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Article

A Static-Sector Construction for the Low-Energy Fine-Structure Constant in Recursive Interval Geometry

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Abstract

We present a limited static-sector construction for the low-energy fine-structure constant within Recursive Interval Geometry (RIG). The recursive substrate itself is not reaxiomatized here; the inherited input is only the canonical principal/structural splitting together with the Ω -weighted norm. The paper introduces four principles: minimal closure with nonempty-state counting, a link-triangle-octahedral carrier hierarchy in one to three dimensions, a display-level readout principle, and a dimension-filtered expansion of the raw structural term. From these assumptions one obtains $D_1 = 3, D_2 = 7, D_3 = 127$, interpreted as nonempty boundary-state counts on minimal one-, two-, and three-dimensional carriers, hence the additive skeleton $N_{\text{sk}} = 137$ and the multiplicative resolution $\Omega = 2667$, together with the display-level form $\alpha^{-1} = \sqrt{N_{\text{sk}}^2 + \Omega^2 \ell_{\text{raw}}^2}$. What is not yet absorbed into deeper internal structure is reduced to three residual bridge coefficients in $\ell_{\text{raw}} = \frac{a_1}{\Omega} + \frac{a_2}{\Omega^2} + \frac{a_3}{\Omega^3}$, which are taken in the working model to be $a_1 = \pi, a_2 = -2$, and $a_3 = 137 + 127/2$. Their strongest current geometric readings are, respectively, the metric normalization of the minimal closed loop, the endpoint subtraction produced when an open interval is closed into a loop, and the sum of the full static skeleton with a third-level shared skeletal load. This yields $\alpha_{\text{RIG}}^{-1} = 137.035999176253147 \dots$, differing from the 2022 Committee on Data for Science and Technology (CODATA) recommended value by 7.47×10^{-10} in α^{-1} , or 5.45×10^{-3} ppb. The claim is therefore not that quantum electrodynamics has been derived, but that a logically explicit substrate model can be tested as a falsifiable interface proposal.

Keywords: fine-structure constant; recursive interval geometry; static sector; reproducibility; foundations of physics; low-energy electromagnetic coupling

1. Introduction

The fine-structure constant α remains one of the most natural objects of foundational reflection. It is measured with extraordinary precision and plays a central role in precision tests of quantum electrodynamics (QED), yet within the Standard Model it still enters as an experimentally supplied low-energy parameter rather than as an output of a deeper structural law [2–12]. That situation continues to motivate attempts to relate α to geometry, symmetry, arithmetic, or vacuum organization. Most such attempts fail for one of two familiar reasons: either the numerical agreement is weak, or the logical status of the proposal is too opaque for serious evaluation.

The present manuscript adopts a deliberately narrower posture. It does not aim to replace QED, reconstruct renormalized dynamics, or explain the running of couplings. The guiding question is instead this: can a recursively organized static substrate define a sharply constrained display-level baseline whose value coincides with the observed low-energy fine-structure constant? Framed in this way, the proposal is neither a completed electrodynamics nor a bare numerological coincidence. It is an interface statement between a substrate model and an observable constant.

The framework used is Recursive Interval Geometry (RIG) [1]. In the broader RIG formalism, recursive states split canonically into principal and structural sectors, and the structural sector is weighted by a resolution scale Ω . The present paper uses only the minimum portion of that background needed for one static-sector application. The recursive substrate itself is not reaxiomatized here. What matters instead is a clean separation among four layers of claims:

1. algebraic structure inherited from RIG,
2. sector-specific principles introduced only for the static low-energy fine-structure-constant problem,
3. consequences derived from those principles,
4. residual hypotheses that have not yet been absorbed into a deeper internal theorem.

This separation fixes which parts of the construction are inherited, which are sector-specific, which are derived, and which remain hypothesis.

The paper is organized accordingly. Section 2 recalls only the inherited principal/structural splitting and the Ω -weighted norm. Section 3 states the additional static-sector principles. Section 4 derives the hierarchy 3, 7, 127, the associated quantities 137 and 2667, and the display-level norm form of the inverse fine-structure constant. Section 5 identifies the residual bridge coefficients and records their strongest current geometric interpretations. Section 6 reports rigidity checks and numerical output. Sections 7 and 8 state the limitations and conclusion.

2. Inherited Structure from RIG

The present paper does not reaxiomatize the recursive substrate. We recall only the part of RIG needed to fix notation and to define the display-level readout [1].

RIG is built over the localized ring $\mathbb{Z}[\Omega^{-1}]$ by the recursive rule

$$\mathcal{I}_{k+1} = \mathcal{I}_k \times \mathcal{I}_k, \quad (1)$$

so that every recursive state admits a canonical principal/structural decomposition

$$x = (x_P, x_L). \quad (2)$$

The corresponding Ω -weighted bilinear form satisfies

$$\langle x, y \rangle_\Omega = \langle x_P, y_P \rangle_\Omega + \Omega^2 \langle x_L, y_L \rangle_\Omega, \quad (3)$$

and induces the real norm

$$\|x\|_\Omega^2 = \|x_P\|_\Omega^2 + \Omega^2 \|x_L\|_\Omega^2. \quad (4)$$

Thus the principal and structural sectors are orthogonal at display level, while structural loading becomes visible only after Ω -weighting is applied.

Equation (4) is the only inherited readout used in what follows. All stronger algebraic properties of RIG belong to the background framework and are not restated here as new assumptions. The purpose of the present paper is narrower: to ask whether a static recursively organized substrate state can be mapped to the observed low-energy value of α^{-1} through the norm (4).

3. Sector-Specific Principles for the Static Fine-Structure-Constant Problem

The additional content of the present paper is confined to the following four static-sector principles.

Principle 1: Minimal Closure and Nonempty-State Counting For each closure dimension relevant to the static problem, one selects the minimal carrier compatible with closure in that dimension.

If the chosen carrier has b independent binary boundary channels, then the number of nonempty states is

$$D = 2^b - 1, \quad (5)$$

where the subtraction of 1 removes the absolutely empty configuration.

Principle 2: static-sector carrier choice in one to three dimensions. In the static vacuum sector considered here, the one-dimensional minimal carrier is a link, the two-dimensional minimal closed carrier is a triangle, and the three-dimensional minimal closed carrier is taken to be the smallest triangular-faced local carrier admitting one global closure relation among its face channels and compatible with a tetrahedral–octahedral space-filling skeleton. In the present working model, this role is assigned to an octahedral carrier. The term “octahedral” is used here structurally rather than metrically: what matters is a closed triangular-faced local carrier with one global redundancy among its face data, not a uniqueness theorem about all possible three-dimensional triangular-faced closures.

Principle 3: display-level readout. Let $F(N, \ell; \Omega)$ denote the display-level value of α^{-1} associated with a principal scalar N and a structural scalar ℓ . We require that the square of this readout be additive under the inherited orthogonal principal/structural decomposition, with normalization

$$F(N, 0; \Omega) = |N|, \quad F(0, \ell; \Omega) = \Omega|\ell|. \quad (6)$$

Equivalently, the display-level readout is assumed to depend on the principal and structural sectors only through an orthogonal square-additive norm law. This encodes the claim that the observed fine-structure constant is a readout of the inherited norm geometry rather than an independent primitive.

Principle 4: dimension-filtered structural expansion. The raw structural term is assumed to admit a lowest-order expansion of the form

$$\ell_{\text{raw}} = \frac{a_1}{\Omega} + \frac{a_2}{\Omega^2} + \frac{a_3}{\Omega^3} + O(\Omega^{-4}), \quad (7)$$

in which a_1 depends only on one-dimensional closure data, a_2 depends only on closure data of dimension at most two, and a_3 depends only on closure data of dimension at most three.

These principles isolate the content that is specific to the present static-sector construction. Nothing in Sections 4–6 should be read as a fresh axiom of the RIG substrate itself.

4. Derived Hierarchy and Display-Level Form

Within the principles of Section 3, the static hierarchy is obtained directly.

Hierarchy. The three numbers 3, 7, 127 admit a uniform interpretation as nonempty boundary-state counts on minimal carriers. A link has two independent endpoint channels, so Principle 1 gives

$$D_1 = 2^2 - 1 = 3. \quad (8)$$

A triangle has three independent edge channels, hence

$$D_2 = 2^3 - 1 = 7. \quad (9)$$

For the three-dimensional carrier, Principle 2 assigns a local octahedral carrier embedded in a tetrahedral–octahedral skeletal picture. This carrier has eight triangular faces together with one global closure relation; therefore the number of independent face channels is $8 - 1 = 7$, and

$$D_3 = 2^7 - 1 = 127. \quad (10)$$

Thus the hierarchy is not introduced as an isolated numerical list but as the successive one-, two-, and three-dimensional nonempty-state count on minimal carriers. From these three counts one obtains the additive skeleton

$$N_{\text{sk}} = D_1 + D_2 + D_3 = 3 + 7 + 127 = 137 \quad (11)$$

and the multiplicative resolution

$$\Omega = D_1 D_2 D_3 = 3 \cdot 7 \cdot 127 = 2667. \quad (12)$$

Display-level interface law. By Principle 3, the square of the display-level readout is additive under the inherited orthogonal principal/structural decomposition. Hence

$$F(N, \ell; \Omega)^2 = P(N) + Q(\Omega \ell) \quad (13)$$

for some nonnegative functions P and Q . The normalization conditions (6) force

$$P(N) = N^2, \quad Q(\Omega \ell) = (\Omega \ell)^2, \quad (14)$$

and therefore

$$F(N, \ell; \Omega) = \sqrt{N^2 + \Omega^2 \ell^2}. \quad (15)$$

Applying (15) to the static vacuum vector

$$\mathbf{V}_{\text{vac}} = (N_{\text{sk}}, \ell_{\text{raw}}) \quad (16)$$

produces the display-level identification

$$\alpha^{-1} = \|\mathbf{V}_{\text{vac}}\|_{\Omega} = \sqrt{N_{\text{sk}}^2 + \Omega^2 \ell_{\text{raw}}^2}. \quad (17)$$

In particular, after substituting (11),

$$\alpha^{-1} = \sqrt{137^2 + \Omega^2 \ell_{\text{raw}}^2}. \quad (18)$$

Expansion shape. Principle 4 does not determine the specific coefficients a_1, a_2, a_3 , but it does determine the hierarchical form of the structural loading. Substituting (7) into (17) gives

$$\alpha^{-1} = \sqrt{137^2 + \left(a_1 + \frac{a_2}{\Omega} + \frac{a_3}{\Omega^2} + O(\Omega^{-3})\right)^2}. \quad (19)$$

Hence three pieces of the proposal are already fixed before the specific numerical assignment is made: the hierarchy 3, 7, 127, the quantities $N_{\text{sk}} = 137$ and $\Omega = 2667$, and the norm form of the display-level inverse fine-structure constant.

5. Interpretation, Residual Hypotheses, and Working Formula

At this stage the remaining freedom has been reduced to three coefficients, a_1, a_2, a_3 , which have not yet been forced by a deeper internal theorem of the substrate. Their status should therefore be stated plainly: they are residual bridge coefficients in the present static-sector model. At the same time, their strongest current readings are not arbitrary numerical decorations. They can be organized by role: a metric normalization, a closure correction, and a skeletal-load term built on the hierarchy 3, 7, 127 itself.

First coefficient. The leading coefficient is taken to be

$$a_1 = \pi. \quad (20)$$

Its strongest current interpretation is metric rather than combinatorial. The one-dimensional carrier is first read as an open link, but the display-level propagation object is a minimal closed loop. Once the unit link is normalized by closure into the smallest closed one-dimensional carrier, the natural geometric normalization is the circumference of the unit-diameter circle. In that sense, $a_1 = \pi$ records the baseline metric cost of minimal loop propagation.

Second coefficient. The next coefficient is taken to be

$$a_2 = -2. \quad (21)$$

This term is interpreted as a closure correction. The open one-dimensional carrier has two endpoint channels, but after closure these endpoints no longer survive as independent boundary freedoms. The negative sign therefore records a topological subtraction, and the magnitude 2 counts the two endpoint contributions that disappear when the open interval is converted into a loop.

Third coefficient. The third coefficient is taken to be

$$a_3 = 137 + \frac{127}{2}. \quad (22)$$

Its strongest current interpretation is skeletal rather than purely local. The term

$$137 = 3 + 7 + 127 \quad (23)$$

is the full static skeleton count obtained by summing the three nonempty-state levels. The additional term $127/2$ is then read as a third-level shared-load correction: the three-dimensional carrier is not treated as an isolated cell, but as a local carrier inside a shared triangular-faced skeletal network, so the highest-level burden is allocated with half weight at the level of one local cell. Thus a_3 combines total skeletal content with a shared third-level load.

Working truncation. With these assignments, and with the present paper truncated at third order, the raw structural term becomes

$$\ell_{\text{raw}} = \frac{\pi}{\Omega} - \frac{2}{\Omega^2} + \frac{137 + 127/2}{\Omega^3}, \quad (24)$$

so that

$$\alpha^{-1} = \sqrt{137^2 + \left(\pi - \frac{2}{\Omega} + \frac{137 + 127/2}{\Omega^2} \right)^2}. \quad (25)$$

This is the baseline working formula; Section 2 evaluates both this baseline case and its nearby self-consistent channel-count controls. Its logical status is deliberately explicit: the hierarchy and norm form are derived within the present framework, whereas the three coefficients remain residual hypotheses supplied with the strongest geometric interpretations currently available.

6. Discrete Rigidity and Numerical Output

The numerical output is therefore supplemented by control tests against nearby discrete alternatives.

Channel-count controls. To test whether the seven-channel three-dimensional carrier is structurally distinguished, we vary the three-dimensional channel count b_3 while keeping the one- and two-dimensional sectors fixed at $D_1 = 3$ and $D_2 = 7$. In this control, every quantity that depends on the third-level carrier is updated self-consistently:

$$D_3(b_3) = 2^{b_3} - 1, \quad N_{\text{sk}}(b_3) = 3 + 7 + D_3(b_3), \quad \Omega(b_3) = 3 \cdot 7 \cdot D_3(b_3). \quad (26)$$

The first two bridge coefficients are kept fixed at $a_1 = \pi$ and $a_2 = -2$, while the third bridge coefficient is reassigned by the same rule used in the baseline case,

$$a_3(b_3) = N_{\text{sk}}(b_3) + \frac{D_3(b_3)}{2}.$$
(27)

Accordingly, the channel-count control is evaluated by

$$\alpha_{\text{ctrl}}^{-1}(b_3) = \sqrt{N_{\text{sk}}(b_3)^2 + \left(\pi - \frac{2}{\Omega(b_3)} + \frac{a_3(b_3)}{\Omega(b_3)^2}\right)^2}.$$
(28)

The present construction corresponds to $b_3 = 7$. The resulting controls are listed in Table 1.

Table 1. Self-consistent channel-count controls for the three-dimensional carrier while the one- and two-dimensional sectors are kept fixed. In each row, D_3 , N_{sk} , Ω , and the third bridge coefficient $a_3 = N_{\text{sk}} + D_3/2$ are updated together. The mismatch is measured relative to the CODATA 2022 recommended value of α^{-1} .

b_3	$D_3 = 2^{b_3} - 1$	N_{sk}	$\Omega = 3 \cdot 7 \cdot D_3$	$\alpha_{\text{ctrl}}^{-1}$	mismatch (ppb)
6	63	73	1323	73.06750634341056	4.67×10^8
7	127	137	2667	137.035999176253147	5.45×10^{-3}
8	255	265	5355	265.01861697599621	9.34×10^8

Table 1 shows that the observed scale of α^{-1} is not reproduced by a broad band of neighboring three-dimensional channel counts. Within the present static template, the seven-channel carrier remains sharply distinguished from its nearest discrete alternatives.

Coefficient controls. A second test keeps the hierarchy fixed but perturbs the coefficients in the effective loop term. Define

$$L_{\text{eff}}(c_2, c_3) = \pi + \frac{c_2}{\Omega} + \frac{c_3}{\Omega^2}, \quad \Omega = 2667,$$
(29)

so that the present model corresponds to $(c_2, c_3) = (-2, 200.5)$. The resulting alternatives are shown in Table 2.

Table 2. Coefficient controls for the effective loop term $L_{\text{eff}}(c_2, c_3)$ at fixed hierarchy $(D_1, D_2, D_3) = (3, 7, 127)$. The mismatch is measured relative to the CODATA 2022 recommended value of α^{-1} .

c_2	c_3	α^{-1}	mismatch (ppb)
−3	200.5	137.03599058282413	6.27×10^1
−2	200.5	137.035999176253147	5.45×10^{-3}
−1	200.5	137.03600777070756	6.27×10^1
−2	137	137.035989716358	1.50
−2	200.5	137.035999176253147	5.45×10^{-3}
−2	264	137.0359993808711	1.49

The sensitivities follow directly from

$$\alpha^{-1} = \sqrt{N_{\text{sk}}^2 + L_{\text{eff}}^2},$$
(30)

which yields

$$\frac{\partial \alpha^{-1}}{\partial c_2} = \frac{L_{\text{eff}}}{\alpha^{-1} \Omega}, \quad \frac{\partial \alpha^{-1}}{\partial c_3} = \frac{L_{\text{eff}}}{\alpha^{-1} \Omega^2}.$$
(31)

At the present values,

$$\frac{\partial \alpha^{-1}}{\partial c_2} = 8.59394 \times 10^{-6}, \quad \frac{\partial \alpha^{-1}}{\partial c_3} = 3.22233 \times 10^{-9}.$$
(32)

Thus the reported agreement is not protected by a broad continuous plateau. It is tied to specific discrete assignments. The reported value is not obtained by continuous parameter fitting: the channel assignments are discrete, and the remaining bridge coefficients are fixed by explicit working hypotheses rather than tuned by numerical optimization.

Numerical value. Equation (24) gives

$$\ell_{\text{raw}} = 1.17767939092353553 \times 10^{-3} \dots, \tag{33}$$

while the corresponding effective structural loading is

$$L_{\text{eff}} = \Omega \ell_{\text{raw}} = 3.140870935593069263 \dots \tag{34}$$

Hence

$$\alpha_{\text{RIG}}^{-1} = 137.035999176253147 \dots \tag{35}$$

For comparison, the CODATA 2022 recommended value is [12]

$$\alpha_{\text{CODATA}}^{-1} = 137.035999177(21), \tag{36}$$

while the value inferred from the 2023 electron magnetic-moment measurement and Standard Model theory is [10]

$$\alpha_{eg-2}^{-1} = 137.035999166(15). \tag{37}$$

The absolute difference from CODATA 2022 is

$$\Delta_{\text{CODATA}} = \left| \alpha_{\text{RIG}}^{-1} - \alpha_{\text{CODATA}}^{-1} \right| = 7.47 \times 10^{-10}, \tag{38}$$

corresponding to

$$\frac{\Delta_{\text{CODATA}}}{\alpha_{\text{CODATA}}^{-1}} = 5.45 \times 10^{-12} = 5.45 \times 10^{-3} \text{ ppb}. \tag{39}$$

The absolute difference from the *eg*-2-based determination is

$$\Delta_{eg-2} = 1.03 \times 10^{-8}, \tag{40}$$

corresponding to 7.48×10^{-2} ppb. The main numerical quantities are collected in Table 3.

Table 3. Main quantities entering the present static-sector formula.

Quantity	Value
D_1	3
D_2	7
D_3	127
N_{sk}	137
Ω	2667
ℓ_{raw}	$1.17767939092353553 \times 10^{-3} \dots$
L_{eff}	$3.140870935593069263 \dots$
α_{RIG}^{-1}	$137.035999176253147 \dots$

7. Limitations and Falsifiability

The present construction is intentionally narrow.

First, it is a static-sector proposal. It does not derive the dynamics of QED, does not include radiative corrections, and does not address the running of α .

Second, the identification of the observed fine-structure constant with the norm (17) is an interface statement. It says that the measured low-energy constant is the display-level readout of a recursively

structured vacuum quantity. Its viability therefore depends on whether that interface can be sharpened, generalized, or independently reproduced in other sectors.

Third, the formula still contains residual hypotheses. The hierarchy, the quantities 137 and 2667, and the norm form for α^{-1} are already fixed within the present framework. What remains unabsorbed at this stage is reduced to three bridge coefficients, namely π , -2 , and $137 + 127/2$. The distinction between derived content and residual hypotheses is therefore kept explicit.

Fourth, the octahedral choice has been presented here as the canonical local carrier for one static-sector closure program, compatible with a tetrahedral–octahedral space-filling skeleton, not as a universal theorem of three-dimensional recursive geometry. Likewise, the coefficient interpretations are presently the strongest available geometric readings, not yet uniqueness theorems.

These limitations also define the falsifiability conditions. The proposal would be weakened if sharper internal derivations fail to emerge for the seven-channel closure rule, for the display-level norm law, or for the coefficient structure of (24). It would be weakened even more if independent observable sectors cannot be generated from the same substrate without introducing uncontrolled extra freedom.

8. Conclusions

We have presented a deliberately limited static-sector construction for the low-energy fine-structure constant within Recursive Interval Geometry. The recursive substrate itself was not reaxiomatized. Instead, the paper separated inherited RIG structure from additional sector-specific principles, then derived from those principles the hierarchy 3, 7, 127, the additive skeleton 137, the multiplicative resolution 2667, and the display-level norm form

$$\alpha^{-1} = \sqrt{N_{\text{sk}}^2 + \Omega^2 \ell_{\text{raw}}^2}.$$

What remains at present is reduced to three residual bridge coefficients, for which explicit geometric interpretations were given: π as minimal-loop metric normalization, -2 as endpoint subtraction under closure, and $137 + 127/2$ as full static skeleton plus shared third-level load.

With the current working assignment

$$\ell_{\text{raw}} = \frac{\pi}{\Omega} - \frac{2}{\Omega^2} + \frac{137 + 127/2}{\Omega^3},$$

the resulting numerical value is

$$\alpha_{\text{RIG}}^{-1} = 137.035999176253147 \dots,$$

which differs from the CODATA 2022 recommended value by 7.47×10^{-10} in α^{-1} . More importantly, nearby discrete alternatives do not reproduce the same result equally well.

The construction isolates inherited structure, sector-specific principles, derived consequences, and residual hypotheses. Within that separation, the hierarchy 3, 7, 127, the quantities 137 and $\Omega = 2667$, and the display-level norm form for α^{-1} are fixed, while the three coefficients in (24) remain to be derived internally. The same static carrier is also being examined in a separate sectorial test on the same recursive substrate. If that independent test also succeeds, the present fine-structure-constant result would no longer stand as an isolated numerical interface, but as one observable readout within a broader recursive program.

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Code Availability Wolfram Language and pure Python code reproducing the numerical results, control tables, sensitivities, and quoted numerical comparisons reported in this manuscript are publicly available in the accompanying Zenodo record at <https://doi.org/10.5281/zenodo.19159715>.

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