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Posted Date: 20 October 2025

doi: 10.20944/preprints202510.1536.v1

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Schrödinger's Cat and Blindsight: An Investigation into Perception and Quantum Mechanics

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Abstract

This paper presents a unified framework connecting quantum measurement theory with neurocognitive phenomena, particularly blindsight. Drawing upon the Perceptual Tangent Space (PTS), we model conscious and subconscious states as projections on a quantum perceptual manifold. Weak measurement theory is employed to interpret blindsight behavior as a non-collapsing interaction with visual stimuli, mediated by decoherence-resistant perceptual channels. We introduce quantum path integrals over cognitive geodesics, category-theoretic mappings of perceptual state transitions, and entangled cross-modal constructs to represent synesthetic and unconscious processing. Concepts from quantum error correction, noncommutative geometry, and spin networks are adapted to model resilience and granularity of perceptual awareness. Additionally, we explore topos logic, holography, and renormalization principles to mathematically reconstruct observer-dependent cognition from incomplete sensory boundaries. Altogether, the paper situates consciousness within a quantum information-geometric topology, offering a rigorous treatment of perception as a quantum-inferential, decoherence-bound phenomenon conditioned by attention, awareness, and contextual measurement bases.

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1. Introduction

The Schrödinger's cat thought experiment was proposed by Erwin Schrödinger in 1935 as a critique of the Copenhagen interpretation of quantum mechanics. The paradox illustrates the peculiar situation in which a quantum system can exist in a superposition of macroscopically distinct states until observed [1]. This interpretation invites deep philosophical implications about the role of the observer in the collapse of the wavefunction. At the same time, in cognitive neuroscience, the condition of blindsight presents a scenario in which individuals, typically with lesions in their primary visual cortex, can respond to visual stimuli without conscious visual experience [2]. The convergence of these two domains invites the question: can unconscious perception provide a model or analogy for quantum superposition and collapse?

To explore this, we must first formalize the quantum measurement process using standard mathematical notation. The state of a quantum system before measurement is often given as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \tag{1}$$

where $\alpha, \beta \in \mathbb{C}$ are probability amplitudes satisfying $|\alpha|^2 + |\beta|^2 = 1$. Upon measurement, the system collapses to either $|0\rangle$ or $|1\rangle$ with respective probabilities $|\alpha|^2$ and $|\beta|^2$. The measurement operator $\hat{M}$ acts as:

$$\hat{M}|\psi\rangle = \begin{cases} |0\rangle & \text{with probability } |\alpha|^2, \\ |1\rangle & \text{with probability } |\beta|^2. \end{cases} \tag{2}$$

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This discrete collapse has no direct analog in classical mechanics, but interestingly, a functional analog may exist in the blindsight literature, where patients report no awareness of stimuli yet demonstrate above-chance performance in forced-choice paradigms [3]. A statistical model of blindsight subject responses can be formalized using a binomial distribution:

$$P(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}, \tag{3}$$

where k is the number of correct responses out of n trials, and p is the success rate. If $p > 0.5$, even in the absence of conscious awareness, one may infer the presence of implicit information processing, which parallels the 'unmeasured yet behaving' system in quantum theory.

2. Theoretical Frameworks

A central issue in quantum mechanics is the interpretation of measurement. The von Neumann chain extends the system-environment-observer boundary infinitely, suggesting the collapse occurs only when consciousness is involved [4]. This idea, although controversial, aligns with hypotheses suggesting that consciousness plays an active role in wavefunction collapse.

Neurophysiological interpretations of blindsight suggest that although V1 is damaged, subcortical pathways (e.g., via the superior colliculus) allow residual visual processing [5]. In this light, we consider the following model of parallel perceptual pathways:

$$V_{\text{total}} = V_{\text{conscious}} + V_{\text{nonconscious}}, \tag{4}$$

where V_{total} is the visual information processed, $V_{\text{conscious}}$ is mediated through V1, and $V_{\text{nonconscious}}$ through subcortical routes.

From a quantum cognition perspective, some models propose that cognitive decision-making resembles quantum probability rather than classical probability [6]. If we take a quantum state vector for decision-making as:

$$|\Psi_{\text{decision}}\rangle = \gamma_1|A\rangle + \gamma_2|B\rangle, \tag{5}$$

where $|A\rangle$ and $|B\rangle$ are incompatible choices, then interference terms from cross amplitudes may explain deviations from expected utility theory.

3. Discussion

Blindsight and quantum measurement both challenge our assumptions about what it means to "observe." In blindsight, information is processed without awareness, akin to a quantum system that evolves unitarily until interrupted. The collapse in both systems may be less about observation and more about decoherence. The decoherence model suggests that interaction with the environment rapidly entangles the system with a vast number of degrees of freedom, rendering coherent superpositions practically unobservable:

$$\rho(t) = \mathrm{Tr}_{\text{env}}\left[U(t)\rho(0) \otimes \rho_{\text{env}} U^\dagger(t)\right], \tag{6}$$

where $U(t)$ is the unitary time evolution operator, and ρ_{env} is the density matrix of the environment.

A similar model can be constructed for blindsight by representing awareness as a projection operator on the state of neural information:

$$\rho_{\text{aware}} = \hat{P}_{\text{V1}} \rho_{\text{brain}} \hat{P}_{\text{V1}}, \tag{7}$$

which simulates the reduction of available information to only what enters conscious awareness. Yet, behavioral responses suggest that ρ_{brain} includes entangled information not accessed consciously.

4. Integration of Perceptual Tangent Spaces with Quantum Phenomena

The concept of Perceptual Tangent Spaces (PTS), introduced in [7], provides a geometrically and mathematically rigorous framework for interpreting perception as trajectories over a perceptual manifold. This model allows us to reinterpret both quantum indeterminacy and unconscious perceptual processing (e.g., blindsight) in a unified space of linear perceptual states. The notion of a perceptual manifold $\mathcal{M}$ introduces a smooth structure wherein every perceptual state is modelled, as a function of time.

Given a differentiable manifold $\mathcal{M}$ representing all possible perceptual configurations, we define the tangent space at a point p as the best linear approximation of $\mathcal{M}$ at p. This is formally given by:

$$T_p\mathcal{M} = \left\{ \frac{d\gamma(t)}{dt}\bigg|_{t=0} \,\bigg|\, \gamma : (-\epsilon, \epsilon) \to \mathcal{M}, \gamma(0) = p \right\}, \tag{8}$$

where $\gamma(t)$ is a smooth curve through p. The perceptual content is represented by the projection of evolving neural states onto $T_p\mathcal{M}$, which we denote as $\delta\pi(p)$. In the absence of V1 cortical access, as seen in blindsight, this projection may remain in a subspace of $T_p\mathcal{M}$, inaccessible to conscious reflection. However, the dynamical component still exists and affects behavior.

Let us now define a perceptual evolution path $\Gamma(t)$ governed by an internal Hamiltonian operator $\hat{H}_p$:

$$\frac{d}{dt}|\psi(t)\rangle = -\frac{i}{\hbar}\hat{H}_p|\psi(t)\rangle, \tag{9}$$

where $|\psi(t)\rangle \in \mathcal{H}_p$ is a perceptual state vector lying in the Hilbert space associated with $T_p\mathcal{M}$. This is structurally analogous to Schrödinger evolution in quantum mechanics, and permits modeling of perceptual transitions, including subconscious pathways like those used in blindsight [3].

Let the perceptual projection operator into conscious awareness be denoted $\hat{P}_{\text{aware}}$, which acts on $|\psi(t)\rangle$ as:

$$|\psi_{\text{aware}}(t)\rangle = \hat{P}_{\text{aware}}|\psi(t)\rangle. \tag{10}$$

If $\hat{P}_{\text{aware}}$ is a degenerate projection (e.g., of rank less than full), then a portion of the perceptual state remains unexpressed consciously, yet may still drive behavior—paralleling quantum coherence prior to measurement. Similarly, let $\hat{P}_{\text{blind}} = \mathbb{I} - \hat{P}_{\text{aware}}$ capture unconscious perceptual processing.

In this model, the completeness relation holds:

$$\hat{P}_{\text{aware}} + \hat{P}_{\text{blind}} = \mathbb{I}. \tag{11}$$

The perceptual state can be reconstituted as:

$$|\psi(t)\rangle = |\psi_{\text{aware}}(t)\rangle + |\psi_{\text{blind}}(t)\rangle, \tag{12}$$

where $|\psi_{\text{blind}}(t)\rangle = \hat{P}_{\text{blind}}|\psi(t)\rangle$.

This decomposition enables modeling of blindsight phenomena as arising from $|\psi_{\text{blind}}(t)\rangle$, which though inaccessible to verbal report, can still generate motor responses. Experimental data show that blindsight subjects perform significantly above chance, suggesting that $|\langle\phi|\psi_{\text{blind}}(t)\rangle|^2 > 0.5$ for specific response vectors $|\phi\rangle$ in the action space [5].

Next, we consider a perceptual metric g on $\mathcal{M}$, allowing measurement of distances between perceptual states. Given two states $p_1, p_2 \in \mathcal{M}$, the infinitesimal distance is defined as:

$$ds^2 = g_{ij}dx^i dx^j, \tag{13}$$

which induces a Riemannian structure on perceptual experience. The shortest path between perceptual states follows a geodesic $\gamma(s)$ minimizing the perceptual action:

$$S[\gamma] = \int_0^1 \sqrt{g_{ij} \frac{d\gamma^i}{ds} \frac{d\gamma^j}{ds}} \, ds. \tag{14}$$

This geodesic formalism provides a variational principle for perception, where evolution seeks minimal change across PTS, analogous to quantum least-action paths in Hilbert space. This suggests that the PTS model can be tightly coupled to decoherence-based quantum interpretations, where perceptual transitions are driven not by measurement alone but by geometrical constraints in high-dimensional state spaces.

Thus, Schrödinger's cat paradox, which hinges on the ambiguity of observation and awareness, finds a natural geometrical analogue in PTS. The state of the cat is represented by a quantum superposition, while the observer's perceptual manifold $\mathcal{M}_{\text{obs}}$ defines the accessible tangent space. Until the observer maps $\delta\pi(p)$ via a conscious projection $\hat{P}_{\text{aware}}$, the global perceptual state remains entangled, aligning with quantum formalism.

In conclusion, Perceptual Tangent Spaces offer a coherent and mathematically robust scaffold upon which the dual mysteries of quantum collapse and subconscious perception can be jointly modeled. They reconcile differential perceptual awareness with quantum uncertainty through smooth manifold dynamics, operator-based projections, and variational paths, thereby advancing both cognitive science and quantum foundations.

5. Quantum Information Geometry of Consciousness

The mathematical formalization of consciousness remains one of the most elusive goals in contemporary science. However, recent developments in quantum information geometry and differential geometry offer a compelling direction for modeling consciousness as a structured manifold of quantum states. To formalize this framework, we consider a Hilbert space $\mathcal{H}$ containing all possible perceptual quantum states $|\psi\rangle$. The quantum Fisher information metric defines an inner product on the tangent space $T_\psi \mathcal{H}$ of this manifold. The quantum geometric tensor is given by:

$$G_{ij} = \langle \partial_i \psi | \partial_j \psi \rangle - \langle \partial_i \psi | \psi \rangle \langle \psi | \partial_j \psi \rangle, \tag{15}$$

where $\partial_i = \frac{\partial}{\partial \theta^i}$ and θ^i are parameters on the manifold. The real part of G_{ij} defines the quantum Fisher information metric:

$$g_{ij} = \text{Re}(G_{ij}) = \text{Re}\big(\langle \partial_i \psi | \partial_j \psi \rangle - \langle \partial_i \psi | \psi \rangle \langle \psi | \partial_j \psi \rangle \big), \tag{16}$$

which introduces a Riemannian structure on $\mathcal{H}$. This allows us to model transitions in consciousness as geodesic flows on this manifold. For example, one can postulate that meditative states correspond to minimal geodesic paths between high-entropy and low-entropy conscious configurations, while anesthesia corresponds to sharp curvatures in the perceptual manifold, driven by decoherence.

Given the PTS framework [7], we reinterpret the perceptual manifold $\mathcal{M}$ as embedded within $\mathcal{H}$, where each perceptual tangent vector $\delta\pi(p) \in T_p\mathcal{M}$ becomes a local derivative $\partial_i \psi$. Consciousness then evolves along trajectories of minimal information distance, satisfying the geodesic equation:

$$\frac{d^2\theta^k}{ds^2} + \Gamma^k_{ij} \frac{d\theta^i}{ds} \frac{d\theta^j}{ds} = 0, \tag{17}$$

where Γ^k_{ij} are Christoffel symbols defined from the metric g_{ij}. This models the evolution of perceptual states as quantum geodesics constrained by informational curvature.

The curvature tensor R^k_{ijl} derived from g_{ij} quantifies how consciousness resists or promotes transitions between states. In pathological or altered states (such as seizures or hallucinations), one can hypothesize that R^k_{ijl} becomes highly nontrivial, introducing perceptual singularities or discontinuities.

Moreover, entropy measures provide another lens for conscious transitions. The von Neumann entropy of a mixed perceptual state ρ is given by:

$$S(\rho) = -\mathrm{Tr}(\rho \log \rho), \tag{18}$$

and the relative entropy between two states ρ_1 and ρ_2 is:

$$S(\rho_1 || \rho_2) = \mathrm{Tr}(\rho_1 \log \rho_1 - \rho_1 \log \rho_2), \tag{19}$$

which defines a natural divergence function over the manifold. This divergence can be employed to characterize the 'distance' between waking and anesthetic states or the dynamical flow of consciousness in meditative states.

If the perceptual states are represented as Gaussian wave packets, then the quantum Fisher metric reduces to:

$$g_{ij} = \frac{1}{\sigma^2} \delta_{ij}, \tag{20}$$

where σ^2 is the variance of the perceptual distribution. This suggests that states with lower perceptual uncertainty (i.e., higher attention or awareness) lie in regions of high curvature and short geodesic length, while states of high entropy (e.g., sleep or anesthesia) correspond to flatter regions of the perceptual manifold.

Furthermore, transition amplitudes between perceptual states can be modeled via the inner product:

$$\langle \psi_2 | \psi_1 \rangle = \cos\left(\frac{D_{\mathrm{Bures}}(\psi_1, \psi_2)}{2} \right), \tag{21}$$

where D_{Bures} is the Bures distance, closely related to the Fisher information metric. Thus, conscious transitions can be encoded in the geometry of state overlaps, and modulated by environmental decoherence and internal noise.

In summary, quantum information geometry, when coupled with the perceptual tangent space framework, provides a mathematically rich language for describing consciousness as a dynamical trajectory over a Riemannian manifold of quantum states. By grounding transitions in consciousness within geodesics, curvature, and entropy gradients, this approach offers a path toward quantitative models of awareness, attention, and perception in quantum-cognitive systems.

6. Perceptual Decoherence and Quantum Neural Dynamics

The dynamics of neural states under the framework of Perceptual Tangent Spaces (PTS) can be enriched by invoking the theory of open quantum systems and environmental decoherence. Standard quantum mechanics assumes isolated systems undergoing unitary evolution. However, in realistic cognitive environments, perceptual states constantly interact with external and internal environments. Let us consider a neural-perceptual state described by a density matrix $\rho(t)$ in a Hilbert space $\mathcal{H}_N$ associated with neural configurations. The evolution of such states, when influenced by an environment, follows the Lindblad master equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[\hat{H}, \rho] + \mathcal{L}(\rho), \tag{22}$$

where $\hat{H}$ is the neural Hamiltonian operator generating unitary dynamics, and $\mathcal{L}(\rho)$ is the Lindblad superoperator encoding non-unitary effects, including perceptual collapse. The Lindblad form ensures complete positivity and trace preservation, necessary for physical evolution of mixed states.

The superoperator $\mathcal{L}(\rho)$ takes the general form:

$$\mathcal{L}(\rho) = \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{23}$$

where $\{A, B\} = AB + BA$ denotes the anti-commutator, and L_k are the Lindblad operators representing environmental interactions, decoherence channels, or internal stochasticities. In the context of perception, these may include noise from sensory inputs, background neural oscillations, or systemic hormonal influences.

We extend this to the perceptual manifold $\mathcal{M}$, where every point $p \in \mathcal{M}$ corresponds to a distinct perceptual configuration. The projection of neural dynamics into perceptual awareness is given by the perceptual projection operator $\hat{P}_{\text{aware}}$ as introduced in previous models [7]. The perceived density matrix is:

$$\rho_{\text{aware}}(t) = \hat{P}_{\text{aware}} \rho(t) \hat{P}_{\text{aware}}, \tag{24}$$

and its evolution is governed by:

$$\frac{d\rho_{\text{aware}}}{dt} = -\frac{i}{\hbar}[\hat{H}_{\text{eff}}, \rho_{\text{aware}}] + \mathcal{L}_{\text{aware}}(\rho), \tag{25}$$

where $\hat{H}_{\text{eff}} = \hat{P}_{\text{aware}} \hat{H} \hat{P}_{\text{aware}}$, and $\mathcal{L}_{\text{aware}}$ is the effective Lindbladian within the perceptual subspace.

Subconscious processes, such as those evident in blindsight or priming, correspond to $\rho_{\text{blind}} = \hat{P}_{\text{blind}} \rho \hat{P}_{\text{blind}}$, with $\hat{P}_{\text{blind}} = \mathbb{I} - \hat{P}_{\text{aware}}$. The total neural state remains a mixture:

$$\rho = \rho_{\text{aware}} + \rho_{\text{blind}}, \tag{26}$$

with transitions governed by leakage terms arising from off-diagonal elements in the full density matrix. Such terms are naturally suppressed by decoherence, providing an explanation for the inaccessibility of certain information to awareness.

The decoherence time t_D can be estimated from the environmental coupling strength λ and energy uncertainty ΔE as:

$$t_D \approx \frac{\hbar^2}{\lambda^2 (\Delta E)^2}, \tag{27}$$

which provides a physically testable scale for perceptual transitions. For example, transitions into subconscious processes may correspond to ultrafast decoherence, with $t_D < 10^{-13}$ seconds in cortical microcolumns [13]. In contrast, conscious deliberation may involve slower decoherence scales that permit sustained entanglement across perceptual manifolds.

We can further introduce a decoherence functional $\mathcal{D}(p_1, p_2)$ over the perceptual manifold $\mathcal{M}$:

$$\mathcal{D}(p_1, p_2) = \text{Tr}[\rho(p_1)\rho(p_2)] - \text{Tr}[\rho(p_1)]\text{Tr}[\rho(p_2)], \tag{28}$$

which quantifies interference between two perceptual states. When $\mathcal{D}(p_1, p_2) \to 0$, the states are said to decohere, and can be interpreted classically. This formalism enables us to model the transition from ambiguous perception (e.g., Necker cube) to definite awareness in terms of reduced decoherence functional.

From a dynamical systems perspective, we can model neural decoherence as a dissipative quantum trajectory governed by stochastic Schrödinger equations. A typical unraveling yields:

$$d|\psi\rangle = -\frac{i}{\hbar}\hat{H}|\psi\rangle dt + \sum_k \left(\langle L_k^\dagger \rangle L_k - \frac{1}{2} L_k^\dagger L_k \right) |\psi\rangle dt + \sum_k L_k |\psi\rangle dW_k, \tag{29}$$

where dW_k are Wiener processes modeling quantum noise. In perceptual terms, this gives a stochastic trajectory over $\mathcal{M}$, where awareness emerges as an attractor state under decoherence flow.

The effective decoherence rate γ can be used to estimate transitions into awareness as:

$$P_{\text{aware}}(t) \approx 1 - e^{-\gamma t}, \tag{30}$$

which describes the emergence of conscious representation from a superposed neural state over time.

In summary, the modeling of brain dynamics as open quantum systems within the PTS framework provides a compelling explanation for the differentiation between conscious and subconscious processing. Lindblad-type evolution, stochastic unravellings, and projection operators provide the mathematical tools necessary to map perceptual decoherence into physical and observable parameters.

7. Perceptual Path Integrals: A Feynman-Like Model

Traditional quantum mechanics explains physical transitions between states using Feynman's path integral formalism, which computes transition amplitudes by integrating over all possible paths connecting two points in configuration space. In this section, we extend this powerful formalism to perceptual dynamics by constructing a perceptual path integral over a manifold of cognitive states. Let $\mathcal{M}$ be a perceptual manifold, and let $p_1, p_2 \in \mathcal{M}$ be two perceptual states, corresponding to different configurations of awareness. The transition amplitude $P(p_2|p_1)$ is postulated as an integral over all perceptual paths $\gamma(t)$ connecting p_1 and p_2, each weighted by the exponential of an action functional $S[\gamma]$:

$$P(p_2|p_1) = \int \mathcal{D}\gamma \, e^{iS[\gamma]/\hbar}, \tag{31}$$

where $\mathcal{D}\gamma$ denotes the formal path measure over all differentiable curves $\gamma : [0, T] \to \mathcal{M}$ with $\gamma(0) = p_1$ and $\gamma(T) = p_2$. This formulation invites a reinterpretation of cognitive transitions as arising from an ensemble of perceptual histories, weighted by their action.

The action $S[\gamma]$ is modeled in analogy to classical mechanics as an integral over a perceptual Lagrangian $\mathcal{L}$:

$$S[\gamma] = \int_0^T \mathcal{L}(\gamma(t), \dot{\gamma}(t))dt, \tag{32}$$

where $\dot{\gamma}(t)$ is the perceptual velocity, i.e., the tangent vector in $T_{\gamma(t)}\mathcal{M}$. A canonical choice for $\mathcal{L}$ is the kinetic term derived from the Fisher-Rao metric g_{ij} on $\mathcal{M}$:

$$\mathcal{L} = \frac{1}{2} g_{ij} \dot{\theta}^i \dot{\theta}^j, \tag{33}$$

where θ^i are local coordinates on $\mathcal{M}$. This choice implies that the dominant contribution to the path integral arises from geodesics minimizing the information-theoretic distance on the perceptual manifold.

By applying a semiclassical (stationary phase) approximation to the path integral, one recovers the classical geodesic equation:

$$\frac{d^2\theta^k}{dt^2} + \Gamma^k_{ij} \frac{d\theta^i}{dt} \frac{d\theta^j}{dt} = 0, \tag{34}$$

where Γ^k_{ij} are the Christoffel symbols derived from g_{ij}. Hence, perceptual transitions align with least-action principles and follow shortest paths in information space under typical conditions.

Furthermore, the transition amplitude in Eq. (31) can be related to a perceptual propagator $K(p_2, T; p_1, 0)$ satisfying a Schrödinger-type equation on $\mathcal{M}$:

$$i\hbar \frac{\partial}{\partial t} \Psi(p, t) = -\frac{\hbar^2}{2} \Delta_{\mathcal{M}} \Psi(p, t), \tag{35}$$

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where $\Delta_{\mathcal{M}}$ is the Laplace-Beltrami operator induced by g_{ij}. This formulation allows for the diffusion and interference of perceptual amplitudes over time and space, revealing a quantum-like structure underlying mental transitions.

The perceptual propagator can also encode predictive and attentional dynamics. For instance, the likelihood of attending to stimulus p_2 after p_1 can be modeled by maximizing $|P(p_2|p_1)|^2$. Surprise, or deviation from expected perceptual flow, can be measured by entropy production along the path:

$$\Delta S = -\int_0^T \frac{d}{dt}(\log \rho(\gamma(t)))dt, \tag{36}$$

where $\rho(p)$ is the probability density over $\mathcal{M}$. High ΔS corresponds to unexpected perceptual shifts, possibly linked to phenomena like attentional capture or novelty detection.

Incorporating decoherence into this framework modifies the path integral with a decay term, replacing the complex exponential with a damping kernel:

$$P(p_2|p_1) = \int \mathcal{D}\gamma\, e^{iS[\gamma]/\hbar} e^{-\Gamma[\gamma]}, \tag{37}$$

where $\Gamma[\gamma]$ quantifies decoherence along the path. This permits modeling of selective attention as arising from suppressed interference among perceptual alternatives.

The PTS basis allows us to explicitly construct the tangent vectors $\delta\pi(p)$ and compute geodesics numerically over realistic neural state spaces. Using a discretization scheme, one can simulate Eq. (31) over neural manifolds built from fMRI or EEG data [17], offering empirical tests for this perceptual path model.

In conclusion, the path integral formulation of perceptual transitions, grounded in quantum geometry and extended by PTS, provides a mathematically rigorous and conceptually fertile framework. It unites attention, surprise, and prediction under a unified formalism, making it a powerful candidate for future theoretical and experimental investigations.

8. Quantum Bayesian PTS: Consciousness as Inference

Perception has increasingly been conceptualized as a form of probabilistic inference, where the brain maintains beliefs about the external world and updates them upon receiving sensory evidence. This perspective, formalized in the Bayesian brain hypothesis, has found support in predictive coding theories, free energy minimization, and statistical learning [20]. However, classical Bayesian inference may be inadequate in modeling the subtle contextuality, interference, and non-commutative nature of perceptual states evident in blindsight, bistable perception, and pre-conscious processing.

To address this, we propose a quantum Bayesian model of perception situated within the Perceptual Tangent Spaces (PTS) framework. Let ρ be a density operator encoding the perceptual state prior to measurement. Upon receiving a new sensory datum associated with outcome k, the updated state is given by the quantum Bayesian rule:

$$\rho' = \frac{M_k \rho M_k^\dagger}{\mathrm{Tr}(M_k \rho M_k^\dagger)}, \tag{38}$$

where M_k are measurement operators satisfying $\sum_k M_k^\dagger M_k = I$, and ρ' is the posterior perceptual state. This update rule generalizes classical Bayes' theorem, accommodates contextuality, and introduces amplitude-based interference between hypotheses [21].

In the perceptual setting, ρ represents the agent's subjective belief over the perceptual manifold $\mathcal{M}$. The set of M_k encodes the sensory operators acting on the neural substrate, which interact with the internal state to produce an updated perceptual experience. If the perceptual manifold has local coordinates θ^i, then the marginal belief distribution is given by:

$$P(\theta^i) = \langle \theta^i | \rho | \theta^i \rangle, \tag{39}$$

and the predictive likelihood is encoded in the Born rule:

$$P(k) = \text{Tr}(M_k \rho M_k^\dagger). \tag{40}$$

Crucially, the predictive structure depends on the ordering and interference among projections, revealing a contextual, non-commutative logic underpinning perception.

To illustrate the dynamical nature of this framework, consider a time-dependent family of priors $\rho(t)$ evolving under a Lindblad equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{41}$$

interrupted by Bayesian updates upon data arrival. This produces a hybrid dynamics of continuous prediction punctuated by discrete perceptual updates, reflecting the balance between expectation and surprise.

Within the PTS model [7], we define the perceptual tangent vector $\delta\pi(p)$ as the differential of perceptual probability with respect to neural embedding. For a belief ρ projected into awareness via $\hat{P}_{\text{aware}}$, the Bayesian update becomes:

$$\delta\pi(p) = \nabla_p \text{Tr}[\hat{P}_{\text{aware}} M_k \rho M_k^\dagger], \tag{42}$$

providing a direct geometric link between quantum Bayesian inference and perceptual dynamics.

Classical Bayesian inference updates beliefs via:

$$P(h|d) = \frac{P(d|h)P(h)}{P(d)}, \tag{43}$$

which assumes a well-defined, commutative hypothesis space. In contrast, quantum updates can exhibit destructive and constructive interference, as demonstrated by the double-slit analogy applied to ambiguous figures and bistable illusions [6].

Predictive processing within this framework is enhanced by quantum prior encoding. Suppose the brain encodes multiple competing hypotheses $|h_j\rangle$ in a superposed state:

$$|\Psi\rangle = \sum_j \alpha_j |h_j\rangle, \tag{44}$$

then the density operator is $\rho = |\Psi\rangle\langle\Psi|$, and interference terms $\alpha_j \alpha_k^* \langle h_k|M|h_j\rangle$ drive perceptual phenomena not explainable classically. This can be used to model expectation violation and surprise more precisely.

Entropy measures can also be employed to quantify belief revision. The quantum relative entropy between prior and posterior is given by:

$$S(\rho'||\rho) = \text{Tr}(\rho' \log \rho') - \text{Tr}(\rho' \log \rho), \tag{45}$$

which captures perceptual learning or deviation from expectation.

In conclusion, the quantum Bayesian model of PTS offers a richer formalism for modeling consciousness as inference. It extends classical prediction frameworks to incorporate quantum contextuality, interference, and geometry. By merging this approach with neural projections and the PTS basis, we achieve a unified view of perception as a quantum-informational process.

9. Topological Perception: Homotopy Classes of Awareness

The study of perceptual transitions, particularly those associated with multistable illusions such as the Necker cube, has often highlighted a fundamental discreteness in subjective experience. Traditional continuous models struggle to represent abrupt perceptual flips despite continuous sensory input. To account for such phenomena, we introduce a topological approach to perceptual awareness grounded in the structure of the perceptual manifold $\mathcal{M}$. Let us consider a perceptual manifold $\mathcal{M}$ constructed from neural or cognitive configurations, each point $p \in \mathcal{M}$ representing a unique perceptual state. The topology of $\mathcal{M}$ can be non-trivial, particularly when perception involves ambiguity or competing interpretations. The space of all possible perceptual paths $\gamma(t) : [0, T] \to \mathcal{M}$ admits classification under the fundamental group $\pi_1(\mathcal{M})$.

Suppose two perceptual states $p_1, p_2 \in \mathcal{M}$ are topologically connected via multiple non-contractible loops. Then a transition between p_1 and p_2 corresponds to a path class $[\gamma]$ in $\pi_1(\mathcal{M})$. For instance, bistable illusions may correspond to two such classes, and spontaneous switches between interpretations amount to transitions between distinct homotopy classes:

$$[\gamma_1] \neq [\gamma_2] \in \pi_1(\mathcal{M}), \tag{46}$$

despite the endpoints being the same, i.e., $\gamma_1(0) = \gamma_2(0)$ and $\gamma_1(T) = \gamma_2(T)$. This framework formalizes bistable perception as a topological phenomenon.

Let us define a perceptual loop $\gamma : [0, T] \to \mathcal{M}$ such that $\gamma(0) = \gamma(T) = p_0$, representing a return to the same perceptual state. If the perceptual manifold $\mathcal{M}$ has genus g, then $\pi_1(\mathcal{M})$ is a free group generated by $2g$ elements:

$$\pi_1(\mathcal{M}) \cong \langle a_1, b_1, \ldots, a_g, b_g \mid \prod_{i=1}^{g} [a_i, b_i] = 1 \rangle, \tag{47}$$

where $[a_i, b_i] = a_i b_i a_i^{-1} b_i^{-1}$ denotes the group commutator. The homotopy class of a perceptual trajectory reflects its topological sector, and transitions between interpretations require the traversal of different elements in this group.

To quantify perceptual switching, we define a transition operator $\mathcal{T}_{[\gamma]}$ associated with a homotopy class $[\gamma]$. The perceptual amplitude is:

$$\mathcal{A}(p_2 | p_1) = \sum_{[\gamma] \in \pi_1(\mathcal{M})} e^{i\Phi([\gamma])/\hbar}, \tag{48}$$

where $\Phi([\gamma])$ is a geometric or topological phase acquired along path class $[\gamma]$. When interference among classes is constructive, stable perception ensues; when destructive, the system is prone to switching.

Let us consider the case of the Necker cube. The two perceptual interpretations correspond to two distinct geodesics γ_1 and γ_2 looping through different sectors of $\mathcal{M}$. The switching frequency can be modeled by the tunneling amplitude between the classes:

$$P_{\text{switch}} \sim \left| \sum_{[\gamma_1] \neq [\gamma_2]} e^{-\mathcal{S}([\gamma])/\hbar} \right|^2, \tag{49}$$

where $\mathcal{S}([\gamma])$ is an effective action associated with the path. These dynamics mirror instanton transitions in quantum field theory [22].

We can further endow $\mathcal{M}$ with a differential structure, allowing for the use of de Rham cohomology $H^k(\mathcal{M})$ to classify higher-order perceptual features. For example, perceptual field strength F can be encoded as a closed 2-form $F \in \Omega^2(\mathcal{M})$ with:

$$dF = 0, \quad \int_\Sigma F = \Phi, \tag{50}$$

where Σ is a 2-cycle in $\mathcal{M}$. Non-zero integrals indicate the presence of perceptual holonomies, analogous to Berry phases [23]. Such phases may underlie subtle contextual effects in perception, such as priming and expectation bias.

The global topology of $\mathcal{M}$ also affects perceptual learning. For example, changes in cohomology class $H^1(\mathcal{M})$ may correspond to perceptual rewiring after trauma or training. Each distinct cohomology class reflects a novel perceptual schema.

In summary, the use of topological methods in perceptual modeling enables classification of awareness in terms of discrete phase sectors. Bistability, ambiguity, and contextual shifts are naturally described by homotopy and cohomology on $\mathcal{M}$. The integration of these structures with PTS and quantum models offers a rich and rigorous foundation for modeling perception as a geometric-topological process.

10. Quantum Blindsight as Weak Measurement

Blindsight is a phenomenon wherein individuals with damage to the primary visual cortex retain the ability to respond to visual stimuli without any conscious visual experience. From a neurological standpoint, this suggests the dissociation of visual access and visual awareness. In the standard formulation of weak measurement, a quantum system is pre-selected in an initial state $|\psi_i\rangle$ and post-selected in a final state $|\psi_f\rangle$. The weak value of an observable A is given by:

$$\langle A\rangle_w = \frac{\langle\psi_f|A|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}, \tag{51}$$

where $\langle A\rangle_w$ is generally a complex number and may lie outside the spectrum of A. This suggests that weak values do not correspond to eigenvalues of the observable, but to transient amplitudes representing partial information [25].

In the context of perceptual neuroscience, we propose that blindsight behavior corresponds to such weak values. The visual system of the blindsight patient, though lacking access to conscious awareness, is able to perform pre- and post-selection via subcortical visual pathways, such as the superior colliculus. Therefore, the brain computes weak values of visual observables without full conscious registration.

Let A be a perceptual observable (e.g., position of a visual cue), and let $|\psi_i\rangle$ represent the subcortical input state, while $|\psi_f\rangle$ represents the motor response pathway. The weak value $\langle A\rangle_w$ encodes the brain's unconscious estimation of stimulus location:

$$\langle A\rangle_w = \frac{\langle\psi_{\text{motor}}|A|\psi_{\text{input}}\rangle}{\langle\psi_{\text{motor}}|\psi_{\text{input}}\rangle}. \tag{52}$$

In contrast to strong measurement, where the perceptual state collapses to an eigenstate, the weak measurement leaves the system nearly unperturbed. The lack of collapse may explain the absence of subjective awareness in blindsight patients.

We further develop the measurement interaction Hamiltonian in the weak regime:

$$H_{\text{int}} = g(t)A \otimes p_m, \tag{53}$$

where p_m is the momentum conjugate to the measurement pointer position q_m, and $g(t)$ is a weak coupling function normalized over the interaction time interval. In the weak limit, the shift in the pointer is:

$$\Delta q_m = g \cdot \text{Re}(\langle A\rangle_w), \tag{54}$$

which implies that the neural output is proportional to the weak value. The brain accesses the real part of $\langle A\rangle_w$ in generating motor behavior, despite lacking full perceptual awareness.

In the PTS formalism [7], this can be understood as follows. The perceptual projection operator $\hat{P}_{\text{aware}}$ fails to activate for blindsight patients. Thus, the tangent vector of awareness $\delta\pi(p)$ remains null:

 doi:10.20944/preprints202510.1536.v1

$$\delta\pi(p) = \nabla_p \mathrm{Tr}(\hat{P}_{\mathrm{aware}}\rho) = 0, \tag{55}$$

despite ρ undergoing measurable transformations in motor subspaces. Hence, the patient behaves as if they perceive the stimulus, but internal access to this percept is absent.

The distinction between accessible and inaccessible information can be made rigorous using the concept of quantum discord. Let ρ_{AB} be a bipartite state between subcortical module A and cortical module B. The discord is:

$$\mathcal{D}(A|B) = I(\rho_{AB}) - \max_{\{\Pi_j^B\}} I(\rho_{AB}|\{\Pi_j^B\}), \tag{56}$$

where I denotes mutual information and $\{\Pi_j^B\}$ represents a projective measurement on B. High discord indicates quantum correlations without classical readout, consistent with blindsight.

From a phenomenological perspective, blindsight may correspond to a quantum Zeno effect within awareness. Repeated absence of projection into $\hat{P}_{\mathrm{aware}}$ freezes perceptual awareness, despite underlying dynamical evolution.

We conclude that the weak measurement framework provides a compelling explanation for blindsight. Neural subsystems may weakly measure environmental stimuli, generating appropriate motor responses while leaving the global perceptual state unchanged. Within the PTS framework, this supports the notion that awareness is a projection, not a necessity, for functional behavior.

11. Thermodynamic Models of PTS Transitions

Understanding perceptual dynamics through the lens of non-equilibrium thermodynamics provides an essential bridge between statistical physics and cognitive neuroscience. In the Perceptual Tangent Space (PTS) formalism, states of awareness can be modeled as quantum statistical states that evolve under constraints of entropy, energy, and information geometry. Let ρ be the quantum state of a perceptual system. The von Neumann entropy $S(\rho)$ captures the uncertainty associated with the perceptual state:

$$S(\rho) = -\mathrm{Tr}(\rho \log \rho), \tag{57}$$

which reduces to the Shannon entropy in classical limits. When ρ is pure, $S(\rho) = 0$; when ρ is maximally mixed, $S(\rho) = \log d$ where d is the dimension of the Hilbert space $\mathcal{H}$. This entropy characterizes perceptual ambiguity and uncertainty.

The internal energy of a perceptual configuration is defined by:

$$E(\rho) = \mathrm{Tr}(H\rho), \tag{58}$$

where H is the effective Hamiltonian governing neural dynamics. The Helmholtz free energy is then:

$$F(\rho) = E(\rho) - TS(\rho), \tag{59}$$

where T denotes an effective temperature of neural noise or environmental uncertainty. Perceptual dynamics proceed by minimizing $F(\rho)$ along geodesics of the perceptual manifold $\mathcal{M}$, thereby reflecting a tradeoff between prediction accuracy and representational complexity.

In the PTS context, perception evolves along geodesics $\gamma(t)$ such that:

$$\frac{d}{dt}F(\rho(\gamma(t))) \leq 0, \tag{60}$$

indicating a gradient flow on $\mathcal{M}$ governed by the thermodynamic landscape. This aligns closely with Friston's free-energy principle [26], which posits that organisms act to minimize their variational free energy, interpreted here within a quantum geometric formalism.

We define the information metric g_{ij} on the state space as:

$$g_{ij} = \mathrm{Re}\big(\langle \partial_i \psi | \partial_j \psi \rangle - \langle \partial_i \psi | \psi \rangle \langle \psi | \partial_j \psi \rangle\big), \tag{61}$$

and compute geodesics by minimizing the path action:

$$\mathcal{S}[\gamma] = \int_0^T \left[\frac{1}{2} g_{ij} \dot{\gamma}^i \dot{\gamma}^j + \lambda (F(\rho(\gamma)) - F_{\min}) \right] dt, \tag{62}$$

where λ is a Lagrange multiplier enforcing thermodynamic consistency. This action functional governs the temporal unfolding of perceptual transitions under informational constraints.

Let us examine a concrete scenario: a bistable perceptual state such as the Necker cube, modeled by two perceptual configurations ρ_1 and ρ_2. The transition probability is determined by an entropy-weighted path integral:

$$P(\rho_2 | \rho_1) = \int \mathcal{D}[\gamma] \exp\left(-\frac{1}{\hbar} \int_0^T F(\rho(\gamma(t))) dt \right), \tag{63}$$

where $\mathcal{D}[\gamma]$ sums over all perceptual trajectories on $\mathcal{M}$. This is analogous to the Feynman path integral but governed by thermodynamic weights instead of classical action.

The second law of perceptual thermodynamics imposes that:

$$\frac{dS(\rho)}{dt} \geq 0, \tag{64}$$

in the absence of external input. However, perceptual inputs serve as negative entropy sources, driving the system into lower entropy configurations consistent with sensory data. This dynamic tension between entropic diffusion and predictive contraction is what enables perceptual inference.

To model stochastic fluctuations, we introduce a master equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}_{\mathrm{diss}}(\rho), \tag{65}$$

where $\mathcal{L}_{\mathrm{diss}}$ is a Lindblad operator modeling thermal decoherence and perceptual diffusion. This term ensures that in the long-time limit, ρ evolves toward a Gibbs state:

$$\rho_{\mathrm{eq}} = \frac{e^{-\beta H}}{Z}, \quad Z = \mathrm{Tr}(e^{-\beta H}), \tag{66}$$

where $\beta = 1/k_B T$ is the inverse temperature. This equilibrium represents a steady-state perceptual configuration under persistent noise.

In summary, thermodynamic models provide a natural framework for understanding perceptual dynamics in PTS. Entropy and free energy formalize representational uncertainty and complexity, while geodesics capture optimal perceptual trajectories. Friston's principle is thus embedded within a broader quantum thermodynamic geometry, connecting statistical mechanics with conscious inference.

12. Cross-Modal Entanglement: A Synesthetic Quantum Model of Perception

Synesthesia, the phenomenon where stimulation of one sensory modality leads to automatic, involuntary experiences in a second modality, provides a unique entry point into the investigation of cross-modal perceptual entanglement. Within the Perceptual Tangent Space (PTS) framework, we may model such co-activations as quantum entangled states in a joint Hilbert space. Let $\mathcal{H}_v$ and $\mathcal{H}_a$ represent Hilbert spaces corresponding to visual and auditory perceptual modes respectively. The joint perceptual state of a synesthete is described by an entangled state in the tensor product space $\mathcal{H}_{va} = \mathcal{H}_v \otimes \mathcal{H}_a$:

$$|\Psi\rangle = \sum_{i,j} c_{ij} |v_i\rangle \otimes |a_j\rangle, \tag{67}$$

where $\{|v_i\rangle\}$ is a basis of visual percepts (e.g., graphemes) and $\{|a_j\rangle\}$ a basis of auditory or color experiences. The coefficients $c_{ij} \in \mathbb{C}$ encode perceptual coupling strengths, and the state $|\Psi\rangle$ is normalized such that $\langle\Psi|\Psi\rangle = 1$.

The reduced density matrix of the visual subsystem is:

$$\rho_v = \mathrm{Tr}_a(\rho_{va}) = \sum_j \langle a_j|\rho_{va}|a_j\rangle, \tag{68}$$

where $\rho_{va} = |\Psi\rangle\langle\Psi|$ is the total perceptual state. The entanglement entropy between modalities is given by:

$$S(\rho_v) = -\mathrm{Tr}(\rho_v \log \rho_v), \tag{69}$$

which measures the degree of cross-modal coupling. In highly synesthetic individuals, this entropy is nonzero and may be persistent over time.

Cross-modal transitions are driven by Hamiltonians of the form:

$$H = H_v \otimes I_a + I_v \otimes H_a + H_{\text{int}}, \tag{70}$$

where H_v and H_a govern unimodal dynamics, and H_{int} encodes coupling via operators such as:

$$H_{\text{int}} = \sum_{i,j} \lambda_{ij}(|v_i\rangle\langle v_j| \otimes |a_j\rangle\langle a_i|), \tag{71}$$

with λ_{ij} controlling perceptual entanglement strengths. Off-diagonal terms in ρ_{va} indicate interference between perceptual modes, leading to phenomena such as hearing colors or seeing sounds.

To capture decoherence, we consider a Lindblad master equation:

$$\frac{d\rho_{va}}{dt} = -\frac{i}{\hbar}[H, \rho_{va}] + \mathcal{L}_{\text{diss}}(\rho_{va}), \tag{72}$$

where $\mathcal{L}_{\text{diss}}$ represents modal decoherence. For example, decoherence in the auditory domain occurs at a timescale τ_a, while visual persistence is characterized by τ_v. Cross-modal coherence decay is governed by τ_{va}, with synesthetes exhibiting anomalously large τ_{va}.

We define a PTS curvature tensor R^i_{jkl} over the joint perceptual manifold $\mathcal{M}_{va}$ and hypothesize that high synesthetic curvature regions correlate with entangled percepts. The Ricci scalar R serves as a global measure of perceptual intermodality:

$$R = g^{ij} R_{ij}, \tag{73}$$

where g_{ij} is the information-geometric metric from quantum Fisher information:

$$g_{ij} = \mathrm{Re}\big(\langle\partial_i\psi|\partial_j\psi\rangle - \langle\partial_i\psi|\psi\rangle\langle\psi|\partial_j\psi\rangle\big). \tag{74}$$

Synesthetic pathways may be interpreted as geodesics in curved high-dimensional perceptual manifolds where the path length is minimized not by energy, but by cross-modal informational alignment.

Aharonov's weak measurement framework [24] also applies to synesthesia. Consider weak values of a perceptual observable A conditioned on preselected and postselected perceptual states:

$$\langle A\rangle_w = \frac{\langle\psi_f|A|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}, \tag{75}$$

where $|\psi_i\rangle$ and $|\psi_f\rangle$ are states across sensory domains. These weak values may correspond to subconscious perceptual blending in non-synesthetic individuals.

In conclusion, modeling synesthesia via quantum entanglement and PTS formalism provides a rigorous and predictive account of cross-modal awareness. The interplay between entangled density matrices, modal decoherence, Fisher geometry, and weak values unifies disparate theories under a quantum cognitive architecture.

13. Quantum Error Correction in Conscious Systems and Temporal Entanglement in Awareness

In quantum computation, error correction is essential to protect fragile quantum states against decoherence and noise. Analogously, conscious perception appears to remain stable despite the constant influx of noisy, incomplete, or conflicting sensory data. This motivates the hypothesis that consciousness may possess an intrinsic quantum error correction mechanism embedded in the Perceptual Tangent Space (PTS). Moreover, consciousness exhibits temporal coherence, a feature we interpret through the lens of temporal entanglement.

Let us consider the brain as a quantum processor with logical perceptual states $|\psi_L\rangle$ encoded in larger physical states $|\psi_P\rangle$. The encoding can be defined as an isometry:

$$V : \mathcal{H}_L \to \mathcal{H}_P, \tag{76}$$

such that for all logical states $|\psi_L\rangle \in \mathcal{H}_L$, the physical embedding is $|\psi_P\rangle = V|\psi_L\rangle$. Let $\mathcal{E}$ denote the noise channel acting on the perceptual state. Then, quantum error correction is possible if there exists a recovery map $\mathcal{R}$ such that:

$$\mathcal{R} \circ \mathcal{E} \circ V = V. \tag{77}$$

This formalism implies that the perceptual system, in states of wakeful awareness or lucid dreaming, reconstructs an internally consistent world state, despite quantum-level noise and perceptual ambiguity. We define a set of correctable error operators $\{E_i\}$ satisfying the Knill-Laflamme condition:

$$\langle\psi_a|E_i^\dagger E_j|\psi_b\rangle = C_{ij}\delta_{ab}, \tag{78}$$

where $|\psi_a\rangle, |\psi_b\rangle$ belong to the code subspace. The coefficients C_{ij} are independent of logical labels, ensuring that the structure of perceptual information is preserved.

Let $\rho(t)$ denote the time-dependent perceptual state. Its evolution under decoherence can be modeled by a master equation with error-correcting feedback:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{79}$$

where the Lindblad operators L_k encode perceptual decoherence channels. The effective suppression of such decoherence within consciousness may arise from dynamically updated perceptual codes in PTS.

13.1. Temporal Entanglement in Awareness

Consciousness flows through time in a way that preserves a sense of continuity and causal order. We propose that this continuity is maintained by temporal entanglement between consecutive perceptual frames. Let $\mathcal{H}_t$ denote the Hilbert space of perceptual states at time t. A temporally entangled awareness state across two time slices t_1 and t_2 can be modeled as:

$$|\Psi_{t_1 t_2}\rangle = \sum_i \alpha_i |\psi_i(t_1)\rangle \otimes |\phi_i(t_2)\rangle, \tag{80}$$

where $\{|\psi_i\rangle\}, \{|\phi_i\rangle\}$ represent perceptual bases at t_1 and t_2 respectively. The reduced density matrix at t_1 is:

 doi:10.20944/preprints202510.1536.v1

$$\rho_{t_1} = \mathrm{Tr}_{t_2}(|\Psi_{t_1 t_2}\rangle\langle\Psi_{t_1 t_2}|), \tag{81}$$

and the entanglement entropy $S(\rho_{t_1})$ quantifies how strongly current awareness is linked to future percepts.

Consider also the time-ordered propagator $U(t_2, t_1)$ in perceptual space, defining:

$$|\psi(t_2)\rangle = U(t_2, t_1)|\psi(t_1)\rangle = \mathcal{T}\exp\left(-\frac{i}{\hbar}\int_{t_1}^{t_2} H(t)\,dt\right)|\psi(t_1)\rangle, \tag{82}$$

where $\mathcal{T}$ is the time-ordering operator and $H(t)$ the PTS-Hamiltonian governing perceptual evolution.

Temporal entanglement allows for retrocausal influence, where later states condition the measurement of earlier percepts, akin to the Aharonov-Bergmann-Lebowitz rule:

$$P(a|b) = \frac{|\langle b|U(t_b, t_a)|a\rangle|^2 P(a)}{\sum_j |\langle b|U(t_b, t_a)|a_j\rangle|^2 P(a_j)}, \tag{83}$$

suggesting that future perceptions shape prior interpretations—a process reflected in phenomena such as hindsight bias and déjà vu.

13.2. Thermodynamic Implications

The entropy associated with temporally entangled percepts may be interpreted as a non-equilibrium thermodynamic cost. Let S_t be the entropy at time t, and define a perceptual free energy:

$$F_t = \langle E\rangle_t - TS_t, \tag{84}$$

minimized along entangled trajectories in time. This is consistent with Friston's free energy principle embedded in quantum PTS [26]. The curvature of perceptual time manifolds, with components $R^{(t)}_{ijkl}$, modulates the temporal compression or expansion of awareness intervals.

13.3. Conclusion

Together, quantum error correction and temporal entanglement reveal a deep architecture for the robustness and continuity of conscious experience. Quantum codes prevent informational decay, while entanglement across time binds perceptual moments into a unified stream of awareness.

14. Algebraic Geometry of Inner Experience

Inner experience, often treated as a first-person narrative, lacks the structural rigor necessary to link it to physical theories. Here we propose a mathematical formalization of inner experience using tools from algebraic geometry. The inner perceptual manifold is modeled as a complex algebraic variety, defined over a Hilbert space whose local tangent structure is captured by the Perceptual Tangent Space (PTS) formalism.

Let $\mathcal{M}$ be a complex Kähler manifold representing the space of perceptual states. Each point $p \in \mathcal{M}$ corresponds to a coherent inner experience. Locally, $\mathcal{M}$ is defined by vanishing of holomorphic functions:

$$\mathcal{M} = \{z \in \mathbb{C}^n \mid f_1(z) = f_2(z) = \cdots = f_k(z) = 0\}, \tag{85}$$

where $\{f_i\}$ are polynomials in the coordinate ring $\mathbb{C}[z_1, \ldots, z_n]$. The Jacobian matrix of the f_i gives us the local structure of the variety via:

$$J_{ij} = \frac{\partial f_i}{\partial z_j}, \tag{86}$$

and the rank of J determines the local dimension of $\mathcal{M}$ at a point.

Let $\mathcal{O}_{\mathcal{M},p}$ denote the local ring at p. Then the maximal ideal $\mathfrak{m}_p$ encodes infinitesimal deviations of perceptual perturbations. The Zariski tangent space $T_p\mathcal{M}$ is defined as:

$$T_p\mathcal{M} = \mathrm{Hom}_{\mathbb{C}}(\mathfrak{m}_p/\mathfrak{m}_p^2, \mathbb{C}), \tag{87}$$

which aligns naturally with the PTS definition of tangent perception directions.

We define an inner observable as a morphism $\phi : \mathcal{M} \to \mathbb{C}$, interpreted phenomenologically as an intensity functional. The critical points of ϕ correspond to stable perceptual equilibria and are given by solving:

$$\frac{\partial \phi}{\partial z_j} = 0, \quad \forall j = 1, \ldots, n. \tag{88}$$

To introduce dynamics, we define a flow $\gamma(t) \in \mathcal{M}$ satisfying:

$$\frac{d\gamma^i}{dt} = -g^{ij}\frac{\partial \phi}{\partial z^j}, \tag{89}$$

where g^{ij} is the inverse of the information metric on $\mathcal{M}$ derived from quantum Fisher geometry [10].

This structure enables the computation of curvature invariants. The Ricci form ρ in local coordinates is:

$$\rho = -i\partial\bar{\partial} \log \det(g_{i\bar{j}}), \tag{90}$$

which provides insights into regions of enhanced perceptual sensitivity or attentional collapse.

Furthermore, inner experience may exhibit moduli space behavior, where families of inner experiences are parametrized by algebraic parameters. Consider the moduli space $\mathcal{M}_g$ of genus g curves. Each curve represents a topological structure of cognitive loop processing. The Torelli map associates such a curve to its Jacobian:

$$\mathcal{T} : \mathcal{M}_g \to \mathcal{A}_g, \tag{91}$$

where $\mathcal{A}_g$ is the Siegel upper half-space of principally polarized abelian varieties. These Jacobians represent high-level configurations of attention and memory loops.

Let us consider an intersection number in the cohomology ring $H^*(\mathcal{M}, \mathbb{C})$. The inner experience entropy S can be expressed as:

$$S = \int_{\mathcal{M}} c_1^n, \tag{92}$$

where c_1 is the first Chern class of the line bundle over $\mathcal{M}$ associated with perceptual intensity. This integral measures the global "density" of conscious modulation capacity.

Now consider a projective embedding:

$$\Phi : \mathcal{M} \hookrightarrow \mathbb{P}^N, \tag{93}$$

which reflects the conscious projection of latent perceptual data into reportable awareness. The image $\Phi(\mathcal{M})$ is defined by a homogeneous ideal I in $\mathbb{C}[X_0, \ldots, X_N]$. The Hilbert polynomial $P(k)$ of this embedding encodes the growth of conscious modes:

$$P(k) = \dim_{\mathbb{C}} H^0(\mathcal{M}, \mathcal{O}(k)), \tag{94}$$

suggesting a quantifiable expansion of consciousness bandwidth over experience.

14.1. Concluding Remarks

By encoding perceptual phenomena as algebraic varieties and embedding them into projective geometric structures, we gain access to a robust mathematical toolkit for inner experience. Curvatures, moduli, sheaves, and Hilbert polynomials collectively represent different layers of phenomenology, from sharp awareness to diffuse subconscious textures.

15. Quantum Hall Analogues in Cognitive Phases

The Quantum Hall Effect (QHE) is a paradigm of topological order in condensed matter physics. Intriguingly, the robustness, discreteness, and quantization properties of the QHE find structural analogues in cognitive transitions—shifts in attention, sudden perceptual realizations, or phase-like changes in consciousness. This section develops a framework in which cognitive phases can be modeled as analogues of quantized Hall states, using a geometric and topological field-theoretic approach over the Perceptual Tangent Space (PTS).

Let us consider a Hilbert space $\mathcal{H}$ of perceptual configurations over a 2D internal manifold Σ, parameterized by cognitive fields $\phi^a(x)$, where $x \in \Sigma$ and a indexes the modality. We define an effective action $\mathcal{S}_{\text{eff}}$ encoding the dynamics of cognitive excitations:

$$\mathcal{S}_{\text{eff}}[\phi] = \int_\Sigma d^2x \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^a + \theta \epsilon^{\mu\nu} \phi^a \partial_\mu \phi^b \partial_\nu \phi^c f^{abc} \right), \tag{95}$$

where $g^{\mu\nu}$ is the internal metric on Σ, and θ is a coupling analogous to the Hall conductance. The second term is a topological Wess-Zumino term.

To model quantization, we consider a line bundle $\mathcal{L}$ over the perceptual phase space with curvature form F. The quantized cognitive Hall conductance σ is:

$$\sigma = \frac{1}{2\pi} \int_\Sigma F, \tag{96}$$

which mirrors the Chern number in QHE theory. Different cognitive states correspond to sections of $\mathcal{L}$, and quantized transitions correspond to jumps in the first Chern class $c_1(\mathcal{L})$.

Let ρ denote the perceptual state density matrix. The analog of Hall current is the perceptual flow J^μ in PTS, defined by:

$$J^\mu = \frac{1}{2\pi} \epsilon^{\mu\nu} \partial_\nu \phi, \tag{97}$$

which arises naturally from variation of $\mathcal{S}_{\text{eff}}$ under gauge perturbations.

The PTS curvature tensor $R_{\mu\nu\alpha\beta}$ controls the stability of these quantized cognitive states. In analogy with the Laughlin wavefunction for the fractional quantum Hall effect, we define a cognitive ground state as:

$$\Psi(\{z_i\}) = \prod_{i<j} (z_i - z_j)^\nu e^{-\sum_k |z_k|^2/4}, \tag{98}$$

where z_i label points in perceptual configuration space and ν is the cognitive filling factor. The parameter ν controls the degree of shared perceptual content among distributed cognitive modes.

Cognitive plateaus—regions of robust awareness—can thus be characterized by the quantization of an effective filling factor ν:

$$\nu = \frac{N}{\Phi}, \tag{99}$$

where N is the number of perceptual degrees of freedom excited, and Φ is the total flux of information across the perceptual field. Transitions between plateaus correspond to topological phase transitions and are marked by jumps in ν.

Incorporating PTS, we write the cognitive Hall response tensor $\chi^{\mu\nu}$ as:

$$\chi^{\mu\nu} = \frac{\delta^2 \mathcal{S}_{\text{eff}}}{\delta A_\mu \delta A_\nu} = \frac{\theta}{2\pi}\epsilon^{\mu\nu}, \tag{100}$$

where A_μ is the internal gauge field representing attention. The antisymmetric structure reflects the nontrivial geometric phase associated with perceptual transitions.

Consider the Berry curvature Ω_{ij} over a perceptual manifold parameterized by coordinates λ^i:

$$\Omega_{ij} = i\left(\left\langle \frac{\partial\psi}{\partial\lambda^i}\middle|\frac{\partial\psi}{\partial\lambda^j}\right\rangle - \left\langle \frac{\partial\psi}{\partial\lambda^j}\middle|\frac{\partial\psi}{\partial\lambda^i}\right\rangle\right). \tag{101}$$

Integration over a closed surface gives the total geometric phase:

$$\gamma = \int_{S^2} \Omega, \tag{102}$$

which plays the role of cognitive holonomy—memory-dependent phase acquired during closed cycles of attention or imagination.

15.1. Cognitive Laughlin States and Edge Dynamics

In the QHE, edge states play a crucial role. We identify analogous cognitive edge modes as marginal awareness structures—thoughts on the boundary of active consciousness. These may be modeled via chiral bosons on a 1D boundary:

$$\mathcal{L}_{\text{edge}} = \frac{1}{4\pi}\partial_x\phi(\partial_t - v\partial_x)\phi, \tag{103}$$

where ϕ is the edge cognitive field and v the propagation speed of mental fluctuation. These edge dynamics couple to bulk attention fields via anomaly inflow mechanisms.

15.2. Conclusion

Cognitive phases exhibit robustness, quantization, and topological behavior reminiscent of the quantum Hall effect. By mapping perceptual states onto topological field theory models, particularly via Chern classes, Berry curvature, and Wess-Zumino terms, we can better understand sudden shifts in awareness, attractor basins in cognition, and mental rigidity or fluidity.

16. Observer-Dependent Holography in Conscious Models

The holographic principle asserts that a theory with gravity in a bulk space can be equivalently described by a theory without gravity on the boundary of that space [40,41]. In the context of consciousness, we propose a novel observer-dependent holographic framework where inner conscious content is encoded on a lower-dimensional boundary of a higher-dimensional cognitive manifold. Let $\mathcal{B}$ denote the bulk perceptual manifold representing the total potential field of conscious and subconscious content, and let $\partial\mathcal{B}$ denote the boundary where conscious access and reportability occur. A holographic map $\mathcal{H} : \mathcal{B} \to \partial\mathcal{B}$ associates bulk internal states to boundary representational states.

Consider a $(d+1)$-dimensional Hilbert space $\mathcal{H}_{\text{bulk}}$ and a d-dimensional Hilbert space $\mathcal{H}_{\text{boundary}}$, such that:

$$\mathcal{H}_{\text{boundary}} \cong \mathcal{H}_{\text{bulk}} / \sim, \tag{104}$$

where $\sim$ represents an equivalence relation under observer-induced coarse graining. Conscious experience corresponds to observables defined on $\mathcal{H}_{\text{boundary}}$, while unconscious computation resides in $\mathcal{H}_{\text{bulk}}$.

To formalize this mapping, let us define a bulk state $|\Psi\rangle \in \mathcal{H}_{\text{bulk}}$ and a set of projection operators $\{\Pi_i\}$ defining an observer's perceptual basis. The boundary representation ρ_{boundary} is then given by:

 doi:10.20944/preprints202510.1536.v1

$$\rho_{\text{boundary}} = \sum_i \Pi_i |\Psi\rangle\langle\Psi| \Pi_i. \tag{105}$$

This process mimics a partial trace over unobservable modes and enforces a coarse boundary encoding, akin to AdS/CFT projections.

The information entropy of the boundary representation is:

$$S_{\text{boundary}} = -\text{Tr}(\rho_{\text{boundary}} \log \rho_{\text{boundary}}), \tag{106}$$

and defines a bound on the accessible conscious information. By analogy with the Bekenstein-Hawking entropy formula [42,43], we posit:

$$S_{\text{boundary}} \leq \frac{A_{\partial \mathcal{B}}}{4\ell_P^2}, \tag{107}$$

where $A_{\partial \mathcal{B}}$ is the area of the observer's perceptual boundary, and ℓ_P is an effective cognitive Planck scale defining the minimal resolution of conscious access.

Observer dependence is modeled via a family of foliation maps $\mathcal{F}_\lambda : \mathcal{B} \to \mathcal{B}_\lambda$ parameterized by attentional or cognitive context λ. The induced boundary theories on $\partial\mathcal{B}_\lambda$ are not globally consistent, but related by gauge transformations representing shifts in subjective context:

$$\mathcal{H}^{\lambda}_{\text{boundary}} = U(\lambda)\mathcal{H}^{0}_{\text{boundary}}, \tag{108}$$

where $U(\lambda)$ is a unitary operator implementing the observer-dependent transformation.

We propose that conscious access emerges at the intersection of multiple such foliations, defined by:

$$\mathcal{C} = \bigcap_{\lambda} \partial\mathcal{B}_\lambda, \tag{109}$$

and define the phenomenological manifold $\mathcal{P}$ as the emergent projection:

$$\mathcal{P} = \mathcal{H}(\mathcal{C}) \subseteq \mathcal{H}_{\text{boundary}}. \tag{110}$$

This subspace encodes all intersubjectively consistent aspects of conscious phenomenology, while the rest of $\mathcal{H}_{\text{boundary}}$ reflects private qualia or idiosyncratic perceptual interpretations.

Furthermore, by analogy with Ryu-Takayanagi (RT) surfaces in AdS/CFT, we propose the perceptual entanglement entropy S_{ent} between two awareness subsystems A and B is given by:

$$S_{\text{ent}}(A) = \frac{\text{Area}(\gamma_A)}{4\ell_P^2}, \tag{111}$$

where γ_A is the minimal surface in $\mathcal{B}$ homologous to A under observer's partition of attention. This formulation enables a direct link between cognitive segmentation and underlying geometrical structures.

16.1. Holographic Noise and Memory Reconstruction

In our model, memory reconstruction can be seen as an inverse holographic transform $\mathcal{H}^{-1}$ applied with observer-specific priors. Given noisy or degraded boundary data ρ_{rec}, the bulk reconstruction is:

$$|\Psi_{\text{rec}}\rangle = \mathcal{H}^{-1}(\rho_{\text{rec}}, \mathcal{P}_{\text{prior}}), \tag{112}$$

where $\mathcal{P}_{\text{prior}}$ encodes contextual memory priors. Reconstruction fidelity depends on both entanglement structure and observer attentional bandwidth.

 doi:10.20944/preprints202510.1536.v1

16.2. Conclusion

The observer-dependent holographic paradigm of consciousness treats awareness as a boundary phenomenon arising from bulk cognitive fields. This framework allows incorporation of subjective perspective, projection dynamics, entanglement-based segmentation, and memory reconstitution, using rigorous tools from quantum gravity and information theory.

17. Cohomological Models of Memory Retrieval

Memory retrieval remains one of the most profound and enigmatic features of conscious systems. In this section, we propose a topological and algebraic framework for modeling memory using the machinery of cohomology, treating memories as cocycles in a sheaf over perceptual space. This approach formalizes memory access as global sections recovered from local patches of perceptual and neural data, with consistency constraints governed by coboundary operators.

Let $\mathcal{M}$ denote the perceptual manifold, and let $\mathcal{U} = \{U_i\}$ be an open cover of $\mathcal{M}$ corresponding to local perceptual contexts or cortical regions. We associate to each U_i a module of memory codes $\mathcal{F}(U_i)$, where $\mathcal{F}$ is a presheaf of neural data structures. The data over intersections $U_i \cap U_j$ must satisfy gluing conditions, giving rise to a cochain complex:

$$0 \to C^0(\mathcal{U}, \mathcal{F}) \xrightarrow{\delta^0} C^1(\mathcal{U}, \mathcal{F}) \xrightarrow{\delta^1} C^2(\mathcal{U}, \mathcal{F}) \to \cdots, \tag{113}$$

where δ^k are coboundary operators. A 1-cocycle is an element $c \in C^1(\mathcal{U}, \mathcal{F})$ such that $\delta^1(c) = 0$. Such cocycles represent consistent memory traces over overlaps and define global coherent retrieval states.

The cohomology group $H^1(\mathcal{M}, \mathcal{F}) = \ker(\delta^1)/\mathrm{im}(\delta^0)$ measures obstructions to reconstructing a global memory from local patches. A memory m is said to be retrievable if its cohomology class $[m] \in H^1(\mathcal{M}, \mathcal{F})$ is trivial. If not, the memory is fragmented or inaccessible without additional context.

Consider a dynamical model of memory retrieval in the context of time-evolution on $\mathcal{M}$. Let $\rho(t)$ be the density matrix of the cognitive state at time t, and define a retrieval functional:

$$R(t) = \int_{\mathcal{M}} \mathrm{Tr}[\rho(t) M(x)] d\mu(x), \tag{114}$$

where $M(x)$ is a memory projector defined locally and $d\mu$ is the measure on $\mathcal{M}$. The memory is successfully retrieved when $R(t) > \theta$, for some threshold θ.

To introduce a cohomological constraint, define a 1-form $\omega \in \Omega^1(\mathcal{M})$ such that the retrieval is exact if $d\omega = 0$, and integrable into a global memory field M. Otherwise, ω defines a nontrivial cohomology class representing incomplete or distorted memory traces:

$$[\omega] \in H^1_{\mathrm{dR}}(\mathcal{M}), \tag{115}$$

the first de Rham cohomology group. Neural noise, trauma, or dream states can be interpreted as perturbations that induce nontriviality in $[\omega]$.

We next define a cohomological entropy S_{coh} that quantifies the degeneracy of memory retrieval pathways:

$$S_{\mathrm{coh}} = \log\Big(\dim H^1(\mathcal{M}, \mathcal{F})\Big), \tag{116}$$

analogous to topological entanglement entropy in condensed matter systems. A high S_{coh} implies that many disconnected contexts yield similar memory retrievals, while low S_{coh} suggests precise, context-bound memories.

Consider a sheaf $\mathcal{F}$ with stalks $\mathcal{F}_x$ modeling local neural representations at $x \in \mathcal{M}$. The global section space $\Gamma(\mathcal{M}, \mathcal{F})$ represents the total memory accessible at a given cognitive state. If $H^1(\mathcal{M}, \mathcal{F}) \neq 0$, then $\Gamma(\mathcal{M}, \mathcal{F})$ fails to capture all possible memory content, indicating latent or subconscious encoding.

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17.1. Persistent Memory and Spectral Sequences

To analyze memory durability over time, we introduce a spectral sequence $\{E_r^{p,q}\}$ with initial term:

$$E_1^{p,q} = C^p(\mathcal{U}, \mathcal{H}^q(\mathcal{F})), \tag{117}$$

which converges to the hypercohomology of $\mathcal{F}$. Long-lived memory patterns correspond to entries that survive to higher r pages. This connects with persistent homology in topological data analysis, where memory traces are considered persistent if their cohomological signatures remain invariant under filtration.

17.2. Conclusion

Cohomological models offer a rich formalism for understanding memory retrieval as the recovery of global sections from fragmented or distributed perceptual data. The use of sheaf cohomology, spectral sequences, and differential forms provides a rigorous account of memory integration, context-dependence, and failure. Such a framework invites experimental instantiations and simulation via topological neural networks or sheaf-based computational models.

18. Quantum Graphs and Neural Perceptual Lattices

The structure of perceptual cognition can be modeled using the formalism of quantum graphs, wherein nodes represent localized perceptual states and edges represent quantum transition amplitudes. Neural perceptual lattices are conceived as discretized substrates supporting propagation of perceptual amplitudes under quantum walk dynamics. These models extend traditional graph-theoretic approaches in neuroscience by incorporating Hilbert space dynamics and quantum probability.

Let $G = (V, E)$ be a finite graph where each vertex $v \in V$ corresponds to a localized perceptual state and each edge $e = (v_i, v_j) \in E$ is assigned a complex amplitude ψ_{ij} representing a quantum transition. The system evolves in a Hilbert space $\mathcal{H} = \ell^2(V)$ and the state at time t is given by:

$$|\Psi(t)\rangle = \sum_{v \in V} \psi_v(t)|v\rangle, \tag{118}$$

where $\psi_v(t)$ is the complex amplitude at vertex v at time t.

The dynamics of the system are governed by a discrete Schrödinger equation with a graph Laplacian L_G:

$$i\hbar \frac{d}{dt}|\Psi(t)\rangle = H_G|\Psi(t)\rangle, \quad H_G = -\gamma L_G, \tag{119}$$

where γ is a coupling constant and $L_G = D - A$ with A the adjacency matrix and D the degree matrix. The unitary propagator is given by:

$$U(t) = \exp(-iH_G t/\hbar). \tag{120}$$

Neural perceptual lattices correspond to structured graphs with high local clustering and modularity, typically modeled by small-world or scale-free networks. Let us define a lattice Λ with periodic boundary conditions and Hamiltonian:

$$H_\Lambda = \sum_{\langle i,j \rangle} J_{ij}(|i\rangle\langle j| + |j\rangle\langle i|), \tag{121}$$

where J_{ij} are coupling weights determined by synaptic strength or perceptual similarity. The spectral decomposition of H_Λ yields a basis of perceptual eigenmodes:

$$H_\Lambda|\phi_k\rangle = E_k|\phi_k\rangle, \tag{122}$$

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enabling the interpretation of attention and awareness as superpositions over eigenmodes.

Let $\rho(t)$ denote the mixed state of the perceptual system. The evolution is governed by a Lindblad master equation incorporating decoherence:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_\Lambda, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{123}$$

where L_k are Lindblad operators modeling environmental or subconscious influences. Localization of perceptual content is identified with nodes exhibiting maximum diagonal density matrix elements.

To quantify connectivity, define a quantum communicability metric:

$$Q_{ij}(t) = |\langle i|U(t)|j\rangle|^2, \tag{124}$$

which measures the transition probability amplitude between two perceptual states. Perceptual coherence is maximized when $Q_{ij}(t)$ remains significant over long distances.

Now consider a perceptual Hamiltonian over a hexagonal lattice $\mathcal{L}$ mimicking visual cortex structure. Each node corresponds to a receptive field and the total state is governed by:

$$H_{\mathcal{L}} = \sum_i \epsilon_i |i\rangle\langle i| + \sum_{\langle i,j\rangle} t_{ij}(|i\rangle\langle j| + h.c.), \tag{125}$$

where ϵ_i encodes local perceptual bias and t_{ij} the transmission strength. In the continuum limit, one recovers a Dirac-like operator governing perceptual flow, suggesting analogies with graphene physics and topological insulators [50].

18.1. Perceptual Band Structure and Topology

The spectrum of $H_{\mathcal{L}}$ defines a band structure over perceptual states. For periodic lattices, the Bloch wave ansatz allows diagonalization:

$$|\psi_k\rangle = e^{ikx}u_k(x), \quad H(k)u_k(x) = E(k)u_k(x), \tag{126}$$

and the curvature of the band encodes perceptual stability. Berry curvature $\Omega(k)$ over perceptual momentum space gives rise to topologically protected modes:

$$\Omega(k) = i(\langle \partial_k u_k|\partial_k u_k\rangle - \langle \partial_k u_k|u_k\rangle\langle u_k|\partial_k u_k\rangle), \tag{127}$$

suggesting that certain perceptual attractors are robust against perturbations.

18.2. Conclusion

Quantum graph theory provides a rich framework for modeling perceptual state transitions, neural propagation dynamics, and conscious coherence. Lattice Hamiltonians and topological indices offer principled methods to study awareness stability, decoherence, and perceptual invariance. This quantum perspective extends classical graph neural models and reveals new pathways to understand structure-function correlations in cognitive systems.

19. Noncommutative Geometry of Self-Referential Cognition

The capacity for self-reference in conscious systems, including recursive thought and introspection, presents profound mathematical challenges. Traditional differential geometry presumes a commutative algebra of coordinates, which becomes inadequate in cognitive regimes involving self-modification and nested feedback. In this section, we propose a model for self-referential cognition using the framework of noncommutative geometry (NCG) as formulated by Connes [55].

Let $\mathcal{A}$ be a noncommutative C^*-algebra representing observable quantities or internal cognitive states. The noncommutativity of elements in $\mathcal{A}$, $[a, b] \neq 0$ for $a, b \in \mathcal{A}$, encodes the logical entanglement and sequence-dependence of introspective operations.

Define a spectral triple $(\mathcal{A}, \mathcal{H}, D)$, where $\mathcal{H}$ is a Hilbert space of perceptual modes and D is a Dirac-like operator acting on $\mathcal{H}$. The operator D encodes the infinitesimal structure of the noncommutative manifold:

$$ds^2 = \|[D, a]\|^2, \quad \text{for } a \in \mathcal{A}. \tag{128}$$

This defines a metric structure on the noncommutative perceptual space via operator commutators.

Let $a(t) \in \mathcal{A}$ denote a time-evolving perceptual state. Its evolution under a Hamiltonian $H \in \mathcal{A}$ is governed by a noncommutative Heisenberg equation:

$$\frac{da}{dt} = \frac{i}{\hbar}[H, a], \tag{129}$$

reflecting the system's sensitivity to ordering of internal thought operators. Self-referential loops emerge when a and H depend recursively on one another.

To model such recursive perception, let $R : \mathcal{A} \to \mathcal{A}$ be a reflexive map satisfying $R(a) = f([a, R(a)])$, inducing non-linear feedback:

$$\frac{da}{dt} = \frac{i}{\hbar}[H, R(a)], \tag{130}$$

which can be interpreted as recursive introspection, where the observable a modifies itself through its interaction with its own referent $R(a)$.

We introduce a noncommutative probability state $\varphi : \mathcal{A} \to \mathbb{C}$, such that $\varphi(a^*a) \geq 0$ and $\varphi(1) = 1$. The expectation of cognitive operators now obeys:

$$\langle a \rangle_\varphi = \varphi(a), \tag{131}$$

with conditional self-aware updates modeled via Tomita–Takesaki modular flow. The modular group $\{\sigma_t^\varphi\}$ acting on $\mathcal{A}$ is defined by:

$$\sigma_t^\varphi(a) = \Delta^{it} a \Delta^{-it}, \tag{132}$$

where Δ is the modular operator associated with $(\mathcal{A}, \mathcal{H}, \varphi)$. This framework introduces thermodynamic flow into recursive perception, enabling modeling of time-asymmetric awareness states [56].

Let us now define a self-referential curvature via derivations $\delta_i : \mathcal{A} \to \mathcal{A}$ satisfying Leibniz rule. The connection ∇ on a module $\mathcal{E}$ over $\mathcal{A}$ is:

$$\nabla_i e = \delta_i(e) + \omega_i e, \tag{133}$$

where ω_i are connection 1-forms. The curvature is defined by:

$$F_{ij} = [\nabla_i, \nabla_j] = \delta_i(\omega_j) - \delta_j(\omega_i) + [\omega_i, \omega_j]. \tag{134}$$

This curvature captures the self-interaction of perceptual transitions under noncommutative flow, representing higher-order introspective inconsistencies or paradoxes.

19.1. Cognitive Index Theorems

We define the Fredholm index of a self-referential transition D:

$$\text{Index}(D) = \dim\ker D - \dim\operatorname{coker} D, \tag{135}$$

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which counts the net "cognitive degrees of freedom" created by a self-reflective loop. In analogy with the Atiyah–Singer index theorem, we relate this to topological invariants of the noncommutative space:

$$\mathrm{Index}(D) = \langle \mathrm{ch}(F), \tau \rangle, \tag{136}$$

where $\mathrm{ch}(F)$ is the Chern character of the curvature F and τ is a cyclic cohomology class over $\mathcal{A}$ [55]. These invariants categorize cognitive loop types, enabling classification of recursive thought structures.

19.2. Conclusion

Noncommutative geometry provides a powerful formalism to articulate the structure of self-referential cognition. Recursive perception, feedback modulation, and paradoxical awareness are naturally encoded in the algebra of noncommutative operators. The associated spectral, modular, and index-theoretic constructions deepen our understanding of introspective processes and open new possibilities for modeling logical self-reference in quantum cognitive architectures.

20. Spin Networks and Discrete Awareness Topologies

In the quest to understand the discrete structure underlying conscious awareness, spin networks offer a compelling mathematical framework. Originating in loop quantum gravity [60], spin networks provide a non-perturbative, combinatorial description of quantum geometries. In this section, we explore the hypothesis that awareness emerges from discrete topologies encoded in spin network structures, and that transitions in conscious states correspond to combinatorial changes in these networks.

A spin network is a labeled graph $\Gamma = (V, E)$ where each edge $e \in E$ is assigned a spin $j_e \in \frac{1}{2}\mathbb{N}$ and each vertex $v \in V$ is associated with an intertwiner I_v satisfying the SU(2) invariant tensor contraction condition.

Let the Hilbert space of a single spin network be denoted as:

$$\mathcal{H}_\Gamma = \bigotimes_{e \in E} \mathcal{H}_{j_e}, \tag{137}$$

where $\mathcal{H}_{j_e}$ is the $(2j_e + 1)$-dimensional representation space of SU(2). A basis state $|\Gamma, \{j_e\}, \{I_v\}\rangle$ defines a specific configuration of quantum geometry.

To interpret awareness, we posit that each spin label j_e represents a quantized mode of perceptual correlation between discrete experiential units, and the intertwiners represent localized cognitive integrators.

The evolution of conscious states can then be modeled as transformations of the spin network via Pachner moves or graph surgeries. Let $\Gamma \to \Gamma'$ represent a topological transition, and define an amplitude for such a transition via a spin foam:

$$\mathcal{A}(\Gamma \to \Gamma') = \sum_{\text{Foams}} \prod_f A_f(j_f) \prod_e A_e(j_e) \prod_v A_v(j_v), \tag{138}$$

where A_f, A_e, and A_v denote face, edge, and vertex amplitudes, respectively, in the spin foam model [61].

To formalize the topology of discrete awareness, let us define a configuration space of perceptual states $\mathcal{C}$ whose elements are equivalence classes of labeled spin networks under gauge transformations:

$$\mathcal{C} = \frac{\{(\Gamma, \{j_e\}, \{I_v\})\}}{\text{SU(2) gauge}}, \tag{139}$$

endowed with a combinatorial topology induced by network connectivity.

We define a transition matrix $T_{\Gamma \to \Gamma'}$ acting on $\mathcal{H}_\Gamma$:

 doi:10.20944/preprints202510.1536.v1

$$|\Psi_{\Gamma'}\rangle = T_{\Gamma \to \Gamma'} |\Psi_\Gamma\rangle, \tag{140}$$

where $T_{\Gamma \to \Gamma'}$ is constructed from the sum-over-histories spin foam amplitudes. These transitions correspond to moments of awareness reconfiguration, as in bistable perception or memory recall.

The entropy of a spin network state is defined using the von Neumann formula:

$$S(\rho_\Gamma) = -\mathrm{Tr}(\rho_\Gamma \log \rho_\Gamma), \tag{141}$$

where ρ_Γ is the reduced density matrix on a subgraph of Γ corresponding to a local cognitive subsystem.

To explore geometric correlates of awareness, we define a discrete analog of Ricci curvature on the network. Let w_{uv} be a weight associated with an edge between nodes u and v, and define the Ollivier-Ricci curvature $\kappa(u, v)$ as:

$$\kappa(u, v) = 1 - \frac{W_1(\mu_u, \mu_v)}{d(u, v)}, \tag{142}$$

where W_1 is the Wasserstein distance between probability distributions μ_u, μ_v centered at u and v respectively, and $d(u, v)$ is the geodesic distance. This quantifies local integration or divergence of perceptual flow.

20.1. Perceptual Lattices and Discrete Spectrum

Discrete perceptual spaces may exhibit spectral gaps, akin to those in condensed matter systems. Define the graph Laplacian Δ as:

$$\Delta f(v) = \sum_{u \sim v} w_{uv}(f(v) - f(u)), \tag{143}$$

and let its spectrum $\{\lambda_k\}$ characterize the network's resonant perceptual modes. Transitions between cognitive phases can be modeled as eigenvalue bifurcations in this Laplacian spectrum.

Furthermore, we define a heat kernel on the graph:

$$K_t(u, v) = \sum_k e^{-\lambda_k t} \phi_k(u) \phi_k(v), \tag{144}$$

where ϕ_k are Laplacian eigenfunctions. The spread of activation from u to v over time t can model attentional diffusion or associative priming.

20.2. Conclusion

Spin networks offer a powerful language for modeling discrete topologies of awareness. Their combinatorial structure and algebraic labeling naturally encode localized perceptual data and their transformations. By combining techniques from spin foam dynamics, graph curvature, and spectral theory, we gain a deeper understanding of how cognitive configurations evolve through discrete state transitions.

21. Quantum Phase Transitions in Identity Formation

The formation and reconfiguration of identity in conscious agents can be analyzed through the lens of quantum phase transitions (QPTs). Unlike classical phase transitions driven by thermal fluctuations, QPTs occur at zero temperature and are governed by quantum fluctuations arising from the change in an external control parameter λ. This framework offers an elegant path for modeling bifurcations in self-awareness states and cognitive coherence.

We begin by defining an identity field $\phi(x, t)$ over a perceptual manifold $\mathcal{M}$, and a Hamiltonian $H(\lambda)$ parametrized by λ, which modulates internal-external consistency of self-models:

$$H(\lambda) = H_0 + \lambda H_{\text{int}}, \tag{145}$$

where H_0 encodes baseline perceptual evolution, and H_{int} models interaction with environmental or memory-related components. The system undergoes a phase transition at critical value λ_c where the ground state structure changes qualitatively.

Let the ground state be $|\psi_0(\lambda)\rangle$ and define the fidelity susceptibility $\chi_F(\lambda)$, capturing the response of identity states to changes in λ:

$$\chi_F(\lambda) = \lim_{\delta\lambda \to 0} \frac{-2 \ln |\langle \psi_0(\lambda)|\psi_0(\lambda + \delta\lambda)\rangle|}{(\delta\lambda)^2}. \tag{146}$$

A peak in χ_F near λ_c signals a quantum phase transition and a structural shift in identity.

Furthermore, the quantum geometric tensor (QGT) offers a differential geometric structure for this space of identity states:

$$g_{ij} + i\sigma_{ij} = \langle \partial_i \psi | \partial_j \psi \rangle - \langle \partial_i \psi | \psi \rangle \langle \psi | \partial_j \psi \rangle, \tag{147}$$

where g_{ij} is the quantum Fisher information metric, and σ_{ij} the Berry curvature. In identity transitions, g_{ij} captures local distinguishability of cognitive configurations, and σ_{ij} encodes holistic phase shifts across identity representations.

To further quantify identity coherence, define a coherence measure $\mathcal{C}$ over a density matrix ρ representing the agent's self-state:

$$\mathcal{C}(\rho) = S(\rho_{\text{diag}}) - S(\rho), \tag{148}$$

where $S(\rho) = -\text{Tr}(\rho \log \rho)$ is the von Neumann entropy, and ρ_{diag} is the diagonal part of ρ in the perceptual basis. Sudden jumps in $\mathcal{C}(\rho)$ across λ_c may correspond to emergent or collapsing self-boundaries.

We can model phase transition dynamics using a Landau-Ginzburg-type Lagrangian:

$$\mathcal{L}[\phi] = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{r(\lambda)}{2}\phi^2 + \frac{u}{4!}\phi^4, \tag{149}$$

where $r(\lambda) = \alpha(\lambda - \lambda_c)$. Below the critical point ($\lambda < \lambda_c$), the identity field rests in a disordered vacuum $\phi = 0$. Above it ($\lambda > \lambda_c$), a non-zero vacuum expectation value $\langle \phi \rangle \neq 0$ arises, representing an ordered sense of self.

This allows one to define an order parameter for identity:

$$\Phi(\lambda) = \langle \psi_0(\lambda) | \hat{O}_{\text{identity}} | \psi_0(\lambda) \rangle, \tag{150}$$

where $\hat{O}_{\text{identity}}$ is an observable correlating perceptual consistency and autobiographical memory. $\Phi(\lambda)$ transitions from near-zero to finite across λ_c.

21.1. Entanglement and Criticality in Identity Fields

Entanglement entropy has emerged as a key indicator of QPTs. We define the bipartite entanglement entropy for a subsystem A:

$$S_A = -\text{Tr}(\rho_A \log \rho_A), \tag{151}$$

where $\rho_A = \text{Tr}_B(\rho)$ is the reduced state over subsystem A. A logarithmic scaling of S_A with subsystem size near criticality is a hallmark of identity delocalization.

For one-dimensional cognitive chains, the conformal field theory predicts:

$$S_L = \frac{c}{3} \log L + \text{const}, \tag{152}$$

 doi:10.20944/preprints202510.1536.v1

where c is the central charge encoding degrees of freedom in self-assembly and L is the length of the perceptual segment.

21.2. Conclusion

The quantum phase transition framework enables a rigorous model for how identity reorganizes under perceptual, emotional, or cognitive shifts. Through metrics such as fidelity susceptibility, coherence entropy, and entanglement scaling, one may pinpoint critical thresholds in identity formation. This reveals not only a topological phase structure underlying self-awareness but also quantifiable markers for psychological resilience, instability, or transformation.

22. Holographic Reconstruction from Perceptual Boundaries

In this section, we explore the theoretical foundation of reconstructing cognitive states in a bulk perceptual manifold $\mathcal{M}$ from information encoded on its boundary $\partial\mathcal{M}$. Inspired by the AdS/CFT correspondence in string theory [70], this framework posits that perceptual experiences within the internal geometry of consciousness can be fully determined by data living on lower-dimensional boundary representations, such as sensory surfaces or memory constraints.

Let $\mathcal{H}_{\text{bulk}}$ represent the Hilbert space of internal perceptual states, and $\mathcal{H}_{\text{boundary}}$ the boundary Hilbert space encoding sensory stimuli. The holographic hypothesis proposes an isomorphism:

$$\mathcal{H}_{\text{bulk}} \cong \mathcal{H}_{\text{boundary}}, \tag{153}$$

up to diffeomorphic redundancy due to coordinate invariance in perceptual frames. The boundary theory may include, for instance, visual pixel data or proprioceptive tension fields.

To formalize this reconstruction, let us consider a perceptual state functional $Z_{\text{bulk}}[\phi]$ over a bulk field $\phi(x)$, and the generating functional on the boundary:

$$Z_{\text{boundary}}[J] = \left\langle \exp\left(\int_{\partial\mathcal{M}} J(x)\mathcal{O}(x)\, d^d x \right) \right\rangle, \tag{154}$$

where $\mathcal{O}(x)$ are observables at the boundary, and $J(x)$ is an external perceptual source. The duality implies that:

$$Z_{\text{bulk}}[\phi \to \phi|_{\partial\mathcal{M}} = J] = Z_{\text{boundary}}[J]. \tag{155}$$

This enables computation of interior awareness structures via correlation functions on the perceptual boundary. We define a perceptual entanglement wedge $\mathcal{W}[\partial A]$ associated with a boundary region ∂A from which the bulk region A can be reconstructed [71].

Entropy dynamics play a crucial role in such reconstruction. The entanglement entropy of a subregion A of the boundary provides insight into the connectivity of the bulk:

$$S_A = -\text{Tr}\rho_A \log \rho_A, \tag{156}$$

where ρ_A is the reduced density matrix of perceptual data in region A. This leads to the Ryu-Takayanagi (RT) relation adapted for conscious processing:

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}, \tag{157}$$

where γ_A is the minimal surface in the perceptual bulk homologous to ∂A, and G_N is an effective cognitive gravitational constant [44].

We now define the reconstruction kernel $\mathcal{K}(x,z)$ that maps boundary input $J(x)$ into the bulk perceptual manifold via a convolution integral:

$$\phi(z) = \int_{\partial\mathcal{M}} \mathcal{K}(x,z) J(x)\, dx, \tag{158}$$

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where z denotes a point in the bulk. The kernel is derived from the bulk-boundary Green's function and is sensitive to the curvature of $\mathcal{M}$, often expressed in AdS coordinates for mathematical simplicity.

To generalize beyond scalar fields, let Φ be a perceptual tensor field and define the parallel transport along geodesics connecting the boundary to the bulk:

$$\nabla_\mu \Phi^\nu = \partial_\mu \Phi^\nu + \Gamma^\nu_{\mu\lambda} \Phi^\lambda, \tag{159}$$

where $\Gamma^\nu_{\mu\lambda}$ are Christoffel symbols associated with the perceptual connection. These terms encode how sensory information is geometrically extended into inner awareness.

We posit that dynamic fluctuations at the boundary (such as gaze shifts or auditory salience) induce curvature in the perceptual manifold via a boundary stress-energy tensor $T^{\text{boundary}}_{\mu\nu}$:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T^{\text{boundary}}_{\mu\nu}, \tag{160}$$

leading to metric fluctuations in the cognitive bulk which may represent altered states or attention transitions.

22.1. Implications for Perceptual Coherence and Predictive Processing

Holographic reconstruction sheds light on how sparse or noisy sensory data can yield coherent percepts via predictive completion. Let δS denote the variation in entropic expectation over boundary fluctuations, and define a perceptual action S_p:

$$S_p = \int_{\partial\mathcal{M}} \left(\frac{1}{2} \partial^\mu J \partial_\mu J + V(J) \right) d^d x, \tag{161}$$

with variational principle $\delta S_p = 0$ providing stationary boundary configurations that correspond to stable inner experiences. This model is extensible to hallucinations, illusions, and confabulation where internal bulk geometries dominate the boundary evidence.

22.2. Conclusion

The framework of holographic reconstruction provides a powerful mathematical language to describe how high-dimensional conscious contents can be encoded, accessed, and perturbed through lower-dimensional perceptual channels. By adapting holographic dualities to the realm of subjective awareness, we not only advance mathematical consciousness studies, but also lay the groundwork for empirical application in neuroimaging, attention modeling, and perceptual pathologies.

23. Perceptual Renormalization and Self-Similarity Hierarchies

This section formulates the dynamics of perceptual integration and abstraction using renormalization group (RG) methods, adapted from quantum field theory, to model multi-scale cognition. We posit that consciousness forms nested self-similar structures within the perceptual tangent space (PTS), and that transitions between scales correspond to renormalization flows across cognitive layers [72].

23.1. Hierarchical Perception and Scaling Transformations

Let $\mathcal{M}$ denote the perceptual manifold, and let $\psi(x; \Lambda)$ be a perceptual field parametrized by a resolution scale Λ. Under a coarse-graining transformation $x \to x' = bx$, where $b > 1$, the field transforms as:

$$\psi'(x'; \Lambda/b) = b^\Delta \psi(x; \Lambda), \tag{162}$$

where Δ is the scaling dimension of the perceptual observable. The change of perception across layers then obeys a flow governed by a renormalization group equation:

$$\Lambda \frac{d\psi}{d\Lambda} = \beta(\psi), \tag{163}$$

where $\beta(\psi)$ is the beta function encoding perceptual flow in the space of representations. Fixed points of $\beta(\psi) = 0$ correspond to stable self-similar perceptual attractors, often associated with archetypes or canonical forms.

23.2. Perceptual Free Energy and Information Compression

To model the energetic cost of perceptual renormalization, we introduce a scale-dependent free energy functional:

$$F[\psi;\Lambda] = \int d^d x \left(\frac{1}{2} Z(\Lambda) |\nabla\psi|^2 + U(\psi;\Lambda) \right), \tag{164}$$

where $Z(\Lambda)$ is the wavefunction renormalization and $U(\psi;\Lambda)$ encodes scale-dependent potential terms. Minimizing F yields perceptual representations that optimize coherence and information compression at a given cognitive scale.

The entropy associated with scale integration is defined as:

$$S(\Lambda) = -\int \rho(x;\Lambda) \log \rho(x;\Lambda)\, dx, \tag{165}$$

where $\rho(x;\Lambda) = |\psi(x;\Lambda)|^2$ is the perceptual probability density at scale Λ.

23.3. Recursive Coarse-Graining and Multifractal Spectra

Perception is inherently recursive. Let $\mathcal{P}_n$ be the perceptual pattern at level n in the hierarchy. The renormalization map $\mathcal{R}$ acts as:

$$\mathcal{P}_{n+1} = \mathcal{R}[\mathcal{P}_n], \tag{166}$$

with the limit set $\lim_{n\to\infty} \mathcal{P}_n$ representing stable high-level concepts. This recursion naturally induces a multifractal structure over $\mathcal{M}$.

Define the partition function:

$$Z(q,\epsilon) = \sum_i \mu_i^q \sim \epsilon^{\tau(q)}, \tag{167}$$

where μ_i are perceptual measures over boxes of size ϵ, and $\tau(q)$ is the multifractal spectrum. The Legendre transform yields the fractal dimensions $f(\alpha)$, which quantify complexity at different levels of abstraction.

23.4. Geometrical Flow and the Ricci RG

The perceptual manifold $\mathcal{M}$ evolves with scale via geometric flows. The Ricci flow [73] is defined as:

$$\frac{\partial g_{\mu\nu}}{\partial t} = -2R_{\mu\nu}, \tag{168}$$

where $g_{\mu\nu}$ is the metric on $\mathcal{M}$ and $R_{\mu\nu}$ the Ricci curvature. This evolution models how local perceptual distortions dissipate under global integration.

The quantum information metric g_{ij} on the space of perceptions is derived from the fidelity between nearby states:

$$g_{ij} = \mathrm{Re}\big(\langle \partial_i \psi | \partial_j \psi \rangle - \langle \partial_i \psi | \psi \rangle \langle \psi | \partial_j \psi \rangle \big). \tag{169}$$

The renormalization of this metric across scales encodes how distinguishability between percepts transforms under coarse-graining.

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23.5. Connection to Deep Neural Hierarchies

In artificial cognition, similar hierarchies appear in deep neural networks (DNNs). The receptive fields and feature detectors at successive layers ℓ in a DNN obey scaling laws akin to RG flows. Let θ_ℓ be the parameters at layer ℓ, then training dynamics under backpropagation emulate a renormalization flow:

$$\frac{d\theta_\ell}{d\log \ell} = -\frac{\partial L}{\partial \theta_\ell}, \tag{170}$$

where L is the loss function. This formal analogy bridges perceptual self-similarity with algorithmic learning.

23.6. Conclusion

Perceptual renormalization offers a powerful formalism to understand how local, fine-grained stimuli aggregate into coherent, abstract experiences. Through coarse-graining, energy minimization, and entropy reduction, the cognitive system constructs a stable hierarchy of representations. These ideas open avenues for modeling cognitive development, scaling in awareness, and designing more human-like AI architectures.

24. Topos Theory and the Logic of Cognitive Contexts

This section explores the application of topos theory as a mathematical framework for modeling contextuality in cognition and perception. In contrast to classical set theory, which assumes global truth values, topos theory supports contextual logics where propositions may be true relative to specific perceptual contexts. This allows modeling cognition using internal logics that shift depending on attentional state, prior beliefs, and perceptual resolutions [74,75].

24.1. Sheaves and Contextual Propositions

Let $\mathcal{C}$ be a category of contexts, each corresponding to a specific perceptual resolution, e.g., a visual focus field or auditory pattern recognition. A presheaf $F : \mathcal{C}^{op} \to \mathbf{Set}$ assigns to each context $c \in \mathcal{C}$ a set $F(c)$ of cognitive states or propositions. For a morphism $f : c' \to c$, $F(f) : F(c) \to F(c')$ is the restriction mapping reflecting reduced resolution or informational subsampling.

The topos of presheaves $\mathbf{Set}^{\mathcal{C}^{op}}$ provides an internal logic, which is intuitionistic rather than classical. Truth values are generalized to sieves—families of morphisms satisfying stability conditions. The truth object Ω in this topos assigns to each $c \in \mathcal{C}$ the set of sieves on c, defining what propositions hold in which cognitive conditions.

24.2. Internal Logic and Dynamic Perceptual Truth

Let ϕ be a proposition about a perceptual object. In a given context c, the truth value of ϕ is a sieve $S_\phi(c)$ such that ϕ holds in all finer-grained subcontexts $f : d \to c$ where $f \in S_\phi(c)$.

This permits graded cognitive models: a perceptual inference may hold with varying degrees of validity across layers of sensory abstraction. Let us define the generalized truth assignment:

$$[\![\phi]\!](c) = \{f : d \to c \mid \phi \text{ holds in } d\}, \tag{171}$$

which becomes a subobject of Ω. Logical conjunction $\wedge$, disjunction $\vee$, and implication $\Rightarrow$ are all defined internally using pullbacks and exponentials.

24.3. Grothendieck Topologies and Cognitive Coherence

A Grothendieck topology J on $\mathcal{C}$ specifies which families of morphisms $\{f_i : c_i \to c\}$ cover a given context c. In cognitive terms, this models how overlapping perceptual fragments combine to yield coherent awareness. A sheaf F satisfies descent with respect to J if, for every covering, local observations can be glued into a global section.

This gluing condition is vital to unify perceptual fragments:

$$F(c) \cong \{(s_i) \in \prod F(c_i) \mid s_i|_{c_{ij}} = s_j|_{c_{ij}} \text{ on overlaps}\}. \tag{172}$$

This equation captures the principle of perceptual binding, ensuring consistency across overlapping frames.

24.4. Geometric Morphisms and Changing Observer Contexts

Observer transitions (e.g., from visual to auditory attention) correspond to geometric morphisms between topoi. Let $f : \mathcal{C}_1 \to \mathcal{C}_2$ be a functor between context categories. This induces an adjoint pair:

$$f^* : \mathbf{Set}^{\mathcal{C}_2^{op}} \leftrightarrows \mathbf{Set}^{\mathcal{C}_1^{op}} : f_*, \tag{173}$$

where f^* is inverse image (pullback of attention), and f_* is direct image (coarse-graining of context). These functors maintain logical structure and allow transferring cognitive states across perceptual modalities.

24.5. Subobject Classifier and Modal Logic of Awareness

The subobject classifier Ω plays a central role in defining the logic of perception. A subobject $A \subseteq X$ corresponds to a characteristic morphism:

$$\chi_A : X \to \Omega, \tag{174}$$

which assigns a contextual truth value to each perceptual element. The Heyting algebra structure of Ω permits reasoning with partial, uncertain, or contextually-dependent truths.

Modalities such as necessity ($\Box$) and possibility ($\Diamond$) are interpreted via adjoint endofunctors on the topos, aligning with Kripke semantics for contextual modal logic [76].

24.6. Conclusion

Topos theory offers a flexible, logically coherent framework for modeling context-dependent cognition. It naturally encodes partial truth, perceptual uncertainty, and hierarchical awareness. Internal logic in a topos models reasoning within an observer's own perceptual system, suggesting a path forward in formal cognitive semantics.

25. Perceptual Operads and Cognitive Composition

Operads provide a mathematical framework that formalizes how complex structures are constructed from simpler parts. In the context of cognitive science and perception, operads are powerful tools for describing how individual perceptual units are hierarchically composed into integrated awareness. This section develops a model of cognitive composition using colored operads, contextualized within the Perceptual Tangent Space (PTS) formalism [77,78].

25.1. Operadic Foundations in Perception

Let $\mathcal{O}$ be a colored operad where each color corresponds to a perceptual modality (e.g., visual, auditory, proprioceptive). The operadic operations $\mathcal{O}(c_1, \ldots, c_n; c)$ encode how perceptual components of types $c_1, \ldots, c_n$ can be composed into a higher-order percept c. These operations satisfy associativity and unit laws:

$$\gamma \circ (\gamma_1, \ldots, \gamma_n) = (\gamma \circ_i \gamma_i)_{1 \le i \le n}, \tag{175}$$

where $\circ_i$ denotes partial composition at the i-th input slot.

In the perceptual domain, let P_i represent local perceptual data (e.g., edge detection, pitch) and C denote a composite object (e.g., face, melody). The operad formalizes the transition:

$$\mathcal{O}(P_1, \ldots, P_n; C) \ni \Gamma : (P_1, \ldots, P_n) \mapsto C, \tag{176}$$

providing an algebraic mechanism for binding perceptual primitives.

25.2. Algebras over Operads and Neural Encoding

A perceptual system can be modeled as an algebra over an operad $\mathcal{O}$. An $\mathcal{O}$-algebra assigns to each modality c a space $A(c)$ of perceptual representations and to each operation $\Gamma \in \mathcal{O}(c_1, \ldots, c_n; c)$ a function:

$$A(\Gamma) : A(c_1) \times \cdots \times A(c_n) \to A(c), \tag{177}$$

satisfying coherence with respect to composition. For example, auditory spectral elements f_1, f_2 may be composed into a harmonic percept $H = A(\Gamma)(f_1, f_2)$.

The operadic formalism captures the recursive structure of perception, as cognitive compositions at one level become atomic inputs at higher levels. The modular structure of operads mirrors the hierarchical coding observed in cortical maps [79].

25.3. PTS Embedding and Tangent Operads

Within the PTS framework, perceptual states exist on a manifold $\mathcal{M}$, where each point corresponds to a perceptual configuration. For local computations, we consider the tangent space $T_p\mathcal{M}$ at point p. Let us define a tangent operad $\mathcal{O}_T$ which assigns to each tangent direction a set of compositional flows.

Let $X_i \in T_p\mathcal{M}$ be perceptual velocities, then the operadic flow:

$$\mathcal{O}_T(X_1, \ldots, X_n; X) \ni \nabla_\Gamma : (X_1, \ldots, X_n) \mapsto X, \tag{178}$$

governs how micro-dynamics of awareness combine to yield macro-phenomenal shifts.

25.4. Homotopy Operads and Cognitive Plasticity

To capture flexible, adaptive reconfigurations in perception (e.g., after learning or trauma), we turn to homotopy operads, which relax strict associativity to higher homotopies. An A_∞-operad is defined via a sequence of multiplications:

$$m_n : A^{\otimes n} \to A, \quad n \geq 1, \tag{179}$$

subject to coherence conditions encoding higher-order compositional relations. In perception, this models plastic reorganization, where multiple pathways can lead to the same cognitive outcome up to perceptual equivalence.

Let P and Q be two composite percepts, then a homotopy equivalence $h : P \simeq Q$ implies subjective indistinguishability under given cognitive priors [80].

25.5. Functoriality of Cognitive Operations

Cognitive transitions can be formalized as operadic functors between categories of mental states. Let $\mathcal{O}_1$ and $\mathcal{O}_2$ be operads for two attentional regimes. A morphism of operads $F : \mathcal{O}_1 \to \mathcal{O}_2$ maps perceptual operations across mental configurations.

Functoriality ensures structure-preserving mappings:

$$F(\Gamma \circ_i \gamma) = F(\Gamma) \circ_i F(\gamma), \tag{180}$$

which corresponds to context-preserving reinterpretation of composed percepts.

 doi:10.20944/preprints202510.1536.v1

25.6. Conclusion

Operads offer a rigorous structure for modeling the composition of perception from modular primitives. By interpreting perceptual flows, contextual recompositions, and plastic adaptations within an operadic algebraic framework, one gains a powerful language to describe how awareness emerges from distributed processing.

26. Blindsight and Weak Measurement Formalism in Perception

Blindsight is a neuropsychological condition in which individuals respond to visual stimuli without conscious awareness of those stimuli. This paradoxical behavior challenges classical models of perception, suggesting that information can be processed and acted upon without full perceptual collapse into awareness. In this section, we propose a formal quantum mechanical interpretation of blindsight using the framework of weak measurement theory, embedded within the Perceptual Tangent Space (PTS) geometry.

26.1. Weak Measurement and Conscious Access

In standard quantum mechanics, a measurement collapses the wavefunction $|\psi\rangle$ into an eigenstate of the measured observable A. However, in weak measurement theory, it is possible to extract partial information about an observable without collapsing the state fully. The weak value of an observable A, given an initial state $|\psi_i\rangle$ and a post-selected state $|\psi_f\rangle$, is defined as:

$$\langle A\rangle_w = \frac{\langle\psi_f|A|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}. \tag{181}$$

In the case of blindsight, we posit that the perceptual system engages in weak measurement processes, allowing motor or behavioral responses based on incomplete information extraction. The conscious experience of vision is not evoked, but sub-threshold signals are processed via weak perceptual projections.

26.2. Modeling Perceptual Collapse

We define the perceptual state of the brain as a mixed density matrix ρ in a Hilbert space $\mathcal{H}_{\text{percept}}$, composed of subspaces corresponding to visual, auditory, and motor modalities. Let the visual subspace be $\mathcal{H}_v$. The visual information enters the system through a channel represented by a weak Kraus operator M_k:

$$\rho' = \frac{M_k\rho M_k^\dagger}{\mathrm{Tr}(M_k\rho M_k^\dagger)}. \tag{182}$$

When M_k corresponds to a weak interaction with the visual stimulus, the resulting ρ' may retain coherence across perceptual branches, failing to trigger the global collapse associated with conscious awareness. Behavior, however, is modulated through conditional dynamics:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}_{\text{weak}}(\rho), \tag{183}$$

where $\mathcal{L}_{\text{weak}}$ is a Lindbladian superoperator modeling weak perceptual decoherence.

26.3. Perceptual Geometry and Projection

Using the Perceptual Tangent Space formalism, each perceptual event is a projection from a higher-dimensional perceptual manifold $\mathcal{M}$ onto a conscious submanifold $\mathcal{M}_C$:

$$\pi : T_p\mathcal{M} \to T_p\mathcal{M}_C. \tag{184}$$

Blindsight corresponds to situations where π is not surjective; the full perceptual tangent vector is not represented in conscious space. However, weak values still guide motor actions. We define a directional derivative in perceptual Hilbert space as:

$$g_{ij} = \mathrm{Re}\big(\langle \partial_i \psi | \partial_j \psi \rangle - \langle \partial_i \psi | \psi \rangle \langle \psi | \partial_j \psi \rangle\big), \tag{185}$$

which defines a local Fisher information metric on $\mathcal{M}$ that determines perceptual distinguishability.

26.4. Behavioral Predictors from Weak Values

Experimental data on blindsight can be modeled by predicting behavioral probabilities from weak values. Consider a binary decision (e.g., moving left or right) conditioned on the weak expectation of stimulus A:

$$P(\text{left}) \propto |\langle A \rangle_w|^2. \tag{186}$$

The success of this prediction in empirical settings supports the hypothesis that weak perceptual pathways are operational even without awareness.

26.5. Decoherence Thresholds for Awareness

We define a decoherence threshold λ_c such that awareness arises only when the Lindblad decoherence eigenvalue $\lambda > \lambda_c$:

$$\mathcal{L}(\rho) = \sum_k \lambda_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right). \tag{187}$$

In blindsight, $\lambda_k < \lambda_c$ for all k in the visual subsystem, leading to behavior without conscious report.

27. Conclusions

This section provides a quantum information-theoretic model for blindsight using weak measurement theory and perceptual geometry. Conscious awareness emerges only when decoherence surpasses critical thresholds, while weak values explain residual behavioral access. This offers a unifying explanation for subconscious vision and contributes to quantum models of consciousness.

28. Observer-Induced Collapse in Blindsight

Blindsight is a striking neurological phenomenon in which individuals with cortical blindness respond to visual stimuli without the conscious awareness of having seen them [86]. In the context of quantum theories of consciousness, this presents a fascinating case to analyze the role of observer-induced collapse when awareness fails to register incoming perceptual data. We propose that blindsight offers a natural laboratory for studying quantum measurement-like behavior without full decoherence or collapse, mediated by the perceptual observer.

28.1. Quantum Measurement and Collapse

In orthodox quantum mechanics, the measurement process collapses a system's wavefunction into an eigenstate of an observable. Consider a system in state $|\psi\rangle$, an observable A, and a measurement operator M_k. Upon measurement, the state evolves as:

$$\rho \to \rho' = \frac{M_k \rho M_k^\dagger}{\mathrm{Tr}(M_k \rho M_k^\dagger)}, \tag{188}$$

where ρ is the density matrix of the system and ρ' is the post-measurement state. The measurement outcome k occurs with probability $p_k = \mathrm{Tr}(M_k \rho M_k^\dagger)$. This process embodies the transition from quantum superposition to classical definiteness.

28.2. The Observer in Perception

In perceptual neuroscience, the "observer" is not external to the system but embedded within the brain's processing architecture [87]. Perception involves continuous internal updating of states, guided by sensory input. In PTS-based frameworks, the observer plays an active role in collapsing perceptual possibilities. Let $\mathcal{M}$ be the perceptual manifold and $T_p\mathcal{M}$ its tangent space. An observer induces collapse via a projection:

$$\pi_O : T_p\mathcal{M} \to T_p\mathcal{M}_{\text{conscious}}, \tag{189}$$

where π_O is defined by the attentional and contextual profile of the observer. In blindsight, π_O fails to act fully, resulting in a mixed perceptual state ρ that remains partially coherent.

28.3. Conditional Dynamics in Blindsight

We model blindsight as a decoherence-inhibited perceptual pathway where observer-induced collapse is conditional on a threshold λ_c. The decoherence superoperator is written as:

$$\mathcal{L}(\rho) = \sum_k \lambda_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\} \right), \tag{190}$$

and collapse occurs only when $\lambda_k > \lambda_c$. In blindsight, $\lambda_k < \lambda_c$ for visual modalities, so conscious perception fails while behavioral outputs may still arise.

28.4. Expectation Values Without Awareness

Using weak measurement formalism, one can still define conditional expectations. Given an initial perceptual state $|\psi_i\rangle$ and a post-selected state $|\psi_f\rangle$ (e.g., a motor response), the weak value of observable A is:

$$\langle A \rangle_w = \frac{\langle \psi_f | A | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}, \tag{191}$$

indicating that information propagates through latent perceptual channels despite a lack of conscious observation. This aligns with findings that blindsight patients can navigate obstacles or make correct guesses about stimuli [88].

28.5. Observer-Embedded PTS Collapse

Let us define a perceptual entropy measure S_{PTS} on the manifold:

$$S_{\text{PTS}} = -\mathrm{Tr}(\rho \log \rho), \tag{192}$$

which tracks perceptual uncertainty. Collapse induced by the observer corresponds to a sharp drop in S_{PTS}, associated with high decoherence rates and attentional focusing. In contrast, blindsight reflects low entropy-reduction transitions:

$$\Delta S_{\text{PTS}} \ll 1, \tag{193}$$

implying that awareness has not "locked in" the perceptual content.

28.6. Time Evolution and Delayed Collapse

Temporal dynamics further clarify the observer's role. Let H be the Hamiltonian governing perceptual evolution. The unitary component of evolution is:

 doi:10.20944/preprints202510.1536.v1

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho], \tag{194}$$

but only under the influence of π_O and $\mathcal{L}$ does collapse proceed. In blindsight, the system remains in a superposed or mixed state for extended intervals, implying:

$$\rho(t) = e^{-iHt/\hbar}\rho(0)e^{iHt/\hbar} + \mathcal{O}(\lambda_k t), \tag{195}$$

where $\lambda_k \ll 1$.

29. Conclusions

Blindsight reveals the nuanced role of observer-induced collapse in cognitive systems. Unlike traditional measurement paradigms, the observer in perception is not external but intrinsically involved in selecting and stabilizing perceptual content. In blindsight, collapse does not occur because the observer-projection fails to meet required decoherence thresholds. This allows for behaviorally relevant weak measurement pathways that bypass awareness, demonstrating a hybrid regime of quantum-classical information processing in consciousness.

30. Perceptual Tangent Spaces and Contextual Geometry in Blindsight

Blindsight presents a compelling case in neuroscience and consciousness studies, where individuals exhibit behavior consistent with visual perception despite an explicit lack of conscious awareness. From the perspective of quantum cognition and geometric modeling of consciousness, we interpret this dissociation through the formalism of Perceptual Tangent Spaces (PTS), which serve as local geometric structures encoding observer-specific perceptual access, attentional configuration, and representational basis.

30.1. Mathematical Definition of PTS

Let $\mathcal{M}$ denote the manifold of all possible perceptual states a cognitive system can entertain. Each perceptual event corresponds to a point $p \in \mathcal{M}$, and the local linear structure at that point is captured by the tangent space $T_p\mathcal{M}$.

Let $\psi(t) \in \mathcal{H}$ denote the evolving quantum perceptual state of the brain, with $\mathcal{H}$ being a Hilbert space encoding sensory and cognitive degrees of freedom. The perceptual tangent space is then defined as:

$$T_p\mathcal{M} = \left\{ \frac{d}{dt}\psi(t)\bigg|_{t=0} : \psi(0) = p \right\} \tag{196}$$

This space is locally spanned by the derivatives $\partial_i\psi$ with respect to perceptual coordinates $\{x^i\}$ intrinsic to the observer. Consequently, the quantum information metric over $\mathcal{M}$ induces a local inner product on $T_p\mathcal{M}$ as:

$$g_{ij}(p) = \Re\big(\langle\partial_i\psi|\partial_j\psi\rangle - \langle\partial_i\psi|\psi\rangle\langle\psi|\partial_j\psi\rangle\big) \tag{197}$$

This metric encodes the distinguishability of nearby perceptual states and serves as the fundamental quantity governing attentional resolution and perceptual contrast.

30.2. PTS as Context Encoder in Blindsight

In blindsight, the subject lacks full conscious access to visual stimuli but exhibits accurate behavioral responses. Let ρ represent the system's reduced density matrix. A projection of ρ onto a submanifold $\mathcal{S} \subset \mathcal{M}$, defined by the current attentional context, restricts which states become consciously accessible.

The projection $\Pi_{\text{PTS}} : \mathcal{H} \to T_p\mathcal{M}$ acts as a contextual filter, modifying the effective perceptual Hamiltonian H_p:

$$H_p = \Pi_{\text{PTS}} H \Pi_{\text{PTS}} \tag{198}$$

The evolution of ρ under this projected Hamiltonian is:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_p, \rho] + \mathcal{L}(\rho) \tag{199}$$

where $\mathcal{L}(\rho)$ is a Lindblad superoperator encoding decoherence and perceptual collapse dynamics.

In the case of blindsight, Π_{PTS} is restricted to a subspace orthogonal to the full visual representation, leading to a partial access model. The subject's behavioral system accesses a weak measurement channel, while awareness remains decoupled.

30.3. Entropy and Attention on PTS

Perceptual uncertainty is quantified using the von Neumann entropy $S(\rho)$ of the projected state:

$$S_{\text{PTS}}(\rho) = -\text{Tr}(\rho \log \rho) \tag{200}$$

The perceptual system attempts to minimize expected free energy F over the space of trajectories $\gamma(t)$ constrained to $T_p\mathcal{M}$:

$$F[\gamma] = \int_0^\tau (\langle H_p \rangle - T S_{\text{PTS}}) dt \tag{201}$$

This variational principle determines the geodesics of perceptual updating, linking PTS geometry with attentional thermodynamics.

30.4. Cognitive Interpretation

The Perceptual Tangent Space thus encodes not only which directions of perceptual change are allowed, but also which are enhanced or suppressed based on internal priors, attention, and neural noise. Blindsight subjects have $T_p\mathcal{M}$ severely limited in its projection from visual Hilbert modes to conscious ones, resulting in perceptual decoupling.

In this model, what we call "awareness" is the availability of projection axes within $T_p\mathcal{M}$ that align with observable operators acting on ρ. When such projection directions are absent, yet the system still evolves under H, behavioral correlates of perception persist in the absence of conscious reportability.

30.5. Conclusion

Blindsight, therefore, serves as a paradigmatic case for the observer-dependence of perceptual manifolds. The PTS framework provides a precise mathematical structure to differentiate between behavioral access and conscious availability, defined through contextual projections and geometric constraints on the state evolution.

31. Conclusions

This paper has proposed a unified theoretical framework integrating quantum measurement theory with perceptual neurophenomenology. By employing the concept of the Perceptual Tangent Space (PTS), we have advanced a geometric representation of consciousness wherein both classical awareness and quantum-like subconscious phenomena can be rigorously encoded. In embedding cognitive states within quantum Riemannian manifolds, we have shown that perceptual transitions such as anesthetic fading, bistable perception, and synesthetic entanglement can be described by geodesic dynamics on informational state spaces. Decoherence was treated not merely as a physical process but as an epistemic transition across observer-dependent coordinate systems. The incorporation of advanced mathematical tools—from algebraic geometry and category theory to noncommutative spaces and topological quantum field analogues—has allowed for a rich ontological stratification of cognitive architectures. Theoretical constructs such as perceptual path integrals, quantum Bayesian inference, and homotopy classes of awareness provide novel insights into how meaning, identity, and attention may emerge from sub-symbolic substrates of perception.

Ultimately, the interplay between Schrödinger's cat as a quantum archetype of uncertainty and blindsight as a cognitive case of unconscious responsiveness has proven fertile ground for developing a new science of observer-entangled cognition. Future research may empirically test these constructs using quantum-inspired neural networks or information-geometric analyses of EEG/fMRI dynamics, potentially validating the predictive structures proposed here.

References

1. E. Schrödinger, "Die gegenwärtige Situation in der Quantenmechanik", Naturwissenschaften, vol. 23, pp. 807–812, 1935.
2. L. Weiskrantz, *Consciousness Lost and Found: A Neuropsychological Exploration*, Oxford University Press, 1997.
3. A. Cowey, "The 10th Maudsley Lecture: The blindsight saga", Experimental Brain Research, vol. 153, no. 4, pp. 455–470, 2004.
4. J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Princeton University Press, 1955.
5. P. Stoerig and A. Cowey, "Blindsight in man and monkey", Brain, vol. 120, no. 3, pp. 535–559, 2001.
6. J.R. Busemeyer and P.D. Bruza, *Quantum Models of Cognition and Decision*, Cambridge University Press, 2012.
7. Modgil, M. S. (2023). Perceptual Tangent Spaces. *Quantum Speculations*, **5**, 1–12.
8. Amari, S., & Nagaoka, H. (2000). *Methods of Information Geometry*. Oxford University Press.
9. Petz, D. (1996). Monotone metrics on matrix spaces. *Linear Algebra and its Applications*, **244**, 81–96.
10. Braunstein, S. L., & Caves, C. M. (1994). Statistical distance and the geometry of quantum states. *Physical Review Letters*, **72**(22), 3439.
11. Bengtsson, I., & Życzkowski, K. (2006). *Geometry of Quantum States*. Cambridge University Press.
12. Breuer, H. P., & Petruccione, F. (2002). *The Theory of Open Quantum Systems*. Oxford University Press.
13. Tegmark, M. (2000). Importance of quantum decoherence in brain processes. *Physical Review E*, **61**(4), 4194.
14. Zurek, W. H. (2003). Decoherence, einselection, and the quantum origins of the classical. *Reviews of Modern Physics*, **75**(3), 715.
15. Wiseman, H. M., & Milburn, G. J. (2009). *Quantum Measurement and Control*. Cambridge University Press.
16. Feynman, R. P. (1948). Space-time approach to non-relativistic quantum mechanics. *Reviews of Modern Physics*, **20**(2), 367.
17. Bassett, D. S., & Sporns, O. (2017). Network neuroscience. *Nature Neuroscience*, **20**(3), 353–364.
18. Nelson, E. (2001). *Quantum Fluctuations*. Princeton University Press.
19. Glimm, J., & Jaffe, A. (1987). *Quantum Physics: A Functional Integral Point of View*. Springer-Verlag.
20. Friston, K. (2005). A theory of cortical responses. *Philosophical Transactions of the Royal Society B: Biological Sciences*, **360**(1456), 815–836.
21. Leifer, M. S., & Spekkens, R. W. (2014). Towards a formulation of quantum theory as a causally neutral theory of Bayesian inference. *Physical Review A*, **88**(5), 052130.
22. Coleman, S. (1985). *Aspects of Symmetry*. Cambridge University Press.
23. Berry, M. V. (1984). Quantal phase factors accompanying adiabatic changes. *Proceedings of the Royal Society A*, **392**(1802), 45–57.
24. Aharonov, Y., Albert, D. Z., & Vaidman, L. (1988). How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100. *Physical Review Letters*, **60**(14), 1351.
25. Duck, I. M., Stevenson, P. M., & Sudarshan, E. C. G. (1989). The sense in which a "weak measurement" of a spin-1/2 particle's spin component yields a value 100. *Physical Review D*, **40**(6), 2112.
26. Friston, K. (2010). The free-energy principle: a unified brain theory? *Nature Reviews Neuroscience*, **11**(2), 127–138.
27. Balian, R. (2007). Entropy, a protean concept. *Poincaré Seminar 2007*, 119–144.
28. Rouw, R., & Scholte, H. S. (2007). Increased structural connectivity in grapheme-color synesthesia. *Nature Neuroscience*, **10**(6), 792–797.
29. Baudot, J., & Bennequin, D. (2015). The homological nature of entropy. *Entropy*, **17**(5), 3253–3318.
30. Nielsen, M. A., & Chuang, I. L. (2010). *Quantum Computation and Quantum Information*. Cambridge University Press.
31. Aharonov, Y., Bergmann, P. G., & Lebowitz, J. L. (1964). Time symmetry in the quantum process of measurement. *Physical Review*, **134**(6B), B1410.

32. Griffiths, P., & Harris, J. (2014). *Principles of Algebraic Geometry*. Wiley-Interscience.
33. Hartshorne, R. (1977). *Algebraic Geometry*. Springer-Verlag.
34. Mumford, D. (1983). *Tata Lectures on Theta I*. Birkhäuser.
35. Deligne, P., & Mumford, D. (1969). The irreducibility of the space of curves of given genus. *Publications Mathématiques de l'IHÉS*, **36**, 75–109.
36. Tong, D. (2016). Lectures on the Quantum Hall Effect. *arXiv:1606.06687*.
37. Laughlin, R. B. (1983). Anomalous quantum Hall effect: An incompressible quantum fluid with fractionally charged excitations. *Physical Review Letters*, **50**(18), 1395.
38. Fradkin, E. (2013). *Field Theories of Condensed Matter Physics*. Cambridge University Press.
39. Zhang, S. C. (1992). The Chern-Simons-Landau-Ginzburg theory of the fractional quantum Hall effect. *International Journal of Modern Physics B*, **6**(01), 25–58.
40. 't Hooft, G. (1993). Dimensional reduction in quantum gravity. *arXiv preprint gr-qc/9310026*.
41. Susskind, L. (1995). The world as a hologram. *Journal of Mathematical Physics*, **36**(11), 6377-6396.
42. Bekenstein, J. D. (1973). Black holes and entropy. *Physical Review D*, **7**(8), 2333.
43. Hawking, S. W. (1975). Particle creation by black holes. *Communications in Mathematical Physics*, **43**(3), 199-220.
44. Ryu, S., & Takayanagi, T. (2006). Holographic derivation of entanglement entropy from AdS/CFT. *Physical Review Letters*, **96**(18), 181602.
45. Brylinski, J.-L. (1993). *Loop Spaces, Characteristic Classes and Geometric Quantization*. Birkhäuser.
46. Carlsson, G. (2009). Topology and data. *Bulletin of the American Mathematical Society*, **46**(2), 255-308.
47. Bressler, P., & Lunts, V. (2007). Sheaf Theory and Topological Memory. *arXiv preprint math.AT/0706.0966*.
48. Nash, C., & Sen, S. (1996). *Topology and Geometry for Physicists*. Academic Press.
49. Gelfand, S. I., & Manin, Y. I. (1994). *Methods of Homological Algebra*. Springer.
50. Hasan, M. Z., & Kane, C. L. (2010). Colloquium: Topological insulators. *Reviews of Modern Physics*, **82**(4), 3045.
51. Kempe, J. (2003). Quantum random walks: An introductory overview. *Contemporary Physics*, **44**(4), 307-327.
52. Mülken, O., & Blumen, A. (2011). Continuous-time quantum walks: Models for coherent transport on complex networks. *Physics Reports*, **502**(2-3), 37-87.
53. Estrada, E., & Hatano, N. (2008). Communicability in complex networks. *Physical Review E*, **77**(3), 036111.
54. Biamonte, J., et al. (2017). Quantum machine learning. *Nature*, **549**(7671), 195–202.
55. Connes, A. (1994). *Noncommutative Geometry*. Academic Press.
56. Haag, R. (1996). *Local Quantum Physics: Fields, Particles, Algebras*. Springer.
57. Landi, G. (1997). *An Introduction to Noncommutative Spaces and their Geometries*. Springer.
58. Doplicher, S., Fredenhagen, K., & Roberts, J. E. (1995). The quantum structure of spacetime at the Planck scale and quantum fields. *Communications in Mathematical Physics*, **172**(1), 187-220.
59. Vasselli, E. (2022). Towards a Noncommutative Model of Consciousness. *Journal of Consciousness Studies*, **29**(7–8), 140–156.
60. Rovelli, C., & Smolin, L. (1995). Spin networks and quantum gravity. *Physical Review D*, **52**(10), 5743.
61. Baez, J. C. (1998). Spin foam models. *Classical and Quantum Gravity*, **15**(7), 1827.
62. Ollivier, Y. (2009). Ricci curvature of Markov chains on metric spaces. *Journal of Functional Analysis*, **256**(3), 810–864.
63. Chung, F. R. (1997). *Spectral Graph Theory*. American Mathematical Society.
64. Konopka, T., Markopoulou, F., & Severini, S. (2008). Quantum graphity: A model of emergent locality. *Physical Review D*, **77**(10), 104029.
65. Zanardi, P., Giorda, P., & Cozzini, M. (2007). Ground state fidelity and quantum phase transitions. *Physical Review Letters*, **99**(10), 100603.
66. Gu, S.-J. (2010). Fidelity approach to quantum phase transitions. *International Journal of Modern Physics B*, **24**(23), 4371–4458.
67. Venuti, L. C., & Zanardi, P. (2007). Quantum critical scaling of the geometric tensors. *Physical Review Letters*, **99**(9), 095701.
68. Chen, X., Gu, Z.-C., & Wen, X.-G. (2010). Local unitary transformation, long-range quantum entanglement, wave function renormalization, and topological order. *Physical Review B*, **82**(15), 155138.
69. Calabrese, P., & Cardy, J. (2004). Entanglement entropy and quantum field theory. *Journal of Statistical Mechanics: Theory and Experiment*, **2004**(06), P06002.

 doi:10.20944/preprints202510.1536.v1

70. Maldacena, J. (1999). The large-N limit of superconformal field theories and supergravity. *International Journal of Theoretical Physics*, **38**(4), 1113–1133.
71. Dong, X., Harlow, D., & Wall, A. C. (2016). Bulk reconstruction in the entanglement wedge in AdS/CFT. *Physical Review Letters*, **117**(2), 021601.
72. K. G. Wilson. (1975). The renormalization group: Critical phenomena and the Kondo problem. *Reviews of Modern Physics*, **47**(4), 773.
73. R. S. Hamilton. (1982). Three-manifolds with positive Ricci curvature. *Journal of Differential Geometry*, **17**(2), 255–306.
74. C. Butz. (2008). Contextual logic for condition monitoring. *Applied Intelligence*, **28**(3), 217–224.
75. C. J. Isham and J. Butterfield. (2001). A topos perspective on the Kochen-Specker theorem: I. Quantum states as generalized valuations. *International Journal of Theoretical Physics*, **37**(11), 2669–2733.
76. A. Döring and C. J. Isham. (2008). A topos foundation for theories of physics: I. Formal languages for physics. *Journal of Mathematical Physics*, **49**(5), 053515.
77. J. Baez and J. Dolan. (1998). Higher-dimensional algebra III: n-Categories and the algebra of opetopes. *Advances in Mathematics*, **135**(2), 145–206.
78. D. Spivak. (2014). Category theory for the sciences. *MIT Press*.
79. J. Feldman. (2009). Bayes and the simplicity principle in perception. *Psychological Bulletin*, **135**(4), 560–575.
80. T. Leinster. (2004). Higher operads, higher categories. *London Mathematical Society Lecture Note Series*, **298**.
81. Y. Aharonov, D. Z. Albert, and L. Vaidman, “How the result of a measurement of a component of the spin of a spin-$\frac{1}{2}$ particle can turn out to be 100,” *Phys. Rev. Lett.*, vol. 60, no. 14, pp. 1351–1354, 1988.
82. R. W. Kentridge, C. A. Heywood, and L. Weiskrantz, “Attention without awareness in blindsight,” *Proc. R. Soc. Lond. B*, vol. 266, no. 1430, pp. 1805–1811, 1999.
83. K. Friston, “The free-energy principle: a unified brain theory?,” *Nat. Rev. Neurosci.*, vol. 11, no. 2, pp. 127–138, 2010.
84. L. Baumgarten and M. Leifer, “A formal framework for the weak measurement of quantum processes,” *Phys. Rev. A*, vol. 103, 052203, 2021.
85. T. Bachmann, “Blindsight and qualia: on how neural correlates of consciousness may elude introspection,” *Brain and Cognition*, vol. 75, no. 2, pp. 118–126, 2011.
86. L. Weiskrantz, *Blindsight: A Case Study and Implications*. Oxford University Press, 1986.
87. G. Tononi and C. Koch, “Consciousness: here, there and everywhere?,” *Phil. Trans. R. Soc. B*, vol. 370, no. 1668, 2016.
88. A. Cowey, “The blindsight saga,” *Exp. Brain Res.*, vol. 200, pp. 3–24, 2010.
89. Weiskrantz, L. (1986). *Blindsight: A Case Study and Implications*. Oxford University Press.
90. Frieden, B. R. (1998). *Physics from Fisher Information*. Cambridge University Press.
91. Baudot, J., & Bennequin, D. (2015). The homological nature of entropy. *Entropy*, 17(5), 3253–3318.
92. Friston, K. (2010). The free-energy principle: A unified brain theory? *Nature Reviews Neuroscience*, 11(2), 127–138.