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Article

Results on Singh–Chatterjea Type Contractive Mappings in b -Metric Spaces

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Abstract

In this paper, we extend the Singh–Chatterjea type contractive mappings from metric spaces to b -metric spaces. Additionally, we introduce extra terms into the mappings to extend and generalize several fixed point results within the framework of b -metric spaces.

Keywords: fixed point; singh-chatterjea type; b -metric spaces; contractive mappings

1. Introduction

Fixed point theory plays a vital role in nonlinear analysis and has found extensive applications in diverse areas such as in computer science, medical science[2], modelling of coronavirus([3–5]), and artistic patterns generation[6].

One of the most fundamental results in fixed point theory is the Banach contraction principle [7]. Over the years, this principle has motivated numerous generalizations that relax the contractive condition or extend the underlying space (metric[7], Banach[8] and Hilbert [9,10]) to enhance its range of applications. For instance, several authors have introduced notable modifications of the classical Banach contraction principle. In 1968, Kannan [11] proposed a contractive condition that does not require continuity of the mapping. Later, in 1972, Chatterjea [12] introduced another type of contraction involving a symmetric condition on the distances between points and their images. In 1977, Singh [13] extended Kannan’s condition to the p -th iterate of a mapping, establishing fixed point results for a wider class of operators. Comprehensive details of various classes of contractive type mappings that generalize the classical Banach contraction(see [14,15]).

In 1989, the b -metric spaces (quasimetric spaces) were introduced by Bakhtin[16] and formally defined by Czerwik[17] in 1993. The b -metric spaces involve relaxing the triangle inequality of standard metric spaces. It is used to generalize Banach’s fixed point theorem. Motivated by these developments and the recent results by Bekri et al [1], the present article investigates fixed point results for Singh–Chatterjea type mappings in complete b -metric spaces. We establish sufficient conditions for the existence and uniqueness of fixed points and examine the convergence behavior of iterative sequences associated with such mappings. Our findings generalize and improve several known results.

2. Preliminaries

For the convenience of the reader, we collect some important definitions and theorems.

Theorem 1 (Banach[7]). *Let (X, d) be a complete metric space and $T : X \rightarrow X$ a mapping for which there exists some $r \in [0, 1)$ such that*

$$d(Tx, Ty) \leq rd(x, y), \quad \forall x, y \in X. \tag{1}$$

Then T has a unique fixed point in X , and for any initial point $x_0 \in X$, the iterative sequence $\{T^n x_0\}$ converges to the fixed point of a mapping T .

Theorem 2 (Kannan[11]). *Let (X, d) be a complete metric space and $T : X \rightarrow X$ a mapping for which there exists some $r \in (0, \frac{1}{2})$ such that*

$$d(Tx, Ty) \leq r(d(x, Tx) + d(y, Ty)), \quad \forall x, y \in X. \tag{2}$$

Then T has a unique fixed point in X , and for any initial point $x_0 \in X$, the iterative sequence $\{T^n x_0\}$ converges to the fixed point of a mapping T .

Theorem 3 (Chatterjea [12]). *Let (X, d) be a complete metric space and $T : X \rightarrow X$ a mapping for which there exists some $r \in (0, \frac{1}{2})$ and*

$$d(Tx, Ty) \leq r(d(x, Ty) + d(y, Tx)), \quad \forall x, y \in X. \tag{3}$$

Then T has a unique fixed point in X , and for any initial point $x_0 \in X$, the iterative sequence $\{T^n x_0\}$ converges to the fixed point of a mapping T .

Theorem 4 (Singh[13]). *Let (X, d) be a complete metric space and $T : X \rightarrow X$ a mapping for which there exists a positive integer p and a number $r \in (0, \frac{1}{2})$ such that*

$$d(T^p x, T^p y) \leq r(d(x, T^p x) + d(y, T^p y)), \quad \forall x, y \in X. \tag{4}$$

Then T has a unique fixed point in X , and for any initial point $x_0 \in X$, the iterative sequence $\{T^n x_0\}$ converges to the fixed point of a mapping T .

Remark 1. *The above theorems have been extended from complete metric spaces to complete b -metric spaces (see [19,20]).*

Definition 1 ([16,17]). *Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d : X \times X \rightarrow \mathbb{R}_+$ is called a b -metric provided that, for all $x, y, z \in X$,*

- 1. $d(x, y) = 0$ if and only if $x = y$,
- 2. $d(x, y) = d(y, x)$,
- 3. $d(x, z) \leq s[d(x, y) + d(y, z)]$.

A pair (X, d) is called a b -metric space.

Lemma 1 (Singh et al[18]). *Let (X, d) be a b -metric space and x_n a sequence in X such that*

$$d(x_{n+1}, x_{n+2}) \leq rd(x_n, x_{n+1}), \quad n = 0, 1, 2, \dots, \tag{5}$$

where $0 \leq r < 1$. Then $\{x_n\}$ is a Cauchy sequence in X provided that $sr < 1$.

Theorem 5 (Chu et al[21]). *If T is a singled valued function defined on a complete metric space X into itself, such that the function T^n is a contraction for some $n \in \mathbb{N}$ (i.e. $d(T^n x, T^n y) \leq \alpha d(x, y)$ for $\alpha \in (0, 1)$), then T has a unique fixed point.*

Remark 2. *An analogous conclusion holds for complete b -metric spaces since the only assumption required in the proof of Theorem 5 is that T^n possesses a unique fixed point.*

3. Main Results

In this section, we extend the result of Bekri et al [1] from a metric spaces to b -metric spaces.

Theorem 6. Let (X, d) be a complete b -metric space and $T : X \rightarrow X$ a mapping. Suppose there exists $p \in \mathbb{N}$ and $\alpha \in (0, \frac{1}{2})$ such that

$$d(T^p x, T^p y) \leq \alpha(d(x, T^p y) + d(y, T^p x)), \quad \forall x, y \in X. \quad (6)$$

Then T has a unique fixed point $x^* \in X$, and for any initial point $x_0 \in X$, the iterative sequence $\{T^n x_0\}$ converges to x^* .

Proof. Let $S = T^p$. It follows from (6) that the mapping $S : X \rightarrow X$ satisfies the Chatterjea-type inequality

$$d(Sx, Sy) \leq \alpha(d(x, Sy) + d(y, Sx)), \quad \forall x, y \in X. \quad (7)$$

Let $x_0 \in X$ and define $x_{n+1} = Sx_n$. Using (7) and Def. 1, we have the following

$$\begin{aligned} d(Sx_n, Sx_{n-1}) &\leq \alpha[d(x_n, Sx_{n-1}) + d(x_{n-1}, Sx_n)] \\ d(x_{n+1}, x_n) &\leq \alpha[d(x_n, x_n) + d(x_{n-1}, x_{n+1})] \\ d(x_{n+1}, x_n) &\leq s\alpha[d(x_{n+1}, x_n) + d(x_n, x_{n-1})] \\ d(x_{n+1}, x_n) &\leq \frac{s\alpha}{1-s\alpha}d(x_n, x_{n-1}). \end{aligned}$$

Since $\alpha \in (0, \frac{1}{2})$ and $s \geq 1$, then $r = \frac{s\alpha}{1-s\alpha} < 1$. It follows immediately that

$$d(x_{n+1}, x_n) \leq rd(x_n, x_{n-1}). \quad (8)$$

Therefore $\{x_n\}$ is Cauchy by lemma 1. The completeness of X implies $x_n \rightarrow x^* \in X$.

Now, we want to show that x^* is a fixed point of S , i.e. $Sx^* = x^*$. Let us consider the inequality with (x^*, x_n) . Using (7), we obtain

$$d(Sx^*, x_{n+1}) \leq \alpha[d(x^*, x_{n+1}) + d(x_n, Sx^*)]$$

Letting $n \rightarrow \infty$ yields

$$d(Sx^*, x^*) \leq \alpha d(x^*, Sx^*)$$

Therefore, $d(x^*, Sx^*) = 0$ since $\alpha \in (0, \frac{1}{2})$. Thus, we have $Sx^* = x^*$. We claim that S has a unique fixed point. Suppose on the contrary that there exist $x_* \in X$ such that $Sx_* = x_*$. It follows from (7) that

$$d(x^*, x_*) = d(Sx^*, Sx_*) \leq \alpha(d(x^*, Sx_*) + d(x_*, Sx^*)) = 2\alpha d(x^*, x_*)$$

Therefore, $d(x^*, x_*) = 0$ since $2\alpha < 1$. Thus $x^* = x_*$. Since x_0 is arbitrary, the above argument holds for any $x_0 \in X$. Hence $S^n x \rightarrow x^*$ for all $x \in X$. Let us recall that we set $S = T^p$, $p \in \mathbb{N}$, so $T^p x^* = x^*$. Then

$$S(Tx^*) = T^p(Tx^*) = T(T^p x^*) = Tx^*. \quad (9)$$

So, Tx^* is also a fixed point of S . By uniqueness of fixed point of S , we have $Tx^* = x^*$. Thus x^* is a unique fixed point of T .

To show the convergence of the full orbit. For each $t = 0, 1, \dots, p-1$

$$T^{pn+r} x_0 = S^n(T^t x_0) \rightarrow x^* \text{ as } n \rightarrow \infty. \quad (10)$$

Hence the entire sequence $(T^n x_0)$ converges to x^* . Therefore T has a unique fixed point x^* , and $T^n x_0 \rightarrow x^*$ for all $x_0 \in X$. $\square$

The following corollaries can be deduced as particular cases of the main Theorem 6.

Corollary 1 (Singh-Chatterjea [1]). *Let (X, d) be a complete metric space and $T : X \rightarrow X$ a mapping. Suppose there exist $p \in \mathbb{N}$ and $\alpha \in (0, \frac{1}{2})$ such that*

$$d(T^p x, T^p y) \leq \alpha(d(x, T^p y) + d(y, T^p x)), \quad \forall x, y \in X. \tag{11}$$

Then T has a unique fixed point $x^ \in X$, and for any initial point $x_0 \in X$, the iterative sequence $\{T^n x_0\}$ converges to x^* .*

Proof. Take $s = 1$ in Theorem 6. $\square$

Corollary 2. *Let (X, d) be a complete b -metric space and $T : X \rightarrow X$ a mapping under the terms $s\alpha \in (0, \frac{1}{2})$ such that*

$$d(Tx, Ty) \leq \alpha(d(x, Ty) + d(y, Tx)), \quad \forall x, y \in X. \tag{12}$$

Then, there exists $x^ \in X$ such that $x_n \rightarrow x^*$ and x^* is unique fixed point of T .*

Proof. Take $p = 1$ in Theorem 6, we get Theorem 3 in [19]. $\square$

Remark 3. *If $p = s = 1$, then Theorem 6 reduces to the classical Chatterjea in Theorem 3 above.*

The example illustrate the effectiveness of our results.

Example 1. *This is an example of a mapping T in a complete b -metric space which satisfies the Singh–Chatterjea condition and admits a unique fixed point, while failing to satisfy the Banach, Kannan, and Chattejea contraction conditions. Let $X = \{0, 1, 2\}$ and $d(x, y) = |x - y|^2$. Consider the mapping $T0 = 0, T1 = 0, T2 = 1$.*

1. *We claim that (X, d) is a complete b -metric space. Indeed,*

$$d(x, z) = |x - z|^2 = |x - y + y - z|^2 \leq 2(d(x, y) + d(y, z)).$$

Since X is finite, (X, d) is a complete b -metric space with $s = 2$.

2. *T is not a Banach contraction. Clearly*

$$\frac{d(T2, T1)}{d(2, 1)} = 1,$$

which means that for $k < 1$, there exist some $x, y \in X$ such that $d(Tx, Ty) > kd(x, y)$.

3. *T is not Kannan contraction. i.e. $d(T2, T1) = 1$ and $d(2, T2) + d(1, T1) = 2$. So, we have $\alpha \geq \frac{1}{2}$.*

4. *T is not Chatterjea contraction. i.e.*

$$\begin{aligned} d(Tx, Ty) &\leq \alpha(d(x, Ty) + d(y, Tx)) \\ d(T1, T0) &\leq \alpha(d(1, T0) + d(0, T1)) \\ 1 &\leq \alpha(0 + 0). \end{aligned}$$

This is impossible. So, T is not Chattejea.

5. *T is Singh-Chatterjea. Indeed, notice that $T^p(X) = \{0\}$ which implies that $d(T^p x, T^p y) = 0$.*

6. *The fixed point of T^p is 0 for $p > 1$. Thus, the unique fixed point of T is 0 as desired.*

4. Generalization of Singh-Chatterjea Contraction

In this section, we generalize the Singh–Chatterjea contraction by replacing the constant α with a monotonically decreasing function. Additionally, we introduce extra terms into the Singh–Chatterjea

contraction framework, allowing us to recover the Banach, Rakotch, Kannan, Chatterjea, Singh, and Singh–Chatterjea contractions as special cases.

Theorem 7. Let (X, d) be a complete b -metric space. Suppose there exist $p \in \mathbb{N}$, a mapping $T : X \rightarrow X$ and α, β, r be monotonically decreasing functions from $(0, \infty)$ to $[0, 1)$ satisfying $\alpha(t) + 2\beta(t) + 2sr(t) < 1$ such that, for each $x, y \in X, x \neq y$,

$$d(T^p x, T^p y) \leq \alpha(t)d(x, y) + \beta(t)[d(x, T^p x) + d(y, T^p y)] + r(t)[d(x, T^p y) + d(y, T^p x)].$$

Then, T has a unique fixed point $x^* \in X$.

Proof. Let $S = T^p$ for some $p \in \mathbb{N}$ and $\alpha(t) = \alpha(d(x, y)), \dots, r(t) = r(d(x, y))$. Then,

$$d(Sx, Sy) \leq \alpha(t)d(x, y) + \beta(t)[d(x, Sx) + d(y, Sy)] + r(t)[d(x, Sy) + d(y, Sx)], \quad \forall x, y \in X.$$

Let $y = Sx$, we obtain

$$d(Sx, S^2 x) \leq \frac{\alpha(t) + \beta(t) + sr(t)}{1 - \beta(t) - sr(t)} d(x, Sx). \quad (13)$$

It follows from the hypothesis that there exists a monotone decreasing function $q(t)$ such that $0 \leq q(t) < 1$ and

$$\begin{aligned} d(Sx, S^2 x) &\leq q(d(x, Sx))d(x, Sx) \\ d(Sx, Sy) &\leq q(d(x, y))d(x, y). \end{aligned}$$

Therefore, $S = T^p$ is a contraction. By Theorem 5, T has a unique fixed point. $\square$

Theorem 8 (Singh-Hardy-Rogers). Let (X, d) be a complete b -metric space. Suppose there exist $p \in \mathbb{N}$, a mapping $T : X \rightarrow X$ and a, b, c, e, f be monotonically decreasing functions from $(0, \infty)$ to $[0, 1)$ satisfying $a(t) + b(t) + c(t) + 2se(t) + f(t) < 1$ such that, for each $x, y \in X, x \neq y$,

$$d(T^p x, T^p y) \leq a(t)d(x, y) + b(t)d(x, T^p x) + c(t)d(y, T^p y) + e(t)d(x, T^p y) + f(t)d(y, T^p x).$$

Then, T has a unique fixed point $x^* \in X$.

Proof. By symmetry property of b -metric spaces, we have

$$\begin{aligned} d(T^p x, T^p y) &\leq a(t)d(x, y) + b(t)d(x, T^p x) + c(t)d(y, T^p y) + e(t)d(x, T^p y) + f(t)d(y, T^p x) \\ d(T^p x, T^p y) &\leq a(t)d(x, y) + b(t)d(y, T^p y) + c(t)d(x, T^p x) + e(t)d(y, T^p x) + f(t)d(x, T^p y). \end{aligned}$$

The sum gives

$$\begin{aligned} d(T^p x, T^p y) &\leq a(t)d(x, y) + \frac{b(t) + c(t)}{2}[d(x, T^p x) + d(y, T^p y)] + \\ &\quad \frac{e(t) + f(t)}{2}[d(x, T^p y) + d(y, T^p x)]. \end{aligned}$$

We obtain the conclusion from Theorem 7 by taking $\alpha(t) = a(t), \beta(t) = \frac{b(t) + c(t)}{2}$ and $r(t) = \frac{e(t) + f(t)}{2}$. $\square$

Example 2. Let $X = [0, 1]$, $d(x, y) = |x - y|$ and consider a mapping $T : X \rightarrow X$ defined by

$$Tx = \begin{cases} 1 - x & \text{if } 0 \leq x \leq \frac{1}{2} \\ \frac{1}{2} & \text{if } \frac{1}{2} < x \leq 1. \end{cases} \quad (14)$$

Clearly, (X, d) is a complete b -metric space with $s = 1$. Also, $T^2x = \frac{1}{2}$. We claim that T is not Hardy-Rogers but T^2 satisfies our contraction. Indeed, take $x = 0$ and $y = \frac{1}{2}$, we have the following

$$\begin{aligned} Tx = \{1\}, Ty = \frac{1}{2}, d(Tx, Ty) = \frac{1}{2}, d(x, y) = \frac{1}{2}, \\ d(x, Tx) = 1, d(y, Ty) = 0, d(x, Ty) = \frac{1}{2}, d(y, Tx) = \frac{1}{2}, \end{aligned}$$

which gives $1 \leq a + 2b + e + f$. By symmetry, we have $1 \leq a + 2c + e + f$. Taking the sum of the two inequalities and divide both side by 2, we obtain $a + b + c + e + f \geq 1$ which contradicts the assumption that $a + b + c + 2se + f < 1$. Notice that $d(T^2x, T^2y) = 0$. Therefore, T^2 satisfies our contraction and have a unique fixed point. It follows from Theorem 8 that T must have a unique fixed point i.e. $T\frac{1}{2} = \frac{1}{2}$.

Example 3. Let $X = [0, 1]$, $d(x, y) = |x - y|^2$ and consider a mapping $T : X \rightarrow X$ defined by

$$Tx = \begin{cases} 1 - 2x & \text{if } 0 \leq x \leq \frac{1}{3} \\ \frac{1}{3} & \text{if } \frac{1}{3} < x \leq 1. \end{cases} \quad (15)$$

This is left for the reader to verify that T is not Hardy-Rogers. Indeed, take $x = 0$ and $y = \frac{1}{3}$, we obtain the following

$$\begin{aligned} Tx = \{1\}, Ty = \frac{1}{3}, d(Tx, Ty) = \frac{4}{9}, d(x, y) = \frac{4}{9}, \\ d(x, Tx) = 1, d(y, Ty) = \frac{1}{9}, d(x, Ty) = \frac{4}{9}, d(y, Tx) = \frac{1}{9}, \end{aligned}$$

which results to $4 \leq 4a + b + c + 4e + f \leq 4(a + b + c + e + f)$. This contradicts the assumption in Theorem 8. Now, we compute $T^2(x) = T(T(x))$ for all $x \in [0, 1]$.

- If $0 \leq x \leq \frac{1}{3}$, then $T(x) = 1 - 2x \in [\frac{1}{3}, 1]$, so $T(T(x)) = T(1 - 2x) = \frac{1}{3}$
- If $\frac{1}{3} < x \leq 1$, then $T(x) = \frac{1}{3}$, so $T(T(x)) = T(\frac{1}{3}) = \frac{1}{3}$

Thus, for all $x \in [0, 1]$, we have $T^2(x) = \frac{1}{3}$. Since $d(T^2x, T^2y) = 0$, we see that T^2 satisfy our contraction and $T\frac{1}{3} = \frac{1}{3}$.

Corollary 3 (Bekri et al [1]). Let (X, d) be a complete metric space and $T : X \rightarrow X$ a mapping. Suppose there exist $p \in \mathbb{N}$ and $r \in (0, \frac{1}{2})$ such that

$$d(T^p x, T^p y) \leq r(d(x, T^p y) + d(y, T^p x)), \quad \forall x, y \in X. \quad (16)$$

Then T has a unique fixed point $x^* \in X$, and for any initial point $x_0 \in X$, the iterative sequence $\{T^n x_0\}$ converges to x^* .

Proof. Take $\alpha(t) = \beta(t) = 0$, $s = 1$, and $r(t)$ to be a number in Theorem 7. $\square$

Corollary 4. If $p(t) = r(t) = 0$ in Theorem 7, then we obtain Rakotch's fixed point theorem [14]. Additionally, If $\alpha(t)$ is a constant, then we have Banach's contraction Principle[7].

Corollary 5. If $\alpha(t) = r(t) = 0$ and $\beta(t)$ is a constant in Theorem 7, then we obtain Singh's fixed point theorem [13].

Corollary 6. If $\alpha(t) = r(t) = 0$ and $p = 1$ in Theorem 7, then we obtain Kannan's fixed point theorem [11].

Corollary 7. If $\beta(t) = \alpha(t) = 0$ and $p = 1$ in Theorem 7, then we obtain Chatterjea's fixed point theorem [12].

Theorem 9. Let (X, d) be a complete b -metric space. Suppose there exist $p \in \mathbb{N}$, a mapping $T : X \rightarrow X$ and $\alpha, \beta, r : [0, \infty) \rightarrow [0, \infty)$ nonnegative functions satisfying $\alpha(t) + 2\beta(t) + 2sr(t) < 1$ and $\limsup_{c \rightarrow t^+} \frac{\alpha(c) + \beta(c) + sr(c)}{1 - \beta(c) - sr(c)}$ for all $t \in [0, \infty)$ such that, for each $x, y \in X$,

$$d(T^p x, T^p y) \leq \alpha(x, y)d(x, y) + \beta(x, y)[d(x, T^p x) + d(y, T^p y)] \\ + r(x, y)[d(x, T^p y) + d(y, T^p x)].$$

Then, T has a unique fixed point $x^* \in X$.

Proof. Let $S = T^p$ for some $p \in \mathbb{N}$, $t = d(x, y)$ and $\alpha(x, y) = \alpha(d(x, y)), \dots, r(x, y) = r(d(x, y))$. Then,

$$d(Sx, Sy) \leq \alpha(t)d(x, y) + \beta(t)[d(x, Sx) + d(y, Sy)] + r(t)[d(x, Sy) + d(y, Sx)].$$

Let $y = Sx$, we obtain

$$d(Sx, S^2x) \leq \frac{\alpha(t) + \beta(t) + sr(t)}{1 - \beta(t) - sr(t)}d(x, Sx). \quad (17)$$

By the hypothesis, there exist $q(t) < 1$ such that

$$d(Sx, Sy) \leq q(d(x, y))d(x, y).$$

Therefore, $S = T^p$ is a contraction. By Theorem 5, T has a unique fixed point. $\square$

Theorem 10. Let (X, d) be a complete b -metric space. Suppose there exist $p \in \mathbb{N}$, a mapping $T : X \rightarrow X$ and $a, b, c, e, f : [0, \infty) \rightarrow [0, \infty)$ nonnegative functions satisfying $a(t) + b(t) + c(t) + 2se(t) + f(t) < 1$ and $\limsup_{c \rightarrow t^+} \frac{\alpha(c) + \beta(c) + sr(c)}{1 - \beta(c) - sr(c)}$ for all $t \in [0, \infty)$ and any $\alpha, \beta, r : [0, \infty) \rightarrow [0, \infty)$ such that, for each $x, y \in X$,

$$d(T^p x, T^p y) \leq a(x, y)d(x, y) + b(x, y)d(x, T^p x) + c(x, y)d(y, T^p y) \\ + e(x, y)d(x, T^p y) + f(x, y)d(y, T^p x).$$

Then, T has a unique fixed point $x^* \in X$.

Proof. Let $S = T^p$ for some $p \in \mathbb{N}$, $t = d(x, y)$ and $\alpha(x, y) = \alpha(d(x, y)), \dots, r(x, y) = r(d(x, y))$. Then,

$$d(Sx, Sy) \leq a(t)d(x, y) + b(t)d(x, Sx) + c(t)d(y, Sy) + e(t)d(x, Sy) + f(t)d(y, Sx).$$

Let $y = Sx$, we obtain

$$d(Sx, S^2x) \leq \frac{a(t) + c(t) + sf(t)}{1 - c(t) - sf(t)}d(x, Sx). \quad (18)$$

By the hypothesis, there exist $q(t) = \min\{\frac{a(t) + c(t) + sf(t)}{1 - c(t) - sf(t)}, \frac{a(t) + b(t) + se(t)}{1 - c(t) - se(t)}\} < 1$ such that

$$d(Sx, Sy) \leq q(d(x, y))d(x, y).$$

Therefore, $S = T^p$ is a contraction. By Theorem 5, T has a unique fixed point. $\square$

5. Open Problems

Recently, some researchers worked on extended b -metric spaces [23], orthogonal b -metric spaces [24] and R -metric spaces [25]. **Questions:** Results on Singh-Chatterjea Type in extended b -Metric Spaces, orthogonal b -metric spaces and R -metric spaces.

6. Conclusions

In this paper, we extended the Singh-Chatterjea type mappings from a metric spaces to the more general setting of a b -metric spaces. Our results unify and generalize several fixed point theorems within the b -metric framework, and illustrative examples are provided to demonstrate the applicability and effectiveness of the proposed results. Furthermore, by incorporating additional terms into the Singh-Chatterjea contraction framework, we recover the Banach, Rakotch, Kannan, Chatterjea, Singh, and Singh-Chatterjea contractions as special cases.

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