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Application of the Bivariate Exponentiated Gumbel Distribution for Extreme Rainfall Frequency Analysis in Contrasting Climates of Mexico

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Abstract

This study proposes a bivariate distribution with Exponentiated Gumbel (BEG) marginals to estimate return levels of annual maximum daily rainfall in Mexico. A dataset of 181 gauging stations from two contrasting climatic regions was analyzed, and the BEG model was compared against the Generalized Extreme Value (GEV), Gumbel (EVI), and Exponentiated Gumbel (EG) distributions. Parameters were estimated using the maximum likelihood method. Goodness-of-fit assessment based on AICc and BIC indicated that 71.8% of the samples were better described by the BEG model than by the univariate alternatives. Moreover, differences in return level estimates became more pronounced at higher non-exceedance probabilities. These results suggest that the BEG distribution provides a robust and reliable tool for the frequency analysis of extreme rainfall.

Keywords: extreme rainfall; frequency analysis; bivariate distribution; exponentiated gumbel; maximum likelihood estimation: AICc and BIC; return levels

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1. Introduction

[1] highlighted that variations in the hydrological cycle have profound impacts on human activities, including an increased risk of flooding as well as the occurrence and severity of droughts. Floods are among the most damaging natural hazards worldwide, and their impacts have intensified due to factors such as irregular human settlements near rivers, deforestation, and continuous land-use changes.

Mexico is especially vulnerable to hydrometeorological events that cause widespread damage across its territory. Between 2007 and 2020, heavy rains, floods, and tropical cyclones affected 27.8 million people, resulting in 1,173 fatalities and economic losses exceeding 24.6 billion USD [2]. Analyses of rainfall patterns in Mexico [3,4] indicate that floods caused by extreme rainfall events are likely to become more frequent and intense in the future. Given the potential consequences of underestimating or overestimating rainfall quantiles—ranging from hydraulic structure failure to unnecessarily inflated construction costs—reliable statistical tools are essential for estimating their magnitude and frequency.

Numerous at-site and regional rainfall frequency analyses have been conducted worldwide, and results consistently demonstrate that no single distribution universally fits annual maximum daily rainfall (AMDR) data. Although the Gumbel distribution (EVI) is widely applied, studies such as [5] in Egypt have shown that it is not always appropriate, while in other regions it has been identified as the most suitable model [6–15].

In Italy, [16] analyzed AMDR records from 297 gauging stations and found that the Fréchet distribution (EVIII) provided the best fit. Their study also revealed significant differences in return periods and return levels compared to results from EVI, reversed Weibull (EVII), Pareto, Lognormal, and Gamma distributions, echoing earlier findings by [17,18]. The EVI, EVII, and EVIII distributions are all special cases of the Generalized Extreme Value (GEV) distribution, which has often been identified as the best-fitting model for AMDR samples in multiple regions [19–28]. Similarly, the Log-Pearson Type III (LPIII) distribution has also proven effective in modeling AMDR series [29–34].

In recent years, the Exponentiated Gumbel (EG) distribution was proposed by [35], who demonstrated its improved performance in climate modeling compared to the EVI and GEV distributions. [36] applied EG, exponentiated Weibull (EW), exponentiated Fréchet (EF), and their mixed forms to AMDR series from 19 Mexican stations, showing that exponentiated and mixed exponentiated distributions provide flexible and reliable alternatives for modeling extreme rainfall. More recently, [37] compared the Gumbel and EG distributions, concluding—based on AIC and BIC criteria—that EG offered a more flexible fit for hydrological datasets, corroborating earlier findings by [35].

Regional frequency analysis has also emerged as a practical approach to reduce uncertainties associated with short or incomplete rainfall records at gauged sites. In this context, multivariate joint estimation models have proven valuable, as they enhance the estimation of marginal distribution parameters and improve regional at-site estimates of return levels by incorporating information from neighboring sites within homogeneous regions. Bivariate approaches have shown promise in flood frequency analysis [38–47].

In this study, a bivariate distribution with Exponentiated Gumbel marginals is proposed to improve the estimation of marginal parameters and corresponding quantiles. The performance of this model is compared with that of univariate probability distributions, namely GEV, EVI, and EG.

2. Materials and Methods

2.1. Study Area

Mexico is characterized by a summer–rainy and winter–dry regime, except in the northwest. Annual precipitation ranges from <500 mm in the north and northwest to >2,000 mm in the humid south and southeast. Two states that illustrate this climatic contrast are Coahuila, in northern Mexico, and Tabasco, in the southeast (Figure 1).

Coahuila (151,563 km²) has a semi-warm summer and cold winter climate, with a mean annual temperature of 20 °C (min. 4 °C, max. 30 °C). Rainfall is scarce, averaging ~400 mm yr^{-1}, concentrated in summer. Maximum daily rainfall varies between 11.3 and 453 mm. Recent extreme events have produced floods affecting Torreon, Monclova, and Piedras Negras.

Tabasco (25,267 km²) is predominantly warm-humid, with abundant summer rains (75.97%), year-round humid conditions (19.64%), and sub-humid summer rains (4.39%). The mean annual temperature is 27 °C (min. 18.5 °C, max. 36 °C). Rainfall occurs throughout the year, peaks from June to October, and averages 2,550 mm yr^{-1}. Maximum daily rainfall ranges from 38.8 to 816.9 mm.

The dataset consists of records from 106 rain gauge stations in Coahuila and 75 stations in Tabasco, obtained from [48]. For each station, annual maximum daily rainfall values were analyzed, with detailed station characteristics provided in Tables 1 and 2.

Table 1. List of 106 rain gauge stations in Coahuila and their corresponding characteristics.

No	(1)	(2)	(3)	(4)	(5)	(6)	(7)	No	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	5001	100.58	27.93	370	64	1950-2013	0.567	54	5068	100.99	29.16	330	27	1987-2013	0.508
2	5002	100.83	28.33	374	61	1950-2010	0.472	55	5069	101.51	27.87	490	34	1980-2013	0.402
3	5003	100.85	25.45	1660	20	1956-1975	0.685	56	5074	100.92	28.49	360	23	1979-2001	0.571
4	5004	102.63	25.12	1300	50	1967-2016	0.475	57	5075	100.85	28.35	380	23	1979-2001	0.570
5	5005	100.76	26.84	582	45	1967-2011	0.662	58	5081	101.11	25.12	2100	45	1969-2013	0.407
6	5006	100.76	26.84	582	41	1970-2010	0.466	59	5085	101.11	25.12	2100	27	1987-2013	0.553
7	5007	103.12	25.78	1105	67	1950-2016	0.428	60	5086	100.96	29.04	340	27	1987-2013	0.685
8	5008	101.38	27.96	380	34	1980-2013	0.331	61	5130	101.32	25.85	1100	45	1969-2013	0.526
9	5009	102.07	26.97	730	33	1981-2013	0.459	62	5133	103.25	25.33	1211	47	1970-2016	0.614
10	5011	101.08	26.13	936	59	1955-2013	0.559	63	5135	103.29	28.09	1080	24	1990-2013	1.424
11	5013	102.95	28.64	1060	35	1979-2013	0.641	64	5136	100.86	24.96	2110	34	1980-2013	0.442
12	5015	102.82	28.80	1040	24	1990-2013	0.516	65	5139	102.94	25.49	1110	67	1950-2016	1.054
13	5016	101.48	25.38	1400	45	1969-2013	0.418	66	5140	100.95	25.54	1400	32	1980-2011	0.773

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14	5018	102.01	25.73	1140	56	1961-2016	0.427	67	5141	101.03	24.96	1920	60	1954-2013	0.508
15	5019	101.42	26.91	615	24	1987-2010	0.567	68	5142	101.40	25.70	1150	45	1969-2013	0.654
16	5020	101.52	27.87	490	35	1979-2013	0.580	69	5144	102.27	26.61	800	33	1981-2013	0.451
17	5021	101.25	27.92	369	34	1980-2013	0.290	70	5145	101.22	25.25	1840	45	1969-2013	0.534
18	5022	102.40	27.31	1100	42	1960-2001	0.552	71	5146	100.83	25.21	2100	60	1954-2013	0.699
19	5023	100.99	29.16	340	27	1987-2013	0.543	72	5147	101.23	27.23	463	34	1980-2013	0.694
20	5024	102.17	25.44	1500	56	1961-2016	0.474	73	5148	100.34	25.28	1740	60	1954-2013	0.904
21	5025	100.52	28.70	250	32	1979-2010	0.390	74	5149	100.53	25.34	2420	60	1954-2013	0.825
22	5026	103.47	25.54	1223	42	1969-2010	0.448	75	5150	101.43	27.18	430	33	1981-2013	0.452
23	5027	103.34	25.70	1120	61	1950-2010	0.557	76	5151	101.25	25.98	920	59	1955-2013	0.500
24	5028	103.05	25.76	1110	67	1950-2016	0.428	77	5152	101.26	26.53	820	33	1981-2013	0.426
25	5029	103.28	25.07	1300	50	1967-2016	0.773	78	5153	101.42	26.77	743	24	1987-2010	0.565
26	5030	100.62	27.52	272	64	1950-2013	0.421	79	5155	101.79	27.05	640	33	1981-2013	0.510
27	5031	101.00	27.42	360	64	1950-2013	0.472	80	5156	101.40	27.89	430	34	1980-2013	0.386
28	5032	100.98	25.53	1470	62	1950-2011	0.588	81	5157	102.73	26.83	900	36	1981-2016	0.457
29	5033	101.12	27.85	339	64	1950-2013	0.446	82	5158	101.31	26.62	920	33	1981-2013	0.414
30	5034	101.12	27.85	339	67	1950-2016	0.474	83	5159	103.04	26.48	1100	61	1950-2010	0.455
31	5035	100.58	25.27	2180	45	1954-1998	0.539	84	5160	100.85	25.45	1660	18	1988-2005	1.008
32	5036	103.00	25.76	1100	67	1950-2016	0.493	85	5162	101.58	25.36	1550	45	1969-2013	0.495
33	5037	102.22	25.62	1170	60	1957-2016	0.462	86	5163	101.73	27.23	640	34	1980-2013	0.487
34	5038	101.35	26.39	1010	33	1981-2013	0.653	87	5164	101.65	27.14	500	33	1981-2013	0.389
35	5039	103.70	27.29	1256	54	1960-2013	0.557	88	5166	101.35	27.75	450	34	1980-2013	0.403
36	5040	103.43	25.52	1123	41	1970-2010	0.572	89	5167	101.36	26.64	660	33	1981-2013	0.371
37	5041	102.81	25.32	1100	45	1972-2016	0.477	90	5168	103.42	28.38	1180	24	1990-2013	0.654
38	5042	100.93	28.49	360	24	1954-1977	0.747	91	5169	101.37	27.20	850	33	1981-2013	0.414
39	5043	100.85	28.34	380	23	1979-2001	0.585	92	5170	101.39	25.52	1680	45	1969-2013	1.137
40	5044	102.07	26.99	740	64	1950-2013	0.482	93	5171	101.72	27.00	1275	33	1981-2013	0.782
41	5045	100.73	27.61	280	64	1950-2013	0.600	94	5174	100.88	25.45	1670	31	1981-2011	0.483
42	5046	103.66	27.29	1383	13	1953-1965	0.870	95	5175	100.89	24.64	1867	41	1973-2013	0.488
43	5047	101.42	26.90	586	35	1951-1985	0.509	96	5176	100.62	25.37	2759	60	1954-2013	0.437
44	5048	101.00	25.43	1700	64	1950-2013	0.579	97	5178	103.21	25.28	1200	47	1970-2016	0.526
45	5049	100.62	25.28	2300	26	1988-2013	0.434	98	5179	102.21	26.93	1091	36	1981-2016	0.550
46	5050	101.53	27.05	510	33	1981-2013	0.344	99	5180	103.26	25.77	1116	67	1950-2016	0.375
47	5051	102.80	25.35	1093	67	1950-2016	0.544	100	5181	102.62	25.62	1109	18	1991-2008	0.597
48	5052	102.80	25.35	1093	24	1987-2010	0.528	101	5182	102.22	26.94	836	33	1981-2013	0.376
49	5058	103.30	28.45	1080	24	1990-2013	0.445	102	5184	102.95	24.81	1460	50	1967-2016	0.328
50	5060	101.25	25.27	1880	45	1969-2013	0.639	103	5185	102.95	24.81	1460	67	1950-2016	0.327
51	5063	100.86	28.35	380	23	1979-2001	0.542	104	5186	101.08	29.04	348	27	1987-2013	0.474
52	5065	100.76	26.84	420	61	1951-2011	0.546	105	5188	101.11	26.85	958	33	1981-2013	0.496
53	5066	101.12	27.84	339	34	1980-2013	0.332	106	5189	100.77	27.59	285	64	1950-2013	0.527

(1) Station code, (2) Longitude (W), (3) Latitude (N), (4) Altitude (masl), (5) Record length (yr.), (6) Period of data, and (7) Coefficient of variation of annual maximum daily rainfall series (Cv).

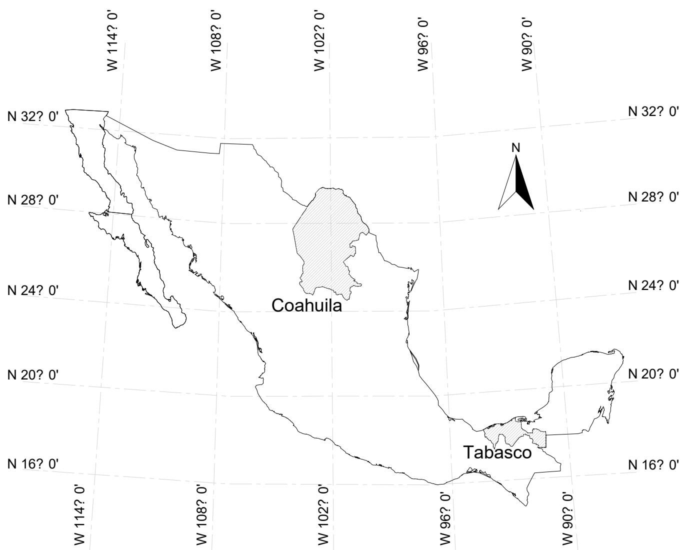

Figure 1. Location of the study areas within Mexico.

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Table 2. List of 75 rain gauge stations in Tabasco and their corresponding characteristics.

No	1	2	3	4	5	6	7	No	1	2	3	4	5	6	7
1	27001	17.81	91.54	18.00	32	1950-1981	0.355	39	27042	17.46	92.78	44.00	58	1960-2017	0.347
2	27002	18.42	92.80	3.00	68	1950-2017	0.495	40	27044	17.55	92.95	51.00	58	1960-2017	0.270
3	27003	18.05	93.97	5.00	56	1961-2016	0.538	41	27045	17.57	92.97	54.00	56	1962-2017	0.283
4	27004	17.45	91.49	14.00	67	1950-2016	0.312	42	27046	17.47	91.43	19.00	63	1954-2016	0.323
5	27006	17.64	91.27	50.00	63	1954-2016	0.415	43	27047	17.47	91.43	22.00	63	1954-2016	0.332
6	27007	18.00	93.62	12.00	57	1961-2017	0.378	44	27048	17.82	92.37	7.00	68	1950-2017	0.373
7	27008	18.00	93.38	25.00	63	1955-2017	0.416	45	27049	17.72	92.81	13.00	67	1951-2017	0.418
8	27009	18.25	93.22	15.00	68	1950-2017	0.415	46	27050	18.38	92.60	2.00	68	1950-2017	0.394
9	27010	18.07	93.18	15.00	63	1955-2017	0.375	47	27051	18.11	93.35	20.00	68	1950-2017	0.353
10	27011	17.61	92.80	20.00	67	1951-2017	0.521	48	27053	18.39	92.88	6.00	68	1950-2017	0.381
11	27012	17.74	91.76	26.00	23	1995-2016	0.254	49	27054	18.00	92.93	24.00	27	1972-1998	0.449
12	27013	18.25	93.55	11.00	15	1965-1979	0.341	50	27055	17.98	92.92	6.00	48	1950-1997	0.407
13	27014	18.02	92.93	7.00	13	1968-1980	0.567	51	27056	17.81	91.54	12.00	31	1986-2016	0.402
14	27015	17.84	93.94	7.00	64	1954-2017	0.312	52	27057	18.27	93.22	13.00	68	1950-2017	0.569
15	27016	18.53	92.63	2.00	68	1950-2017	0.409	53	27059	17.94	91.18	41.00	64	1954-2017	0.407
16	27017	17.83	93.39	36.00	68	1950-2017	0.337	54	27060	17.97	93.77	11.00	57	1961-2017	0.352
17	27018	17.87	93.47	29.00	68	1950-2017	0.335	55	27061	17.51	92.92	86.00	56	1962-2017	0.345
18	27019	17.72	92.81	14.00	67	1951-2017	0.303	56	27065	17.99	92.83	14.00	48	1950-1997	0.400
19	27020	18.17	93.05	10.00	68	1950-2017	0.500	57	27068	17.53	92.93	78.00	56	1962-2017	0.314
20	27021	17.76	91.29	29.00	63	1954-2016	0.356	58	27069	17.86	91.78	5.00	31	1986-2016	0.242
21	27022	17.63	92.54	24.00	50	1950-1999	0.350	59	27070	17.38	92.75	63.00	58	1960-2017	0.329
22	27024	17.52	92.93	80.00	56	1962-2017	0.364	60	27071	17.80	92.48	11.00	67	1950-2016	0.361
23	27026	18.10	94.05	10.00	63	1954-2016	0.484	61	27073	18.17	93.49	7.00	46	1972-2017	0.449
24	27027	17.59	92.70	22.00	67	1951-2017	0.370	62	27075	18.11	93.57	10.00	46	1972-2017	0.434
25	27028	18.09	92.14	6.00	68	1950-2017	0.398	63	27076	18.11	93.50	13.00	46	1972-2017	0.548
26	27029	18.14	92.86	10.00	68	1950-2017	0.369	64	27077	18.07	93.62	12.00	46	1972-2017	0.392
27	27030	17.76	92.61	11.00	67	1950-2016	0.283	65	27078	18.02	93.50	19.00	46	1972-2017	0.439
28	27031	17.75	92.60	10.00	67	1950-2016	0.436	66	27079	18.05	93.44	21.00	11	1969-1979	0.369
29	27032	17.65	93.40	44.00	68	1950-2017	0.319	67	27080	17.97	93.50	21.00	57	1961-2017	0.370
30	27033	17.73	93.63	50.00	68	1950-2017	0.329	68	27083	17.98	92.93	27.00	12	2005-2016	0.468
31	27034	18.40	93.21	6.00	68	1950-2017	0.369	69	27084	18.17	93.02	10.00	68	1950-2017	0.457
32	27035	17.76	93.38	36.00	68	1950-2017	0.303	70	27087	17.99	91.39	54.00	31	1986-2016	0.320
33	27036	18.07	93.18	15.00	68	1950-2017	0.439	71	27088	17.62	91.55	67.00	17	1983-1999	0.432
34	27037	17.85	93.88	21.00	68	1950-2017	0.385	72	27090	17.97	91.60	10.00	31	1986-2016	0.331
35	27038	18.30	93.07	7.00	68	1950-2017	0.524	73	27091	17.94	91.81	5.00	31	1986-2016	0.303
36	27039	18.00	93.28	23.00	68	1950-2017	0.375	74	27092	17.85	92.93	18.00	18	2000-2017	0.313
37	27040	17.79	91.16	44.00	65	1952-2016	0.403	75	27093	18.01	91.56	14.00	31	1986-2016	0.295
38	27041	18.09	93.35	20.00	63	1955-2017	0.337								

(1) Station code, (2) Longitude (W), (3) Latitude (N), (4) Altitude (masl), (5) Record length (yr.), (6) Period of data, and (7) Coefficient of variation of annual maximum daily rainfall series (Cv).

2.2. Delineation of Homogeneous Regions

The joint parameter estimation model requires that all samples belong to the same homogeneous region. In this study, regions were delineated using the coefficient of variation (Cv) as the measure of comparability among stations.

In the first stage, three initial zones were identified by setting confidence limits defined as the mean Cv ± one standard deviation. At this stage, most stations were concentrated in region 2 (Figure 2), leaving only the stations in regions 1 and 3 distinctly separated.

In the second stage, the stations within region 2 were reclassified by recalculating new confidence limits and repeating the same procedure. This subdivision resulted in four regions (1, 2, 4, and 5) (Figure 3).

The process was repeated iteratively, each time focusing on the stations remaining in the central region (closer to the mean Cv value). In this case, region 3 was progressively subdivided until a sufficient distribution of stations across regions was achieved. After the final iteration, seven homogeneous regions were identified (Figure 4).

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It is important to note that the total number of stages required depends on the number of stations analyzed, since the central group—located around the mean Cv—will ultimately determine how many subdivisions are necessary to achieve adequately homogeneous regions.

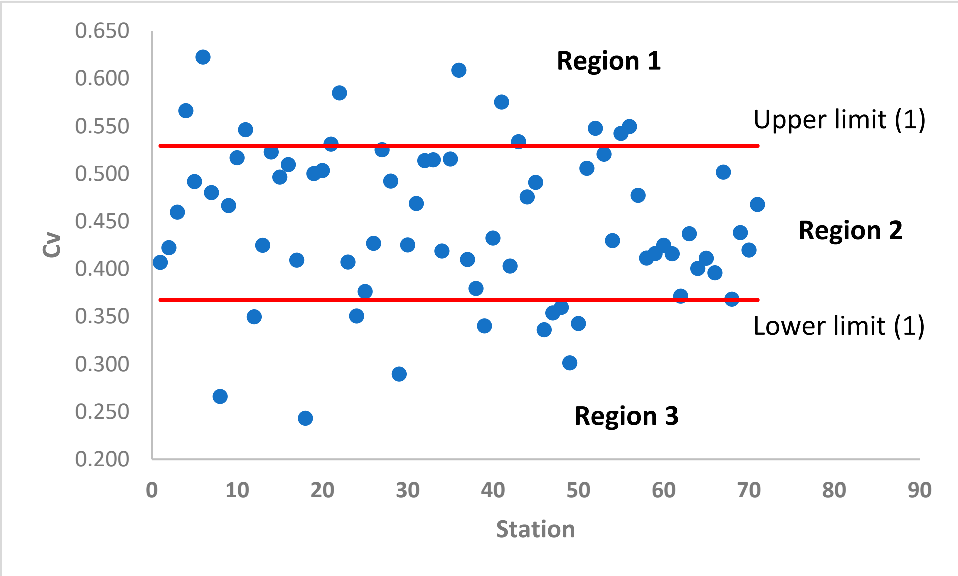

Figure 2. First stage for delineation of homogeneous regions.

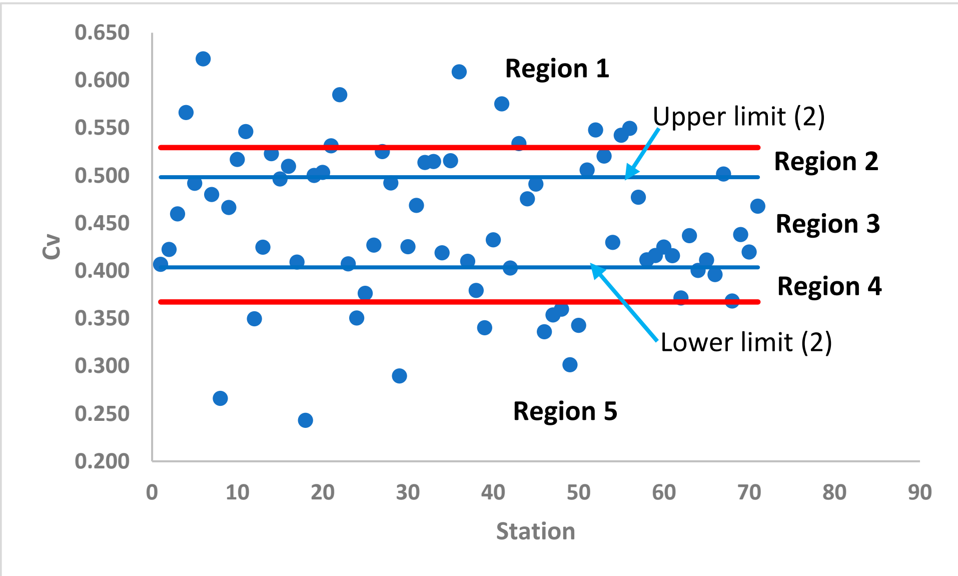

Figure 3. Second stage for delineation of homogeneous regions.

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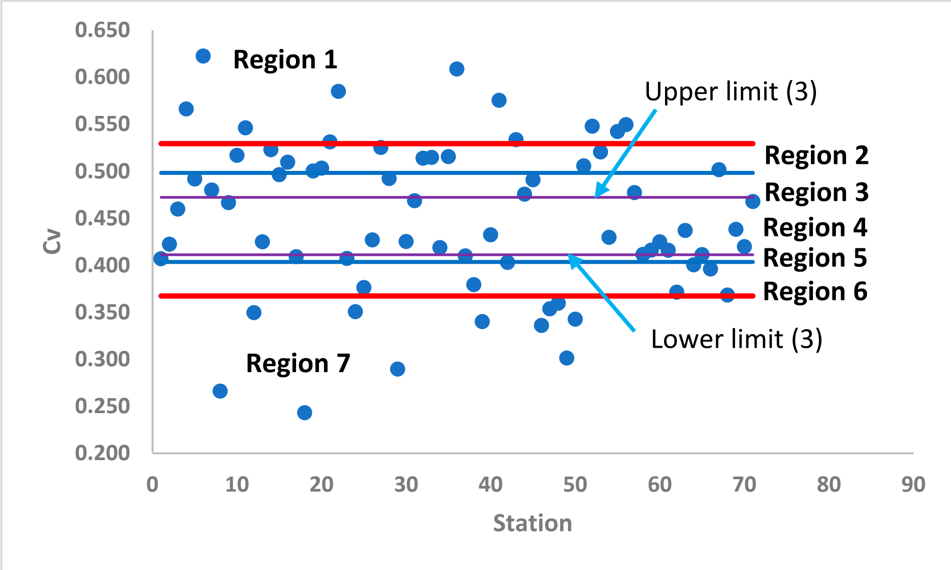

Figure 4. Final stage for delineation of homogeneous regions.

2.3. Univariate Distributions

a. GEV distribution

The cumulative distribution function is [49]:

$$F(x) = exp\left\{-\left[1-\left(\frac{x-\varepsilon}{\lambda}\right)\beta\right]^{\frac{1}{\beta}}\right\} \tag{1}$$

where $\varepsilon, \lambda, \beta$ are the location, scale, and shape parameters, respectively.

And

$$f(x) = \frac{1}{\lambda} exp\left\{-\left[1-\left(\frac{x-\varepsilon}{\lambda}\right)\beta\right]^{\frac{1}{\beta}}\right\}\left[1-\left(\frac{x-\varepsilon}{\lambda}\right)\beta\right]^{\frac{1}{\beta}-1} \tag{2}$$

The log-likelihood for the GEV distribution is:

$$\ln L(x;\varepsilon,\lambda,\beta) = -n\ln\lambda + \sum_{i=1}^{n}\left\{-\left[1-\left(\frac{x_i-\varepsilon}{\lambda}\right)\beta\right]^{\frac{1}{\beta}}\right\} + \left(\frac{1}{\beta}-1\right)\sum_{i=1}^{n}\ln\left[1-\left(\frac{x_i-\varepsilon}{\lambda}\right)\beta\right] \tag{3}$$

The quantile $\hat{x}_T$ corresponding to a given return period T is:

$$\hat{x}_T = \hat{\varepsilon} + \hat{\lambda}\left\{\frac{1-\left[-\ln\left(1-\frac{1}{T}\right)\right]^{\hat{\beta}}}{\hat{\beta}}\right\} \tag{4}$$

b. EVI (Gumbel) distribution

$$F(x) = exp\left\{-exp\left[-\left(\frac{x-\nu}{\alpha}\right)\right]\right\} \tag{5}$$

$$f(x) = \frac{1}{\alpha} exp\left\{-exp\left[-\left(\frac{x-\nu}{\alpha}\right)\right]\right\} exp\left[-\left(\frac{x-\nu}{\alpha}\right)\right] \tag{6}$$

where ν, α are the location and scale parameters $\alpha > 0$

$$\ln L(x;\nu,\alpha) = -n\ln\alpha - \sum_{i=1}^{n} exp\left[-\left(\frac{x-\nu}{\alpha}\right)\right] + \sum_{i=1}^{n}\left[-\left(\frac{x_i-\nu}{\alpha}\right)\right] \tag{7}$$

$$\hat{x}_T = \hat{\nu} - \hat{\alpha}\ln\left[-\ln\left(1-\frac{1}{T}\right)\right] \tag{8}$$

c. Exponentiated Gumbel distribution (EG) [50].

$$F(x) = 1 - \left\{1 - exp\left\{-exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right]\right\}\right\}^{\kappa} \tag{9}$$

$$f(x) = \frac{\kappa}{\sigma}\left\{1 - exp\left\{-exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right]\right\}\right\}^{\kappa-1} exp\left\{-exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right]\right\} exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right] \tag{10}$$

$$\ln L(x;\mu,\sigma,\kappa) = n\ln\kappa - n\ln\sigma + (\kappa-1)\sum_{i=1}^{n}\ln\left\{1-exp\left\{-exp\left[-\left(\frac{x_i-\mu}{\sigma}\right)\right]\right\}\right\} - \sum_{i=1}^{n} exp\left[-\left(\frac{x_i-\mu}{\sigma}\right)\right] + \sum_{i=1}^{n}\left[-\left(\frac{x_i-\mu}{\sigma}\right)\right] \tag{11}$$

$$\hat{x}_T = \hat{\mu} - \hat{\sigma}\ln\left[-ln\left(1-\frac{1}{T}\right)^{\frac{1}{\hat{\kappa}}}\right] \tag{12}$$

2.4. Biexponentiated Gumbel Distribution (BEG)

As already mentioned, multivariate extreme value distributions have shown to be a reliable option for fitting hydrological variables. [51] compared the mixed and logistic bivariate models and concluded that the logistic model (LM) is the best option in flood frequency analysis.

The LM is [52]:

$$[-\log F(x,y,m)]^m = [-\log F(x)]^m + [-\log F(y)]^m \tag{13}$$

where x and y denote the extreme events gauged in a couple of neighboring sites.

and

$$f(x,y,m) = \frac{\partial^2 F(x,y,m)}{\partial x \partial y}; \quad f(x,y,m) \geq 0 \tag{14}$$

The bivariate likelihood function is:

$$L(x,y,\theta) = \prod_{i=1}^{n} f(x_i, y_i, \theta) \tag{15}$$

The log-likelihood function is:

$$\ln L(x,y,\theta) = \sum_{i=1}^{n} \ln f(x_i, y_i, \theta) \tag{16}$$

The expression (16) is used when both samples share a common length of record, however, the general form for considering different sizes is [53]:

$$\ln L(x,y,\theta) = I_1\left[\sum_{i=1}^{n_1} \ln f(r_i,\theta)\right] + I_2\left[\sum_{i=1}^{n_2} \ln f(x_i,y_i,\theta)\right] + I_3\left[\sum_{i=1}^{n_3} \ln f(s_i,\theta)\right] \tag{17}$$

Marginals F(x) and F(y) are proposed to be EG distributions. The BEG log-likelihood function to maximize in the parameter estimation procedure is:

$$\begin{aligned} \ln L(x,y,\theta) = {} & I_1\left[n_1 \ln \kappa_p - n_1 \ln \sigma_p + (\kappa_p - 1)\sum_{i=1}^{n_1} \log\left\{1 - exp\left\{-exp\left[-\left(\frac{r_i-\mu_p}{\sigma_p}\right)\right]\right\}\right\} - \right. \\ & \left. \sum_{i=1}^{n_1} exp\left[-\left(\frac{r_i-\mu_p}{\sigma_p}\right)\right] + \sum_{i=1}^{n_1}\left[-\left(\frac{r_i-\mu_p}{\sigma_p}\right)\right]\right] + \\ & I_2\left[\sum_{i=1}^{n_2} \ln\left\{\frac{f(x)}{F(x)}\frac{f(y)}{F(y)}(-\ln F(x))^{m-1}(-\ln F(y))^{m-1} \exp\left\{-\left[(-\ln F(x))^m + (-\ln F(y))^m\right]^{1/m}\right\}\left[(-\ln F(x))^m + \right.\right.\right. \\ & \left.\left.\left. (-\ln F(y))^m\right]^{\frac{1}{m}-2}\left\{(m-1) + \left[(-\ln F(x))^m + (-\ln F(y))^m\right]^{1/m}\right\}\right\}\right] + I_3\left[n_3 \ln \kappa_q - n_3 \ln \sigma_q + (\kappa_q - \right. \\ & \left. 1)\sum_{i=1}^{n_3} \log\left\{1 - exp\left\{-exp\left[-\left(\frac{s_i-\mu_q}{\sigma_q}\right)\right]\right\}\right\} - \sum_{i=1}^{n_3} exp\left[-\left(\frac{s_i-\mu_q}{\sigma_q}\right)\right] + \sum_{i=1}^{n_1}\left[-\left(\frac{s_i-\mu_q}{\sigma_q}\right)\right]\right] \end{aligned} \tag{18}$$

where x and y are the variables in the common period (CP) with length n_2, r denotes to the variable x or y with length n_1 before the CP, s is the variable x or y with length n_3 after the CP; $I_j = 1$ if $n_j > 0$ or $I_j = 0$ if $n_j = 0$ for j = 1, 2, 3; p =1 and q = 1 if r or s represent to variable x; p = 2 and q = 2 if r or s characterize to variable y, and

$$F(x) = 1 - \left\{1 - exp\left\{-exp\left[-\left(\frac{x_i-\mu_1}{\sigma_1}\right)\right]\right\}\right\}^{\kappa_1} \tag{19}$$

$$f(x) = \frac{\kappa_1}{\sigma_1}\left\{1 - exp\left\{-exp\left[-\left(\frac{x_i-\mu_1}{\sigma_1}\right)\right]\right\}\right\}^{\kappa_1-1} exp\left\{-exp\left[-\left(\frac{x_i-\mu_1}{\sigma_1}\right)\right]\right\} exp\left[-\left(\frac{x_i-\mu_1}{\sigma_1}\right)\right] \tag{20}$$

$$F(y) = 1 - \left\{1 - exp\left\{-exp\left[-\left(\frac{y_i-\mu_2}{\sigma_2}\right)\right]\right\}\right\}^{\kappa_2} \tag{21}$$

$$f(y) = \frac{\kappa_2}{\sigma_2}\left\{1 - exp\left\{-exp\left[-\left(\frac{y_i-\mu_2}{\sigma_2}\right)\right]\right\}\right\}^{\kappa_2-1} exp\left\{-exp\left[-\left(\frac{y_i-\mu_2}{\sigma_2}\right)\right]\right\} exp\left[-\left(\frac{y_i-\mu_2}{\sigma_2}\right)\right] \tag{22}$$

The Rosenbrock optimization algorithm for constrained variables [54] was selected for estimating the univariate and bivariate parameters by the direct maximization of equations (3), (7), (11), and (18).

2.5. Selection of Best Fit

The selection of fit between the Empirical and Theoretical distribution of the AMDR was based on the AICc and BIC goodness of fit tests.

The AIC was proposed by Akaike [55]:

$$\text{AIC} = -2\ln L(\hat{\theta}) + 2p \tag{23}$$

while the BIC is [56]:

$$\mathrm{BIC} = -2\ln L(\hat{\theta}) + p\ln(n) \quad (24)$$

where $\ln L(\hat{\theta})$ represents the log-likelihood of empirical distribution, p the number of maximum-likelihood estimates of the parameters, and n the length of record.

For small n, a corrected version of AIC is proposed as [57]:

$$\mathrm{AICc} = \mathrm{AIC} + \frac{2p(p+1)}{n-p-1} \quad (25)$$

Distribution having least value of AICc, or BIC is considered as best model.

2.6. Reliability of Estimated Quantiles

It is very important to evaluate whether the BEG distribution provides more accurate and reliable quantile estimates than traditional univariate approaches. This evaluation is essential in hydrological frequency analysis, since underestimation of quantiles can increase the risk of hydraulic structure failure, while overestimation may result in unnecessarily high construction costs. To assess reliability, quantile estimates obtained from the BEG model were compared with those from univariate distributions using two statistical criteria: bias (BIAS) and mean squared error (MSE). This framework allowed us to determine not only the accuracy of the estimated quantiles but also the extent to which the joint estimation procedure enhances the transfer of information across sites, thereby improving the robustness of extreme rainfall frequency analysis.

To formalize this evaluation, the reliability of estimated quantiles was expressed in terms of bias and mean squared error, which are defined as follows:

Let $\hat{\psi}$ be the quantile to be computed by using the BEG distribution:

$$\mathrm{BIAS} = m(\hat{\psi}) - \psi \quad (26)$$

$$\mathrm{MSE} = S^2(\hat{\psi}) + [m(\hat{\psi}) - \psi]^2 \quad (27)$$

For the number of simulated samples “ n_s”

$$m(\hat{\psi}) = \frac{1}{n_s}\sum_{i=1}^{n_s}\hat{\psi}_i \quad (28)$$

And

$$S^2(\hat{\psi}) = \frac{1}{n_s}\sum_{i=1}^{n_s}[\hat{\psi}_i - m(\hat{\psi})]^2 \quad (29)$$

When estimating quantiles is desirable to have unbiased and minimum MSE estimators.

3. Results

A procedure of data generation was performed to show if the quantiles obtained through of the bivariate joint estimation of parameters are more reliable with those obtained by its univariate counterpart.

For the EG distribution, data were generated using population parameters $\mu_1 = 15.8$, $\sigma_1 = 2.5$ and $\kappa_1 = 1.3$ with sample sizes n = 10, 20 and 50. A total of 1,000 simulated samples were considered for each n.

For the BEG distribution, quantiles were obtained by combining samples with sizes n_1-n_2: 10-10, 10-20, 20-20, 20-50, 50-50 and 50-100. Comparisons were performed for non-exceedance probabilities of 0.50, 0.80, 0.90. 095 0.98 and 0.99. The associated site has population parameters $\mu_2 = 18.7$, $\sigma_2 = 3.0$ and $\kappa_2 = 1.1$.

Results (Tables 3 and 4) indicate that as n_2 increased relative to n_1, both BIAS and MSE of the shorter series decreased. This demonstrates an effective transfer of information when parameters are jointly estimated, supporting the conclusion that quantiles computed using the BEG distribution are more reliable than those obtained from the univariate case.

Table 3. Quantile biases obtained for the EG marginal with length n_1.

Sample Sizes		Non exceedance probability					
n_1	n_2	0.50	0.80	0.90	0.95	0.98	0.99
10	10	-0.0394	0.1823	0.3623	0.5524	0.8167	1.0247
10	20	-0.0944	0.0571	0.1929	0.3415	0.5531	0.7220

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20	20	-0.0849	0.0387	0.1474	0.2657	0.4336	0.5673
20	50	-0.1037	-0.0427	0.0300	0.1167	0.2466	0.3533
50	50	-0.0764	-0.0112	0.0596	0.1418	0.2630	0.3617
50	100	-0.0618	-0.0554	0.0470	0.0612	0.1644	0.2510

Table 4. Quantile MSE's obtained for the EG marginal with length n_1.

Sample Sizes		**Non exceedance probability**					
n_1	n_2	**0.50**	**0.80**	**0.90**	**0.95**	**0.98**	**0.99**
10	10	0.9016	2.1029	3.3334	4.8397	7.2743	9.4667
10	20	0.8650	1.8796	3.0143	4.4338	6.7517	8.8482
20	20	0.4006	0.8913	1.4458	2.1516	3.3257	4.4039
20	50	0.3771	0.8721	1.4153	2.0979	3.2235	4.2517
50	50	0.1683	0.3966	0.6547	0.9854	1.5414	2.0573
50	100	0.1659	0.3916	0.6339	0.9398	1.4491	1.9196

The rain gauge stations in Coahuila and Tabasco were classified into homogeneous regions by applying the delineation procedure previously described. This methodological approach yielded nine regions in Coahuila (Table 5) and seven regions in Tabasco (Table 6), ensuring consistency with the adopted regionalization framework.

Table 5. Stations by homogeneous region "R" in Coahuila.

R1	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**R8**	**R9**
5029	5003	5001	5022	5015	5002	5009	5007	5008
5042	5005	5011	5023	5036	5004	5026	5016	5021
5046	5013	5019	5035	5044	5006	5033	5018	5050
5135	5038	5020	5051	5047	5024	5037	5025	5066
5139	5045	5027	5063	5052	5031	5049	5028	5167
5140	5060	5032	5065	5068	5034	5058	5030	5184
5148	5086	5039	5085	5130	5041	5136	5069	5185
5149	5133	5040	5179	5141	5186	5144	5081	
5160	5142	5043		5145		5150	5152	
5170	5146	5048		5151		5157	5156	
5171	5147	5074		5155		5159	5158	
	5168	5075		5162		5176	5164	
	5181	5153		5163			5166	
				5174			5169	
				5175			5180	
				5178			5182	
				5188				
				5189				

Table 6. Stations by homogeneous region "R" in Tabasco.

R1	R2	R3	R4	R5	R6	R7
27002	27006	27016	27001	27013	27004	27012
27003	27008	27028	27007	27017	27015	27019
27011	27009	27040	27010	27018	27032	27030
27014	27031	27050	27021	27041	27033	27035
27020	27036	27055	27022	27042	27046	27044
27026	27049	27056	27024	27061	27047	27045
27038	27054	27059	27027		27068	27069
27057	27073	27065	27029		27070	27091

27076	27075	27034	27087	27093
27083	27078	27037	27090	
27084	27088	27039	27092	
		27048		
		27051		
		27053		
		27060		
		27071		
		27077		
		27079		
		27080		

The analysis of the Coahuila dataset considered all possible pairwise station combinations within each homogeneous region to evaluate the dependence structure of extreme rainfall. Tables 7 and 8 illustrate representative examples for selected stations, showing the BEG model combinations applied (Table 7) and the corresponding return level estimates (Table 8). When extended to the complete set of stations, the procedure enabled a systematic assessment of return levels using the BEG framework. For comparison, alternative univariate models—Gumbel, Generalized Extreme Value (GEV), and Exponentiated Gumbel—were also fitted to the same stations, and the resulting return levels were ranked according to goodness-of-fit statistics, including the Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) (Table 9). Notably, in 65% of the stations in Coahuila the best fit was achieved with the BEG distribution, highlighting its reliability for modeling extreme rainfall in this region.

Table 7. Examples of BEG model combinations applied to selected stations in Coahuila.

	Station	Neighboring	Relative sample sizes			Bivariate parameters							Marginal (1)	
Region	base	station	n_1	n_2	n_3	Loc 1	Sca 1	Shp 1	Loc 2	Sca 2	Shp 2	m	AICc	BIC
3	5020	5032	29	33	2	67.54	37.27	1.22	31.88	12.98	0.83	1.15	342.0	345.9
		5040	9	32	3	64.77	37.85	1.11	35.08	18.13	0.83	1.14	347.5	351.4
		5153	8	24	3	57.38	36.47	0.93	39.76	23.10	0.96	1.04	353.4	357.3
		5019	8	24	3	55.72	32.19	0.81	32.52	16.02	0.92	1.09	356.7	360.6
		5001	29	35	0	64.38	37.87	1.07	51.19	30.33	1.00	1.04	358.3	362.1
		5074	0	23	12	59.85	36.82	0.94	70.66	29.77	0.92	1.20	360.9	364.7
		5075	0	23	12	63.11	37.02	1.01	71.22	37.63	0.84	1.21	371.8	375.7
		5043	0	23	12	64.17	37.22	1.03	69.69	36.79	0.81	1.21	373.6	377.5
5	5163	5044	30	34	0	52.47	26.04	1.08	28.28	10.82	0.54	1.12	307.7	311.5
		5145	11	34	0	42.86	21.58	0.70	37.11	12.00	0.83	1.29	311.6	315.4
		5178	10	34	3	45.83	21.41	0.69	37.77	14.68	0.89	1.37	313.8	317.5
		5162	11	34	0	50.50	24.56	0.97	35.09	14.31	0.72	1.02	314.6	318.4
		5141	26	34	0	49.01	24.53	0.98	38.17	13.42	0.80	1.05	314.7	318.4
		5052	7	24	3	47.41	21.63	0.88	35.32	17.94	0.86	1.14	323.2	327.0
		5174	1	31	2	52.21	23.84	1.09	31.13	11.25	0.73	1.16	324.3	328.1
		5188	1	33	0	44.00	19.64	0.70	46.18	22.24	1.12	1.07	327.1	330.9
		5155	1	33	0	50.34	24.18	1.00	43.41	23.08	0.98	1.37	327.7	331.5
		5047	29	6	28	49.54	23.72	1.00	47.53	22.96	0.90	2.31	332.0	335.7
		5189	30	34	0	50.33	24.55	1.00	59.26	24.58	0.87	1.16	380.6	384.4
		5068	7	27	0	51.23	24.55	1.00	79.24	35.89	0.83	1.03	391.8	395.6
7	5009	5136	1	33	0	31.74	15.29	0.89	33.32	11.26	0.85	1.10	282.7	286.4
		5144	0	33	0	32.04	15.94	0.91	28.97	14.07	0.82	1.50	290.3	294.0
		5159	31	30	3	31.51	15.30	0.84	29.56	14.07	0.91	1.05	295.3	299.0
		5176	27	33	0	36.07	16.46	1.09	28.56	9.34	0.59	1.08	296.9	300.5
		5150	0	33	0	31.53	15.29	0.88	33.95	16.04	0.87	1.34	297.6	301.3
		5037	24	33	3	40.51	21.42	1.65	26.68	9.73	0.54	1.06	302.1	305.8
		5049	7	26	0	33.41	16.48	1.04	40.12	15.08	0.81	1.08	308.0	311.7
		5033	31	33	0	31.54	15.67	0.96	60.91	27.60	0.93	1.17	495.1	498.7
8	5018	5158	20	33	3	26.77	12.18	0.80	27.47	11.31	0.98	1.29	453.3	458.9
		5182	20	33	3	27.72	12.21	0.87	28.99	11.06	0.91	1.17	456.1	461.7
		5016	8	45	3	28.37	12.21	0.92	33.40	14.35	0.86	1.19	493.7	499.3
		5081	8	45	3	28.21	12.21	0.94	34.29	11.25	0.71	1.06	498.0	503.6

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5152	20	33	3	26.92	12.22	0.81	42.30	18.92	0.89	1.05	514.9	520.5
5164	20	33	3	28.36	12.20	0.91	46.12	13.97	0.87	1.18	516.2	521.8
5166	19	34	3	28.07	12.21	0.91	58.84	26.49	1.08	1.15	601.4	607.0
5156	19	34	3	26.96	12.22	0.83	64.71	24.54	1.03	1.11	607.6	613.2
5069	19	34	3	27.29	12.22	0.82	64.32	26.19	0.92	1.08	625.7	631.3
5025	18	32	6	28.18	12.22	0.88	75.66	28.81	1.01	1.18	674.3	679.9
5030	11	53	3	27.96	12.22	0.88	59.95	24.26	0.96	1.54	726.5	732.1

Loc = location parameter; Sca = Scale parameter; Shp = Shape parameter; Marginal (1) = Station base.

Table 8. Examples of return levels (mm) estimated with the BEG distribution for selected base stations in Coahuila.

Region	Station base	Neighboring Station	Return period T(years) 1.1	2	5	10	20	50	100	500	1000	5000	10000
3	5020	5032	32.0	74.1	110.9	134.6	157.1	185.8	207.3	256.6	277.8	326.8	347.9
		5040	30.2	74.9	114.9	141.1	166.0	198.1	222.1	277.4	301.1	356.2	379.9
		5153	26.5	73.3	116.7	145.7	173.7	209.9	237.2	300.1	327.2	390.1	417.2
		5019	30.5	75.0	117.8	146.8	175.1	212.0	239.8	304.2	331.9	396.2	423.9
		5001	30.3	75.9	116.9	143.9	169.6	202.8	227.7	285.1	309.7	366.9	391.5
		5074	28.5	75.6	119.1	148.2	176.2	212.5	239.7	302.7	329.8	392.7	419.8
		5075	30.5	76.2	117.8	145.3	171.6	205.7	231.2	290.2	315.5	374.4	399.7
		5043	31.2	76.7	118.0	145.2	171.3	204.9	230.2	288.4	313.5	371.6	396.6
5	5163	5044	28.9	60.0	88.0	106.3	123.8	146.3	163.2	202.1	218.8	257.6	274.3
		5145	27.2	59.4	91.3	113.3	134.9	163.2	184.6	234.1	255.5	305.0	326.3
		5178	30.4	62.5	94.3	116.4	138.0	166.4	187.8	237.4	258.8	308.4	329.8
		5162	29.3	60.1	88.5	107.3	125.4	148.8	166.4	207.0	224.5	265.0	282.5
		5141	27.7	58.4	86.5	105.1	123.0	146.2	163.6	203.8	221.1	261.2	278.5
		5052	29.6	58.3	85.2	103.4	120.9	143.7	160.9	200.6	217.7	257.4	274.5
		5174	30.5	58.9	84.3	101.0	116.8	137.3	152.6	187.9	203.0	238.2	253.3
		5188	29.8	59.1	88.2	108.4	128.1	154.0	173.5	218.8	238.3	283.5	303.0
		5155	29.2	59.2	86.6	104.7	122.1	144.6	161.5	200.5	217.2	256.1	272.9
		5047	28.8	58.1	84.9	102.7	119.7	141.7	158.2	196.3	212.7	250.8	267.1
		5189	28.9	59.4	87.4	105.9	123.6	146.6	163.8	203.6	220.8	260.5	277.6
		5068	29.8	60.3	88.1	106.5	124.2	147.1	164.3	204.0	221.0	260.6	277.6
7	5009	5136	19.1	39.2	58.0	70.6	82.8	98.7	110.7	138.3	150.2	177.8	189.7
		5144	18.7	39.5	58.9	71.9	84.5	100.8	113.1	141.4	153.7	182.0	194.2
		5159	19.2	39.9	59.5	72.8	85.7	102.4	115.0	144.3	156.9	186.1	198.7
		5176	21.1	40.7	58.3	69.8	80.7	94.9	105.5	129.8	140.3	164.6	175.1
		5150	19.0	39.2	58.3	71.1	83.6	99.7	111.9	140.0	152.1	180.2	192.3
		5037	17.8	39.0	56.5	67.3	77.4	90.1	99.5	120.7	129.8	150.7	159.7
		5049	18.7	38.7	56.9	68.8	80.2	95.0	106.0	131.5	142.4	167.8	178.8
		5033	18.1	37.9	56.1	68.3	79.9	95.1	106.4	132.7	143.9	170.1	181.4
8	5018	5158	17.2	34.1	50.4	61.5	72.2	86.2	96.8	121.3	131.9	156.4	166.9
		5182	17.7	33.9	49.2	59.5	69.4	82.3	92.1	114.6	124.3	146.8	156.5
		5016	18.1	33.9	48.5	58.4	67.8	80.1	89.3	110.7	119.9	141.2	150.4
		5081	17.8	33.4	47.8	57.5	66.7	78.7	87.8	108.6	117.6	138.4	147.4
		5152	17.3	34.2	50.4	61.5	72.2	86.2	96.7	121.1	131.6	156.0	166.5
		5164	18.1	34.0	48.7	58.6	68.1	80.5	89.8	111.3	120.5	142.0	151.3
		5166	17.8	33.7	48.4	58.3	67.8	80.2	89.5	111.0	120.3	141.8	151.0
		5156	17.2	33.8	49.6	60.3	70.7	84.3	94.5	118.1	128.3	151.9	162.1
		5069	17.6	34.3	50.3	61.2	71.7	85.4	95.8	119.8	130.1	154.0	164.3
		5025	18.1	34.3	49.5	59.7	69.6	82.4	92.1	114.5	124.1	146.5	156.1
		5030	17.9	34.1	49.3	59.6	69.5	82.4	92.1	114.6	124.3	146.7	156.4

Table 9. Return levels (mm) from alternative univariate distributions (Gumbel, GEV, and Exponentiated Gumbel), ordered by best fit for selected Coahuila stations.

Station base	Distribution	Return period T(years) 1.1	2	5	10	20	50	100	500	1000	5000	10000	AICc	BIC
5020	BEG	32.0	74.1	110.9	134.6	157.1	185.8	207.3	256.6	277.8	326.8	347.9	342.0	345.9
	EVI	31.3	75.5	116.0	142.7	168.4	201.7	226.6	284.1	308.9	366.3	391.0	366.2	369.0
	EG	31.7	72.6	116.9	148.7	180.3	221.9	253.4	326.6	358.0	431.2	462.7	368.1	372.0

	GEV	31.8	73.8	115.9	145.8	176.3	218.3	251.9	336.8	376.7	478.1	525.8	368.3	372.2
	BEG	28.9	60.0	88.0	106.3	123.8	146.3	163.2	202.1	218.8	257.6	274.3	307.7	311.5
5163	EVI	29.2	59.2	86.5	104.6	122.0	144.5	161.3	200.3	217.0	255.9	272.6	329.2	331.9
	EG	29.6	57.1	87.0	108.6	129.9	158.1	179.4	228.9	250.2	299.7	321.0	331.1	334.9
	GEV	29.5	58.4	86.5	106.1	125.6	152.0	172.7	223.5	246.8	304.1	330.4	331.5	335.3
	BEG	19.1	39.2	58.0	70.6	82.8	98.7	110.7	138.3	150.2	177.8	189.7	282.7	286.4
	EVI	19.0	39.6	58.3	70.7	82.7	98.1	109.6	136.3	147.8	174.4	185.9	292.6	295.2
5009	GEV	18.7	40.9	58.2	68.3	77.0	87.1	93.9	107.3	112.2	121.9	125.5	294.3	298.0
	EG	18.8	40.8	58.1	68.5	78.1	90.0	98.6	118.1	126.4	145.3	153.4	294.6	298.3
	BEG	17.2	34.1	50.4	61.5	72.2	86.2	96.8	121.3	131.9	156.4	166.9	453.3	458.9
	EVI	17.9	34.1	49.0	58.8	68.2	80.4	89.5	110.6	119.7	140.8	149.8	467.6	471.4
5018	BEG	17.2	34.1	50.4	61.5	72.2	86.2	96.8	121.3	131.9	156.4	166.9	453.3	458.9
	EV1	17.9	34.1	49.0	58.8	68.2	80.4	89.5	110.6	119.7	140.8	149.8	467.6	471.4
	GEV	17.7	35.0	48.9	57.2	64.7	73.4	79.5	91.8	96.5	106.2	109.9	469.1	474.7
	EG	17.8	34.7	48.8	57.7	65.9	76.4	84.1	101.6	109.1	126.5	134.0	469.5	475.2

A similar procedure was applied in Tabasco, where all possible station pairs within each homogeneous region were analyzed. Table 10 presents the parameters obtained for the best BEG combination at each station. Return levels derived from the BEG distribution were then compared with those obtained from alternative univariate models (Gumbel, GEV, and Exponentiated Gumbel), and the results were ranked according to statistical criteria (AIC and BIC) (Table 11). In this case, 81% of the stations achieved the best fit with the BEG distribution, confirming the robustness of this approach under the markedly different climatic conditions of southeastern Mexico.

Table 10. Estimated parameters from the best BEG combination for each station in Tabasco.

	Station	Neighboring	Relative sample sizes			Bivariate parameters							Marginal (1)	
	Base	station	n_1	n_2	n_3	Loc 1	Sca 1	Shp 1	Loc 2	Sca 2	Shp 2	m	AICc	BIC
1	27001	27079	19	11	2	114.46	36.67	1.01	138.75	41.45	0.87	1.29	336.32	339.86
2	27002	27014	18	13	37	114.10	40.40	0.94	119.45	70.32	0.86	2.42	702.44	708.72
3	27003	27014	7	13	36	98.38	25.70	0.40	119.80	65.22	0.91	1.24	589.90	595.51
4	27004	27090	36	31	0	110.67	22.38	0.51	101.05	22.37	0.57	1.15	666.42	672.65
5	27006	27088	29	17	17	105.94	30.81	0.95	76.97	27.88	0.66	1.29	619.05	625.07
6	27007	27029	11	57	0	131.48	42.00	0.77	113.91	39.99	0.95	1.13	600.03	605.71
7	27008	27054	17	27	19	97.77	18.65	0.26	127.41	56.93	1.00	1.52	660.34	666.36
8	27009	27054	22	27	19	135.70	44.31	1.00	127.42	52.15	0.91	1.34	718.75	725.03
9	27010	27079	14	11	38	123.25	38.53	0.86	147.76	45.13	1.16	1.38	649.82	655.84
10	27011	27002	1	67	0	160.24	70.47	1.09	107.46	36.17	0.80	1.05	763.06	769.30
11	27012	27035	45	22	1	126.26	33.52	1.11	132.06	31.98	0.90	1.22	289.12	291.26
12	27013	27018	15	15	38	132.03	35.31	0.89	123.42	33.51	0.89	1.52	191.90	191.84
13	27014	27026	14	13	36	120.04	53.19	0.70	122.18	41.49	0.95	1.32	166.50	165.53
14	27015	27092	46	18	0	113.40	35.76	0.96	146.17	57.30	1.12	1.30	670.54	676.62
15	27016	27056	36	31	1	97.75	32.77	0.59	118.04	47.29	1.07	1.21	708.77	715.06
16	27017	27013	15	15	38	134.17	37.78	1.02	132.36	41.62	0.92	2.36	681.26	687.54
17	27018	27013	15	15	38	123.07	32.78	0.87	132.16	35.09	0.90	1.52	675.41	681.69
18	27019	27012	44	22	1	147.20	45.75	0.99	127.08	32.93	0.96	1.04	696.27	702.50
19	27020	27083	55	12	1	99.36	24.17	0.40	103.24	36.31	0.83	1.11	699.38	705.67
20	27021	27079	15	11	37	110.44	31.37	1.03	132.83	39.30	0.64	1.11	629.32	635.34
21	27022	27079	19	11	20	138.90	39.08	0.88	146.16	50.81	1.17	1.15	518.07	523.28
22	27024	27048	12	56	0	147.36	44.74	0.87	125.87	39.16	0.85	1.34	594.03	599.65

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23	27026	27014	14	13	36	102.51	27.73	0.52	120.69	59.99	0.71	1.35	655.62	661.65
24	27027	27079	18	11	38	164.74	50.78	1.01	141.09	38.69	0.73	1.14	717.74	723.98
25	27028	27056	36	31	1	107.84	36.34	0.80	116.02	45.31	0.95	1.20	710.27	716.55
26	27029	27001	0	32	36	113.41	37.90	0.93	112.69	37.73	0.96	1.53	695.63	701.91
27	27030	27035	0	67	1	135.14	34.48	0.96	120.84	28.57	0.69	1.03	687.77	694.01
28	27031	27054	22	27	18	135.85	40.70	0.89	129.95	50.62	1.21	1.14	704.26	710.49
29	27032	27092	50	18	0	138.39	40.98	0.92	151.05	55.36	1.25	1.44	713.04	719.33
30	27033	27092	50	18	0	127.12	39.22	0.96	150.66	57.95	1.23	1.52	710.24	716.52
31	27034	27001	0	32	36	130.19	43.33	0.95	108.22	33.75	0.77	1.14	710.84	717.12
32	27035	27012	45	22	1	130.55	31.62	0.87	119.88	31.18	0.92	1.23	669.85	676.13
33	27036	27054	22	27	19	121.60	42.68	0.87	130.54	46.83	1.01	1.37	717.07	723.36
34	27037	27001	0	32	36	113.79	33.10	0.83	108.77	29.76	0.75	1.17	689.19	695.48
35	27038	27083	55	12	1	108.25	17.06	0.31	102.66	40.07	1.11	1.06	701.69	707.98
36	27039	27001	0	32	36	127.49	43.59	0.97	110.16	38.57	0.96	1.21	708.04	714.32
37	27040	27059	2	63	1	101.73	30.48	0.94	92.14	27.89	0.80	1.36	668.66	674.79
38	27041	27013	10	15	38	131.52	36.35	0.85	130.35	36.10	0.92	1.74	641.99	648.01
39	27042	27013	5	15	38	189.07	60.80	0.99	131.24	41.42	1.03	1.09	642.96	648.69
40	27044	27019	9	58	0	165.06	45.79	0.93	144.81	45.73	0.93	1.39	607.95	613.69
41	27045	27019	11	56	0	171.69	45.19	0.87	159.11	55.12	1.30	1.21	586.91	592.52
42	27046	27087	32	31	0	114.53	32.19	0.80	106.56	32.46	0.91	1.09	632.34	638.36
43	27047	27087	32	31	0	136.95	49.48	1.60	114.04	33.95	1.09	1.07	641.85	647.88
44	27048	27079	19	11	38	129.63	42.09	0.95	141.41	50.56	1.02	1.06	707.42	713.70
45	27049	27054	21	27	19	136.06	52.22	0.98	126.19	47.96	0.86	1.17	722.10	728.33
46	27050	27056	36	31	1	100.21	31.08	0.75	114.85	45.25	0.98	1.03	699.77	706.05
47	27051	27079	19	11	38	127.81	36.80	0.80	147.84	43.43	1.17	1.45	706.42	712.70
48	27053	27001	0	32	36	109.84	30.78	0.63	108.50	30.98	0.84	1.42	699.12	705.40
49	27054	27006	18	27	18	123.00	40.40	0.74	106.39	30.60	0.96	1.08	292.45	295.30
50	27055	27050	0	48	20	127.77	40.52	0.73	109.71	35.29	0.91	1.15	508.39	513.46
51	27056	27050	36	31	1	115.72	45.14	0.99	107.11	33.31	0.84	1.03	350.36	353.77
52	27057	27014	18	13	37	108.13	26.53	0.40	120.66	82.01	1.00	1.54	721.47	727.75
53	27059	27040	2	63	1	89.09	24.26	0.67	99.56	27.99	0.84	1.35	654.61	660.69
54	27060	27079	8	11	38	127.35	36.45	0.90	148.72	50.87	1.17	1.20	583.95	589.62
55	27061	27013	3	15	38	173.95	60.17	1.11	129.08	32.22	0.85	1.07	607.09	612.71
56	27065	27059	4	44	20	114.55	37.51	0.86	92.14	28.56	0.91	1.10	495.03	500.10
57	27068	27032	12	56	0	172.13	51.08	0.88	139.05	41.32	0.95	1.13	603.21	608.83
58	27069	27091	0	31	0	83.00	21.67	1.00	85.76	29.72	0.96	2.50	312.04	315.46
59	27070	27032	10	58	0	171.56	51.39	1.02	139.73	41.01	0.97	1.06	620.65	626.38
60	27071	27001	0	32	35	112.73	33.34	0.77	108.48	32.72	0.75	1.38	683.69	689.93
61	27073	27054	0	27	19	114.61	53.32	0.91	126.83	44.87	0.86	1.03	506.40	511.31
62	27075	27054	0	27	19	135.38	51.13	0.89	127.35	48.05	0.94	1.14	505.01	509.93
63	27076	27002	22	46	0	123.89	61.90	0.80	116.78	43.72	1.05	1.07	513.88	518.79
64	27077	27029	22	46	0	145.53	54.39	1.05	109.64	35.79	0.83	1.02	488.76	493.67
65	27078	27006	18	45	1	125.15	43.05	0.89	102.24	26.92	0.80	1.05	482.09	487.01
66	27079	27001	19	11	2	143.02	40.46	0.88	113.26	37.61	0.97	1.30	125.55	123.32
67	27080	27079	8	11	38	111.78	30.78	0.77	140.76	50.34	1.18	1.76	581.62	587.29
68	27083	27084	55	12	1	104.37	43.51	1.03	129.71	44.99	0.95	1.16	199.03	197.48
69	27084	27083	55	12	1	122.92	39.23	0.78	105.16	30.80	0.81	1.16	705.33	711.61
70	27087	27090	0	31	0	107.69	31.62	0.97	101.84	24.43	0.63	1.19	322.52	325.94
71	27088	27006	29	17	17	73.91	20.77	0.70	106.45	31.01	0.96	1.04	256.86	257.51
72	27090	27087	0	31	0	105.47	26.44	0.73	110.24	32.47	0.99	1.18	320.20	323.61

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73	27091	27069	0	31	0	81.64	26.70	0.82	82.85	21.46	1.01	2.50	297.36	300.77
74	27092	27015	46	18	0	147.27	57.00	1.11	111.85	35.49	0.94	1.29	197.55	198.50
75	27093	27030	36	31	0	101.74	27.05	1.07	135.53	34.75	1.00	1.12	433.86	437.27

Loc = location parameter; Sca = Scale parameter; Shp = Shape parameter; Marginal (1) = Station base.

Table 11. Return levels (mm) from alternative univariate distributions (Gumbel, GEV, and Exponentiated Gumbel), ordered by best fit for selected Tabasco stations.

Station		Return period T(years)												
Base	**Distribution**	**1.1**	**2**	**5**	**10**	**20**	**50**	**100**	**500**	**1000**	**5000**	**10000**	**AICc**	**BIC**
27001	BEG	82.3	127.6	168.9	196.2	222.4	256.3	281.7	340.3	365.5	424.1	449.2	336.32	339.86
27001	EG	82.5	118.9	173.1	214.1	255.6	252.6	252.6	252.6	252.6	252.6	252.6	336.98	340.52
27001	EV1	82.5	128.0	169.5	196.9	223.3	257.4	283.0	342.0	367.4	426.4	451.7	337.54	340.06
27001	GEV	83.9	124.3	169.0	203.4	240.6	295.7	342.8	474.2	542.2	732.6	831.2	339.44	342.98
27002	BEG	79.7	131.2	178.9	210.7	241.3	281.0	310.9	379.8	409.4	478.3	507.9	702.44	708.72
27002	GEV	79.6	121.4	170.6	210.3	254.9	323.8	385.2	567.5	667.3	964.9	1128.1	722.81	729.10
27002	EG	79.1	123.1	175.8	214.9	253.9	305.4	344.4	434.9	473.9	564.4	603.4	724.77	731.05
27002	EV1	77.7	126.9	171.9	201.7	230.3	267.3	295.0	359.1	386.6	450.5	478.0	728.60	732.85
27003	BEG	87.2	140.7	202.0	247.0	291.8	350.9	395.7	499.6	544.3	648.2	692.9	589.90	595.51
27003	GEV	85.0	132.4	188.8	234.7	286.6	367.4	439.8	657.2	777.4	1139.3	1339.7	611.28	616.89
27003	EG	84.2	134.1	195.6	241.5	287.3	347.9	393.7	500.1	545.9	652.3	698.1	613.46	619.08
27003	EV1	82.6	139.2	190.9	225.2	258.0	300.5	332.4	406.0	437.7	511.2	542.8	616.38	620.20
27004	BEG	97.8	137.6	180.4	211.0	241.3	281.3	311.6	381.8	412.0	482.2	512.4	666.42	672.65
27004	EV1	97.8	140.8	180.0	206.0	230.9	263.2	287.4	343.2	367.2	423.0	447.0	692.85	697.07
27004	EG	98.2	138.0	180.9	211.6	242.1	282.2	312.6	383.1	413.4	483.9	514.3	694.06	700.29
27004	GEV	98.3	139.2	179.9	208.9	238.1	278.4	310.4	390.9	428.6	523.8	568.3	694.55	700.78
27006	BEG	79.7	119.0	155.3	179.6	202.9	233.2	255.9	308.4	331.0	383.5	406.1	619.05	625.07
27006	GEV	81.2	109.3	147.8	183.0	226.5	301.6	375.9	634.7	798.6	1370.8	1733.6	632.93	638.95
27006	EG	81.6	111.7	153.5	185.1	216.7	258.5	290.1	340.8	340.8	340.8	340.8	633.03	639.05
27006	EV1	78.5	115.9	150.0	172.6	194.3	222.4	243.4	292.0	312.9	361.4	382.3	642.86	646.95
27007	BEG	99.3	158.7	216.3	255.7	294.1	344.2	382.0	469.7	507.4	595.0	632.7	600.03	605.71
27007	EV1	101.7	160.4	214.0	249.5	283.6	327.6	360.7	437.0	469.8	546.0	578.7	626.05	629.91
27007	EG	102.1	157.9	215.4	255.9	295.8	348.4	388.1	480.2	519.9	612.0	651.7	627.53	633.21
27007	GEV	102.3	158.2	214.1	253.8	294.0	349.5	393.7	505.2	557.5	689.9	752.1	627.72	633.40
27008	BEG	94.5	146.2	211.8	260.9	310.1	375.0	424.1	538.2	587.3	701.3	749.6	660.34	666.36
27008	EG	94.8	143.9	212.8	264.8	316.9	385.6	437.7	498.7	498.7	498.7	498.7	692.92	698.94
27008	GEV	96.1	146.5	207.1	256.9	313.6	402.4	482.8	726.6	862.8	1277.6	1509.8	695.79	701.81
27008	EV1	93.5	153.5	208.2	244.5	279.2	324.2	357.9	435.9	469.4	547.1	580.6	697.10	701.19

Together, these results demonstrate that the BEG distribution consistently outperforms classical models, providing the best fit in both the arid to semi-arid conditions of Coahuila and the humid tropical environment of Tabasco. This consistency across regions and evaluation metrics underscores the versatility of the BEG framework for regional frequency analysis in contrasting hydroclimatic settings.

4. Discussion

The results obtained in Coahuila and Tabasco demonstrate the advantages of adopting flexible probability models such as the Bivariate Exponentiated Gumbel (BEG) distribution for extreme rainfall analysis. In both regions, the BEG framework provided the best fit for a majority of stations—65% in Coahuila and 81% in Tabasco—when compared to the classical Gumbel, Generalized Extreme Value (GEV), and Exponentiated Gumbel distributions. This superior performance, confirmed by goodness-of-fit criteria including AIC and BIC, highlights the ability of the BEG model to capture the dependence structure of extreme events more effectively than univariate alternatives.

The simulation experiments reinforce this conclusion. By jointly estimating parameters across paired samples, the BEG approach significantly reduced bias and mean squared error in quantile estimation, particularly as one sample length increased relative to the other. This result demonstrates the effective transfer of information between series, a feature that is especially relevant in hydrological contexts where records are often short, fragmented, or incomplete. In contrast, univariate models produced higher estimation errors under identical conditions, confirming the greater robustness of BEG.

An important aspect of these findings is the consistency of the BEG performance across contrasting climatic contexts. In Coahuila, a predominantly arid to semi-arid region, rainfall extremes are generally short-lived and spatially heterogeneous, which complicates regionalization and frequency analysis. Conversely, Tabasco is located in a humid tropical environment where rainfall extremes are often more spatially extensive and influenced by large-scale atmospheric dynamics. Despite these marked hydroclimatic differences, the BEG distribution consistently outperformed traditional models, underscoring its robustness and adaptability.

The systematic evaluation of all possible station pairs within homogeneous regions further strengthens the validity of the results. This approach ensures that spatial dependence is explicitly considered, rather than assuming independence between stations—a limitation common in univariate frequency analysis. The strong performance of the BEG distribution suggests that bivariate or multivariate models may be more appropriate for regions with high spatial variability, particularly where water resource planning and hydraulic design require accurate estimation of joint extremes.

From a practical perspective, the improved fit obtained with the BEG distribution has direct implications for risk management and infrastructure design. Underestimation of return levels, especially for long return periods, can lead to inadequate sizing of hydraulic structures and increased vulnerability to extreme events. By providing more reliable estimates, the BEG framework contributes to reducing uncertainty in hydrological design, supporting more resilient adaptation strategies in the face of climate variability and change.

Finally, these results align with recent studies emphasizing the importance of moving beyond stationary univariate models in hydrology. The incorporation of flexible, bivariate approaches not only improves statistical performance but also offers a more realistic representation of rainfall extremes, particularly in regions with complex climatic dynamics. The outcomes from Coahuila and Tabasco therefore provide empirical evidence supporting the broader adoption of BEG-based regional frequency analysis in Mexico and comparable hydroclimatic contexts.

5. Conclusions

This study applied the Bivariate Exponentiated Gumbel (BEG) distribution to extreme rainfall analysis in Coahuila and Tabasco, two Mexican states with contrasting climatic conditions. The main conclusions are as follows:

Across the stations analyzed, the BEG distribution provided the best fit in 65% of the cases in Coahuila and 81% in Tabasco, outperforming classical alternatives such as the Gumbel, Generalized Extreme Value (GEV), and Exponentiated Gumbel distributions. This performance was consistently confirmed by statistical indicators including AIC and BIC.

Simulation experiments showed that BEG reduced bias and mean squared error when estimating quantiles, especially for short or heterogeneous samples. The joint estimation of parameters allowed effective transfer of information, resulting in more reliable quantiles than those obtained from univariate approaches.

The BEG model demonstrated strong adaptability in both arid to semi-arid conditions (Coahuila) and humid tropical conditions (Tabasco), underscoring its versatility as a tool for regional frequency analysis of extreme rainfall.

The systematic evaluation of all possible pairwise station combinations within homogeneous regions captured spatial dependence more effectively than univariate models. This methodological strength enhances the reliability of rainfall return level estimates, particularly for long return periods.

The improved accuracy of return level estimates obtained with the BEG distribution can reduce underestimation of design events, thereby contributing to safer and more resilient hydraulic infrastructure, as well as supporting climate adaptation planning in water management.

In summary, the BEG distribution proves to be a reliable and flexible alternative for regional extreme rainfall analysis in Mexico. Its demonstrated robustness suggests that it can be extended to other regions with similar hydroclimatic variability, providing a valuable tool for both scientific research and applied hydrological practice. Future research should explore the application of the BEG model under non-stationary conditions, test its performance in multivariate settings that include variables such as streamflow or temperature, and assess its potential for integration into climate change impact studies.

Data Availability Statement: Data will be made available at a reasonable request. Note: Please confirm or update DAS at revision which version we should follow next. DAS type which you selected in the system as follow: Data available in a publicly accessible repository; The original data presented in the study are openly available in [repository name, e.g., FigShare] at [DOI/URL] or [reference/accession number]. Or select an alternative template from the options available through the following link: https://www.mdpi.com/ethics#_bookmark21.

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Abbreviations

The following abbreviations are used in this manuscript:

BEV	Bivariate Extreme Value Distribution
FFA	Flood Frequency Analysis
GEV	Generalized Extreme Value
R	Homogeneous Region
T	Return Period

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