation between (i,j) and ϕ will be specified by the variational structure below.

A4 (Stability / Lipschitz Continuity). $\mathcal{P}$ and the induced dynamics are stable: for suitable norms $\|\cdot\|_{\mathcal{H}}, \|\cdot\|_{\mathcal{F}}$,

$$\|\delta\phi\|_{\mathcal{F}} \;\leq\; L\,\|\delta\Phi\|_{\mathcal{H}},$$

with $L < \infty$. Time evolution on Σ forms a strongly continuous semigroup, and solutions do not blow up in finite time under admissible sources s.

A5 (Minimum Distortion Principle under Capacity Constraints). The actual projected field ϕ minimizes a distortion functional $\mathcal{L}$ subject to the capacity bounds (A3). Concretely, over a time window $[t_0, t_1]$,

$$\min_{\phi} \;\; \mathcal{L}[\phi;\Phi] \;:=\; \frac{1}{2}\int_{t_0}^{t_1}\!\!\int_{\mathbb{R}^3} \Big(\alpha\,|\partial_t\phi|^2 + \beta\,|\nabla\phi|^2 + \mu\,|\phi|^2\Big)\,dx\,dt \;-\; \int J_\Phi\,\phi\,dx\,dt, \tag{2.3}$$

subject to the pointwise capacity constraints in (A3). Here:

- $\alpha > 0$ penalizes rapid temporal updates (temporal capacity);
- $\beta > 0$ penalizes steep spatial gradients (spatial capacity);
- $\mu \geq 0$ models retention/accumulation on the interface (vanishing for "pure boundary modes");
- J_Φ is the effective source induced by Φ via the kernel K (precise form given in §2.3).

2.3. The Kernel-Induced Source J_Φ

Equation (2.1) implies that the influence of Φ on ϕ can be represented, at the level of the Euler–Lagrange equations, by an effective source J_Φ that is a linear functional of Φ:

$$J_\Phi(x,t) \;:=\; \int_{\mathcal{M}_H} \mathcal{K}(x,t;\xi)\,\Phi(\xi)\,d\xi,$$

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where $\mathcal{K}$ may coincide with K or arise from a preconditioning of K by the variational structure (e.g., after integrating by parts in the action). We assume:

- $J_\Phi \in L^2_{\text{loc}}(\Sigma)$;
- For localized perturbations of Φ, J_Φ is compactly supported in a neighborhood compatible with (A2).

These conditions ensure that the source term in the governing PDE (derived in §2.5) is well-defined, causal, and consistent with finite propagation.

2.4. Capacity Constraints and Dualization

The pointwise constraints $i \le I_{\max}$, $\|j\| \le J_{\max}$ are enforced via dual variables (Lagrange multipliers) or, equivalently, penalty functionals that renormalize α and β. A standard convex dualization yields an unconstrained functional equivalent to (2.3):

$$\widetilde{\mathcal{L}}[\phi;\Phi] \;=\; \frac{1}{2}\int \Big(\alpha_{\text{eff}}\,|\partial_t\phi|^2 + \beta_{\text{eff}}\,|\nabla\phi|^2 + \mu_{\text{eff}}\,|\phi|^2\Big)\,dx\,dt \;-\; \int J_\Phi\,\phi\,dx\,dt, \tag{2.4}$$

with $\alpha_{\text{eff}}, \beta_{\text{eff}}, \mu_{\text{eff}}$ increasing functions of the tightness of the capacity bounds $(I_{\max}, J_{\max})$. Crucially, the ratio

$$\frac{\beta_{\text{eff}}}{\alpha_{\text{eff}}}$$

is governed by the relative capacity (spatial resolution vs. temporal update), not by their absolute magnitudes. This ratio will determine the characteristic speed c in §2.6.

2.5. Variational Principle and Euler–Lagrange Equations

Taking first variations of (2.4) with respect to ϕ (fixing endpoints or imposing standard vanishing boundary variations), we obtain the Euler–Lagrange equation:

$$\alpha_{\text{eff}}\,\partial_{tt}\phi \;-\; \beta_{\text{eff}}\,\Delta\phi \;+\; \mu_{\text{eff}}\,\phi \;=\; J_\Phi(x,t). \tag{2.5}$$

We assume coefficients are positive and piecewise smooth (constant for the simplest homogeneous interface; slowly varying for inhomogeneous capacity). Equation (2.5) is hyperbolic in its principal part whenever $\alpha_{\text{eff}}, \beta_{\text{eff}} > 0$.

2.6. Hyperbolicity, Characteristic Cones, and the Speed Limit

The principal symbol of (2.5) is

$$\sigma(\omega,k) \;=\; \alpha_{\text{eff}}\,\omega^2 \;-\; \beta_{\text{eff}}\,\|k\|^2.$$

Hyperbolicity implies the existence of non-trivial (ω, k) satisfying $\sigma = 0$, defining characteristic cones. The corresponding characteristic speed is

$$c \;:=\; \sqrt{\frac{\beta_{\text{eff}}}{\alpha_{\text{eff}}}} \;>\; 0. \tag{2.6}$$

Solutions to (2.5) exhibit finite propagation: disturbances produced by compactly supported $J_\Phi(\cdot, t_0)$ at time t_0 are confined within $\|x - x_0\| \le c\,(t - t_0)$. This realizes the causality postulate (A2) and simultaneously derives the same c from the capacity-weighted variational structure—i.e., the speed limit is not postulated but emerges from the interface's finite informational capacity.

2.7. Energy Balance, Information Flux, and a Local Speed Bound

Define the energy-like density and Poynting-like flux associated with (2.5):

$$\mathcal{E} := \frac{1}{2}\Big(\frac{1}{c^2}|\partial_t\phi|^2 + \|\nabla\phi\|^2\Big) + \frac{\mu_{\text{eff}}}{2\beta_{\text{eff}}}|\phi|^2, \qquad \mathcal{S} := \frac{1}{c^2}(\partial_t\phi)\,\nabla\phi,$$

with c given by (2.6). A standard multiplier argument yields the local balance law

$$\partial_t\mathcal{E} + \nabla\cdot\mathcal{S} = \frac{1}{\beta_{\text{eff}}}(\partial_t\phi)\,J_\Phi.$$

By Cauchy–Schwarz,

$$\|\mathcal{S}\| \le c\,\mathcal{E},$$

so the local transport speed $\|\mathcal{S}\|/\mathcal{E}$ is bounded by c. This provides an information-flux interpretation of the speed limit: the interface cannot convey information faster than c without violating either the energy balance or the capacity constraints encoded in $\alpha_{\text{eff}}, \beta_{\text{eff}}$.

2.8. Massless Boundary Modes vs Retentive Modes

The parameter μ_{eff} quantifies retention/accumulation on the interface. Considering plane-wave ansatz $\phi \sim e^{i(k\cdot x-\omega t)}$ for the homogeneous case, (2.5) yields the dispersion relation

$$\omega^2 = c^2\|k\|^2 + \omega_0^2, \qquad \omega_0^2 := \frac{\mu_{\text{eff}}}{\alpha_{\text{eff}}}.$$

- *Massless (boundary) modes:* $\mu_{\text{eff}} = 0 \Rightarrow \omega = c\|k\|$, group velocity $v_g = \partial\omega/\partial\|k\| = c$. These correspond to non-retentive excitations (no net accumulation on the interface), i.e., "photons" in the projectional sense.
- *Massive (retentive) modes:* $\mu_{\text{eff}} > 0 \Rightarrow v_g = \dfrac{c^2\|k\|}{\sqrt{c^2\|k\|^2+\omega_0^2}} < c$. Retention reduces the maximal group velocity, aligning with subluminal propagation.

Thus, the speed limit and the massless/massive split are not independent assumptions: both descend from the same capacity-weighted projectional variational structure.

2.9. Parameter Identification and Physical Interpretability

We now connect $\alpha_{\text{eff}}, \beta_{\text{eff}}, \mu_{\text{eff}}$ to operational capacity measures. Suppose the interface admits:

- a maximal temporal update rate R_t (effective sampling per unit time per unit volume), and
- a maximal spatial resolvability R_x (effective samples per unit length).

A dimensional analysis consistent with (2.3)–(2.5) leads to the scalings

$$\alpha_{\text{eff}} \propto R_t^{-2}, \qquad \beta_{\text{eff}} \propto R_x^{-2}, \quad \Rightarrow \quad c = \sqrt{\frac{\beta_{\text{eff}}}{\alpha_{\text{eff}}}} \propto \frac{R_t}{R_x}.$$

Hence,

- Increasing temporal update capacity (larger R_t) raises c;
- Increasing spatial resolution capacity (larger R_x) lowers c for fixed R_t;
- The ratio R_t/R_x fixes the universal limiting speed perceived on Σ.

The retention parameter μ_{eff} can be tied to storage-like or dissipative mechanisms (e.g., coarse-grain accumulation, effective frictional pinning of modes on Σ), thereby controlling the mass gap ω_0.

2.10. Regularity, Well-Posedness, and Compatibility with Axioms

With $\alpha_{\text{eff}}, \beta_{\text{eff}} > 0$ and $\mu_{\text{eff}} \ge 0$ piecewise smooth, (2.5) is a second-order strictly hyperbolic PDE. Standard energy methods yield:

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- *Well-posedness (Hadamard):* Given $(\phi, \partial_t\phi)|_{t=t_0} \in H^1(\mathbb{R}^3) \times L^2(\mathbb{R}^3)$ and $J_\Phi \in L^2_{\text{loc}}$, there exists a unique weak solution with continuous dependence on data.
- *Finite Propagation:* Support of solutions respects the characteristic cones determined by c (consistency with A2).
- *Stability (A4):* Energy estimates show no finite-time blow-up under bounded source J_Φ, consistent with Lipschitz continuity of the solution operator.

These properties ensure that the projectional dynamics are mathematically controlled and physically interpretable.

2.11. Summary of Section

- We defined a projection operator $\mathcal{P}$ with kernel K mapping higher-dimensional information Φ to interface fields ϕ.
- We posited finite capacity on the interface (A3) and a minimum-distortion variational principle (A5) under these constraints.
- Dualization yields an effective quadratic action with coefficients $\alpha_{\text{eff}}, \beta_{\text{eff}}, \mu_{\text{eff}}$ that encode temporal capacity, spatial capacity, and retention.
- The Euler–Lagrange equation is hyperbolic, with a characteristic speed $c = \sqrt{\beta_{\text{eff}}/\alpha_{\text{eff}}}$ that both enforces and explains the finite influence domain (A2).
- Massless (non-retentive) and massive (retentive) modes arise as $\mu_{\text{eff}} = 0$ and $\mu_{\text{eff}} > 0$ cases of the same structure, fixing their propagation speeds relative to c.
- Energy/flux identities provide a local information-speed bound $\|\mathcal{S}\|/\mathcal{E} \leq c$, furnishing an information-theoretic interpretation of the speed limit.

This completes the structural groundwork needed for the formal derivation in the next section, where we prove finite-speed propagation and identify photons with massless boundary modes ($\mu_{\text{eff}} = 0$) that travel at the capacity-imposed limit c.

3. Formal Derivation of the Speed Limit

3.1. Governing Equation Revisited

From the variational principle in Section 2.5, we obtained the effective Euler–Lagrange dynamics for the projected field $\phi(x,t)$:

$$\alpha_{\text{eff}}\, \partial_{tt}\phi \;-\; \beta_{\text{eff}}\, \Delta\phi \;+\; \mu_{\text{eff}}\, \phi \;=\; J_\Phi(x,t), \tag{3.1}$$

with constants (or slowly varying coefficients) $\alpha_{\text{eff}} > 0,\ \beta_{\text{eff}} > 0,\ \mu_{\text{eff}} \geq 0$. Here J_Φ encodes the influence of the higher-dimensional state Φ. Equation (3.1) is strictly hyperbolic in its principal part. The following theorem makes precise the finite propagation speed inherent to its solutions. *3.2. Characteristic Speed and Hyperbolicity*

Theorem 3.1 (Characteristic Speed of Projectional Dynamics)**.** *Let ϕ solve (3.1) with compactly supported initial data $(\phi, \partial_t\phi)|_{t=t_0}$ and compactly supported source $J_\Phi(x,t)$ for $t \geq t_0$. Then disturbances propagate within a finite domain bounded by the cone*

$$\|x - x_0\| \;\leq\; c\,(t - t_0), \qquad c := \sqrt{\frac{\beta_{\mathit{eff}}}{\alpha_{\mathit{eff}}}}.$$

In particular, if ϕ and J_Φ vanish outside a ball of radius R at $t = t_0$, then $\phi(x,t) = 0$ whenever $\|x\| > R + c(t - t_0)$.

Proof. *1. Principal symbol.* The Fourier transform in space gives

$$\alpha_{\text{eff}}\, \partial_{tt}\hat{\phi}(k,t) + \beta_{\text{eff}}\, \|k\|^2 \hat{\phi}(k,t) + \mu_{\text{eff}}\hat{\phi}(k,t) = \hat{J}_\Phi(k,t).$$

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The homogeneous dispersion relation is

$$\omega^2 = c^2\|k\|^2 + \omega_0^2, \qquad c^2 = \frac{\beta_{\mathrm{eff}}}{\alpha_{\mathrm{eff}}}, \quad \omega_0^2 = \frac{\mu_{\mathrm{eff}}}{\alpha_{\mathrm{eff}}}.$$

2. *Group velocity.* The group velocity is

$$v_g = \frac{\partial\omega}{\partial\|k\|} = \frac{c^2\|k\|}{\sqrt{c^2\|k\|^2 + \omega_0^2}} \le c.$$

3. *Finite propagation domain.* Standard results for strictly hyperbolic PDEs (see e.g. Hörmander, Taylor) guarantee that disturbances propagate within the characteristic cone defined by c. This follows from energy estimates and the domain-of-dependence property. □

3.3. Massless and Massive Modes

Corollary 3.1 (Massless Boundary Modes)**.** *If $\mu_{eff} = 0$, the dispersion relation reduces to*

$$\omega^2 = c^2\|k\|^2,$$

so all plane-wave excitations propagate at group velocity $v_g = c$. These excitations correspond to massless modes, identified physically as photons in the projectional framework.

Note (Mathematical Rigor). The finite propagation speed property invoked in Theorem 3.1 is a classical result of strictly hyperbolic PDEs. Rigorous proofs can be found in Hörmander's *Linear Partial Differential Operators* and Taylor's *Partial Differential Equations*. We include this note here as a reminder to cite the relevant references in the final bibliography.

Corollary 3.2 (Massive Retentive Modes)**.** *If $\mu_{eff} > 0$, then the group velocity satisfies $v_g < c$. Such modes accumulate information on the projection surface, corresponding to massive excitations.*

3.4. Energy Identity and Flux Bound

Define the energy density and flux associated with ϕ:

$$\mathcal{E}(x,t) = \frac{1}{2}\left(\frac{1}{c^2}|\partial_t\phi|^2 + \|\nabla\phi\|^2\right) + \frac{\mu_{\mathrm{eff}}}{2\beta_{\mathrm{eff}}}|\phi|^2, \qquad \mathcal{S}(x,t) = \frac{1}{c^2}(\partial_t\phi)\,\nabla\phi.$$

Lemma 3.1 (Local Energy Balance)**.** *Solutions to* (3.1) *satisfy*

$$\partial_t\mathcal{E} + \nabla\cdot\mathcal{S} = \frac{1}{\beta_{eff}}(\partial_t\phi)\,J_\Phi.$$

Proof. Multiply (3.1) by $\partial_t\phi/\beta_{\mathrm{eff}}$, integrate by parts in space, and reorganize terms. □

Theorem 3.2 (Information Flux Bound)**.** *The energy flux satisfies the inequality*

$$\|\mathcal{S}\| \le c\,\mathcal{E}.$$

Thus, the local transport speed of information, defined as $\|\mathcal{S}\|/\mathcal{E}$, never exceeds c.

Proof. Apply the Cauchy–Schwarz inequality:

$$\|\mathcal{S}\| = \frac{1}{c^2}|\partial_t\phi|\,\|\nabla\phi\| \le \frac{1}{2c^2}\left(|\partial_t\phi|^2 + \|\nabla\phi\|^2\right) \le c\,\mathcal{E}.$$

□

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3.5. Compatibility with Projectional Axioms

- **A1 (Idempotency):** Once projected, ϕ evolves solely under (3.1), consistent with $\mathcal{P}^2 = \mathcal{P}$.
- **A2 (Causality):** The domain-of-dependence result (Theorem 3.1) guarantees finite influence bounded by c.
- **A3 (Finite Capacity):** Bounds on $|\partial_t \phi|$ and $\|\nabla \phi\|$ feed into $\alpha_{\text{eff}}, \beta_{\text{eff}}$, fixing c.
- **A4 (Stability):** Energy estimates (Lemma 3.1) guarantee no blow-up under finite sources.
- **A5 (Minimum Distortion):** The governing PDE is exactly the Euler–Lagrange equation from minimizing the penalized functional under capacity constraints.

Thus, the formal derivation validates the axioms and shows that the finite speed limit is a necessary consequence of projectional capacity.

3.6. Summary

We have shown that:

- The governing dynamics of the projected field are hyperbolic with characteristic velocity $c = \sqrt{\frac{\beta_{\text{eff}}}{\alpha_{\text{eff}}}}$.
- This finite speed emerges from capacity constraints, not as an external postulate.
- Massless excitations $(\mu = 0)$ propagate at c; massive excitations $(\mu > 0)$ propagate subluminally.
- An energy/flux inequality provides a local information-theoretic bound, further confirming that no signal exceeds c.

This establishes the projectional origin of the universal speed limit, and identifies the photon as the natural boundary mode of the projection process.

4. Interpretation of the Emergent Universal Speed Limit

The derivations presented in Section 3 established that the projected dynamics on the interface are governed by a hyperbolic equation with a characteristic velocity

$$c = \sqrt{\frac{\beta_{\text{eff}}}{\alpha_{\text{eff}}}}.$$

This result allows us to reinterpret the universal speed limit not as a primitive constant of nature, but as the emergent ratio of two fundamental informational capacities: temporal update versus spatial resolution. The existence and invariance of c therefore follow not from axiomatic stipulation, but from the finite informational structure of the projection surface itself.

4.1. From Postulate to Derived Constant

In special relativity, c is posited as an invariant constant for all inertial observers, forming the backbone of Lorentz symmetry. In quantum field theory, massless particles such as photons are guaranteed to propagate at c due to gauge invariance, but the constant's existence itself is taken as given. In cosmology, c is embedded in the metric structure of spacetime and used to define causal cones. In all these approaches, the universality of c is employed as a foundational axiom.
By contrast, TAP derives c directly from the structure of the projection. The informational surface can only sustain changes at a finite rate per unit time and per unit space; it is the balance of these capacities that defines the characteristic propagation speed. Hence, c is not an arbitrary parameter fixed by experiment, but an emergent consequence of projectional geometry.

4.2. Photon as Boundary Mode

Within this picture, the photon acquires a clear ontological role. Modes with $\mu_{\text{eff}} = 0$ correspond to non-retentive excitations: they leave no accumulation on the projection surface, and therefore propagate at the absolute capacity limit. These modes naturally align with the photon, which in conventional theory is defined as massless and luminal. TAP thus removes the arbitrariness of associating "massless"

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with "luminal": the photon is luminal precisely because, as a boundary mode, it does not burden the interface with retention.
By contrast, modes with $\mu_{\text{eff}} > 0$ accumulate information on the surface and are therefore constrained to group velocities strictly below c. What in field theory is described as a "mass term" corresponds, in TAP, to a quantitative measure of retention. This reinterpretation connects mass, subluminal propagation, and capacity constraints within a unified structure.

4.3. Local Invariance and Universality

The universality of c in relativity is expressed through the invariance of causal cones under Lorentz transformations. In TAP, an analogous result follows from the invariance of the ratio $\beta_{\text{eff}}/\alpha_{\text{eff}}$ under conformal rescaling. If both coefficients scale by the same factor, their ratio—and hence the speed limit—remains unchanged. Thus, even in the presence of global inhomogeneities or slow variations of capacity across the projection surface, local observers will measure the same c.
This interpretation provides a structural explanation for why the limiting velocity is not only finite but also universal: it is anchored in the conformal invariance of the capacity ratio. The local constancy of c ceases to be an independent axiom and becomes a direct corollary of projectional stability.

4.4. Absence of Dispersion in Vacuum

A final point of interpretation concerns dispersion. For $\mu_{\text{eff}} = 0$ and constant $\alpha_{\text{eff}}, \beta_{\text{eff}}$, the dispersion relation is strictly linear, $\omega = c\|k\|$. The group velocity equals c for all wave numbers, ensuring that massless modes are nondispersive in vacuum. This property, empirically confirmed for photons, here follows as a direct consequence of the frequency-independence of the capacity ratio. Should a deviation be observed, it would imply that $\beta_{\text{eff}}/\alpha_{\text{eff}}$ varies with frequency, signaling a breakdown of the assumption of scale-free capacity.

5. Comparison with Existing Frameworks

5.1. Reframing the Universal Speed Limit

Conventional physics frameworks treat c as an axiom: relativity embeds it in spacetime geometry, QFT encodes it in the commutator structure of fields, and cosmology employs it in metric-based causal cones. TAP reframes c as an emergent ratio of finite informational capacities on the projection surface. Rather than assuming c, we derive it, and then reinterpret photons and massive particles within this structure.

5.2. Comparison with Special Relativity

In special relativity, the universality of c underlies Lorentz invariance and the relativity of simultaneity. TAP agrees on the invariance but attributes it to the structural constancy of $\beta_{\text{eff}}/\alpha_{\text{eff}}$. Lorentz symmetry thus arises as a secondary mathematical manifestation of projectional invariance, not as a primitive symmetry of nature.

5.3. Comparison with Quantum Field Theory

QFT explains luminal photon propagation by gauge invariance: the masslessness of the photon ensures $v = c$. TAP reproduces this without invoking gauge postulates. Instead, $\mu_{\text{eff}} = 0$ defines boundary modes that do not retain information on the projection surface, and hence propagate at the capacity limit. Gauge invariance may then be reinterpreted as a mechanism ensuring $\mu_{\text{eff}} = 0$.

5.4. Information-Theoretic Analogies

Information-theoretic approaches, such as Shannon capacity bounds, emphasize that finite channels limit the rate of information transmission. TAP provides a physical instantiation: the projection surface *is* the channel, with finite update and resolution capacities. The universal speed limit c is precisely the information velocity implied by this channel capacity. Photons, as non-retentive boundary modes, achieve this limit, while massive modes are retentive and thus slower.

5.5. Relation to Holography and Quantum Gravity

The holographic principle posits that spacetime degrees of freedom are encoded on a lower-dimensional boundary. TAP is compatible with this view, identifying the observable universe as precisely such a boundary projection. The emergence of a finite speed limit from capacity constraints resonates with holographic bounds on entropy and information flow. This suggests TAP may offer a concrete microphysical mechanism underlying holography.

5.6. Cosmological Implications

Cosmology assumes the constancy of c across epochs. TAP predicts that c reflects the ratio of temporal to spatial capacities, which could in principle vary slowly over cosmic history if these capacities evolve. Observational limits on the time variation of c thus constrain the stability of the capacity ratio. TAP also hints that apparent "horizons" may be artifacts of projectional limitations rather than true causal barriers.

5.7. Summary

TAP does not contradict established frameworks but reinterprets them. Relativity's Lorentz symmetry, QFT's gauge invariance, and holography's boundary encoding each appear as higher-level consequences of a more primitive fact: the projection surface has finite capacity, and its ratio of temporal to spatial update scales sets a universal velocity c.

6. Consolidated Perspective on c

6.1. Unified Limiting Velocity

The derivations in Sections 2–4 show that $c = \sqrt{\beta_{\text{eff}}/\alpha_{\text{eff}}}$ is the sole characteristic velocity permitted by projectional dynamics. No signal, regardless of composition, can exceed this bound without violating informational constraints.

6.2. Vacuum Nondispersion

Massless modes exhibit nondispersive propagation: $\omega = c\|k\|$ across all frequencies. This reproduces the observed vacuum nondispersion of light, normally assumed in relativity, as a derived property of projectional balance.

6.3. Local Conformal Invariance

Scaling both α_{eff} and β_{eff} leaves c invariant. This mirrors conformal invariance and ensures that local observers always measure the same limiting speed, regardless of rescalings or inhomogeneities.

6.4. Preparing for Discussion

This consolidated perspective reveals c not as a contingent parameter but as a structural ratio of interface capacities. The photon is identified as the massless boundary mode that saturates this limit. These results prepare the ground for a broader discussion of implications and testable predictions.

7. Discussion

The TAP framework advances several novel insights:

- **Emergent c:** The universal speed limit arises not from postulation but from projectional geometry and finite informational capacity.
- **Photon as Boundary Mode:** Photons are naturally identified as non-retentive excitations that propagate at c, while massive particles correspond to retentive modes with $\mu_{\text{eff}} > 0$.
- **Unified Interpretation:** Relativity, QFT, and holography are reinterpreted as emergent structures consistent with, but not primary to, projectional capacity.
- **Observational Consequences:** Any frequency dependence of c, or slow temporal drift in its value, would indicate scale-dependent or evolving capacity ratios. Thus TAP offers concrete falsifiability.

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- **Philosophical Import:** The framework connects physics with information theory and even metaphysical notions: the observable world is not a self-contained arena but a projection of deeper informational structures.

Future work must address the microphysical origin of capacity bounds, explore links to quantum entanglement, and develop observational tests sensitive to capacity variation.

8. Conclusion

We have derived the universal speed limit c as the emergent ratio of temporal and spatial informational capacities on a projection surface. This limit is not an arbitrary axiom but a structural necessity of projectional dynamics. Photons appear as non-retentive boundary modes that saturate the limit, while massive particles are retentive excitations that propagate subluminally. Lorentz invariance, gauge invariance, and holographic principles can all be reinterpreted as higher-level consequences of this structure.

The TAP framework thus offers a unified, information-geometric account of why c is finite and universal. It opens a path toward reconciling relativity, quantum field theory, and cosmology within a single principle: finite informational capacity under projection.

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