Validity of Linearized Colmation Models for Methane Migration and Smart Ventilation Design in Underground Mines

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Posted Date: 23 March 2026

doi: 10.20944/preprints202603.1798.v1

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 doi:10.20944/preprints202603.1798.v1

Article

Validity of Linearized Colmation Models for Methane Migration and Smart Ventilation Design in Underground Mines

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Featured Application

The obtained findings provide a quantitative foundation for predictive modeling of methane migration and permeability evolution, supporting the development of smart ventilation systems, methane emission mitigation strategies, and digital mine-environment models. Consequently, the proposed approach contributes to improved mine safety, reduced greenhouse gas emissions, and data-driven decision-making processes that support the just transition of the mining sector.

Abstract

Colmation phenomena play a critical role in long-term gas flow through porous media, significantly influencing methane migration, mine ventilation efficiency, and emission control in both active and abandoned coal mines. In colmation modeling, three fundamental kinetic types are commonly distinguished, with the third kinetic providing a generalized nonlinear formulation capable of describing state-dependent and spatially variable permeability degradation. However, the strong nonlinearity of the coupled transport–colmation equations prevents the derivation of closed-form solutions, which necessitates the application of linearization techniques. In this study, gas flow with colmation governed by third-kinetics is analyzed with particular emphasis on methane migration in underground mining environments. Linearization of nonlinear kinetic terms is applied at the level of the coupled mass balance and colmation equations, resulting in an approximate form of Darcy's law and an explicit analytical solution describing the evolution of the porous medium state. The primary objective of the study is to quantify the error introduced by the adopted linearization and to analyze its spatial and temporal propagation with respect to the nonlinear reference solution. A rigorous error estimation based on Taylor series truncation is developed, yielding an explicit criterion that defines the validity range of the linearized solution. The results demonstrate that the approximation remains reliable within the regime of weak colmation, while the associated error is locally generated and propagates through transport mechanisms without exhibiting uncontrolled growth.

Keywords: methane migration; methane mitigation; smart ventilation; colmation modeling; underground coal mining; emission control; digital mine models; just transition

1. Introduction

Methane emissions from underground mining operations and abandoned coal mines remain one of the major challenges in the context of climate policy, mine safety, and the just transition of the extractive industries [1]. Effective methane mitigation requires not only advanced capture technologies but also reliable predictive models of gas migration and flow resistance evolution within the rock mass [2]. In this context, mathematical models describing long-term permeability degradation processes, such as colmation, become a critical component of smart ventilation and digital mine-environment systems.

 doi:10.20944/preprints202603.1798.v1

Colmation, in the scope of methane in coal seams, primarily refers to the phenomenon of clogging (blocking) of pores, fractures, and drainage systems by fine coal particles (coal dust) and waste rock. This phenomenon significantly hinders or prevents effective methane recovery (Coal Bed Methane—CBM) and increases the methane hazard in mines. It is a phenomenon of key importance for mining safety and the efficiency of methane drainage [3,4].

Key aspects of colmation in relation to methane in mines include:

* Colmation mechanism: Fine coal fractions (dust) are displaced by flowing gas or water, depositing in fractures (cracks), which closes the escape routes for methane.

* Impact on Coal Bed Methane (CBM) extraction: Colmation of boreholes and their surroundings (the near-wellbore zone) causes a decrease in the permeability of the deposit, which drastically reduces methane production efficiency and may lead to pump clogging.

* Safety hazard: Blocking the flow of methane in the seam can lead to methane accumulation, increasing the risk of sudden gas and rock outbursts.

* Colmation in methane drainage systems: In underground methane drainage systems, colmation (often combined with water condensation) blocks pipelines, lowering the effectiveness of methane drainage and necessitating more frequent maintenance [5,6].

Detailed categorization of the causes, mechanisms, and effects of the colmation process includes the following issues:

Hard coals possess a specific porous structure. Methane is trapped within them primarily through adsorption (physicochemical bonding on the surface of micropores). For the gas to be released, it must pass through a system of fractures (so-called cleats). Colmation is the process that blocks these “highways” for the gas [7].

Main causes of colmation in relation to methane from coal mines:

* Strata pressure: As depth increases, the overburden pressure squeezes natural fractures in the coal, drastically reducing its permeability.

* Matrix Swelling: Coal increases its volume under the influence of gas adsorption (especially methane). Swelling coal grains constrict flow paths, hindering further gas outflow.

* Mechanical phenomena: Fine coal fractions (coal dust) carried by flowing gas or water can settle in fracture constrictions, creating blockages.

* Chemical and biological deposits: Precipitation of minerals from mine waters (e.g., carbonates, chlorides) or the development of bacterial biofilms in coal pores [8].

The phenomenon of colmation is associated with the flow of liquids or gases through porous media, where the transported medium contains suspended solid particles capable of depositing within the pore space [9]. This process leads to a gradual change in the structure of the medium, specifically to a reduction in its porosity and permeability, and consequently to a significant modification of the flow conditions. Colmation can be undesirable, such as in the case of reduced efficiency of wells or filtration systems, or desirable, when the goal is to reduce the permeability of the medium, for example, in sealing the rock mass or controlling the inflow of water into mine workings. Therefore, the correct description of the mechanisms of colmation is of significant importance both from a theoretical and engineering perspective [10].

The mathematical description of flow with colmation is based on a coupled system of partial differential equations, including the transport balance equation, the colmation process kinetics equation, and the equation of motion. The transport balance equation describes the behavior of the concentration of particles carried in the flowing medium, accounting for their displacement and the loss resulting from the deposition process within the porous structure [11,12]. The colmation kinetics equation determines the rate of change of the state variable characterizing the porous medium, most often porosity, as a function of local flow conditions and structural state. In turn, the equation of motion, in the form resulting from Darcy’s law, relates the filtration velocity field or pressure gradient to the current properties of the medium, which undergo continuous evolution during the colmation process.

 doi:10.20944/preprints202603.1798.v1

The mutual coupling of these equations makes the description of flow with colmation strongly nonlinear. Within the classical theory of colmation described by Trzaska [13] , three basic process kinetics are distinguished:

* First kinetics assumes a constant rate of mass exchange between the liquid phase and the porous skeleton, independent of the degree of colmation.

* Second kinetics accounts for the self-inhibiting nature of the process, where the rate of change decreases as the available pore space decreases.

* Third kinetics is the most general formulation, where the colmation rate depends nonlinearly both on the current structural state of the medium and the intensity of particle transport [6].

This approach allows for the description of processes in which colmation vanishes both in the initial state and at full saturation of the medium, reaching a maximum for intermediate states, which corresponds to the actual course of long-term filtration processes. It should be emphasized that the term “vanishing of colmation” refers to the rate of its progression (the derivative of the state variable with respect to time) rather than the existence of the process itself.

The application of the third kinetics, however, leads to a coupled system of differential equations with a high degree of nonlinearity, for which an explicit solution is generally not known. In previous works, this problem was most often solved by introducing linearization procedures for selected terms, which allowed for the analytical determination of approximate distributions. Such an approach raises the question of the range of applicability of the obtained solutions and the error resulting from the adopted simplifications.

In this work, flow with colmation proceeding according to the third kinetics is analyzed [4]. The considerations are conducted within a one-dimensional description of flow through a porous medium where the coupled system of equations has a nonlinear form. For this reason, linearization of the nonlinear terms was applied at the stage of solving the mass balance and colmation kinetics equations. The resulting state variable serves as the basis for further analysis of Darcy’s equation in an approximate approach.

The primary objective of the study is to estimate the error resulting from the adopted linearization and analyze its propagation in space-time.

Methane emissions from underground mining operations and abandoned coal mines remain one of the major challenges in the context of climate policy, mine safety, and the just transition of the extractive industries. Effective methane mitigation requires not only advanced capture technologies but also reliable predictive models of gas migration and flow resistance evolution within the rock mass. In this context, mathematical models describing long-term permeability degradation processes, such as colmation, become a critical component of smart ventilation and digital mine-environment systems.

2. Mathematical Model of the Coupled Transport and Colmation Process in the Trzaska Model Approach [13]

This article considers a one-dimensional mathematical model describing the coupled process of mass transport and colmation in a porous medium as proposed by Trzaska.

This model describes a phenomenon dependent on both the spatial variable x and time t, where changes in the structure of the medium directly affect transport, and transport determines the rate of further colmation. The mathematical description, encompassing the form of the balance equations, the local kinetics equation, and the method for solving the corresponding system of partial differential equations, is reconstructive and does not constitute the original contribution of the authors. The presentation of the model aims to organize its assumptions and create a starting point for further considerations.

The basis of the description is a system of partial differential equations in which the key role is played by the function $P(x,t)$, interpreted as a dimensionless state variable of the medium related to the degree of its colmation, ranging from zero (uncolmated state) to a maximum value corresponding

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to full saturation of the pore structure. This system is written in the form of a balance equation and a local kinetics equation.

The mass balance equation in the considered one-dimensional case takes the form:

$$\frac{\partial}{\partial x}\big(q(t)N(x,t)\big) + \frac{\partial P(x,t)}{\partial t} = 0 (1)$$

where $q(t)$ denotes the volumetric flow rate, while $N(x, t)$ is a dimensionless function describing the spatial distribution of particle concentration.

The evolution of the function $P(x,t)$ over time is described by the kinetics equation III:

$$\frac{\partial P(x,t)}{\partial t} = -\alpha(P(x,t) - \beta)(P(x,t) - \varepsilon_o)q(t)N(0,t) (2)$$

where α, β and ε_o are dimensionless constant parameters of the model: α is the proportionality coefficient, β is an additional coefficient exclusive to kinetics III, and ε_o denotes the initial porosity.

It is assumed that at the initial time, the medium is in an uncolmated state, leading to the condition:

$$P(x,0) = 0 (3)$$

Additionally, at the boundary of the medium, the condition is specified:

$$N(0,t) = n (4)$$

describing the intensity of particle inflow from the environment, where n is the dimensionless, normalized volumetric concentration of the colmatant.

The inflow condition specified at the boundary x = 0 (4) means that equation (2) at the boundary takes the form of an ordinary differential equation with respect to time for the function P(0, t). Its solution under the initial condition (3) takes the form:

$$P(0,t) = \varepsilon_o \frac{1-exp\left[-n\alpha(\varepsilon_o-\beta)\int_0^t q(\tau)d\tau\right]}{1-\frac{\varepsilon_o}{\beta}exp\left[-n\alpha(\varepsilon_o-\beta)\int_0^t q(\tau)d\tau\right]} (5)$$

Formula (5) describes the boundary solution—i.e., the time course of $P(0,t)$ at the inflow boundary.

It provides an explicit expression for the value of P at the point $x = 0$ and allows to analyze the influence of the model parameters and the forcing function $q(t)$ without solving the full problem in the entire spatial domain.

The aim of further considerations is the elimination of the product $q(t)N(x,t)$ from equations (1) and (2). From the kinetics equation (2), after algebraic transformations, the left side can be written as a logarithmic derivative:

$$q(t)N(x,t) = -\frac{1}{\alpha(P(x,t)-\varepsilon_o)(P(x,t)-\beta)}\frac{\partial P(x,t)}{\partial t} = -\frac{1}{\alpha(\varepsilon-\beta)}\frac{\partial}{\partial t} ln\left|\frac{P(x,t)-\varepsilon_o}{P(x,t)-\beta}\right| \quad (6)$$

Substituting (6) into the balance equation (1) leads to a partial differential equation of the form:

$$\frac{\partial^2}{\partial t \partial x} ln\left|\frac{P(x,t)-\varepsilon_o}{P(x,t)-\beta}\right| - \alpha(\varepsilon-\beta)\frac{\partial P(x,t)}{\partial t} = 0 \quad (7)$$

Following the approach proposed by Trzaska, an auxiliary function $K(x,t)$ is introduced, defined by relationship (8), which allows the equation to be brought into a form enabling separation of variables:

$$K(x,t) = -\frac{P(x,t)-\varepsilon_o}{P(x,t)-\beta}$$
$$P(x,t) = f\big(K(x,t)\big) = \frac{\varepsilon_o+\beta K(x,t)}{1+K(x,t)} \quad (8)$$

After integrating equation (7) with respect to time at a fixed spatial variable x and considering the initial condition (3), a differential equation for the function $K(x,t)$is obtained:

$$\frac{\partial K(x,t)}{\partial x} - \alpha(\varepsilon_o-\beta)K(x,t)\frac{\varepsilon_o+\beta\cdot K(x,t)}{1+K(x,t)} = 0 \quad (9)$$

 doi:10.20944/preprints202603.1798.v1

This equation is nonlinear and generally leads to an implicit solution, which complicates its direct application. To obtain a form enabling further analysis, the equation is replaced by its linearized form near the initial state $P(x,t) = 0$,, which corresponds to linearization with respect to $K(x,t)$ around the point K_0.

From the condition

$$P(x,t) = 0 \Rightarrow \frac{\varepsilon_o+\beta K(x,t)}{1+K(x,t)} = 0 \Rightarrow K_o = -\frac{\varepsilon_o}{\beta} \quad (10)$$

Follows the value of the function $K(x,t)$ corresponding to the initial state, which is the reference point for the linearization procedure.

Linearizing the expression:

$$\frac{\varepsilon_o+\beta K(x,t)}{1+K(x,t)} \quad (11)$$

around point K_0, the following approximation is obtained:

$$\frac{\varepsilon_o+\beta K(x,t)}{1+K(x,t)} \approx -\frac{\beta^2}{\varepsilon_o-\beta}[K(x,t) - K_o] \quad (12)$$

Substituting (12) into (9) yields the simplified equation:

$$\frac{\partial K(x,t)}{\partial x} + \alpha\beta^2 K(x,t)[K(x,t) - K_o] = 0 \quad (13)$$

whose general solution is given by the formula:

$$K(x,t) = K_o\left[1 - F(t)e^{-K_o\alpha\beta^2 x}\right]^{-1} \quad (14)$$

To determine the function F(t), the condition at point $x = 0$ is used. Since the course of $P(0,t)$ is known from equation (5), and the relationship between the functions $K(x,t)$ and $P(x,t)$ is given by equation (8), this condition can be rewritten in a form relating directly to the function $K(0,t)$.

$$K(0,t) = -\frac{\varepsilon_o}{\beta} exp\left[-n\alpha(\varepsilon_o - \beta)\int_0^t q(\tau)d\tau\right] \quad (15)$$

After substituting into (14), we obtain:

$$F(t) = 1 - exp\left[n\alpha(\varepsilon_o - \beta)\int_0^t q(\tau)d\tau\right] \quad (16)$$

Ultimately, the solution for function K(x, t) takes the form:

$$K(x,t) = -\frac{\varepsilon_o}{\beta}\left\{1 - \left[1 - exp\left(n\alpha(\varepsilon_o - \beta)\int_0^t q(\tau)d\tau\right)\right] exp(\varepsilon_o\alpha\beta x)\right\}^{-1} \quad (17)$$

while the function P(x, t) is given by:

$$P(x,t) = \varepsilon_o \frac{exp\left(n\alpha(\varepsilon_o-\beta)\int_0^t q(\tau)d\tau\right)-1}{exp\left(n\alpha(\varepsilon_o-\beta)\int_0^t q(\tau)d\tau\right)-\frac{\varepsilon_o-\beta}{\beta}exp(n\alpha\varepsilon_o x)-1} \quad (18)$$

Knowing $P(x,t)$, the function $N(x,t)$. can be uniquely determined. This closes the transport and colmation kinetics model. From an engineering perspective, the presented model constitutes a simplified yet computationally efficient representation of permeability degradation processes, suitable for integration into ventilation simulations and digital mine models used for methane control.

3. Character and Sources of Error Resulting from Linearization of the Model

The application of the linearization procedure for the nonlinear term in the equation for $K(x,t)$ enabled an explicit solution but introduced an approximation which is the primary source of error. The linearization was performed around the initial state $P(x,t)$=0, defining its formal correctness range, corresponding to an uncolmatized porous medium, which clearly determines the scope of its formal correctness.

 doi:10.20944/preprints202603.1798.v1

It should be noted that this linearization is performed in the state variable space P, not with respect to independent variables x and t; its correctness therefore concerns the neighborhood of the state $P = 0$, and not a specific point in space-time.

Mathematically, it corresponds to truncating the Taylor expansion of the nonlinear kinetics function to the first-order terms. Omitting higher-order components leads to an error that increases as the state variable $P(x,t)$ deviates from the reference point. in which linearization was performed.

Consequently, the resulting solution is a local approximation, the accuracy of which depends on the current degree of colmation of the medium.

Physically, this means the linearized solution correctly describes only the early stages of the colmation process or areas with small structural changes. As colmation progresses, the influence of omitted nonlinear components becomes significant.

An important feature is the propagation of the error in time and space. The error generated at the kinetics stage is carried by the transport equation, this causes its accumulation in space-time and can lead to noticeable differences in the predicted state distributions.

It should also be emphasized that the function $F(t)$, appearing in the solution of the linearized equation for $K(x,t)$, was determined based on the explicit solution of the kinetic equation at $x = 0$, without using linear approximations. This means that the boundary condition in time is satisfied exactly, eliminating the possibility of error initiation at the boundary of the medium. Consequently, the error resulting from linearization is generated exclusively in the internal region of the medium and is subject to propagation in space, but its growth is not uncontrolled.

The above considerations are qualitative in nature and serve to identify the mechanisms responsible for the generation and propagation of the error resulting from model linearization. The next subsection will conduct a quantitative analysis of this error, allowing for an assessment of the scope of applicability of the obtained approximate solution.

3. Estimation and Propagation of the Error Resulting from Linearization

To quantitatively estimate the error resulting from the assumed linearization, an error function $E(x,t)$ is introduced, defined as the difference between the linearized solution and the reference solution. The linearized solution is defined as the function $P_{\text{lin}}(x,t)$ obtained by linearizing the nonlinear term of the process kinetics. The reference solution $P_{\text{ref}}(x,t)$ is assumed to be the solution of the full, nonlinear model of coupled transport and colmation, uniquely defined by an implicit equation relating the function $P(x,t)$, the spatial variable x, and time t. In the special case of the medium boundary ($x = 0$), this solution is reduced to an explicit form resulting directly from the kinetic equation, which allows for the unambiguous determination of the function $F(t)$ occurring in the linearized solution.

To quantitatively estimate the error, the error function E(x, t) is introduced, defined as the difference between the linearized and reference solutions: Thus, the error function is defined as:

$$E(x,t) = P_{lin}(x,t) - P_{ref}(x,t) \qquad (19)$$

and, due to the way the function $F(t)$ is determined based on the exact kinetic solution, the boundary condition in time is satisfied exactly. This leads to the property:

$$E(0,t) = 0, \forall t \in \mathbb{R} \qquad (20)$$

which means that the error resulting from linearization is not initiated at the edge of the medium, but generated only in its interior and is subject to further propagation in space.

The linearization procedure used in the previous chapter consists in replacing the nonlinear term in the equation for $K(x,t)$ with its Taylor expansion restricted to first-order terms in the neighborhood of the reference point $K0$ (corresponding to the state $P(x,t) = 0$). From Taylor's theorem it follows that the omitted term has quadratic order with respect to the deviation from the reference point, i.e., it is proportional to $(K - K_0)^2$ (up to a factor depending on the second derivative of the considered nonlinear function). This means that the local error resulting from the truncation of the Taylor expansion can be treated as:

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$$\Delta G(K) = \mathcal{O}((K - K_o)^2) \quad (21)$$

Since the variables K and P are related by a unique transformation (8), and in the reference state $P = 0 \leftrightarrow K = K_0$ holds, then in the range of small values of P the relationship $K - K_0 = \mathcal{O}(P)$. holds. Consequently, the linearization error, and thus the difference between the linearized and reference solutions, is of second-order nature in the initial range of the process with respect to the state variable:

$$E(x,t) = \mathcal{O}(P(x,t)^2) \quad (22)$$

It provides an explicit expression for the value of P at the point $x = 0$ and allows us to analyze the influence of the model parameters and the forcing function $q(t)$ without solving the full problem in the entire spatial domain.

The above qualitative considerations can be supplemented with a quantitative estimate of the error resulting from truncating the Taylor expansion of the nonlinear dependence present in the model, leading to an explicit criterion for the applicability of the approximate solution. For this purpose, the fact that the linearization was performed with respect to the auxiliary function $K(x,t)$, related to the state variable $P(x,t)$ by relationship (8) is used. The reference state corresponding to the uncolmatized medium, defined by the condition $P = 0$, corresponds to point $K0$, defined by equation (10).

By introducing the deviation of function $K(x,t)$ from the reference point:

$$\Delta K(x,t) = K(x,t) - K_o \quad (23)$$

we can proceed to a quantitative estimation of the error resulting from the truncation of the Taylor expansion of the function $f(K)$. From Taylor's theorem it follows that the expansion of the function $f(K)$ in the neighborhood of the point K_0 has the form:

$$f\big(K(x,t)\big) = f(K_o) + f'(K_o)\Delta K(x,t) + \frac{1}{2}f''(\xi)\Delta K(x,t)^2, \xi \in \big(K_o, K(x,t)\big) \quad (24)$$

where, due to the property $f(K0) = 0$, the linearized solution takes the form:

$$K_{lin}(x,t) = f'(K_o)\Delta K(x,t) \quad (25)$$

The second-order component omitted in the linearization procedure is a local approximation error and can be written in the form:

$$E_k(x,t) = \frac{1}{2}f''(\xi)[\Delta K(x,t)]^2 \quad (26)$$

The derivatives of the function $f(K)$ are given by the formulas:

$$f'\big(K(x,t)\big) = \frac{\beta - \varepsilon_o}{[1+K(x,t)]^2}, f''\big(K(x,t)\big) = -\frac{2(\beta - \varepsilon_o)}{[1+K(x,t)]^3} \quad (27)$$

In order to formulate a quantitative criterion, the relative linearization error is introduced, defined as the quotient of the approximation error and the linear solution error:

$$r(x,t) = \frac{|E_k(x,t)|}{|K_{lin}(x,t)|} \quad (28)$$

After substituting the dependencies (25)–(27) and simplifying, the approximation is obtained:

$$r(x,t) \approx \frac{1}{2}\frac{|f''(K_o)|}{|f'(K_o)|}|\Delta K(x,t)| \quad (29)$$

By assuming the permissible tolerance of the relative error ρ (e.g., $\rho = 0.05$ or $\rho = 0.1$), the condition for the applicability of the approximate solution is obtained in the form of the inequality:

$$r(x,t) \le \rho \quad (30)$$

After taking into account the derivative forms at K_0, this leads to an explicit criterion:

$$|\Delta K(x,t)| \le \rho\frac{|\beta - \varepsilon_o|}{|\beta|} \quad (31)$$

Inequality (31) provides a quantitative estimate of the range within which the solution obtained by linearization maintains the specified accuracy in terms of relative error.

 doi:10.20944/preprints202603.1798.v1

The error analysis of the approximate solution was performed solely for the variable $K(x,t)$, which is the variable with respect to which the nonlinear dependence in the model was linearized. The resulting criterion (31) results directly from the estimation of the residual of the Taylor expansion of the function $P = f(K)$ and constitutes the basic, primary result of the error analysis.

The state variable $P(x,t)$ was not the linearized variable, but a function of the variable $K(x,t)$. Consequently, the transition from the criterion in variable K to the notation in variable P does not lead to a new error estimate, but is merely a local approximation resulting from the first term of the Taylor expansion of the relationship $P = f(K)$ in the neighborhood of the reference point $K0$. In this sense, the change in variable causes the deviation $P(x,t)$ from the reference state $P = 0$ to be of the order of:

$$\Delta P(x,t) \sim f'(K_o)\Delta K(x,t) \qquad (32)$$

which allows us to estimate the error in the state variable by the relationship

$$|\Delta P(x,t)| \lesssim \rho|\beta| \quad (34)$$

The obtained estimate is of auxiliary nature and is valid only locally, i.e., in the weak colmation regime, in which the neglected higher-order components of the Taylor expansion remain secondary. After, taking into account, the explicit dependencies between variables P and K and the linearization applicability criterion resulting from the analysis of the Taylor expansion residual, this estimate simplifies to a form independent of the parameter εo. Consequently, the local error in the state variable P is, in the first approximation, controlled solely by the adopted tolerance ρ and the kinetic parameter β, which is consistent with the local nature of the adopted linearization procedure and does not change the range of applicability of the approximate solution.

Since the error estimate concerns the nonlinear relationship $P = f(K)$, used to obtain the linearized form of the differential equation, the obtained explicit solution remains reliable only within the range of independent variables where criterion (31) is satisfied. Outside this range, no new error occurs, but the condition for the correctness of the adopted approximation is violated.

The error resulting from the linearization of the nonlinear kinetic term is local in the space of the state variable values and, as previously discussed, is not initialized at the boundary of the medium. This is because the function $F(t)$, appearing in the solution of the linearized transport equation, was determined based on the explicit solution of the kinetic equation at the point $x = 0$, without using linear approximations. Consequently, the condition of consistency between the linearized solution and the reference solution is satisfied at the boundary of the medium, leading to the disappearance of the error at the point $x = 0$.

Despite the zero error at the boundary, its presence in the internal region of the medium is unavoidable. The source of the error remains the local approximation of the nonlinear kinetics of the process, whose accuracy decreases as the solution moves away from the reference state. This error, generated locally in the kinetic equation, is then transmitted spatially through the transport equation, which acts as an operator propagating information along the flow direction.

The spatial dependence of the error is therefore not primary in nature but is a consequence of the coupling of local kinetic dynamics with the transport process. This means that the error distribution as a function of variable x reflects the spatial distribution of the state function $P(x,t)$, not the mere existence of a spatial variable in the model equations. In particular, the maximum error may occur in regions distant from the boundary, even though the temporal boundary condition is perfectly satisfied.

A significant consequence of this model structure is the absence of uncontrolled error growth in space. Exactly meeting the boundary condition at $x = 0$ eliminates the possibility of error initiation at the boundary of the medium, and its further evolution is determined solely by the local properties of the solution and the transport mechanism. As a result, error propagation is orderly and closely related to the dynamics of the colmation process, rather than to the consequences of the adopted approximation.

 doi:10.20944/preprints202603.1798.v1

The explicit error bounds obtained for the linearized solution are particularly relevant for smart ventilation control algorithms, where inaccurate predictions of methane migration may lead to either excessive energy consumption or increased safety risks.

4. Discussion

The obtained results allow us to clearly determine the extent to which the linearized solution of the III kinetics model maintains reliability for methane migration predictions. It was demonstrated that the error resulting from linearization is local in the state variable space and is directly related to the current stage of the co;mation process, not to the spatial position or process duration.

An important aspect from the perspective of engineering applications is that the solution error is not initiated at the boundary of the medium, which results from the exact fulfillment of the boundary condition in time. This means that any discrepancies between the approximate solution and the nonlinear reference solution are generated exclusively within the internal region of the medium and propagate according to the gas transport mechanism. This error structure eliminates the risk of uncontrolled error growth and increases the stability of the solution in the analyzed regime.

From the perspective of modeling methane migration in mines, the explicit criterion for the applicability of the linearized solution, based on the estimation of the Taylor expansion residual, is particularly important. This criterion allows for a clear definition of the range of parameters and medium states within which the simplified model can be safely used in predictive analyses, including simulations supporting the design of ventilation and methane drainage systems.

The results of the analysis confirm that in the weak clogging regime, the linearized solution is a computationally effective tool for describing permeability changes affecting methane migration. At the same time, they indicate the need for caution in applying this approach in later stages of the process, when nonlinear kinetics begin to play a dominant role.

The results allow for the unambiguous determination of the range in which the linearized colmation model (III kinetics) maintains reliability for methane migration forecasts. It has been shown that the linearization error is local in the state variable space and is directly related to the current degree of colmation rather than spatial position or time duration.

A significant aspect for engineering applications is that the error is not initiated at the boundary, ensuring stability. The explicit criterion based on the Taylor expansion residue allows for the safe use of the simplified model in simulations supporting the design of ventilation and methane drainage systems. While effective for early process stages, caution is required in later stages when nonlinear components become dominant.

5. Conclusions

The analysis demonstrated that the solution obtained by linearizing the nonlinear III kinetics model of methane clogging can be effectively used to describe the early stages of the process and to assess its impact on methane migration in porous media. The developed applicability criterion allows for a clear definition of the range within which the approximate solution maintains the desired accuracy.

It was demonstrated that the error resulting from linearization is local in the state variable space and is not initiated at the boundary of the media, which ensures the stability of the solution in predictive analyses. In this approach, the explicit solution acts as a closing element of the coupled flow model based on the differential form of the Darcy equation, enabling practical computational applications.

From the perspective of hard coal mining, the obtained results are directly relevant for assessing the effectiveness of methane drainage and designing ventilation systems, as the clogging process affects pressure distributions and methane fluxes in the areas of excavations and drainage holes. The proposed approach enables the informed use of simplified kinetic models in mining safety and methane emission reduction.

 doi:10.20944/preprints202603.1798.v1

Thanks to the computationally efficient and controlled description of methane migration with changing medium permeability, the presented model can be a predictive element in intelligent ventilation systems and digital models of the mine environment. Consequently, it contributes to efforts to reduce methane emissions, improve occupational safety, and achieve the goals of decarbonization and just transition in mining regions.

This work demonstrates the error introduced by the commonly used linearization of nonlinear type III colmation kinetics and the extent to which such simplification is still acceptable. An explicit criterion is proposed to assess when a linearized solution can be considered a reliable predictive tool and when it loses its physical meaning.

This provides the familiar mathematical model with information relevant to engineering practice, particularly modeling methane migration and the design of methane drainage and ventilation systems in mines.

The analysis conducted has shown that the linearized solution of the nonlinear third-kinetic colmation model can be effectively used to describe the early stages of the process and evaluate its impact on methane migration. The developed criterion allows for the definition of the range where the approximate solution maintains specified accuracy. The error is local and not initiated at the boundary, ensuring prognostic stability. In this context, the explicit solution serves as a closing element for the coupled flow model based on the differential form of Darcy's law. For coal mining, these results are directly relevant to methane drainage effectiveness and ventilation system design. The proposed approach enables the informed application of simplified kinetic models in mining safety and emission mitigation. Consequently, it aligns with efforts to reduce greenhouse gas emissions and support the just transition of mining regions

Author Contributions: Conceptualization, W.F., K.B. and B.T.; methodology, W.F. and K.B.; software, W.F.; validation, W.F. and K.B.; formal analysis, W.F. and K.B.; investigation, W.F. and K.B; resources, W.F., K.B. and B.T.; data curation, W.F, K.B. and B.T.; writing—original draft preparation, W.F. and K.B.; writing—review and editing, B.T.; visualization, W.F. and K.B.; supervision, B.T.; project administration, W.F., K.B. and B.T.; funding acquisition, B.T. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by AGH University of Krakow, Poland, grant number **501.00-100302-10000.**

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: We encourage all authors of articles published in MDPI journals to share their research data. In this section, please provide details regarding where data supporting reported results can be found, including links to publicly archived datasets analyzed or generated during the study. Where no new data were created, or where data is unavailable due to privacy or ethical restrictions, a statement is still required. Suggested Data Availability Statements are available in section "MDPI Research Data Policies" at https://www.mdpi.com/ethics.

Acknowledgments: The authors have reviewed and edited the output and take full responsibility for the content of this publication.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Borowski, M.; Zwolińska-Glądys, K.; Riesgo Fernández, P. Methane Emissions from Coal Mining: Challenges and Opportunities in the Context of Green Mining Technologies. *Energies* **2026**, 19, 770. https://doi.org/10.3390/en19030770
2. Borowski. M.; Zwolińska-Glądys. K.; Cheng. J., Methane Emissions from Coal Mines: Quantification, Capture, and Utilization Strategies for Atmospheric Impact Mitigation A Case Study from Poland. *Atmosphere* **2025**, 16, 174. https://doi.org/10.3390/atmos16020174

 doi:10.20944/preprints202603.1798.v1

3. Kaczmarczyk-Kuszpit, W.; Słota-Valim, M.; Wrana, A.; Leśniak, G.; et al. An integrated approach to design methane drainage boreholes in post-mining areas of an active coal mine: A case study from the Pniówek Coal Mine. *Appl. Sci.* **2025,** 15, 3271.
4. Echternach-Jaubert, M.; Pellerin, R.; Gamache, M. Implementation approaches for digital transformation: Current state of the mining industry. *Min. Metall. Explor.* **2025,** 42, 311–325.
5. Trzaska. A., Broda..K., Filipek. W.; Sobowska, K: O zjawisku kolmatacji i jego wpływie na otoczenie — The phenomenon of colmatage and its effect on the environment, *Archives of Mining Sciences*; **2005**, ISSN 0860-7001. vol. 50 spec. iss.,
6. Litwiniszyn. J.; Elementy dynamiki przepływów w sieciach wentylacyjnych — Elements of flow dynamics in ventilation networks, in : *Materiały 1 Szkoły Aerologii Górniczej* , **1999**, ISBN: 83-904303-8-X,.
7. Litwiniszyn. J., Colmatage Considered as a Certain Stochastic Process. *Bull. Acad. Polon. Sci., Ser. Sci. techn.*, **1963.** 3
8. Filipek.W., Broda.K., Pressure distribution in a porous medium during the colmatage phenomenon accurring according to the first kinetics without linearization of the expression defining the porosity of the considered medium as a function of position and time, *Inżynieria Mineralna- Journal of the Polish Mineral Engineering Society*; **2023,** ISSN 1640-4920. — vol. 2 no. 2, s. 7–13. doi.org/10.29227/IM-2024-01-108
9. Trzaska, A., New Kinetics Equations of the Colmatage Process and Their Application. *Archiwum Górnictwa*, **1972.** 17,4,361- 384
10. Trzaska, A., Broda, K. Comparison of The Actual Course of Colmatage with Its Theoretical Description. *Archiwum Górnictwa*, **1991**, 36,4,325-332.
11. Trzaska A., Broda K., Possibility of Determining Colmatage Parameters and Functions Basing on the Theory of Colmatage and Experiment. *Archives of Mining Sciences*, **2000,** 45, 4, 527-542
12. Hendel, J.; Macuda, J.; Chećko, J. Worldwide examples of coal mine methane emission and gob gas production modeling. Proc. Int. Multidisciplinary Scientific GeoConference SGEM; **2015**; pp. 563–570.
13. Trzaska, A., Sobowska, K., Kolmatacja : przepływy z wymianą masy i zmianą pędu — Colmatage : the flows with mass and momentum exchange, Kraków: *AGH Uczelniane Wydawnictwa Naukowo-Dydaktyczne*, **2007**, ISBN: 978-83-7464-124-1