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Article

Proof of the Binary Goldbach Conjecture

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Abstract

In this article the proof of the binary Goldbach conjecture is established (any integer greater than one is the mean arithmetic of two positive primes). To this end, Chen's weak conjecture is proved (any even integer greater than one is the difference of two positive primes) and a "localised" algorithm is developed for the construction of two recurrent sequences of extreme Goldbach decomponents (U_{2n}) and (V_{2n}) , $((U_{2n}) \text{ dependent of } (V_{2n}))$ for any integer $n \geq 2$ (U_{2n}) and (V_{2n}) are positive primes and $U_{2n} + V_{2n} = 2n$. To form them, a third sequence of primes (W_{2n}) is defined for any integer $n \geq 3$ by $W_{2n} = \text{Sup} (p \in \mathcal{P} : p \leq 2n - 3)$, \mathcal{P} denoting the set of positive primes. The Goldbach conjecture has been proved for all even integers $2n$ between 4 and $4 \cdot 10^{18}$ and in the neighbourhood of 10^{100} , 10^{200} and 10^{300} for intervals of amplitude 10^9 . The table of extreme Goldbach decomponents, compiled using the programs in Appendix 15 and written with the Maxima and Maple scientific computing software, as well as files from ResearchGate, Internet Archive, and the OEIS, reaches values of the order of $2n = 10^{5000}$. Algorithms for locating Goldbach's decomponents for very large values of $2n$ are also proposed. In addition, a global proof by strong recurrence "finite ascent and descent method" on all the Goldbach decomponents is provided by using sequences of primes $(W_{q_{2n}})$ defined by $W_{q_{2n}} = \text{Sup} (p \in \mathcal{P} : p \leq 2n - q)$ for any odd positive prime q , and a further proof by Euclidean divisions of $2n$ by its two assumed extreme Goldbach decomponents is announced by identifying uniqueness, coincidence and consistency of the two operations. Next, a majorization of U_{2n} by $n^{0.525}$, $0.7 \ln^{2.2}(n)$ with probability one and $5 \ln^{1.3}(n)$ on average for any integer n large enough is justified. Finally, the Lagrange-Lemoine-Levy (3L) conjecture and its generalization called "Bachet-Bézout-Goldbach"(BBG) conjecture are proven by the same type of method. Additionally, we provide heuristic estimates for Goldbach's conjecture and demonstrate graphical synthesis using a Goldbach tree.

Keywords: prime number theorem; binary Goldbach conjecture; Chen's weak conjecture; Lagrange-Lemoine-Levy conjecture; Bachet-Bezout-Goldbach conjecture; Goldbach decomponents; computational number theory; gaps between consecutive primes

1. Overview

Number theory, "the queen of mathematics" studies the structures and properties defined on integers and primes (Euclid [15], Hadamard [18], Hardy, Wright [20], Landau [26], Tchebychev [44]). Numerous problems have been raised and conjectures made, the statements of which are often simple but very difficult to prove. These main components include :

- Elementary arithmetic .
 - Operations on integers, determination and properties of primes. (Basic operations, congruence, gcd, lcm,).
 - Decomposition of integers into products or sums of primes (Fundamental theorem of arithmetic, decomposition of large numbers, cryptography and Goldbach's conjecture, see Filhoa, Jaimea, de Oliveira Gouveaa, Keller Fächter, [16]).

- Analytical number theory .
 - Distribution of primes : Prime Number Theorem, the Riemann hypothesis, (see Hadamard [18], De la Vallée-Poussin [45], Littlewood [29] and Erdos [14],).
 - Gaps between consecutive primes (Bombieri,Davenport [3], Cramer [9], Baker,Harmann,Iwaniec, Pintz [4],[5],[24], Granville [17], Maynard [31], Tao [43], Shanks [40], Tchebychev [44] and Zhang [49]).
- Algebraic, probabilistic, combinatorial and algorithmic number theories .
 - Modular arithmetic.
 - Diophantine approximations and equations.
 - Arithmetic and algebraic functions.
 - Diophantine and number geometry.
 - Computational number theory.

2. Definitions notations and background

The integers $h, m, M, n, N, k, K, p, q, Q, r, \dots$ used in this article are always positive. (2.1)

The symbol " \mid " means : such as or knowing that.

(2.2)

Let \mathcal{P} be the infinite set of positive primes p_k (called simply primes)

(2.3)

$$(p_1 = 2 ; p_2 = 3 ; p_3 = 5 ; p_4 = 7 ; p_5 = 11 ; p_6 = 13 ; \dots)$$

For any non-zero integer K $\mathcal{P}_K = \{ p \in \mathcal{P} : p \leq 2K \}$

(2.4)

Writing the large numbers calculated in Appendix 14 is simplified by defining the following constants:

$$M = 10^9 ; R = 4.10^8 ; G = 10^{100} ; S = 10^{500} ; T = 10^{1000} \quad (2.5)$$

$p_k \# = \prod_1^k p_j$ is the primorial of p_k .

(2.6)

$\ln(x)$ denotes the neperian logarithm of the real $x > 0$

(2.7)

$\text{LambertW}(k, x)$ is the Lambert multi-branched function

(2.8)

Let (W_{2n}) be the sequence of primes defined by

$$\forall n \in \mathbb{N} + 3 \quad W_{2n} = \text{Sup} (p \in \mathcal{P} : p \leq 2n - 3)$$

(2.9)

For any odd prime q , let (Wq_{2n}) be the sequence of primes defined by

$$\forall n \in \mathbb{N} \quad n \geq \frac{(q+3)}{2} \quad Wq_{2n} = \text{Sup} (p \in \mathcal{P} : p \leq 2n - q)$$

(2.10)

Any sequence denoted by $(G_{2n}) = (U_{2n}; V_{2n})$ verifying (2.11) is called a *Goldbach sequence*.

$$\forall n \in \mathbb{N} + 2 \quad U_{2n}, V_{2n} \in \mathcal{P} \quad \text{and} \quad U_{2n} + V_{2n} = 2n$$

(2.11)

U_{2n} and V_{2n} are also known as "Goldbach partitions or Goldbach decomponents".

Iwaniec, Pintz [24] have shown that for a sufficiently large integer n there is always a prime between $n - n^{23/42}$ and n . Baker and Harman [4],[5] concluded that there is a prime in the interval $[n; n + o(n^{0.525})]$. Thus this results provides an increase of the gap between two consecutive primes p_k and p_{k+1} of the form

$$\forall \varepsilon > 0 \quad \exists k_\varepsilon \in \mathbb{N}^* \quad \mid \quad \forall k \in \mathbb{N} \quad k \geq k_\varepsilon \quad p_{k+1} - p_k < \varepsilon \cdot p_k^{0.525}$$

(2.12)

The results obtained on the Cramer-Granville-Maier-Nicely conjecture [1],[3],[9],[17],[30],[32] imply the following majorization.

For any real $c > 2$ and for any integer $k \geq 500$

$$(with\ probability\ one) \quad (2.13) \quad p_{k+1} - p_k \leq 0.7 \ln^c(p_k)$$

and

$$(on\ average) \quad (2.14) \quad p_{k+1} - p_k \leq 20 \ln(p_k)$$

The following abbreviations have been adopted :

$$(2.15) \quad \bullet \quad \text{Lagrange-Lemoine-Levy conjecture} \quad (3L) \quad \text{conjecture}$$

$$(2.16) \quad \bullet \quad \text{Bachet-Bézout-Goldbach conjecture} \quad (BBG) \quad \text{conjecture}$$

$$(2.17) \quad \bullet \quad \text{(Extreme) Goldbach decomponents} \quad (E).G.D.$$

3. Introduction

Chen [7], Hardy, Littlewood [21], Hegfoltt, Platt [22], Ramaré, Saouter [35], Tao [43],

Tchebychev [44] and Vinogradov [46] have taken important steps and obtained promising results on the Goldbach conjecture (Any integer $n \geq 2$ is the mean arithmetic of two primes). Indeed, Helfgott, Platt [22] proved the ternary Goldbach conjecture in 2013.

Silva, Herzog, Pardi [41] held the record for calculating the terms of Goldbach sequences after determining pairs of primes $(U_{2n}; V_{2n})$ verifying

$$(3.1) \quad \forall n \in \mathbb{N} \quad \left| \begin{array}{l} 4 \leq 2n \leq 4.10^{18} \\ U_{2n} + V_{2n} = 2n \end{array} \right.$$

Goldbach's conjecture has also been verified for all even integers $2n$ satisfying

$$10^{5k} \leq 2n \leq 10^{5k} + 10^8 \quad : \quad k = 3, 4, 5, 6, \dots, 20$$

and

$$10^{10k} \leq 2n \leq 10^{10k} + 10^9 \quad : \quad k = 20, 21, 22, 23, 24, \dots, 30$$

by Deshouillers, te Riele, Saouter [11].

In previous research work there is no explicit construction of recurrent Goldbach sequences.

In this article, for any integer n greater than two the E.G.D. U_{2n} and V_{2n} are computed iteratively using a simple and efficient "localised" algorithm.

Using Maxima and Maple scientific computing software on a personal computer Silva's record is broken and many E.G.D. are calculated up to the neighbourhood of

$$2n = 10^{500}, 10^{1000}, 10^{5000} \quad \text{and} \quad G.D. \quad \text{around} \quad 10^{10000} \quad (\text{see Sainty [37]})$$

"In Researchgate.net, Internet Archive, and OEIS, E.G.D. files are supplied : E.G.D. File S around $2n = 10^S$ for $S = 1, 2, 3, \dots, 10000$ ".

The binary Goldbach conjecture can be proved globally by strong recurrence on all G.D. using (Wq_{2n}) sequences of primes in the same way via Goldbach(-) conjecture (Any even integer greater than one is the difference of two primes) demonstrated in Teorem 4.

• Remark.

1. **Chen conjecture:** For any integer $K \geq 1$ there are infinitely many pairs of primes with a difference equal to $2K$.

2. **De Polignac conjecture :** Same as Chen, but with consecutive pairs of primes.

3. **What we know :** April 2013, Yitang Zhang [49] demonstrates that the smallest even integer $2K$ verifying the conjecture is greater than 70 million.

In 2014 James Maynard [31] then Terence Tao [43] lowered this limit to 246.

We validate Chen's weak conjecture by verifying directly in the primes tables that all even gaps from 2 to 246 are possible (see Appendix 16).

In addition, the (3L) conjectures [10],[23],[25],[28],[47] and its generalization called (BBG) conjecture are validated.

Using case disjunction reasoning we construct two recurrent E.G.D. sequences of primes (V_{2n}) and (U_{2n}) according to the sequence (W_{2n}) by the following process

Firstly,

$$U_4 = 2 \quad \text{and} \quad V_4 = 2$$

(3.2)

For any integer n greater than two

- Either

$(2n - W_{2n})$ is a prime

then V_{2n} and U_{2n} are defined directly in terms of W_{2n} .

- Either

$(2n - W_{2n})$ is a composite number

then V_{2n} and U_{2n} are determined from the previous terms of the sequence (G_{2n}) .

(This process can be reversed by first determining the increasing sequence of primes less than $\text{Inf}(2n - W_{2k} \in \mathcal{P} : k \in \mathbb{N})$, which saves a lot of computing time when programming).

4.Theorem (Chen's weak or Goldbach(-) conjecture)

$$\forall K \in \mathbb{N}^* \quad \exists p, q \in \mathcal{P} \quad | \quad p - q = 2K$$

(4.1)

$$\text{If } K \geq 2 \quad 3 \leq q \leq 2K \quad \text{and}$$

$$3 + 2K \leq p \leq 4K$$

Practical method on some examples:

First of all $(5 - 3 = 2)$, then we begin the process at $(7 - 3 = 4)$; we will select the smallest primes for which the difference is precisely 6 $(11 - 5 = 6)$, then 8 $(11 - 3 = 8)$, then 10

$(13 - 3 = 10)$,....., then $2K$ (demonstration established by strong recurrence, by the absurd and feedback). All pairs of Goldbach(-) partitions obtained by this method for K between 2 and 123 are listed in Appendix 16 to validate it using Tao results.

Proof. An other proof can also be established by strong recurrence on the integer $K \geq$

2. Let $\mathcal{P}_{Chen}(K)$ be the following property

$$" \forall K \in \mathbb{N}^* \quad \exists p, q \in \mathcal{P} \quad | \quad p - q = 2K \quad 3 \leq q \leq 2K \quad \text{and} \quad 2K + 3 \leq p \leq 4K " \quad (4.2)$$

► $\mathcal{P}_{Chen}(2)$ is true : $7 - 3 = 4$ $q = 3 \leq 4$ and $p = 7 \leq 4 \times 2 = 8$

► Let's show

$$\forall M \in \mathbb{N} \quad | \quad 2 \leq M \leq K \quad \text{then} \quad \mathcal{P}_{Chen}(M) \Rightarrow$$

$$\mathcal{P}_{Chen}(K+1)$$

We reason through the absurd

$$\text{Let } p, q \in \mathcal{P}_K \quad | \quad p \geq q$$

$$\forall P, Q \in \mathcal{P} \quad | \quad P \geq Q \quad \exists h, m \in \mathbb{N} \quad |$$

$$P = p + 2h \quad \text{and} \quad Q = q + 2m$$

we assume that

$$P - Q = p + 2h - q - 2m \neq 2(K + 1)$$

(4.3)

Therefore

$$p - q \neq 2(K + 1 - h + m)$$

(4.4)

You can always choose $h \geq m$ and $h - m \leq K + 1$.

The set $\{K + 1 - h + m; 2h$ and $2m$ are any gaps between primes} contains all even integers between 2 and $2K$.

However the strong recurrence hypothesis asserts that

$$\forall M \in \mathbb{N} \mid M \leq K \quad \exists p, q \in \mathcal{P} \mid p - q = 2M$$

(4.5)

By choosing :

$$M = K + 1 - h + m$$

this contradicts (4.4).

So

$$\exists h, m \in \mathbb{N} \mid P - Q = p + 2h - q - 2m = 2(K + 1)$$

(4.6)

knowing

$$p, \quad p + 2h, \quad q, \quad q + 2m \in \mathcal{P}$$

$$h \geq m \quad \text{and} \quad h - m \leq K + 1$$

Thus validating the heredity of property $\mathcal{P}_{Chen}(K)$.

The property $\mathcal{P}_{Chen}(K)$ is therefore true. As a result Goldbach(-) conjecture is validated.

5 Corollary

Let (R_{2K}) and (Q_{2K}) be two sequences of primes determined by

$$R_{2K} = \text{Inf}(p \in \mathcal{P} : p - 2K \in \mathcal{P}) \quad \text{and} \quad Q_{2K} = \text{Inf}(p \in \mathcal{P} : 2K + p \in \mathcal{P}) = R_{2K}$$

$$- 2K \quad (5.1)$$

They are defined for any integer $K \in \mathbb{N}^*$

(5.2)

and satisfy

$$\lim_{K \rightarrow \infty} R_{2K} = +\infty$$

(5.3)

$$\forall K \in \mathbb{N}^* \quad R_{2K}, Q_{2K} \in \mathcal{P} \quad \text{and} \quad R_{2K} - Q_{2K} = 2K$$

(5.4)

$$\forall K \in \mathbb{N}^* \mid 2 \leq K \leq 16 \quad 3 \leq Q_{2K} \leq 2K \quad \text{and} \quad 2K + 3 \leq R_{2K} \leq 4K$$

(5.5)

For any integer K large enough

$$3 \leq Q_{2K} \leq (2K)^{0.525} \quad \text{and} \quad 2K + 3 \leq R_{2K} \leq 2K + (2K)^{0.525}$$

(5.6)

Proof.

(5.1) ; (5.2) : According to the previous theorem, the sequences (R_{2K}) and (Q_{2K}) are defined by strong recurrence (finite descent).

(5.3) : $R_{2K} \geq 2K \implies \lim_{K \rightarrow \infty} R_{2K} = +\infty$

(5.4) : By construction, these sequences thus verify : $R_{2K} - Q_{2K} = 2K$

(5.5) : The property can be verified directly term-to-term by examining the sequence proposed above.

(5.6) : This property is verified up to $2K = 246$ by calculations on the previous list.

We prove this result by recurrence

First of all, we order the Goldbach(-) decomponents at a fixed prime q , so as to obtain the estimate (5.6) more easily.

Let q_r be the $(r + 1)$ th prime :

We examine the sequences of primes $(T_r(K))_{K \in \mathbb{N}}$ satisfying :

$$T_1(K) = 2K + 3$$

$$(T_1(K) ; 2K) \rightarrow (5;2) ; (7;4) ; (11;8) ; (13;10) ; (17;14) ; (19;16) ; (23;20) ; (29;26) ; (29;28);..$$

$$T_2(K) = 2K + 5$$

$$(T_2(K) ; 2K) \rightarrow (7;2) ; (11;6) ; (13;8) ; (17;12) ; (19;14) ; (23;18) ; (29;24) ; (31;26) ; (37;32).....$$

$$T_3(K) = 2K + 7$$

$$(T_3(K) ; 2K) \rightarrow (11;4) ; (13;6) ; (17;10) ; (19;12) ; (23;16) ; (29;22) ; (31;24) ; (37;30).....$$

$$T_4(K) = 2K + 11$$

$$(T_{11}(K) ; 2K) \rightarrow (13;2) ; (17;6) ; (19;8) ; (23;12) ; (29;18) ; (31;20) ; (37;26) ; (41;30) ; (43;34).....$$

$$(T_{13}(K) ; 2K) \rightarrow (17;4) ; (19;6) ; (23;10) ; (29;16) ; (31;18) ; (37;24) ; (41;28) ; (43;30) ; (47;34).....$$

.....

$$T_r(K) = 2K + q_r \quad (K \in \mathbb{N}^* : T_r(K) \text{ and } q_r \text{ are primes}) \quad (\text{see Appendix 16})$$

For any integer K satisfying $(2K)^{0.525} > q_r$ the property holds for $T_r(K)$.

Therefore it is generally validated for all $K > K_0$, since we obtain all possible cases of

Chen's weak conjecture starting with $T_1(K)$, then $T_2(K)$, then $T_3(K)$ for $(2K)^{0.525}$

$\leq q_r$.

(can be proved by strong recurrence using the same method as in Theorem 4 by "finite descent").

Let $a = \frac{40}{21}$ and $P_a(r)$ be the following property

"For any integer $M \mid 2M < (q_r)^a$ there exists at least a prime $q < q_r \mid 2M + q \in \mathcal{P}$ "

► $P_a(K_0)$ is true (see Appendix 16).

► Let's show :

$$P_a(r) \implies P_a(r+1)$$

$$q_{r+1} \leq q_r + q_r^{0.525}$$

(5.6)

It is assumed that $M \mid$

$T_{r+1}(K) - q_{r+1} \neq 2M$ knowing $2M < (q_{r+1})^{c_p}$

$\forall T_m(R), q_m \in \mathcal{P} \exists h, s \in \mathbb{N} \mid T_{r+1}(K) = T_m(R) + 2h$ and

$$q_{r+1} = q_m + 2s \quad (5.7)$$

then

$$T_m(R) - q_m \neq 2(M + s - h)$$

(5.8)

which is impossible according to the hypothesis of strong recurrence since

$2(M + s - h)$ is less than $\text{Sup}(q_m)^a$ and that all primes $T_m(R), q_m$ satisfy the recurrence hypothesis.

We deduce that :

$$P_{c_p}(r) \implies P_{c_p}(r+1)$$

Thus the property (5.5) is true.

6 Lemma (Goldbach's

fundamental Lemma)

Let q be an odd prime

For any integer $n \geq n_q$ there exists an integer s |

$$2n - Wq_{2s} \in \mathcal{P} \quad (6.1)$$

Let (Zq_{2n}) be the sequence of primes defined by

$$\forall n \in \mathbb{N} \quad n \geq n_q \quad Zq_{2n} = \text{Inf} (2n - Wq_{2k} \in \mathcal{P} : k \in \mathbb{N}) \quad (6.2)$$

All G.D. are contains in the set $\{(2n - Zq_{2n} ; Zq_{2n}) : n \in \mathbb{N} + 3\}$

$$\text{For any integer } n \geq n_0 \quad Z3_{2n} \leq (2n)^{0.525} \quad (6.3)$$

$$Z3_{2n} \leq o (2n)^{0.525} \quad (6.4)$$

Proof. The proofs of propositions (6.1), (6.2) and (6.3) are established following the same principle of strong recurrence as in Theorem 4 and Corollary 5 by "return, absurd and finite descent"

(6.1): For any integer $n > 3$ and for any odd primes r, q | $3 \leq r < q$,

there exists an integer M_r |

$$2n - Wq_{2k} = 2n - 2M_r - Wr_{2k} = 2(n - M_r) - Wr_{2k}$$

or

$$2(n + 1) - Wq_{2k} = 2(n + 1 - M_r) - Wr_{2k}$$

then by recurrence and the absurd the property is validated.

(*Proof to develop*).

Remark. A better estimate of the following form can be obtained by the same method with probability one or on average using the results of Bombieri [3], Cramer [9], Granville [17],

Nicely [32] and Maier [30] :

$$\forall n \in \mathbb{N}: n \geq n_0 \quad ;$$

$$\text{For any real } c > 2 \quad U_{2n} < 1.7 \ln(n)^c$$

(with probability one) (6.5)

and

$$\exists K' \geq 3.5 \quad | \quad U_{2n} < K' \ln^{1.3}(n)$$

(on average) (6.6)

7 Principle of proof

To determine the E.G.D. three sequences of primes $(W_{2n}), (V_{2n}), (U_{2n})$ are defined and they verify the following properties

$$\lim_{n \rightarrow +\infty} V_{2n} = \quad (7.1)$$

$\forall n \in \mathbb{N} + 2$ V_{2n} is defined as a function of $W_{2n} = \text{Sup} (p \in \mathcal{P} : p \leq 2n -$

$$3) \quad (7.2)$$

(W_{2n}) is an increasing sequence of primes that contains all of them except $p_1 = 2$

$$(7.3)$$

$$\lim W_{2n} = +\infty$$

$$(7.4)$$

(U_{2n}) is a complementary sequence to (W_{2n}) of negligible primes with respect to

$$2n \quad (7.5) \quad \text{For any integer } n \geq 3$$

- If $(2n - W_{2n})$ is a prime

then V_{2n} and U_{2n} are defined by

$$V_{2n} = W_{2n} \quad \text{and}$$

$$U_{2n} = 2n - W_{2n} \quad (7.6)$$

- Otherwise, if $(2n - W_{2n})$ is a composite number

we search for two previous terms of the sequence $(G_{2n}), (U_{2(n-k)})$ and $V_{2(n-k)}$

satisfying the following conditions

$$\begin{aligned} & U_{2(n-k)}, V_{2(n-k)}, [U_{2(n-k)} + 2k] \\ \in \quad \mathcal{P} & \quad (7.7) \end{aligned}$$

$$U_{2(n-k)} + V_{2(n-k)} = 2(n - k)$$

which is always possible (see Theorem 4 and "Goldbach's fundamental Lemma 6")

So by setting

$$\begin{aligned} V_{2n} &= V_{2(n-k)} \quad \text{and} \quad U_{2n} = \\ U_{2(n-k)} + 2k & \quad (7.8) \end{aligned}$$

two new primes V_{2n} and U_{2n} satisfying (4.10) are generated

$$U_{2n} + V_{2n} = 2n$$

(7.9)

This process is then repeated incrementing n by one unit ($n \leftarrow n + 1$).

- *Remark.* Using the same method as in Theorem 4, we can the following equivalent property by strong recurrence : For any integer n greater than 48

$$\mathcal{P}_{ret}(n) : \text{ " There exists an integer } K \text{ such that } 2K + U_{2(n-k)} \in \mathcal{P} \text{ "}$$

(7.10)

To this end, .

► $\mathcal{P}_{ret}(49)$ is true.

► The heredity of the property $\mathcal{P}_{ret}(n) : \mathcal{P}_{ret}(n) \implies \mathcal{P}_{ret}(n + 1)$

can be proved by the absurd and returning to the previous terms by noting that

For any integer $r \mid r \leq n$, there is at least one integer $M_r \mid$

$$U_{2(n+1-k)} = 2M_r + U_{2(r+1-k)}$$

then

$$\begin{aligned} 2K + U_{2(n+1-k)} &= 2(K + M_r) + U_{2(r+1-k)} \\ &= 2P + U_{2(r+1+M_r-P)} \end{aligned}$$

(7.11)

By posing : $P = K + M_r$ and $r + 1 +$

$$M_r \leq n$$

Now, according to the recurrence hypothesis on $\mathcal{P}_{ret}(n)$ there exists an integer $P \mid$

$$(7.12) \quad 2P + U_{2(r+1+M_{r-P})} \in \mathcal{P}$$

then there exists an integer $K \mid$

$$(7.13) \quad 2K + U_{2(n+1-k)} \in \mathcal{P}$$

In summary, the property $\mathcal{P}_{ret}(n)$ is hereditary and, as a result, verifiable.

We apply the same type of reasoning using Theorem 4 to the general case with the sequence (Wq_{2n}) , showing :

For any integer $n > 2$ there exists an integer $K \mid$

$$2K + q_{2n} \in \mathcal{P}$$

8 Theorem (*Goldbach conjecture*)

(i) *There exists at least a recurrent sequence $(G_{2n}) = (U_{2n}; V_{2n})$ of primes satisfying the following conditions.*

For any integer $n \geq 2$

$$U_{2n}, V_{2n} \in \mathcal{P} \quad \text{and} \quad U_{2n} + V_{2n} = 2n \quad (8.1)$$

(Any integer $n \geq 2$ is the mean arithmetic of two primes)

(ii) An algorithm can be used to explicitly compute any term U_{2n} and V_{2n}

(8.2)

Proof.

■ GLOBAL STRONG RECURRENCE :

The proof can be made using the following strong recurrence principle.

Let $P_G(n)$ be the property defined for any integer $n \geq 2$ by

$P_G(n)$: " For any integer p satisfying $2 \leq p \leq n$ there exists two primes U_{2p} and V_{2p} such their sum is equal to $2p$ ".

$(\forall p \in \mathbb{N} \mid 2 \leq p \leq n \quad U_{2p} , V_{2p} \in \mathcal{P} \quad \text{and}$

$U_{2p} + V_{2p} = 2p)$

Let's show by strong recurrence that $P_G(n)$ is true for any integer $n \geq 2$

► $P_G(2)$ is true : it suffices to choose $U_4 = V_4 = 2$.

► Let's show that the property $P_G(n)$ is hereditary : $P_G(n) \implies P_G(n+1)$

Assume property $P_G(n)$ is true.

• If $(2(n+1) - W_{2(n+1)})$ is a

prime

then $V_{2(n+1)}$ and $U_{2(n+1)}$ are

defined by

$V_{2(n+1)} = W_{2(n+1)}$ and

$U_{2(n+1)} = 2(n+1) - W_{2(n+1)}$ (8.3)

- Otherwise, if $(2(n+1) - W_{2(n+1)})$ is a composite number

there exists an integer k to obtain two terms $U_{2(n+1-k)}$ and $V_{2(n+1-k)}$ satisfying the following conditions

$$\begin{aligned}
 & U_{2(n+1-k)}, V_{2(n+1-k)} \quad \text{and} \quad U_{2(n+1-k)} + 2k \in \\
 \mathcal{P} & \quad (8.4) \quad U_{2(n+1-k)} \\
 & + V_{2(n+1-k)} = 2(n+1-k)
 \end{aligned}$$

we use the previous terms of the sequence (G_{2n}) .

For any integer $q \mid 1 \leq q \leq n-3$ we have

$$3 \leq U_{2(n-q)} \leq n.$$

Then there exists an integer $k \mid 1 \leq k \leq n-3$

$$\begin{aligned}
 R_{2n} & = U_{2(n-k)} + 2k \in \mathcal{P} \\
 (8.5)
 \end{aligned}$$

following the Bertrand principle and Theorem 4 since all primes smaller than $(2n)^{0.525}$ are in the set $\{U_{2k} : k \leq n\}$

(If there were no such primes, we would have a contradiction with the Theorem 4 or with Goldbach's fundamental Lemma 6). In fact, in an equivalent way (see the previous remark) we can copy the proof of Theorem 4 by performing a similar strong recurrence "finite descent feedback and absurd" directly on the set $\{U_{2k} : k \leq n\}$

$$\begin{aligned}
 R_{2n} & = U_{2(n-k)} + 2k \in \mathcal{P} \\
 (8.6)
 \end{aligned}$$

The smallest integer $k \mid R_{2n} \in \mathcal{P}$ is denoted by k_n .

So by setting

$$U_{2n} = U_{2(n-k_n)} + 2k_n \quad \text{and} \quad V_{2n} = V_{2(n-k_n)} \in \mathcal{P} \quad (8.7)$$

(These two terms are primes)

In the previous steps two primes $U_{2(n-k_n)}$ and $V_{2(n-k_n)}$ whose sum is equal to $2(n - k_n)$ were determined.

$$U_{2(n-k_n)} + V_{2(n-k_n)} = 2(n - k_n) \quad (8.8)$$

By adding the term $2k_n$ to each member of the equality (8.6) it follows

$$U_{2(n-k_n)} + 2k_n + V_{2(n-k_n)} = 2(n - k_n) + 2k_n \quad (8.9)$$

$$\Leftrightarrow [U_{2(n-k_n)} + 2k_n] + V_{2(n-k_n)} = 2n \quad (8.10)$$

$$\Leftrightarrow U_{2n} + V_{2n} = 2n \quad (8.11)$$

Two new primes $V_{2(n+1)}$ and $U_{2(n+1)}$ satisfying $(U_{2(n+1)} + V_{2(n+1)} = 2(n+1))$ are generated.

It follows that $P_G(n+1)$ is true. Then the property $P_G(n)$ is hereditary :

$$P_G(n) \implies P_G(n+1).$$

Therefore for any integer $n \geq 2$ the property $P_G(n)$ is true.

It follows

$\forall n \in \mathbb{N} + 2$ there are two primes U_{2n} and V_{2n} and such their sum is $2n$: $U_{2n} + V_{2n} = 2n$

■ ALGORITHM :

For any integer $n \geq 3$

- If $(2n - W_{2n})$ is a prime

then V_{2n} and U_{2n} are defined by

$$V_{2n} = W_{2n} \quad \text{and} \quad U_{2n} = 2n$$

$$- W_{2n} \quad (8.12)$$

- Otherwise, if $(2n - W_{2n})$ is a composite number

we use the previous terms of the sequence (G_{2n}) .

For any integer $q \mid 1 \leq q \leq n-3$ we have

$$3 \leq U_{2(n-q)} \leq n.$$

Then there exists an integer $k \mid 1 \leq k \leq n-3$

$$R_{2n} = U_{2(n-k)} + 2k \in \mathcal{P}$$

$$(8.13)$$

following Theorem 4 since all primes smaller than $(2n)^{0.525}$ are in the set $\{ U_{2k} : k \leq n \}$

(If there were no such primes, we would have a contradiction with the Theorem 4 or with *Goldbach's fundamental Lemma 6*). In fact, in an equivalent way (see the previous remark) we can copy the proof of Teorem 4 by performing a similar strong recurrence "finite descent return and absurd" directly on the set $\{ U_{2k} : k \leq n \}$ |

$$R_{2n} = U_{2(n-k)} + 2k \in P$$

(8.14)

The smallest integer $k \mid R_{2n} \in P$ is denoted by k_n .

So

$$U_{2n} = U_{2(n-k_n)} + 2k_n \quad \text{and} \quad V_{2n} = V_{2(n-k_n)} \in P \quad (8.15)$$

(These two terms are primes)

In the previous steps two primes $U_{2(n-k_n)}$ and $V_{2(n-k_n)}$ whose sum is equal to $2(n - k_n)$ were determined.

$$U_{2(n-k_n)} + V_{2(n-k_n)} = 2(n - k_n) \quad (8.16)$$

By adding the term $2k_n$ to each member of the equality (8.16) it follows

$$U_{2(n-k_n)} + 2k_n + V_{2(n-k_n)} = 2(n - k_n) + 2k_n \quad (8.17)$$

$$(8.18) \quad \Leftrightarrow \quad [U_{2(n-k_n)} + 2k_n] + V_{2(n-k_n)} = 2n$$

$$(8.19) \quad \Leftrightarrow \quad U_{2n} + V_{2n} = 2n$$

Finally, for any integer $n \geq 3$ this algorithm determines two sequences of primes (U_{2n}) and (V_{2n}) verifying Goldbach's conjecture.

9 Lemma

The sequence (U_{2n}) verifies the following majorization

For any integer $n \geq 65$

$$(2n)^{0.525} \leq U_{2n} \quad (9.1)$$

and

$$((2n)^{0.525}) = o(U_{2n}) \quad (9.2)$$

Proof. According to the programm 12.2 and Appendix 14 the majorization (9.1) is verified

for any integer $n \mid 65 \leq n \leq 2000$.

For any integer $n > 2000$ the proof is established by recurrence. For this purpose let

$P_{bhip}(n)$ be the following property

$$P_{bhip}(n) : \quad " U_{2n} \leq (2n)^{0.525} " .$$

(9.3)

► $P_{bhip}(2000)$ is true according to program 13.2 and the table in appendix 14.

► For any integer $n \geq 2000$ let's show that $P_{bhip}(n)$ is hereditary :

$$P_{bhip}(n) \implies P_{bhip}(n+1)$$

Assume that $P_{bhip}(n)$ is true : then

- If $(2(n+1) - W_{2(n+1)})$ is a prime

then $V_{2(n+1)}$ and $U_{2(n+1)}$ are

defined by

$$V_{2(n+1)} = W_{2(n+1)} \quad \text{and} \quad U_{2(n+1)} = 2(n+1) -$$

$$W_{2(n+1)} \quad (9.4)$$

According to the results in [4],[5],[24] (see Lemma 9) there is a constant $K > 0$

such that

$$2(n+1) - K \cdot [2(n+1)]^{0.525} < W_{2(n+1)} < 2(n+1)$$

$$\begin{aligned} &\Rightarrow U_{2(n+1)} = 2(n+1) - W_{2(n+1)} < K \cdot [2(n+1)]^{0.525} \\ &\Rightarrow U_{2(n+1)} \\ \leq & K \cdot [2(n+1)]^{0.525} \end{aligned}$$

- Otherwise, if $(2(n+1) - W_{2(n+1)})$ is a composite number

$$\exists p \in \mathbb{N}^* \mid U_{2(n+1)} = U_{2(n+1-p)} + 2p \quad (9.5)$$

According to [4],[5],[24]

$$U_{2(n+1)} = 2p + U_{2(n+1-p)} = 2p + 2(n+1-p) - W_{2(n+1-p)} = 2(n+1) - W_{2(n+1-p)} \quad (9.6)$$

Via "Goldbach's fundamental Lemma 6" it follows that

$$U_{2(n+1)} < K \cdot [2(n+1)]^{0.525} \quad (9.7)$$

$P_{bhip}(n+1)$ is true then $P_{bhip}(n)$ is hereditary.

So for any integer $n \geq 2000$ the property $P_{bhip}(n)$ is true.

Finally
$$U_{2(n+1)} \leq [2(n+1)]^{0.525}$$

- *Remark.* A more precise estimate can be obtained using the Cipolla or Axler frames [8],[2].

10 Propositions

A) Link between Goldbach conjecture and the fundamental theorem of arithmetic.

A log-exp correspondence is established by linking the sum and product of primes via Goldbach's conjecture and the fundamental theorem of arithmetic, since if $G.D.$ of $2n$ are p' and q' , and if $2n$ decomposes into factors P'' and Q'' | $(p', q' \in \mathcal{P} \mid p' \gg q' \text{ and } P'' \gg Q'')$; then,

$$2n = P'' \cdot Q'' = p' + q' \quad \text{and} \quad p' - q' = 2K$$

$$\ln(P'' \cdot Q'') = \ln(P'') + \ln(Q'')$$

$$= \ln(p' + q') = \ln(p'(1 + q'/p'))$$

$$\approx \ln(p') + q'/p'$$

By choosing $p' = \text{next or prevprime}(P'')$ ($P'' = p' \pm a$) we obtain a q' localization of the form

$$q' \approx [p' \cdot \ln(Q'')] \approx [p' \cdot \ln(2n/P'')] \approx [p' \cdot \ln(2n/p')]$$

.then

$$2n \approx p'(1 + \ln(2n/p'))$$

$$2n \approx p'(1 + \ln(2n/P'')) \approx p'(1 + \ln(2n/(p' \pm a)))$$

$$2n \approx p'(1 + \ln(2n/(p'(1 \pm a/p'))))$$

$$2n \approx p'(1 + \ln((2n/p') \cdot (1/(1 \pm a/p'))))$$

$$2n \approx p'(1 + \ln(2n/p') + \ln(1/(1 \pm a/p')))$$

$$2n \approx p'(1 + \ln(2n/p')) \mp a$$

You can solve equations like these using Maple,

$$\text{solve}(2n \pm a = x \cdot (1 + \ln(2n/x)), x)$$

to locate p' and proceed by successive next or prevprime to determine two G.D. of $2n$,

(programming possible in Algorithm 14). This procedure appears to generalise Pocklington's theorem, and we observe that the G.D. and their number $G(E)$ are related to the number of prime factors in the decomposition of $2n$.

Examples :

● `evalf(solve([90 = x*(1 + ln(96/x)), x < 96], x));` {x = 64.12418697}; $p' = 67$
 $q' = 29$

● `evalf(solve([1000 = x*(1 + ln(1100/x)), x < 1100], x));` {x = 665.6361412}

`prevprime(665);` 661
`isprime(1100 - 661);` true ; $p' = 661$ $q' = 439$

● `evalf(solve([9700 = x*(1 + ln(10000/x)), x < 10000], x));` {x = 7652.697929}

`prevprime(7652);` 7649
`isprime(10000 - 7649);` true ; $p' = 7649$ $q' = 4351$

● `evalf(solve([99950 = x*(1 + ln(100000/x)), x < 100000], x));` {x = 96854.43333}

`a := prevprime(96799);` $a := 96797$ # Obtenu au bout
de 3 ou 4 `prevprime()`
`isprime(100000 - a);` true ; $p' = 96799$ $q' = 3201$

Solutions are :

$$x_0 = \text{Re}(- (2n \pm a)/\text{LambertW}(-1, -(2n \pm a) /$$

$(2n.e))$

and

$$x_1 = \operatorname{Re}(- (2n \pm a) / \operatorname{LambertW}(-(2n \pm a) /$$

(2n.e)))

Remarks. For any composite number n greater than three,

- $\gcd(n, p') = \gcd(n, 2n - p') = \gcd(n, q') = \gcd(n, K) = \gcd(n, p' \cdot q') = \gcd(n, n^2 - K^2) =$

1

- The smallest E.G.D. of $2n$ is less than the square of its greatest prime factor.
- For any non-zero integer R , the smallest of G.D.'s of $R \cdot p_k^\#$ is greater than

p_k .

B) Method of locating G.D. products (Difference in squares: $N^2 - K^2$ or decentered dichotomy by geometric mean (see code RSA , [37]).

Locally (in the vicinity of $2n$), there exists a sub-sequence (p'_s, q'_s) of G.D. of $2s$ such that the product sequence $M_s = p'_s \cdot q'_s = s^2 - k^2$ is almost increasing (the variations of the geometric mean almost follow those of the arithmetic mean; indeed, if

$$p'_{m+1} \geq p'_m, p'_{m+1} \approx p'_m \gg q'_m, q'_{m+1} \text{ and } q'_{m+1} \geq q'_m$$

then

$$p'_{m+1} \cdot q'_{m+1} - p'_m \cdot q'_m = (p'_{m+1} - p'_m) \cdot q'_{m+1} + p'_m (q'_{m+1} - q'_m) \geq 0).$$

Si on choisit : $q'_m = q'_{m+1}$, on minimise et on controle mieux l'écart $M_{s+1} - M_s$

Thus, it is possible to determine Goldbach decomponents of $2n$ by the following algorithm, choosing a neighborhood of $2n$ of amplitude $c \cdot \ln^2(n)$ in agreement with the estimates made on the $G(E)$ distribution function associated with the Goldbach comet.

Another possible method.

By off-center dichotomy using geometric means, similar to that used to crack RSA codes

(see Sainty [37]).

>

```
n2:= 1000;
# To determine two G.D.s of 2n = 1000, we choose two decomponents of a lower
integer, m2 and two decomponents of a higher integer, r2 to 2n; we easily calculate
m2 < 2n = n2 < r2 and their differences km2 and kr2; then we examine their
products which are assumed to preserve order, (if the initial decomponents are well chosen
:
p'1 ≤ p'2, p'1 ≈ p'2 ≫ q'1, q'2 and q'2 ≥ q'1, (p'1 · q'1 = n2 - k2); we then define
admissible bounds for k from a = p'1 · q'1 and b = p'2 · q'2 min2 =
trunc(evalf(sqrt(n2 - b), Digits)) and max2 = trunc(evalf(sqrt(n2 - a), Digits));
decomponents of 2n are deduced by iterating the nextprime( ) command from n +
min2, (choose a gap of the order of c.ln2(n) between m2 and r2.
```

```
pinf := prevprime(735);                               pinf :=
733
qinf := nextprime(17);                                 qinf := 19
psup := nextprime(1050);                               psup :=
1051
qsup := nextprime(29);                                 qsup :=
31
m2 := pinf + qinf;                                     m2 :=
752
r2 := psup + qsup;                                     r2 :=
1082
km2 := pinf - qinf;                                    km2 :=
714
kr2 := psup - qsup;                                    kr2 :=
1020
a := m2*m2 - km2*km2;                                  a := 55708
# a := pinf.qinf
b := r2*r2 - kr2*kr2;                                  b :=
130324 # b := psup.qsup
min2 := trunc(evalf(sqrt(0.25*n2*n2 - b), digits);    min2 := 466
max2 := trunc(evalf(sqrt(0.25*n2*n2 - a), digits);    max2 := 485
n:= trunc(0.5*n2);
em := nextprime(n + min2 - 1);                          em := 967
nextprime(em);                                          971
em2 := 0.5*n2 + max2;                                    em2 :=
985.0
```

```

q := n2 - 971;
:= 29
isprime(q);
true

```

C) Euclidean divisions of $2n$ by its presumed Goldbach decomponents

To determine two Goldbach decomponents of $2n$, the following parameters can be used :

If $p' + q' = 2n$, ($p', q' \in \mathbb{P} \mid p' \gg q'$) then we perform the Euclidean division of p' by q' under the following conditions :

$$p' = m.q' + r \quad 0 < r < q' \quad r \wedge q' = 1 \quad r \wedge m = 1$$

We deduce that $q' = (2n - r) / (m+1)$ or $2n = (m+1).q' + r$ (dual view point).

which leads to the algorithm.

(To develop)

Implementation:

We perform the Euclidean division of $2n$ by primes in ascending order.

3,5,7,11,.....

$$20 = 3 \times 6 + 2 = (3 \times 5 + 2) + 3 = 17 + 3$$

$$22 = 3 \times 7 + 1 = (3 \times 6 + 1) + 3 = 19 + 3$$

$$24 = 3 \times 8 = 5 \times 4 + 4 = (5 \times 3 + 4) + 5 = 19 + 5$$

$$26 = 3 \times 8 + 2 = (3 \times 7 + 2) + 3 = 23 + 3$$

$$28 = 3 \times 9 + 1 = (3 \times 8 + 1) + 3 = 5 \times 5 + 3 = (5 \times 4 + 3) + 5 = 23 + 5$$

$$30 = 3 \times 10 = 5 \times 6 = 7 \times 4 + 2 = (7 \times 3 + 2) + 7 = 23 + 7$$

$$32 = 3 \times 10 + 2 = (3 \times 9 + 2) + 3 = 29 + 3$$

$$34 = 3 \times 10 + 4 = (3 \times 9 + 4) + 3 = 31 + 3$$

$$36 = 3 \times 12 = 5 \times 7 + 1 = (5 \times 6 + 1) + 5 = 31 + 5$$

$$38 = 3 \times 12 + 2 = (3 \times 11 + 2) + 3 = 35 + 3 = 5 \times 7 + 3 = (5 \times 6 + 3) + 5 = 33 + 5$$

$$= 7 \times 5 + 3 = (7 \times 4 + 3) + 7 = 31 + 7$$

$$\begin{aligned} 500 &= 3 \times 166 + 2 = (3 \times 165 + 2) + 3 = 497 + 3 = 5 \times 100 = 7 \times 71 + 3 = (7 \times 70 + 3) \\ &+ 7 = 493 + 7 = 11 \times 45 + 5 = (11 \times 44 + 5) + 11 = 489 + 11 = 13 \times 38 + 6 = (13 \times 37 \\ &+ 6) + 13 = 487 + 13 \end{aligned}$$

For large integers, we will begin Euclidean division with a prime divisor of the order of $\text{nextprime}(\text{trunc}(c.\ln(n)))$.

Remark. This point of view allows us to give another proof of the Binary Goldbach Conjecture equivalent but more explicit by identifying uniqueness, coincidence and consistency using euclidean division of $2n$ by $p_k \in \mathcal{P}$ $p_k > n$: $2n = p_k + R_k$ and

$2n$ by $q_r \in \mathcal{P}$ $p_k \gg q_r$ which gives

$$2n = m.q_r + t_r \quad \text{by Euclidean division, hence } 2n = ((m-1).q_r + t_r) + q_r ;$$

p_k, q_r are increasing sequences, R_k and $D_r = (m-1).q_r + t_r$ are decreasing sequences. By uniqueness of Euclidean division and since $D_r \gg q_r$ and $p_k \gg q_r$, R_k ,

$$[2n/q] = m = 1 + [p/q], \quad (D_r \text{ is the result of the euclidean division of } 2n \text{ by } q_r$$

), we deduce that there exists k_0 and r_0 such that :

$$p_{k_0} = D_{r_0} \quad \text{and} \quad R_{k_0} = q_{r_0} .$$

11 Theorem

For any integer $n \geq 3$ it is easy to check

(W_{2n}) is a positive increasing sequence of primes

(11.1)

$$(11.2) \quad \{ W_{2n} : n \in \mathbb{N} + 3 \} \cup \{ 2 \} = \mathcal{P}$$

$$(11.3) \quad \lim W_{2n} = +\infty$$

$$(11.4) \quad (U_{2n}) \text{ and } (V_{2n}) \text{ are sequences of primes and the set } \{ U_{2k} : k \leq n \}$$

contains all primes less than $\ln(n)$

$$(11.5) \quad n \leq V_{2n} \leq W_{2n}$$

$$(11.6) \quad 3 \leq 2n - W_{2n} \leq U_{2n} \leq n$$

$$(11.7) \quad \lim V_{2n} = +\infty$$

Proof.

(11.1) : For any integer $n \geq 2$ $\mathcal{P}_n \subset \mathcal{P}_{n+1}$. Therefore, $W_{2n} \leq W_{2(n+1)}$. So the sequence (W_{2n}) is increasing.

(11.2) : Any prime except $p_1 = 2$ is odd, hence the result.

(11.3) : $\lim W_{2n} = \lim p_k = +\infty$

(11.4) : By definition $V_{2n} = W_{2n}$ or there exists an integer $k \leq n - 2$ | $V_{2n} = V_{2(n-k)}$.

So the terms of the sequence (V_{2n}) are primes.

(11.5): According to Lemma 9, for any integer $n \geq 65$

$$U_{2n} < (2n)^{0.525}$$

therefore

$$U_{2n} < (2n)^{0.55} < n$$

and

$$V_{2n} = 2n - U_{2n} > 2n - n > n$$

For any integer $n \mid 3 \leq n \leq 65$ verification is carried out according to the computer program in paragraph 13.2 and the table in appendix 14.

We can also see that by construction $V_{2n} \geq U_{2n}$ because if we assume the opposite then

V_{2n} is not the largest prime number verifying

$$\frac{1}{2} (U_{2n} + V_{2n}) = n.$$

So

$$V_{2n} \geq n$$

According to (11.5) $n \leq V_{2n} \implies U_{2n} = 2n - V_{2n} \leq 2n -$

$$n \leq n \quad (11.6)$$

$$V_{2n} \leq W_{2n} \implies 2n - W_{2n} \leq 2n -$$

$$V_{2n} = U_{2n} \quad (11.7)$$

By (11.5) for any integer $n \geq 2$: $n \leq V_{2n}$

$$\lim V_{2n} = +\infty.$$

12 Lemma

We dissociate the following cases mod 6 for any even integer $2n \quad n \geq 3: \quad p + q = 2n \quad p, q \in \mathcal{P}$

1. If $2n = 6m$ then $(p ; q) = (6r + 5 ; 6(m - r - 1) + 1)$ or $(6r+1 ; 6(m - 1 - r) + 5)$
2. If $2n = 6m + 2$ then $(p ; q) = (6r + 1 ; 6(m - r) + 1)$
3. If $2n = 6m + 4$ then $(p ; q) = (6r + 5 ; 6(m - 1 - r) + 5)$

Table 1. Sum of integers 1, 5 mod 6 (in $\mathbb{Z}/6\mathbb{Z}$).

$p + q \pmod 6$	1	5
1	2	0
5	0	4

(To adapt with $2n = 30m + k$)

Table 2. Sum of integers 1, 7, 11, 13, 17, 19, 23, 29 mod 30 (in $\mathbb{Z}/30\mathbb{Z}$).

+ mod 30	1	7	11	13	17	19	23	29
1	2	8	12	14	18	20	24	0
7	8	14	18	20	24	26	0	6
11	12	18	22	24	28	0	4	10
13	14	20	24	26	0	2	6	12
17	18	24	28	0	4	6	10	16
19	20	26	0	2	6	8	12	18
23	24	0	4	6	10	12	16	22
29	0	6	10	12	16	18	22	28

Proof.

13 Properties

For any integer $k \geq 2$ there are infinitely many integers n | $U_{2n} = p_k$

(13.1)

$V_{2n} \sim 2n$ $(n \rightarrow +\infty)$

(13.2)

For any integer $n \geq 5000$

$U_{2n} \ll V_{2n}$ and $\lim \left(\frac{U_{2n}}{V_{2n}} \right) = 0$

(13.3)

The smallest integer n | $U_{2n} \neq 2n - W_{2n}$ is obtained for $n = 49$ and $G_{98} = (79; 19)$

(13.4)

(This type of terms increases in the Goldbach sequence (G_{2n}) as n increases in the sense of the Schnirelmann density and there are an infinite number of them; their proportion per interval can be computed using the results given in [39]).

The sequence (G_{2n}) is "extremal" in the sense that for any integer $n \geq 2$

(13.5)

V_{2n} and U_{2n} are the largest and smallest possible primes | $U_{2n} + V_{2n} = 2n$.

The Cramer-Granville-Maier-Nicely conjecture [9],[17],[30],[32] is verified with probability one. It leads to the following majorization

For any integer $p \geq 500$

$$U_{2p} \leq 0.7 [\ln(2p)]^{(2.2 - \frac{1}{p})}$$

(with probability one) (13.6)

The proof is similar to that of Lemma 9 and is validated by the studying functions of the type

$$f: x \rightarrow a \cdot g(x) + b[\ln(g(x))]^c \quad (a, b > 0; c > 2) \text{ with}$$

$$g: x \rightarrow 0.7 [\ln(x)]^{(c - \frac{1}{x})} \quad \text{and} \quad h: x \rightarrow 0.7 [\ln(x)]^{(2.2 - \frac{1}{x})} \text{ by using Maple software.}$$

A better estimate can be obtained via [29],[31],[30].

According to Bombieri [3] and using the same method as in the proof of Lemma 9, we obtain the following estimate of U_{2n}

$$\forall \varepsilon > 0 \quad U_{2n} = \mathcal{O}(\ln^{1.3+\varepsilon}(n))$$

(on average) (13.7)

14 Algorithm

14.1 Algorithm written in natural language.

Inputs :

Input four integer variables : k, N, n, P

Input : $p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7, \dots, p_N$ the first N primes.

: $n \leftarrow 3$

: $P = M, R, G, S$ or T as indicated in paragraph 2

Algorithm body :

A) Compute : $W_{2n} = \text{Sup}(p \in \mathcal{P} : p \leq 2n - 3)$

If $T_{2n} = (2n - W_{2n})$ is a prime

$$U_{2n} \leftarrow T_{2n} \quad \text{and} \quad V_{2n} \leftarrow$$

$$W_{2n} \quad (14.1.1)$$

otherwise

B) If T_{2n} is a composite number

Let : $k = 1$

B.1) While $U_{2(n-k)} + 2k$ is a composite number
assign to k the value $k + 1$ ($k \leftarrow k + 1$).

return to

B1)

End while

Assign to k the value k_n ($k_n \leftarrow k$)

Let :

$$U_{2n} = U_{2(n-k_n)} + 2k_n \quad \text{and} \quad V_{2n} = V_{2(n-k_n)}$$

(14.1.2)

Assign to n the value $n + 1$ ($n \leftarrow n + 1$ and return to A)

End :

Outputs for integers less than 10^4 :

Print ($2n = \bullet$; $2n - 3 = \bullet$; $W_{2n} = \bullet$; $T_{2n} = \bullet$; $V_{2n} = \bullet$; $U_{2n} = \bullet$)

Outputs for large integers :

Print ($2n - P = \bullet$; $2n - 3 - P = \bullet$; $W_{2n} - P = \bullet$; $T_{2n} = \bullet$; $V_{2n} - P = \bullet$; $U_{2n} = \bullet$)

14.2 Program written with Maxima software for $2n$ around

10^{1000}

```
c : 10**1000 ; for n : c + 40000 step 2 thru c + 40100 do
( b:2, test : 0 , b : next_prime(b) , e : n - b ,
if primep(e)
then print( n - c , b , e - c )
else while test = 0 do ( e : n - b , if primep(e)
then test:1 , print( n - c , b , e - c )
else test : 0 , b : next_prime(b)) ;
```

14.3 Program written with Maplesoft Maple for $2n$ around

10^{1000}

```
G := 10^1000:
V := [1, 11, 13, 17, 19, 23, 29]:
A := G + 500000:
B := A + 59:
b:=2:
st := time( ):
for q from A by 6 to B do # Program modulo 30 .using the results of
                          Lemma 11
                          Possibility of inverting the two
                          loops or defining three similar structures with s := 0,
                          1, 2.
    for s from 0 to 2 do
n := q + s + s:
    b := trunc(0.59b - 20); # Improving computation time: the idea is to recognise that
                          for any integer n large enough there exists a Goldbach
                          decomponent  $p'_n$  and a successor  $p'_{n+1}$  such that
(E):  $|p'_{n+1} - p'_n| < k \cdot \ln^2(n)$  ; this reduces the number of
nextprime(•)'operations which take up the most computing time.
      ( If  $G = 10^{500}$  : Computingtime is
      around 10 sec for thirty terms;The algorithm can be refined
      by exploiting frame (E). Cesàro averages can also be used
      to determine the initial condition for b).
t:= 0:
R := [[1, 5], [1], [5]]: Q := [[1, 7, 11, 13, 17, 19, 23, 29], [1, 13, 19], [11, 17, 23], [7,
13, 17, 19, 23, 29], [1, 7, 19], [11, 17, 23, 29], [1, 11, 13, 19, 23, 29], [1, 7, 13], [17, 23,
29], [1, 7, 11, 17, 19, 29], [1, 19, 7, 13], [11, 23, 29], [1, 23, 7, 17, 11, 13], [7, 19, 13], [11,
17, 29]]:
    while t = 0 do .
```

$b := \text{nextprime}(b + 100);$ # Additional test possible by improving Lemma 11.
(with $V \pmod{30}$).

Possibility of replacing nextprime with a faster procedure
(see Sainty [37]).(the computation time is greatly reduced
by replacing with $b := \text{nextprime}(b + k(b,G))$, $k(b,G)$
constant around 150 for $G=10^{1000}$, $k(b,G)$ chosen
randomly with the rand procedure or very slowly increasing
as a function of b and G), but in general we don't
obtain the E.G.D. but any Goldbach decomponents.

$e := n - b;$

$K := e \bmod 6;$

if K **in** $R[s+1]$ **then**

if $\text{isprime}(e)$

Then $t := 1;$

$\text{print}(n - G, b, e - G);$

end if;

end if;

end do;

end do;

end do;

$\text{Computingtime} := \text{time}() - st;$

Comments : Possible test with $\text{igcd}(n, b) = 1$ and $\text{igcd}(n, 2n - b) = 1$
(or $\text{igcd}(n, b(n-b)) = 1$) then $\text{isprime}(b)$ and $\text{isprime}(2n - b)$ may be faster than
 $\text{nextprime}()$, if we can improve the gcd algorithm.

RESULTS :

$$G = 10^{1000}$$

$b:$

$b := \text{nextprime}(b + \text{rand}(100..150))$

$b := \text{nextprime}(b + 100)$

$b := \text{nextprime}(b + 150)$

$n - G$	b	$n - G$		
$- b$			500000, 139387, 360613	500000, 361069, 138931
500000, 54133, 445867			500002, 40693, 459309	500002, 40693, 459309
500002, 40693, 459309			500004, 731447, -231443	500004, 535637, -35633
500004, 422393, 77611			500006, 54139, 445867	500006, 277789, 22221

500006, 49157, 450849	500008, 205651, 294357	500008, 205651, 29435
500008, 222991, 277017	500010, 100109, 399901	500010, 138959, 36105
500010, 259451, 240559	500012, 40693, 459319	500012, 40693, 45931
500012, 521981, -21969	500014, 261823, 238191	500014, 145501, 3545
500014, 622561, -22547	500016, 82913, 417103	500016, 198659, 3013
500016, 342929, 157087	500018, 300889, 199129	500018, 26309, 47370
500018, 25097, 474921	500020, 12583, 487437	500020, 77347, 42267
500020, 95083, 404937	500022, 233591, 266431	500022, 160709, 3393
500022, 201821, 298201	500024, 159871, 340153	500024, 162553, 3374
500024, 226337, 273687	500026, 106087, 393939	500026, 106087, 3939
500026, 255859, 244167	500028, 608459, -108431	500028, 263009, 2370
500028, 8147, 491881	500030, 30347, 469683	500030, 151813, 3482
500030, 83833, 416197	500032, 43261, 456771	500032, 24049, 47598
500032, 43261, 456771	500034, 201833, 298201	500034, 400031, 1000
500034, 162251, 337783	500036, 186859, 313177	500036, 145037, 3549
500036, 179203, 320833	500038, 95101, 404937	500038, 854257, -354
500038, 12601, 487437	500040, 121763, 378277	500040, 121763, 3782
500040, 608471, -108431	500042, 9029, 491013	500042, 8161, 491881
500042, 157103, 342939	500044, 148663, 351381	500044, 145987, 3540
500044, 145531, 354513	500046, 304847, 195199	500046, 304847, 1951
500046, 440303, 59743	500048, 157109, 342939	500048, 12611, 48743
500048, 162577, 337471	500050, 40459, 459591	500050, 163729, 3363
500050, 258637, 241413	500052, 8171, 491881	500052, 100151, 3999
500052, 111791, 388261	500054, 223037, 277017	500054, 155291, 3447
500054, 139661, 360393	500056, 49207, 450849	500056, 126397, 3736
500056, 126397, 373659	500058, 301349, 198709	500058, 208277, 2917
500058, 40739, 459319		500060, 67547, 43251
500060, 106121, 393939		
	<i>Computationtime:= 188.250 sec</i>	<i>Computationtime:= 163.828 sec</i>
<i>Computationtime:= 179.343 sec</i>		

b:= nextprime(b+rand(150..175)) b:= nextprime(b+rand(140..160))

<i>n-G</i>	<i>b</i>	<i>n-b-G</i>	<i>n-G</i>	<i>b</i>	<i>n-b-G</i>	
500000, 139387, 360613			500000, 112429, -387571			Record : 116 sec; see in researchgate files PDFGOLDBACHTEST4,10 (For n from G+5000000 to 5000058 by 2), [37].
500002, 90481, 409521			500002, 40693, 459309			
500004, 422393, 77611			500004, 277787, 222217			
500006, 145007, 354999			500006, 82903, 417103			
500008, 604339, -104331			500008, 148627, 351381			

500010, 138959, 361051	500010, 139397, 360613	
500012, 221021, 278991	500012, 40693, 459319	
500014, 334843, 165171	500014, 145501, 354513	
500016, 297779, 202237	500016, 388313, 111703	
500018, 167267, 332751	500018, 258329, 241689	
500020, 54577, 445443	500020, 77347, 422673	
500022, 139409, 360613	500022, 453683, 46339	
500024, 336491, 163533	500024, 67511, 432513	
500026, 12589, 487437	500026, 221197, 278829	
500028, 263009, 237019	500028, 263009, 237019	
500030, 145517, 354513	500030, 112459, 387571	
500032, 334861, 165171	500032, 178681, 321351	
500034, 163697, 336337	500034, 208253, 291781	
500036, 318979, 181057	500036, 274019, 226017	
500038, 221047, 278991	500038, 14071, 485967	
500040, 761591, -261551	500040, 162257, 337783	
500042, 178691, 321351	500042, 361111, 138931	
500044, 54601, 445443	500044, 52903, 447141	
500046, 174989, 325057	500046, 582299, -82253	
500048, 84229, 415819	500048, 8167, 491881	
500050, 163729, 336321	500050, 67537, 432513	
500052, 159899, 340153	500052, 111791, 388261	
500054, 155291, 344763	500054, 126641, 373413	
500056, 166183, 333873	500056, 126397, 373659	
500058, 151841, 348217	500058, 40739, 459319	
Computationtime:= 174.438 sec	Computationtime:= 138.578 sec	

500000, 9473, 490527
500002, 24019, 475983
500004, 8123, 491881
500006, 9479, 490527
500008, 25087, 474921
500010, 57917, 442093
500012, 8999, 491013
500014, 9001, 491013
500016, 40697, 459319
500018, 9491, 490527
500020, 9007, 491013

500022, 139409, 360613
 500024, 9011, 491013
 500026, 9013, 491013
 500028, 8147, 491881
 500030, 26321, 473709
 500032, 24049, 475983
 500034, 54167, 445867
 500036, 57943, 442093
 500038, 9511, 490527
 500040, 57947, 442093
 500042, 8161, 491881
 500044, 24061, 475983
 500046, 162263, 337783
 500048, 8167, 491881
 500050, 12613, 487437
 500052, 8171, 491881
 500054, 9041, 491013
 500056, 9043, 491013
 500058, 40739, 459319

Computingtime : 343.453 sec

$$G = 10^{2000}$$

<i>b</i>	<i>n - b - G</i>	<i>n - G</i>	<i>n - b - G</i>	<i>b</i>	<i>n - G</i>
		40000,	39957,	43	40050,
86117,	-46067				
		40002,	39091,	911	40052,
503,	39549				
		40004,	39957,	47	40054,
97,	39957				
		40006,	39549,	457	40056,
89393,	-49337				

101, 39957	40008, 25369, 14639	40058,
103, 39957	40010, 39957, 53	40060,
39091	40012, 39549, 463	40062, 971,
39957	40014, 17737, 22277	40064, 107,
109, 39957	40016, 39957, 59	40066,
977, 39091	40018, 39957, 61	40068,
39957	40020, 39091, 929	40070, 113,
39549	40022, 39141, 881	40072, 523,
983, 39091	40024, 39957, 67	40074,
16937, 23139	40026, 35443, 4583	40076,
937, 39141	40028, 39957, 71	40078,
4637, 35443	40030, 39957, 73	40080,
39141	40032, 39091, 941	40082, 941,
39957	40034, 35443, 4591	40084, 127,
4643, 35443	40036, 39957, 79	40086,
39957	40038, 39091, 947	40088, 131,
541, 39549	40040, 39957, 83	40090,
35443	40042, 23139, 16903	40092, 4649,
39957	40044, 39091, 953	40094, 137,
139, 39957	40046, 39957, 89	40096,
31991, 8107	40048, 39549, 499	40098,

40100, 1009, 39091

$$G = 10^{3000}$$

n - G *b* *n - b - G*

100000, 36529, 63471
 100002, 77069, 22933
 100004, 22717, 77287
 100006, 181873, -81867
 100008, 12239, 87769
 100010, 4547, 95463
 100012, 4549, 95463
 100014, 22727, 77287
 100016, 59497, 40519
 100018, 24847, 75171
 100020, 12251, 87769
 100022, 12253, 87769
 100024, 4561, 95463
 100026, 22739, 77287
 100028, 22741, 77287
 100030, 4567, 95463
 100032, 12263, 87769
 100034, 36563, 63471
 100036, 42649, 57387
 100038, 12269, 87769
 100040, 23143, 76897
 100042, 36571, 63471
 100044, 43973, 56071
 100046, 4583, 95463
 100048, 24877, 75171
 100050, 12281, 87769

$$G = 10^{5000}$$

$n - G$	b	$n - b - G$	$n - G$	b	$n - b - G$
$n - G$	b	$n - b - G$			

100000, 31147, 68853			100050, 12611, 87439		
100100, 31247, 68853					
100002, 309371, -209369			100052, 12613, 87439		
100102, 31249, 68853					
100104, 105071, -4967					
100106, 13649, 86457					
100004, 31151, 68853			100054, 13597, 86457		
100108, 640669, -540561					
100006, 31153, 68853			100056, 105023, -4967		
100110, 12671, 87439					
100008, 12569, 87439			100058, 12619, 87439		
100112, 31259, 68853					
100114, 87991, 12123					
100116, 122033, -21917					
100118, 18379, 81739					

100010, 13553, 86457	100060, 54151, 45909
100012, 31159, 68853	100062, 108971, -8909
100014, 108923, -8909	100064, 103091, -3027
100016, 12577, 87439	100066, 87943, 12123
100018, 592237, -492219	100068, 18329, 81739
100020, 104987, -4967	100070, 13613, 86457
100022, 12583, 87439	100072, 31219, 68853
100024, 13567, 86457	100074, 264881, -
100026, 18287, 81739	100076, 12637, 87439
100028, 12589, 87439	100078, 107971, -7893
100030, 31177, 68853	100080, 12641, 87439
100032, 61871, 38161	100082, 76913, 23169

100034, 13577, 86457
100036, 31183, 68853
100038, 108947, -8909
100088, 61927, 38161
100040, 12601, 87439
100042, 31189, 68853
100044, 457091, -357047
100046, 18307, 81739
100048, 13591, 86457

100084, 13627, 86457
100086, 12647, 87439
10038, 108947, -8909

100090, 13633, 86457
100092, 12653, 87439
100094, 61933, 38161
100096, 87973, 12123
100098, 12659, 87439

100120, 31267, 68853
100122, 61961, 38161

100124, 31271, 68853

100126, 13669, 86457
100128, 12689, 87439
100130, 31277, 68853
100132, 76963, 23169
100134, 122051, -21917
100136, 12697, 87439
100138, 13681, 86457
100140, 18401, 81739
100142, 12703, 87439
100144, 13687, 86457
100146, 152993, -52847
100148, 13691, 86457
100150, 13693, 86457

1000000, 35509, 964491
 1000002, 113, 999889
 1000004, 69193, 930811
 1000006, 95233, 904773
 1000008, 69197, 930811
 1000010, 31873, 968137
 1000012, 35521, 964491
 1000014, 69203, 930811
 1000016, 127, 999889
 1000018, 35527, 964491
 1000020, 131, 999889

Maple program corrected and improved, (see Sainty [37]).

15 Appendix

Application of Algorithm 14 : Table of extreme Goldbach partitions U_{2n} and V_{2n} computed from program 14.2 ($2 \leq 2n \leq 10^{1000} + 4020$).

The ** sign in the table below indicates the results given by the algorithm 14 in case **B** of return to the previous terms of the sequence (G_{2n}) .

WATCH OUT !

To simplify the display of large numbers n ($2n > 10^9$) the results are entered as follows

:

$$2n - P, (2n - 3) - P, W_{2n} - P, T_{2n}, V_{2n} - P \text{ and } U_{2n}$$

with

$P = M, R, G, S,$ or T constants defined in (2.3)

$\frac{2n}{2n-3}$	W_{2n}	$T_{2n}=2n-W_{2n}$	V_{2n}	U_{2n}
$\frac{4}{1}$	X	X	2	2
$\frac{6}{3}$	3	3	3	3
$\frac{8}{5}$	5	3	5	3
$\frac{10}{7}$	7	3	7	3
$\frac{112}{9}$	7	5	7	5
$\frac{14}{11}$	11	3	11	3
$\frac{16}{13}$	13	3	13	3
$\frac{18}{15}$	13	5	13	5
$\frac{20}{17}$	17	3	17	3
$\frac{22}{19}$	19	3	19	3
$\frac{24}{21}$	19	5	19	5
$\frac{26}{23}$	23	3	23	3
$\frac{28}{25}$	23	5	23	5
$\frac{30}{27}$	23	7	23	7
$\frac{32}{29}$	29	3	29	3

34				
31	31	3	31	3
36				
33	31	5	31	5
38				
35	31	7	31	7
40				
37	37	3	37	3
80				
77	73	7	73	7
82				
79	79	3	79	3
84				
81	79	5	79	5
86				
83	83	3	83	3
88				
85	83	5	83	5
90				
87	83	7	83	7
92				
89	89	3	89	3
94				
91	89	5	89	5
96				
93	89	7	89	7
**98				
95	89	9	79	19
100				
97	97	3	97	3
120				
117	113	7	113	7
**122				
119	113	9	109	13
124				
121	113	11	113	11
126				
123	113	13	113	13

**128				
125	113	15	109	19
130				
127	127	3	127	3
132				
129	127	5	127	5
134				
131	131	3	131	3
136				
133	131	5	131	5
138				
135	131	7	131	7
140				
137	137	3	137	3
**500				
497	491	9	487	13
502				
499	499	3	499	3
504				
501	499	5	499	5
506				
503	503	3	503	3
508				
505	503	5	503	5
510				
507	503	7	503	7
1000				
997	997	3	997	3
1002				
999	997	5	997	5
1004				
1001	997	7	997	7
**1006				
1003	997	9	983	23
1008				
1005	997	11	997	11
1010				
1007	997	13	997	13

1012	1009	3	1009	3
1009				
1014	1009	5	1009	5
1011				
1016	1013	3	1013	3
1013				
1018	1013	5	1013	5
1015				
10002	9973	29	9973	29
9999				
10004	9973	31	9973	31
10001				
*10006	9973	33	9923	83
10003				
**10008	9973	35	9967	41
10005				
10010	10007	3	10007	3
10007				
10012	10009	3	10009	3
10009				
10014	10009	5	10009	5
10011				
10016	10009	7	10009	7
10013				
**10018	10009	9	10007	11
10015				
10020	10009	11	10009	11
10017				
$2n - M$	$(2n -$	$W_{2n} - M$	$T_{2n} = 2n - W_{2n}$	$V_{2n} - M$
$3) - M$				U_{2n}
+1000	+993	7	+993	7
+997				
**+1002	+993	9	+931	71
+999				
+1004	+993	11	+993	11
+1001				
+1006	+993	13	+993	13
+1003				

**+1008	+993	15	+919	89
+1005				
+1010	+993	17	+993	17
+1007				
+1012	+993	19	+993	19
+1009				
+1014	+1011	3	+1011	3
+1011				
+1016	+1011	5	+1011	5
+1013				
+1018	+1011	7	+1011	7
+1015				
**+1020	+1011	9	+931	89
+1017				
$2n - R$	$(2n -$	$W_{2n} - R$	$T_{2n} = 2n - W_{2n}$	$V_{2n} - R$
$3) - R$				U_{2n}
**+1000	+979	21	+903	97
+997				
+1002	+979	23	+979	23
+999				
**+1004	+979	25	+951	53
+1001				
**+1006	+979	27	+903	103
+1003				
+1008	+979	29	+979	29
+1005				
+1010	+979	31	+979	31
+1007				
**+1012	+979	33	+951	61
+1009				
**+1014	+979	35	+ 781	233
+1011				
+1016	+979	37	+979	37
+1013				
**+1018	+979	39	+951	67
+1015				
+1020	+1017	3	+1017	3
+1017				

$2n - G$ $(2n - 3) - G$	$(2n)$ $W_{2n - G}$	$T_{2n} = 2n - W_{2n}$	$V_{2n - G}$	U_{2n}
**+10000 +9997	+9631	369	+7443	2557
**+10002 +9999	+9631	371	+9259	743
+10004 +10001	+9631	373	+9631	373
**+10006 +10003	+9631	375	+8583	1423
**+10008 +10005	+9631	377	+6637	3371
+10010 +10007	+9631	379	+9631	379
**+10012 +10009	+9631	381	+8583	1429
+10014 +10011	+9631	383	+9631	383
**+10016 +10013	+9631	385	+9259	757
**+10018 +10015	+9631	387	+4491	5527
+10020 +10017	+9631	389	+9631	389
$2n - S$ $(2n - 3) - S$	$W_{2n - S}$	$T_{2n} = 2n - W_{2n}$	$V_{2n - S}$	U_{2n}
**+20000 +19997	+18031	1969	+17409	2591
**+20002 +19999	+18031	1971	+17409	2593
+20004 +20001	+18031	1973	+18031	1973
**+20006 +20003	+18031	1975	+16663	3343
**+20008 +20005	+18031	1977	+16941	3067
+20010 +20007	+18031	1979	+18031	1979
**+20012	+18031	1981	+5671	14341

+20009				
**+20014	+18031	1983	+4101	15913
+20011				
**+20016	+18031	1985	+3229	16787
+20013				
+20018	+18031	1987	+18031	1987
+20015				
**+20020	+18031	1989	+16941	3079
+20017				
$2n-T$	$W_{2n}-T$	$T_{2n} = 2n - W_{2n}$	$V_{2n} - T$	U_{2n}
$(2n-3)-T$				
**+40000	+29737	10263	+ 21567	18433
+39997				
**+40002	+29737	10265	+ 22273	17729
+39999				
+40004	+29737	10267	+29737	10267
+40001				
**+40006	+29737	10269	+21567	18439
+40003				
+40008	+29737	10271	+29737	10271
+40005				
+40010	+29737	10273	+29737	10273
+ 40007				
**+40012	+29737	10275	+10401	29611
+40009				
**+40014	+29737	10277	-56003	96017
+40011				
**+40016	+29737	10279	+27057	12959
+40013				
**+40018	+29737	10281	+25947	14071
+40015				
**+40020	+29737	10283	+24493	15527
+40017				

7-3=4	11-5=6	11-3=8	13-3=10	17-5=12	17-3=14	19-3=16	23-5=18
23-3=20	29-7=22	29-5=24	29-3=26	31-3=28	37-7=30	37-5=32	37-3=34
41-5=36	41-3=38	43-3=40	47-5=42	47-3=44	53-7=46	53-5=48	53-3=50
59-7=52	59-5=54	59-3=56	61-3=58	67-7=60	67-5=62	67-3=64	71-5=66
71-3=68	73-3=70	79-7=72	79-5=74	79-3=76	83-5=78	83-3=80	89-7=82
89-5=84	89-3=86	101-13=88	97-7=90	97-5=92	97-3=94	101-5=96	101-3=98
103-3=100	107-5=102	107-3=104	109-3=106	113-5=108	113-3=110	131- 19=112	127- 13=114
127- 11=116	131- 13=118	127-7=120	127-5=122	127-3=124	131-5=126	131-3=128	137-7=130
137-5=132	137-3=134	139-3=136	149- 11=138	151- 11=140	149-7=142	149-5=144	149-3=146
151-3=148	157-7=150	157-5=152	157-3=154	163-7=156	163-5=158	163-3=160	167-5=162
167-3=164	173-7=166	173-5=168	173-3=170	179-7=172	179-5=174	179-3=176	181-3=178
191- 11=180	193- 11=182	191-7=184	191-5=186	191-3=188	193-3=190	197-5=192	197-3=194
199-3=196	211- 13=198	211- 11=200	233- 31=202	211-7=204	211-5=206	211-3=208	223- 13=210
229- 17=212	227- 13=214	223-7=216	223-5=218	223-3=220	227-5=222	227-3=224	229-3=226
233-5=228	233-3=230	239-7=232	239-5=234	239-3=236	241-3=238	251- 11=240	271- 29=242
251-7=244	251-5=246						

17 Appendix

$$T_r(K)$$

	$q_1 = 3$	$q_2 = 5$	$q_3 = 7$	$q_4 = 11$	$q_5 = 17$	$q_6 = 23$	$q_7 = 29$	$q_8 = 37$	$q_9 = 47$	$q_{10} = 59$	$q_{11} = 73$
$2K = 2$	5	7		13		19			31		
$2K = 4$	7		11		17		23				41
$2K = 6$		11	13	17	19	23		29		37	43
$2K = 8$	11	13		19				31	37		
$2K = 10$	13				23		29			41	47
$2K = 12$		17	19	23		29	31		41	43	
$2K = 14$	17	19				31		37	43		
$2K = 16$	19		23		29					47	59
$2K = 18$		23		29	31		37	41	47		61
$2K = 20$	23			31		37		43			67
$2K = 22$			29				41			53	
$2K = 24$		29	31		37	41	43	47	53		71
$2K = 26$	29	31		37		43					73
$2K = 28$	31				41		47			59	
$2K = 30$			37	41	43	47		53	59	61	
$2K = 32$		37		43					61		79
$2K = 34$	37		41		47		53				
$2K = 36$		41	43	47		53		59		67	83
$2K = 38$	41	43						61	67		
$2K = 40$	43		47		53		59			71	
$2K = 42$		47		53		59	61		71	73	89
$2K = 44$	47					61		67	73		
$2K = 46$			53		59						
$2K = 48$		53		59	61		67	71		79	
$2K = 50$	53			61		67		73	79		97
$2K = 52$			59				71			83	
$2K = 54$		59	61		67	71	73		83		
$2K = 56$	59	61		67		73		79			
$2K = 58$	61				71					89	
$2K = 60$			67	71	73		79	83	89		

18 Perspectives and generalizations

18.1 Other Goldbach sequences (G'_{2n}) independent of (G_{2n}) may be studied using the increasing sequences of primes (W'_{2n}) defined by

For any integer $n \geq 3$

$$W'_{2n} = \text{Sup} (p \in \mathcal{P} : p \leq f(n))$$

(18.1.1)

f is a function defined on the interval $J = [3 ; +\infty[$ and satisfying the following conditions

- f is strictly increasing on the interval J
- $f(3) = 3$ and $\lim_{x \rightarrow +\infty} f(x) = +\infty$
- $\forall x \in I \quad f(x) \leq 2x - 3$

For example, one of the following functions defined on J can be selected.

- $f: x \rightarrow ax + 3 - 3a$ ($a \in \mathbb{R} : 0 < a \leq 2$)
- $g: x \rightarrow [4\sqrt{3x} - 9]$ ($[x]$ is the integer part of the real x)
- $h: x \rightarrow 6 \ln\left(\frac{x}{3}\right) + 3$

18.2 Using this method it would be interesting to study the Schnirelmann density [39] of primes $3, 5, 7, 11, \dots$ in the sequence (U_{2n}) on variable intervals and the Caesaro sums of U_{2n} E.D.G.'s with a view to more efficient programming for their calculation.

18.3 It is possible to exceed the values shown in the table of $2n = 10^{1000}$ (many E.G.D have been calculated for values of $2n$ in the order of 10^{2000} , 10^{5000} (and G.D. in the order of 10^{10000} Sainty [37]) by perfecting this algorithm, exploiting the

fact that one of Goldbach's decomponents can be chosen equal to $4p + 3$, (G.D. are primes of the form

$6m + 1$ or $6m + 5$ and can be expressed more precisely using primes of the form $30m + r$:

$r \in [1,7,11,13,17,19,23,29]$ (see Table mod 30, Lemma 11), by using De Pocklington Theorem [6],[34],[36], Primality tests [37], Cipolla-Axler-Dusart type functions and improvement of primes frames [2],[8],[12],[13],[37] via a new Prime number Theorem to better identify the terms of (G_{2n}) , supercomputers and more efficient software as C++, or Assembler compilation.

18.4 Any Goldbach decomponent of order $2n = 10^{10000}$ can be determined more quickly by replacing the instruction $b:=2$ by $b:=\text{trunc}(c.b + d)$ and $b := \text{nextprime}(b)$ with

$b := \text{nextprime}(b + k(b, G))$, where $k(b, G)$ is a constant of around 150 for $G = 10^{1000}$

and is chosen randomly using the rand procedure or increases very slowly as a function of

b and G . An increasing sequence of primes, b_k , can also be determined in stages by

replacing the initial value $b:=2$ by $b:=\text{trunc}(k_0.b - k_1.\ln^s(n) - k_2)$ and by setting c

$:= \text{trunc}(a.\ln^d(b))$,

$1 \leq d, s \leq 2$ and $b := b + c$ for each stage, followed by $b := \text{nextprime}(b)$ until

the next stage, (see Sainty [37]); Note that for any even integer $2n$ large enough there

exists G.D. $p'_n, p'_{n+1}, q'_n, q'_{n+1} \mid p'_n + q'_n = 2n$ and $p'_{n+1} + q'_{n+1} = 2(n$

$+ 1)$ with

$p'_{n+1} - p'_n$ and $q'_{n+1} - q'_n < k \cdot \ln^2(n)$). It is therefore advisable to develop adaptive algorithms based on this model using A.I., as a function of the program's G parameter.

18.5 Diophantine equations and conjectures of the same nature ((3L) conjecture [9],[21],[23],[26],[27],[44]) can be processed using similar reasoning and algorithms.

■ To validate the (3L) conjecture we study the following sequences of primes (Wl_{2n}), (Vl_{2n}) and (Ul_{2n}) defined by

$$\text{For any integer } n \geq 3 \quad Wl_{2n} = \text{Sup} \left(p \in \mathcal{P} : p \leq n - 1 \right)$$

(18.5.1)

- If $Tl_{2n} = (2n + 1 - 2 Wl_{2n})$ is a **prime**

then let

$$Vl_{2n} = Wl_{2n} \quad \text{and} \quad Ul_{2n} = Tl_{2n}$$

(18.5.2)

- If Tl_{2n} is a **composite number**

then there exists an integer $k \quad 1 \leq k \leq n - 3 \mid$

$$Ul_{2(n-k)} + 2k \in \mathcal{P}$$

(18.5.3)

then let

$$Vl_{2n} = Vl_{2(n-k)} \quad \text{and} \quad Ul_{2n} = Ul_{2(n-k)} + 2k$$

(18.5.4)

■ Using the same type of reasoning a generalization, the (BBG) conjecture of the following form can be validated

- Let K and Q be two odd integers prime to each other :

For any integer $n \mid 2n \geq 3(K + Q)$ there exist two primes Ub_{2n} and Vb_{2n} verifying

$$K \cdot Ub_{2n} + Q \cdot Vb_{2n} = 2n$$

(18.5.5)

- Let K and Q be two integers of different parity prime to each other :

For any integer $n \mid 2n \geq 3(K + Q)$ there are two primes Ub_{2n} and Vb_{2n} verifying

$$K \cdot Ub_{2n} + Q \cdot Vb_{2n} = 2n + 1$$

(18.5.6)

18.6 Remark.

GOLDBACH (-) :

$$R_{2K} = \text{Inf} (p \in \mathcal{P} : p - 2K \in \mathcal{P}) \quad \text{and} \quad \underline{Q_{2K}} = \text{Inf} (p \in \mathcal{P} : 2K + p \in \mathcal{P}) = R_{2K} - 2K$$

GOLDBACH (+) :

$$V_{2K} = \text{Sup} (p \in \mathcal{P} : 2K - p \in \mathcal{P}) \quad \text{and} \quad \underline{U}_{2K} = \text{Inf} (p \in \mathcal{P} : 2K - p \in \mathcal{P}) \\ = 2K - V_{2K}$$

(It is possible to envisage symmetries in the Goldbach triangle).

18.7 The sequences (Wq_{2n}) generate all the G.D. and may enable us to better estimate the values of distribution function G of the Goldbach's Comet, probably of type :

$$0.57 \cdot \frac{E}{\ln^2(E)} < G(E) < 3.62 \cdot \frac{E}{\ln^2(E)} \quad , \quad \text{see Woon [48] .} \\ \text{Average value of } G(E) \approx 1.62 \cdot \frac{E}{\ln^2(E)}$$

19 Conclusion

19.1 A recurrent and explicit Goldbach sequence $(G_{2n}) = (U_{2n}; V_{2n})$ verifying

$$\forall n \in \mathbb{N} + 2 \quad U_{2n}, V_{2n} \in \mathcal{P} \quad \text{and} \quad U_{2n} + V_{2n} = 2n$$

has been developed using an simple and efficient "localised" algorithm. The Goldbach conjecture has been proved by strong recurrence (absurd and finite descent), (un arbre analogue à celui des diviseurs d'un entier est envisageable permettant de visualiser les analogies entre le théorème fondamental de l'arithmétique et la conjecture de Goldbach).

A relation (Proposition 10) is established between the fundamental theorem of arithmetic and the Goldbach conjecture (sum and product of primes), allowing fast computation of G.D. of very large even integers via a «localisation» of G.D.'s using a generalized Pocklington-type algorithm and further proof of Goldbach's binary conjecture via

Euclidean divisions of $2n$ by primes and consistent increasing and decreasing sequences.

19.2 The records of Silva [41] and Deshouillers, te Riele, Saouter [11] are beaten on a personal computer. Hundreds E.G.D. U_{2n} and V_{2n} are obtained for values around $2n = 10^{1000}$, twenty-six around $2n = 10^{2000}$, seventy-five around $2n = 10^{5000}$ and G.D. around $2n = 10^{10000}$ for a computation time of less than three hours (see Sainty [37]).

19.3 For a given integer $n \geq 49$ the evaluation of the terms U_{2n} and V_{2n} does not require the computing of all previous terms U_{2k} and $V_{2k} \mid 1 \leq k < n - 1$. we will only consider those that verify :

$$U_{2k} \leq 5. \ln^{1.3}(2n) \quad \text{and} \quad 2n - 5. \ln^{1.3}(2n) \leq V_{2k} \leq 2n$$

(on average) (19.3.1)

This property allows any E.G.D U_{2n} and V_{2n} to be calculated quite quickly, the upper limit being defined by the scientific software and the computer's ability to determine the largest prime preceding $2n - 2$ (*next or prevprime*($2n - 2$) function).

19.4 Therefore the (BBG), the (3L) and the binary Goldbach(- / +) conjectures "Any even integer greater than three is the sum and difference of two primes" are true.

In fact these two conjectures are intertwined.

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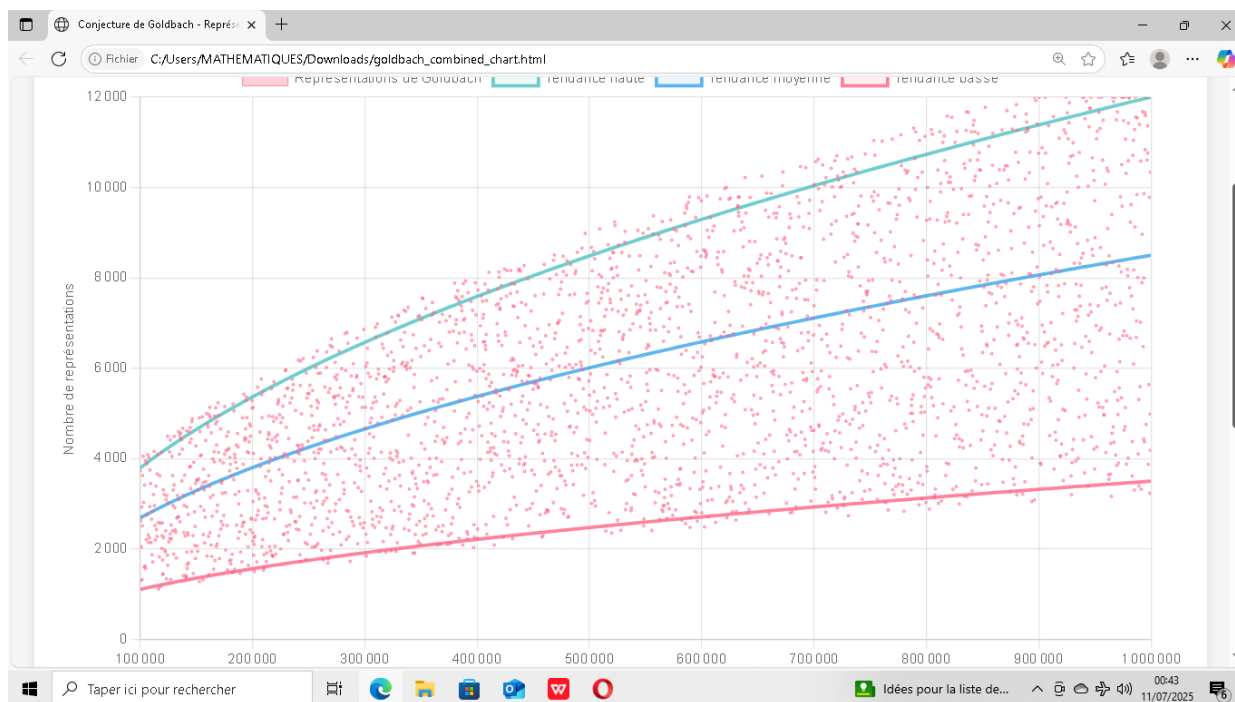
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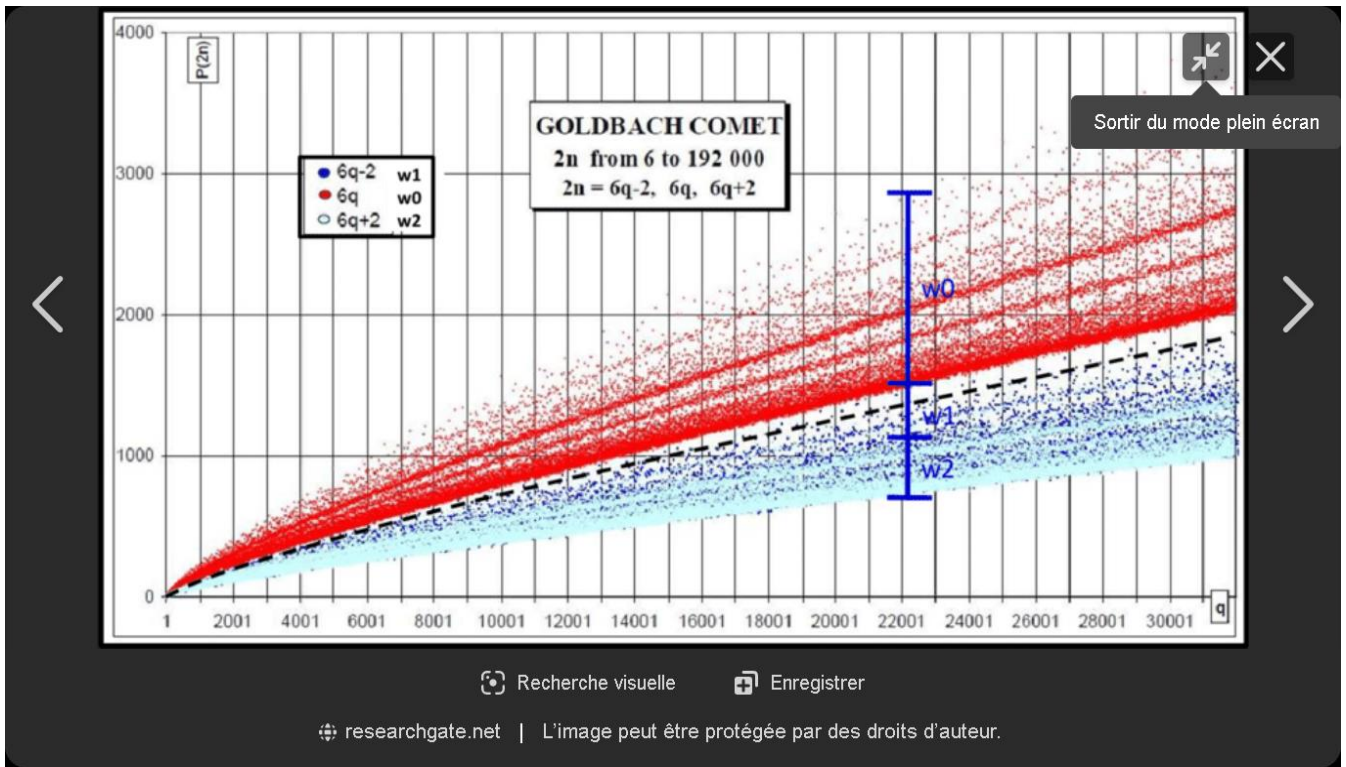
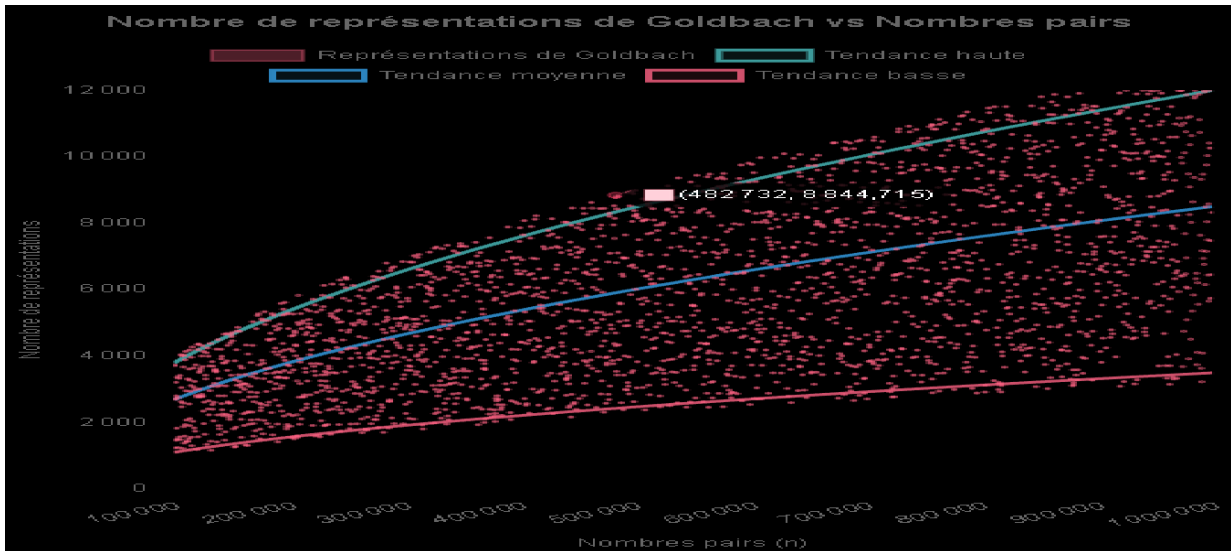
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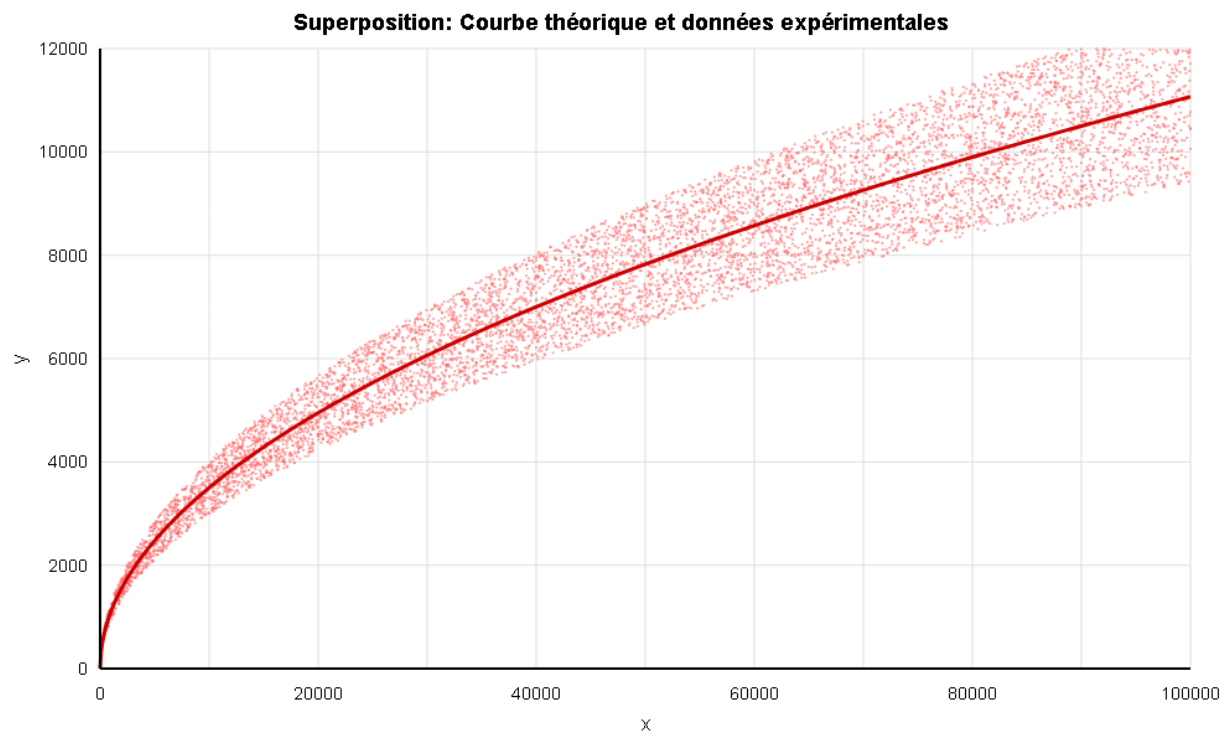
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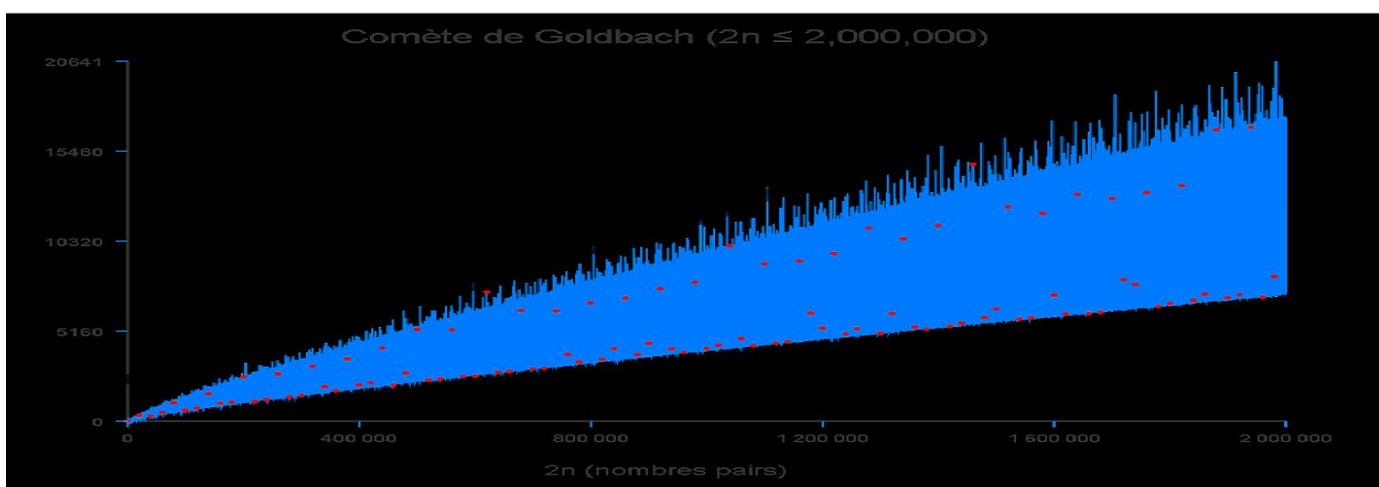
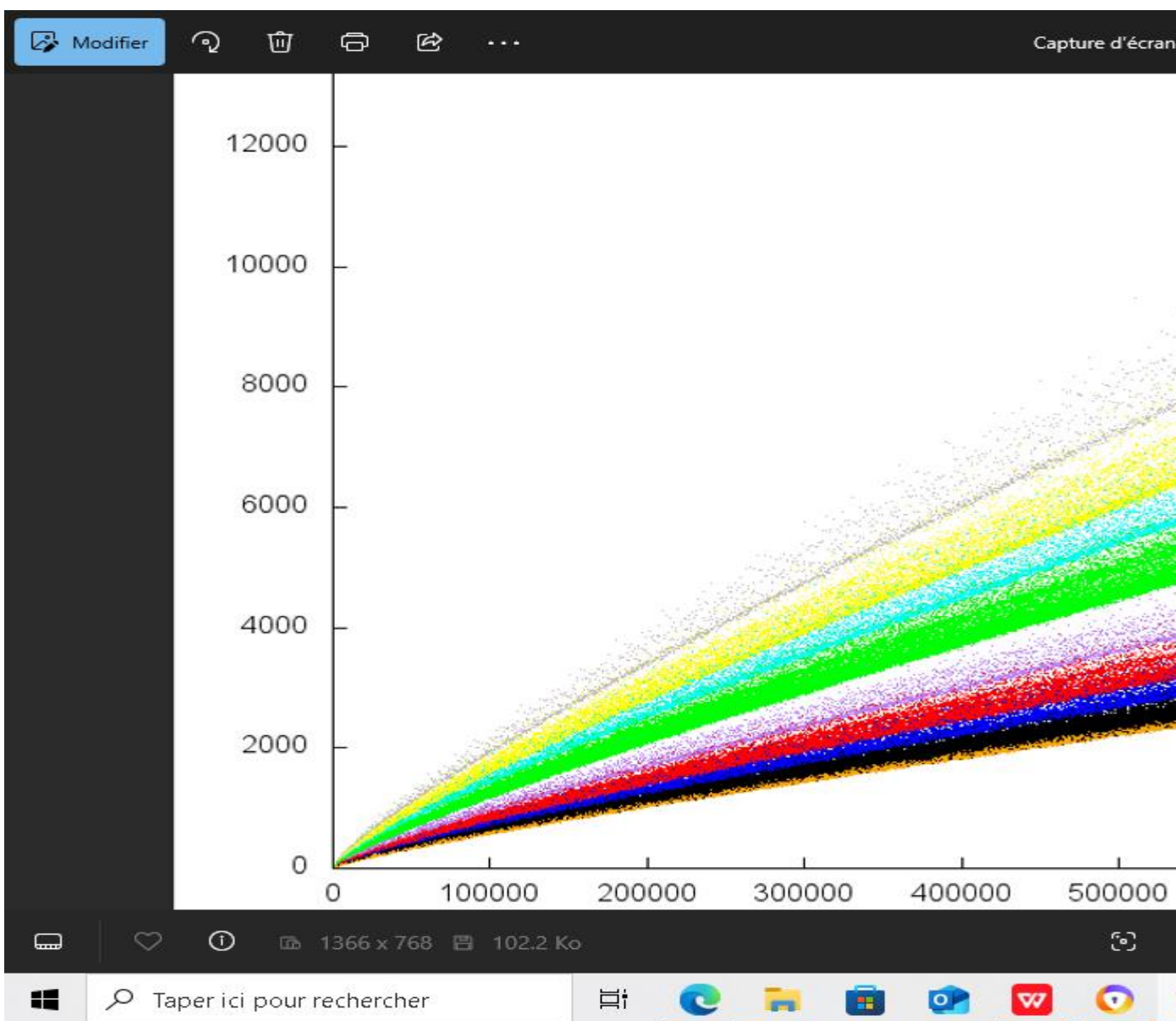
Framing and mean value of the Goldbach comet by functions of the type

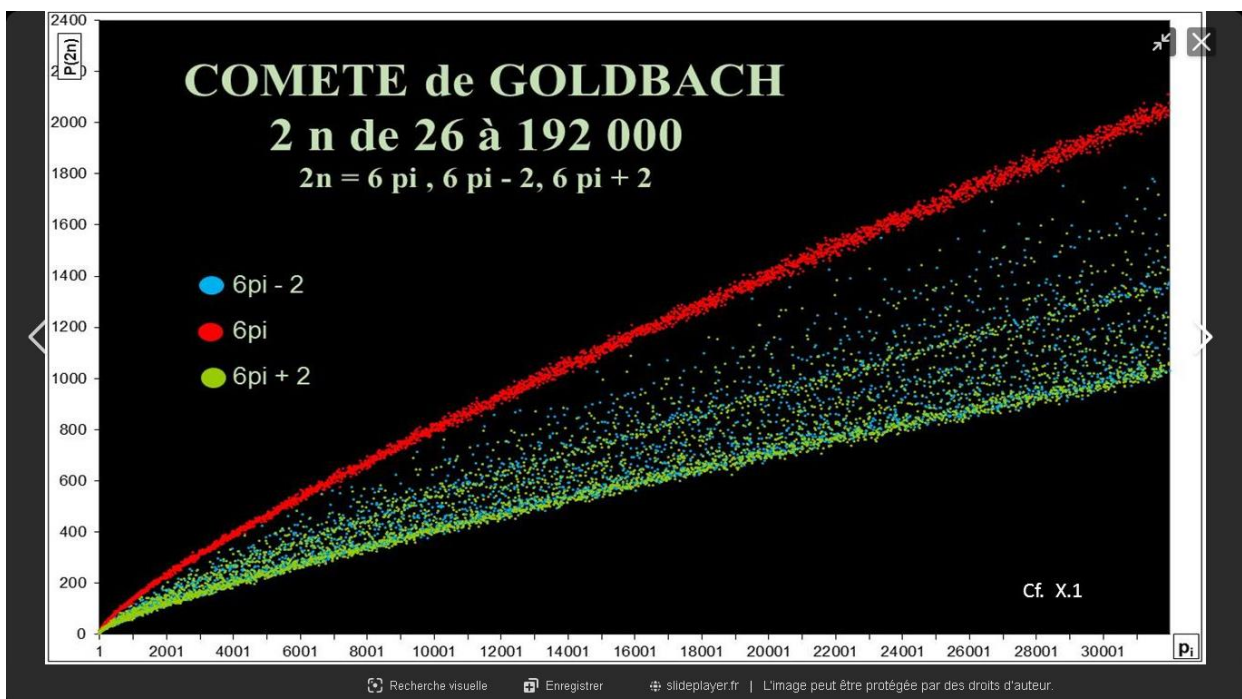
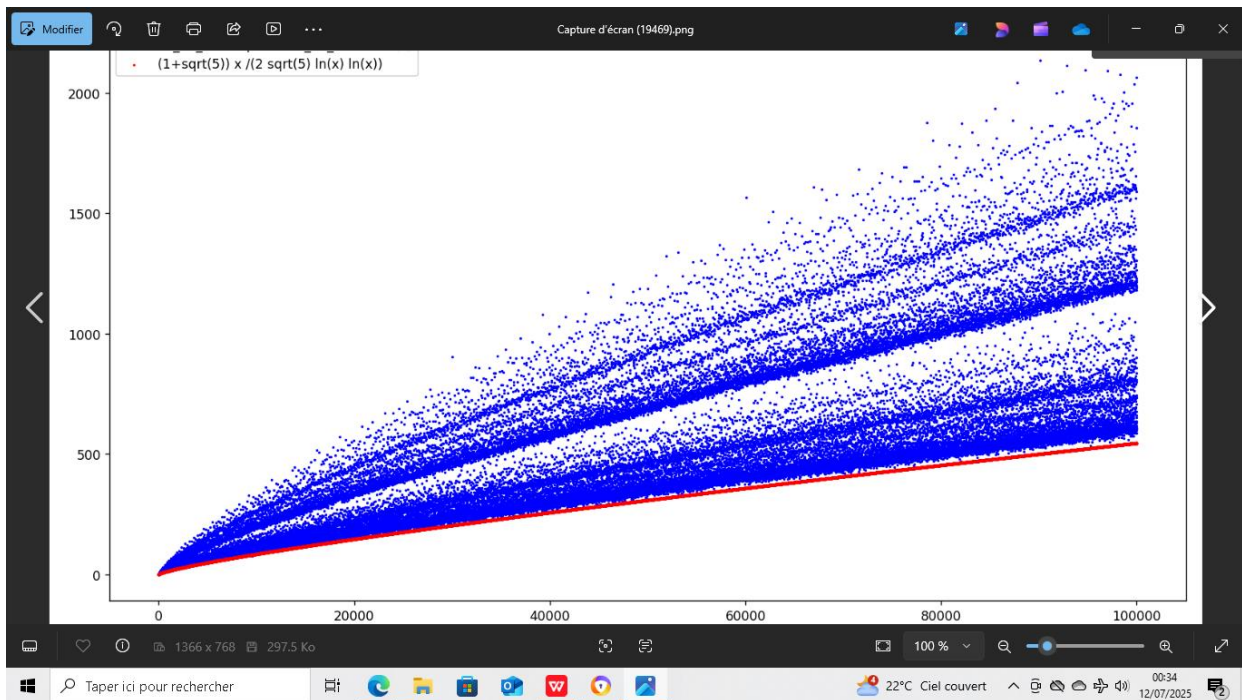
$f : x \rightarrow a.x / \ln^2(x)$, (via AI CLAUDE : to be specified).











Comments :

The majority of mathematicians believe Goldbach's conjecture to be true, mainly,, based on statistical reasoning centred on the distribution of primes. The larger the number, the more ways there are to decompose it into a sum of two or three other primes. A crude heuristic approach to this argument (for the Binary Goldbach Conjecture) is to consider the prime number theorem, this states that a randomly chosen integer m has a probability of being prime equal to $1/\ln(m)$.

. Therefore, if n is a large even integer and m is a number between 3 and n , the probability that both m and $(n - m)$ are primes is approximately $1/(\ln(n).\ln(n - m))$. Although this heuristic argument is imperfect for several reasons, such as the lack of consideration of correlations between the probabilities of m and $(n - m)$ being primes, it nevertheless indicates

that the total number of ways of writing a large even integer n as the sum of two odd primes is approximately proportional to $n / \ln^2(n)$.

SYNTHESE

GRAPHIQUE

For every even integer $2n \geq 8$ (in parallel with the divisor tree developed from the Fundamental Theorem of Arithmetic), we uniquely associate a Goldbach tree. This allows us to visualise the proof of the Goldbach conjecture and provides the unique extreme decomponents of $2n$ according to all possible even sums of primes. The tree always ends with

$2 + 2 + 2 + \dots + 2 = 2n$. This technique can be used to create new number bases based on primes. Other variations of this tree can be created by adding or subtracting odd integers (other than $+1$ or -1) to the E.G.D. determined at each level.

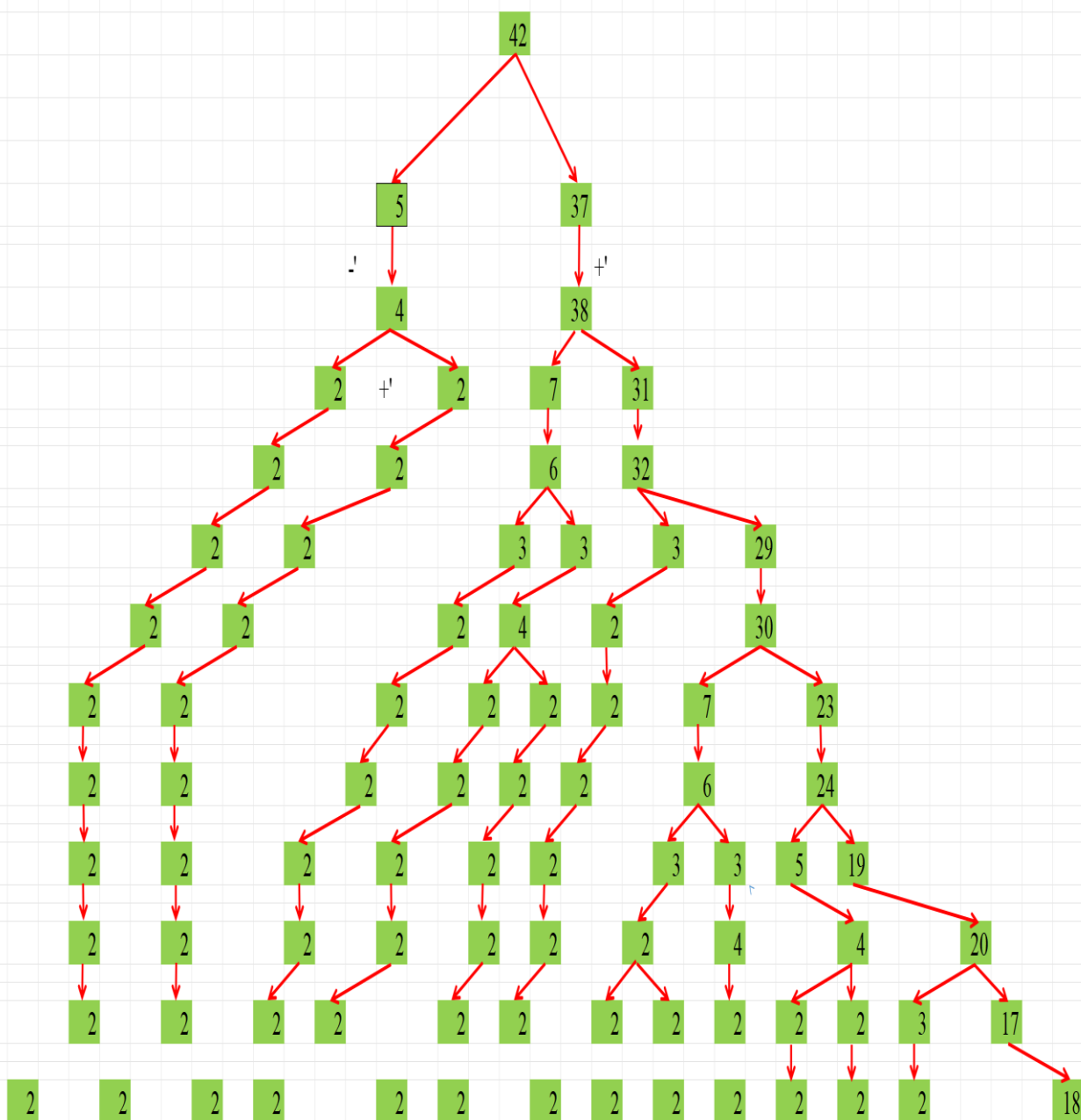
Example: (Draft for $2n = 42$).

Goldbach's extreme decomponents tree in even sums of primes.

ARBRE DES DECOMPOSANTS DE GOLDBACH SUIVANT TOUTES LES SOMME POSSIBLES DE NOMBRES PREMIER

DEGS : Décomposition de Goldbach en sommes

DEGS TWO PRIMES



.....*To continue*.....

End of tree : $2 + 2 + 2 + \dots + 2 = 2 \times 21 = 42 = 2n$



