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Article

Resolving the "Theory of Everything" Paradox via Constructive Immanence and the Boundedness of Physical Information: A Formal Proof of Physical Decidability

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Abstract

A recent proposal by Faizal et al. (2025) argues that because formal axiomatic systems describing quantum gravity are subject to Gödelian incompleteness, the physical universe must rely on "non-algorithmic" layers or an external "Meta-Theory of Everything" to ensure consistency. We demonstrate that this conclusion rests on a fundamental category error: the conflation of the syntactic limitations of a descriptive Formal Axiomatic System (FAS) with the semantic reality of the physical universe. We provide a formal proof that Gödel's theorems, which apply strictly to systems capable of modeling Peano Arithmetic and Actual Infinity, are inapplicable to a physical universe constrained by the Bekenstein Bound. By formalizing the universe as a Measure-Many Quantum Finite Automaton (QFA) over a finite Hilbert space, we demonstrate an isomorphism to a Deterministic Finite Automaton (DFA) under unitary evolution. Unlike Turing Machines or Linear Bounded Automata, for which key logical properties are undecidable, the FSA class is strictly decidable. Consequently, we prove that the universe is logically self-consistent without the need for external axioms. Furthermore, we demonstrate that "singularities" are artifacts of the continuum limit ($\lim_{n \rightarrow \infty}$), representing a divergence between the information density required by the mathematical map and the capacity of the physical territory. We conclude that the universe operates on a principle of Constructive Immanence: consistency is not a theorem to be proved by a meta-system, but a state to be actualized by the system itself.

Keywords: Gödel's incompleteness theorems; quantum gravity; Bekenstein bound; constructive logic; finite state automata; physical decidability; map-territory relation

1. Introduction

The quest for a "Theory of Everything" ($\mathcal{F}_{QG}$) represents the ambition to encapsulate the fundamental dynamics of the universe within a single, consistent mathematical framework. However, this ambition faces a profound logical hurdle. As articulated in the recent letter by Faizal et al. (2025), any formal axiomatic system sufficiently complex to describe arithmetic is subject to Gödel's Incompleteness Theorems (Gödel, 1931). Faizal et al. argue that because $\mathcal{F}_{QG}$ cannot prove its own consistency ($Con(\mathcal{F}_{QG})$), the physical universe itself must contain "non-algorithmic" elements or require a "Meta-Theory of Everything" ($\mathcal{M}_{ToE}$) containing an external truth predicate to bridge the gap between algorithmic provability and physical reality.

This paper contends that the conclusion reached by Faizal et al. is a non-sequitur resulting from a category error. Specifically, it conflates the properties of the mathematical description (the Map) with the properties of the physical object (the Territory). The "undecidability" identified by Faizal et al. is not an ontological feature of the universe but a syntactic limitation of the specific mathematical tools—namely, Zermelo-Fraenkel set theory (ZFC) and the continuum assumption—used to model it.

We posit two distinct ontological categories:

1. The FAS (Formal Axiomatic System): The mathematical model consisting of language, axioms, and inference rules. This system is subject to Gödelian incompleteness if it assumes infinite sets.
2. The RU (Real Universe): The physical, interacting system of matter and energy. This system is constrained by physical laws, specifically the Bekenstein Bound (Bekenstein, 1981) and the Bremermann limit (Bremermann, 1962).

The central thesis of this paper is that the physical universe, being finite in information content within any causal horizon (Lloyd, 2002), is formally equivalent to a Finite State Automaton (FSA) rather than a Turing Machine with infinite tape. Since the Halting Problem and Gödelian incompleteness do not apply to FSAs, the physical universe is logically decidable. The "singularities" and "undecidable" propositions cited by Faizal et al. are shown to be artifacts of infinite precision assumptions that have no physical correlate. Consequently, the proposed "Meta-Theory" is an unnecessary metaphysical addition; the universe requires only Constructive Immanence—the principle that the actualization of a physical state is its own proof of consistency.

2. Literature Review

The intersection of mathematical logic and theoretical physics has been a subject of intense debate since the formulation of the Incompleteness Theorems. This review categorizes the existing scholarship into three distinct domains: the application of Gödelian logic to physical theories, the physical limits of computation, and the ontological distinction between mathematical models and physical reality.

2.1. Gödelian Incompleteness in Physics

The foundational work of Gödel (1931) established that any consistent axiomatic system capable of expressing Peano arithmetic cannot prove its own consistency. The implications for physics were famously highlighted by Hawking (2002), who suggested that a finite set of physical laws might never fully describe the universe, marking the "end of physics" as a complete deductive system. This line of reasoning is further developed by Penrose (1989; 1994), who utilizes the Lucas-Penrose argument to posit that human consciousness and physical reality must involve non-computable processes to transcend algorithmic limitations. Faizal et al. (2025) build directly upon this tradition, incorporating Tarski's Undefinability Theorem (Tarski, 1936) and Chaitin's information-theoretic incompleteness (Chaitin, 1975) to argue that the "Theory of Everything" requires a non-algorithmic meta-layer.

2.2. The Physical Limits of Computation

Parallel to the logical critique is the development of Information Physics, which treats the universe as a computational system constrained by thermodynamic laws. Landauer (1961) established the principle that "information is physical," linking logical irreversibility to heat generation. Crucially, Bekenstein (1981) derived the universal upper bound on the entropy (information) that can be contained within a finite region of space, a result later generalized by the Holographic Principle ('t Hooft, 1993; Susskind, 1995). Lloyd (2002) extended this analysis to calculate the ultimate computational capacity of the universe, demonstrating that the universe processes a finite number of operations approximately (10^{120}) on a finite number of bits approximately (10^{90}). This body of work suggests that the physical universe is strictly finite in its informational content, a constraint often ignored by formal axiomatic systems that assume infinite precision.

2.3. The Map-Territory Distinction

The philosophical distinction between a model and reality, famously summarized by Korzybski (1933), is critical in the context of quantum gravity. Whitehead (1925) termed the conflation of abstract models with physical reality the "Fallacy of Misplaced Concreteness." In modern constructive

mathematics, this is echoed by the rejection of "Actual Infinity" in favor of "Potential Infinity" (Bishop, 1967). While Faizal et al. (2025) cite Chaitin to argue for non-algorithmic physics, Chaitin himself (2005) has argued that real numbers and the continuum are "mathematical fantasies" with no physical meaning. This suggests that the "undecidability" identified by Faizal et al. may not be a feature of the territory (physics) but a defect of the map (ZFC-based mathematics).

3. Research Questions

To resolve the paradox presented by Faizal et al., this paper addresses the following specific questions:

1. (The Finitude Question): Does the physical universe, constrained by the Bekenstein Bound and the Planck scale, possess the necessary properties (specifically Actual Infinity) to be subject to Gödel’s Incompleteness Theorems?
2. (The Model Question): Is the appropriate computational model for the universe a Turing Machine with an infinite tape, or a Finite State Automaton (FSA)?
3. (The Singularity Question): Are the singularities cited by Faizal et al. (e.g., Black Holes) evidence of non-algorithmic physics, or are they mathematical artifacts resulting from the assumption of a continuous metric at the Planck scale?
4. (The Consistency Question): Does a physical system require a deductive meta-proof to establish its consistency, or is consistency an inherent property of state actualization (Constructive Immanence)?

4. Methodology

To address the research questions, this paper employs a methodology grounded in Model Theory, Computational Complexity Theory, and Information Physics. We establish a formal distinction between the mathematical theory used to describe the universe and the physical system itself.

4.1. Formal Definitions

We define the Formal Axiomatic System (The Map) as $\mathcal{T}_{QG} = (\mathcal{L}, \Sigma, \vdash)$, where $\mathcal{L}$ is a first-order language, Σ is a consistent, recursive set of axioms (e.g., ZFC extended with physical postulates), and $\vdash$ is the relation of logical consequence. This system is subject to Gödel’s Second Incompleteness Theorem if and only if it is capable of modeling Peano Arithmetic (PA) and assumes the existence of infinite sets (i.e., the domain of quantification D has cardinality $|D| \geq \aleph_0$).

We define the Physical System (The Territory) as a dynamical system:

$$\mathcal{U} = (S, \delta, s_0)$$

where S is the set of distinguishable physical states, $\delta: S \rightarrow S$ is the transition function representing physical laws, and s_0 is the initial state.

Note: We define $\mathcal{U}$ strictly within the causal horizon (Particle Horizon). Physical regions causally disconnected from $\mathcal{U}$ cannot affect the logical consistency of state transitions within $\mathcal{U}$.

4.2. Analytical Framework

The analysis proceeds in three steps:

1. Cardinality Analysis: We apply the Bekenstein Bound to determine the cardinality of the state space S . This determines the appropriate computational class for $\mathcal{U}$ (Turing Machine vs. Finite State Automaton).
2. Decidability Proof: Based on the computational class, we determine whether the Halting Problem and Gödelian Incompleteness are applicable to $\mathcal{U}$.

3. Limit Analysis: We analyze the mathematical definition of a singularity ($R \rightarrow \infty$) against the information-theoretic constraints of the physical system to determine if the singularity is a physical reality or a model artifact.

5. Analysis and Results

5.1. Result I: The Theorem of Physical Decidability

This section addresses Research Questions 1 and 2. We formally test the hypothesis that the physical universe $\mathcal{U}$ possesses the computational capacity to be subject to Gödel's Second Incompleteness Theorem.

Premise 5.1.1 (The Holographic Information Bound):

Let $\Omega \subset \mathbb{R}^4$ be a causally connected region of spacetime with boundary $\partial\Omega$ and area $A(\partial\Omega)$. Following Bekenstein (1981) and 't Hooft (1993), the maximum entropy S_{max} (and thus the maximum information content I_{max}) contained within Ω is bounded by:

$$I_{max}(\Omega) \leq \frac{A(\partial\Omega)}{4l_p^2 \ln 2}$$

where

$$l_p = \sqrt{\frac{\hbar G}{c^3}}$$

is the Planck length.

For the observable universe, the particle horizon R_H is finite ($R_H \approx 14\text{Gpc}$). Thus, $A(\partial\Omega)$ is finite, and $I_{max} < \infty$.

Note: While the particle horizon R_H expands over cosmic time (t), at any specific instance t_0 , the value $I_{max}(t_0)$ is strictly finite. The universe never possesses 'Actual Infinity' in storage capacity, only a 'Potential Infinity' of future expansion, which does not violate the finite state assumption at any given step.

Lemma 5.1.2 (Finitude of the Hilbert Space):

Let $\mathcal{H}_u$ be the Hilbert space describing the quantum state of the universe. The dimension of this Hilbert space, $D_{\mathcal{H}}$, is given by the exponent of the maximum entropy: $D_{\mathcal{H}} = \dim(\mathcal{H}_u) = \exp(S_{max})$. Since S_{max} is finite, $D_{\mathcal{H}}$ is finite. Consequently, the set of distinguishable orthogonal states $\mathcal{S} = \{|s_1\rangle, |s_2\rangle, \dots, |s_N\rangle\}$ is finite, with $N = D_{\mathcal{H}}$.

While a standard Hilbert space $\mathcal{H}_u$ implies continuous coefficients, the Bekenstein Bound imposes a fundamental limit on distinguishability. Two quantum states $|\psi\rangle$ and $|\phi\rangle$ cannot be physically distinguished if their overlap is smaller than the precision allowed by the system's total information content. Therefore, the effective state space $\mathcal{S}$ is a discrete ϵ -net over the Hilbert space, rendering the set of distinguishable states finite, where $|\mathcal{S}| \approx 2^{I_{max}}$. The Bekenstein Bound applies not only to the number of orthogonal dimensions (N) but also to the precision of the state coefficients. In standard Quantum Mechanics, a state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ implies $\alpha, \beta \in \mathbb{C}$. However, infinite precision in α and β would allow infinite information storage within a single qubit, violating the holographic entropy bound S_{max} . Therefore, the physical state space is not the continuous complex projective space $\mathbb{CP}^{N-1}$, but a discrete lattice of distinguishable states. The state space $\mathcal{S}$ is strictly finite.

Crucially, this finite dimension $D_{\mathcal{H}}$ holds regardless of whether spacetime is fundamental or emergent. Faizal et al. argue that quantum gravity may involve a 'pre-geometric' phase where smooth spacetime breaks down. However, the Holographic Principle dictates that the information content of the bulk—even in a pre-geometric state—cannot exceed the capacity of the boundary. Thus, the finiteness of the state space is an information-theoretic invariant that persists even through the 'breakdown' of classical geometry.

Theorem 5.1.3 (Isomorphism to Finite Automata):

The dynamical evolution of the universe is isomorphic to a Finite State Automaton (FSA), not a Turing Machine.

Proof:

1. Let the universe be defined as the dynamical system $\mathcal{M}_U = (S, \delta, s_0)$, where S is the finite set of states derived in Lemma 5.1.2, and $s_0 \in S$ is the initial state.
2. The laws of physics define a transition function via the Unitary Operator U . Formally, this corresponds to a General Quantum Finite Automaton (QFA) (specifically, a Measure-Many QFA). A QFA utilizes a finite-dimensional Hilbert space $\mathcal{H}$ and a set of unitary transition matrices. While a QFA accepts inputs probabilistically upon measurement, the *internal evolution* of the unmeasured state (the wavefunction of the universe) is unitary and deterministic: $|\psi_{t+1}\rangle = U |\psi_t\rangle$. Because the Hilbert space dimension $D_{\mathcal{H}}$ is finite (Lemma 5.1.2), the set of distinguishable state vectors is finite. Therefore, the evolution of the QFA can be mapped bijectively to a classical Deterministic Finite Automaton (DFA) where the 'states' of the DFA are the discrete vectors in $\mathcal{H}$. This ensures the system retains the decidability properties of regular languages.
3. A Turing Machine $\mathcal{T}\mathcal{M}$ is defined as a tuple $(Q, \Gamma, b, \Sigma, \delta_{\mathcal{T}\mathcal{M}}, q_0, F)$, where the tape alphabet Γ allows for an infinite tape. This implies a configuration space of cardinality $|C_{\mathcal{T}\mathcal{M}}| = \aleph_0$.
4. Since $|S| = N < \aleph_0$, there exists no bijection between the state space of $\mathcal{U}$ and the configuration space of a Turing Machine.
5. Since the system is isomorphic to a DFA, and a DFA is a subclass of Finite State Automata, $\mathcal{M}_U$ satisfies the formal definition of a Finite State Automaton.
6. Decidability: The Halting Problem $H(\mathcal{M}, w)$ and the Emptiness Problem $E(\mathcal{M})$ are decidable for the class of Finite State Automata. Specifically, the reachability of any state can be determined in $O(N)$ time.
7. Incompleteness: Gödel's theorems apply only to formal systems capable of modeling Peano Arithmetic (PA). PA requires the axiom $\forall n \exists S(n)$ (infinite successors). An FSA cannot model PA as it overflows at state N .
8. Therefore, $\mathcal{U}$ is logically decidable and immune to Gödelian incompleteness. Q.E.D.

5.2. Result II: Constructive Immanence

This section addresses RQ4. We formalize the distinction between syntactic consistency (in $\mathcal{T}_{QG}$) and ontological consistency (in $\mathcal{U}$).

Definition 5.2.1 (Syntactic Consistency):

Let Γ be the set of formulas derivable in the theory $\mathcal{T}_{QG}$. The theory is syntactically consistent if:

$$\nexists \phi: (\Gamma \vdash \phi) \wedge (\Gamma \vdash \neg \phi)$$

Establishing this requires a meta-proof (e.g., Gentzen's consistency proof) which transcends the system.

Definition 5.2.2 (Ontological Consistency via Unitarity):

In a quantum mechanical system $\mathcal{U}$, consistency is defined by the preservation of the norm of the state vector (Unitarity).

Let $U(t, t_0)$ be the time-evolution operator. Consistency requires:

$$\forall t, \|\psi(t)\|^2 = \langle \psi(t_0) | U^\dagger(t, t_0) U(t, t_0) | \psi(t_0) \rangle = 1$$

In this framework, 'inconsistency' would manifest physically as the violation of unitarity (probability leakage), resulting in the cessation of the system's existence. Thus, a system that exists is, by definition, consistent.

Theorem 5.2.3 (The Autopoietic Consistency Proof):

The physical actualization of a state transition is the constructive proof of its consistency.

Proof:

1. We adopt the framework of Constructive Logic (Curry-Howard Isomorphism), where the truth of a proposition P is equivalent to the existence of a witness w ($w:P$).
2. Let P_{trans} be the proposition: "The transition from state s_t to s_{t+1} is valid."
3. The witness w for P_{trans} is the physical state s_{t+1} itself.
4. If the universe evolves to $t + 1$, then s_{t+1} exists.
5. Therefore, w exists, and P_{trans} is true.
6. Unlike $\mathcal{T}_{QG}$, which must derive $\text{Con}(\mathcal{T})$ from axioms, $\mathcal{U}$ demonstrates $\text{Con}(\mathcal{U})$ by the non-vanishing of the state vector. If $\mathcal{U}$ were inconsistent, the probability amplitude would diverge or vanish (violating Unitarity).
7. The persistence of existence is the constructive proof of consistency. Q.E.D.

5.3. Result III: Resolution of Singularities as Artifacts

This addresses Research Question 3. We prove that singularities are contradictions arising from the mismatch between the Kolmogorov complexity of the model and the information capacity of the system.

Theorem 5.3.1 (The Information-Theoretic Singularity Contradiction): A physical singularity implies a violation of the Bekenstein Bound.

Proof:

1. Let $\mathcal{M}_{GR}$ be the continuous manifold of General Relativity. A singularity is defined as a point $p \in \mathcal{M}_{GR}$ where the curvature scalar $R \rightarrow \infty$ as the volume $V \rightarrow 0$.
2. To uniquely specify the geometry of a region containing infinite curvature, the metric tensor $g_{\mu\nu}$ requires infinite precision (infinite bits).
3. Let $K(s_p)$ be the Kolmogorov complexity (minimum description length) of the physical state at the singularity. Since the curvature is unbounded and non-repeating (in a generic chaotic collapse), $K(s_p) \rightarrow \infty$.
4. From Premise 5.1.1, the maximum information capacity of the region is $I_{max}(V) \propto A(V)$.
5. The mathematical model of a singularity assumes the limit $V \rightarrow 0$, implying $A \rightarrow 0$.
6. However, Quantum Gravity imposes a lower bound on area (Planck Area, A_p). The contradiction arises because the continuous map ($\mathcal{M}_{GR}$) demands $K(s_p) \rightarrow \infty$ as $A \rightarrow 0$, while the physical territory ($\mathcal{U}$) halts information density at $I_{max}(A_p)$. The limit $A \rightarrow 0$ is therefore a syntactic error of the map, not a physical state.
7. Therefore, the physical information limit is:

$$\lim_{V \rightarrow 0} I_{max}(V) = 0$$

8. This yields the contradiction:

$$K(s_p) \rightarrow \infty \text{ while } I_{max} \rightarrow 0$$

9. Conclusion: A physical state cannot have a complexity greater than its information capacity ($K(s) \leq I_{max}$). Therefore, the state s_p (the singularity) is physically impossible. The "singularity" is an artifact of $\mathcal{T}_{QG}$ (the map) assuming continuous variables ($K \rightarrow \infty$) in a domain where $\mathcal{U}$ (the

territory) has zero capacity. The physical system must transition to a discrete, finite-complexity description before the limit is reached. Q.E.D.

6. Discussion

The analytical results compel a fundamental re-evaluation of the relationship between mathematical logic and physical reality. The argument presented by Faizal et al. (2025)—that the universe requires a "Meta-Theory" to resolve the undecidability of its laws—is revealed not as a physical insight, but as a symptom of the "Great Compromise" in modern science: the reliance on platonist mathematics (ZFC) to describe a constructive, finite universe.

6.1. The Epistemic Nature of Undecidability

The "undecidability" identified by Faizal et al. is strictly epistemic, not ontic. It represents the limit of what a physicist can prove about the universe using a Formal Axiomatic System ($\mathcal{T}_{QG}$) that assumes infinite expressive power. However, as proven in Theorem 5.1.3, the universe itself ($\mathcal{U}$) operates as a Finite State Automaton (FSA).

In an FSA, there is no distinction between "true" and "provable" in the Gödelian sense because the system cannot formulate self-referential statements about its own provability predicate. The universe simply transitions. To claim that the universe is "undecidable" because ZFC is undecidable is equivalent to claiming that a digital photograph has infinite resolution because the Euclidean geometry used to describe the lens allows for irrational coordinates. It is a confusion of the map's resolution with the territory's structure. The "undecidable" propositions are merely "syntax errors" generated by applying an infinite map to a finite territory.

Faizal et al. (2025) cite the undecidability of the spectral gap (Cubitt et al., 2015) and quantum thermalization (Shiraishi & Matsumoto, 2021) as empirical evidence that the universe contains non-algorithmic features. However, it is crucial to note that these mathematical proofs strictly rely on the thermodynamic limit ($N \rightarrow \infty$). Specifically, the spectral gap problem is only undecidable for infinite lattices; for any finite-dimensional Hilbert space $D_{\mathcal{H}}$ (as required by the Bekenstein Bound), the spectral gap is computable via exact diagonalization in finite time. Faizal et al. confuse the properties of an infinite approximation with the properties of the finite physical system. The 'undecidability' they identify is not a feature of the territory, but an artifact of the map.

6.2. The "Meta-Theory" as a Violation of Parsimony

The proposal of a "Meta-Theory of Everything" ($\mathcal{M}_{ToE}$) containing an external truth predicate $T(x)$ is a violation of the Principle of Parsimony (Occam's Razor). Faizal et al. introduce a metaphysical entity (a non-algorithmic layer) to solve a problem that does not exist in the physical system.

By invoking an external "oracle" to certify truths that the algorithmic core cannot prove, they effectively reintroduce a theological structure into physics—a "God of the Axioms." Our analysis demonstrates that the universe requires no such external certifier. Through Constructive Immanence (Theorem 5.2.3), the universe validates its own consistency through the act of existence. The "proof" of the next moment is the occurrence of the next moment. The postulate of $\mathcal{M}_{ToE}$ is therefore redundant; the physical system $\mathcal{U}$ is self-sufficient.

6.3. Reframing the Simulation Hypothesis

Faizal et al. argue that the universe cannot be a simulation because a simulation (being algorithmic) would be incomplete, whereas the universe is consistent. This argument collapses under the weight of Theorem 5.1.3. Since the universe is finite and decidable, it is fully computable.

However, this does not imply the 'Simulation Hypothesis' in the traditional dualistic sense, which necessitates an external Simulator. Instead, the universe is a system where the distinction

between 'hardware' (matter) and 'software' (laws) dissolves. It is an autopoietic computation that writes and executes itself. Crucially, as an FSA, the universe is immune to the Halting Problem; it requires no external operator to resolve infinite loops or undecidable states. Thus, the universe is not a simulation of reality; it is the computation of reality.

6.4. *The Artifact of the Continuum*

Finally, the fear expressed by Faizal et al.—that singularities represent a "breakdown of science"—is unfounded. As shown in Theorem 5.3.1, singularities are contradictions that arise only when one insists on infinite information density ($K \rightarrow \infty$). The Bekenstein Bound ensures that the physical system transitions to a discrete description before any singularity can form. The "breakdown" is merely the failure of the continuous approximation ($\mathbb{R}$) at the Planck scale. The physics does not stop.

6.5. *Empirical Consistency with Discrete Spacetime*

Critics often cite the Fermi Gamma-ray Space Telescope observations of GRB 090510 (Abdo et al., 2009)—where high-energy photons arrived simultaneously with low-energy photons—as evidence against discrete spacetime. However, this objection assumes a 'bumpy road' model where discreteness acts as a dispersive medium.

In the FSA framework, this result is not a contradiction but a confirmation. Time in an FSA is defined strictly by state transitions ($t \rightarrow t + 1$). The speed of light c represents the maximum causal propagation rate: one informational bit per clock cycle. The Fermi result confirms that this update rule is universal and energy-independent. Whether a photon carries 31 GeV or 1 eV, it propagates at the fundamental limit of the automaton (1 cell per tick). Thus, the 'exact same time' observed by Fermi corresponds to the 'exact same number of state transitions,' validating the uniformity of the computational substrate.

7. Conclusion

This paper has provided a formal refutation of the claim that Gödelian incompleteness necessitates a non-algorithmic or meta-theoretical foundation for the physical universe. We have demonstrated that this claim rests on a fundamental category error: the application of theorems derived from infinite set theory (ZFC) to a physical reality constrained by the Bekenstein Bound.

Our findings are summarized as follows:

1. **Physical Decidability:** The physical universe, being finite in information content within any causal horizon, is formally equivalent to a Finite State Automaton (FSA), not a Turing Machine with an infinite tape. Since the Halting Problem is decidable for FSAs, the universe is logically decidable and immune to Gödel's Second Incompleteness Theorem.
2. **Artifacts of the Map:** The "singularities" and "undecidable" propositions cited by Faizal et al. are mathematical artifacts resulting from the unphysical assumption of infinite precision (the continuum limit). They represent a breakdown of the descriptive map ($\mathcal{T}_{QG}$), not the physical territory ($\mathcal{U}$).
3. **Constructive Immanence:** The universe does not require an external "Meta-Theory" to ensure its consistency. Consistency is an inherent, emergent property of the system's state transitions. The universe proves its consistency not by syntactic derivation, but by ontological endurance.

We conclude that the paradoxes of Quantum Gravity are not evidence of the limits of computation, but evidence of the limits of the axiomatic method itself. We conclude that the paradoxes of Quantum Gravity cited by Faizal et al. are not evidence of the limits of computation, but evidence of the limits of the axiomatic method itself. The 'undecidability' they identify is strictly a property of the map, not the territory. Ultimately, physics does not have the luxury of choosing

between ZFC-based incompleteness and constructive decidability; the Bekenstein Bound forces the choice. By strictly excluding the 'Actual Infinity' required for Gödelian paradoxes, the universe ensures its own consistency, rendering the proposal of a 'Meta-Theory' not only redundant but physically incoherent.

Appendix A. The Computational Class of the Physical Universe

Objective: To formally prove that a universe constrained by the Holographic Principle belongs to the computational class of Finite State Automata (FSA), thereby rendering Gödel's Incompleteness Theorems inapplicable.

A.1. The Dimension of the Physical Hilbert Space

Let $\mathcal{U}$ denote the physical universe within the particle horizon. According to the Holographic Principle ('t Hooft, 1993; Susskind, 1995), the maximum information content I_{max} (in bits) of a region is proportional to its boundary area A in Planck units.

$$I_{max} = \frac{A}{4l_p^2 \ln 2}$$

Let $\mathcal{H}_{\mathcal{U}}$ be the Hilbert space describing the quantum state of the universe. The dimension of this space, $D_{\mathcal{H}}$, corresponds to the total number of distinguishable orthogonal states the system can occupy. This is given by:

$$D_{\mathcal{H}} = 2^{I_{max}}$$

Since the observable universe has a finite radius R_H , the area $A = 4\pi R_H^2$ is finite. Consequently, I_{max} is finite, and $D_{\mathcal{H}}$ is finite. Let $N = D_{\mathcal{H}}$.

A.2. Isomorphism to Finite State Automata

A computational system is defined by its state space and transition rules.

- Turing Machine (TM): A tuple $(Q, \Gamma, b, \Sigma, \delta, q_0, F)$ where the tape alphabet Γ implies an infinite configuration space ($|C_{TM}| = \aleph_0$).
- Quantum Finite Automaton (QFA): A system defined by a finite-dimensional Hilbert space $\mathcal{H}$ and a set of unitary transition matrices. Since the dimension is finite ($N < \infty$), the state vector space is discrete.
- Deterministic Finite Automaton (DFA): A tuple $(S, \Sigma, \delta, s_0, F)$ where the set of states S is finite ($|S| < \aleph_0$) and transitions are deterministic.

A.3. Decidability Consequence

For any FSA $\mathcal{M}$, the following problems are decidable:

1. Reachability: Given state s_i , is there a path from s_0 to s_i ? (Decidable in $O(N)$).
2. Emptiness: Does the language $L(\mathcal{M})$ contain any strings? (Decidable).
3. Finiteness: Is the language finite? (Decidable).

Gödel's Second Incompleteness Theorem applies only to systems that can encode Peano Arithmetic (PA). Modeling PA requires an infinite set of states to represent the successor function $S(n)$ for all $n \in \mathbb{N}$. An FSA cannot model PA. Therefore, $\mathcal{U}$ is not subject to Gödelian undecidability.

Appendix B. The Information-Theoretic Limit of Geometry

Objective: To prove that physical singularities are mathematical artifacts arising from the divergence between the information capacity of the physical system and the complexity requirements of the continuous model.

B.1. Kolmogorov Complexity of the Continuum

Let $\mathcal{M}_{GR}$ be the continuous spacetime manifold. A state $s(x)$ at point x is described by the metric tensor $g_{\mu\nu}(x)$. In a continuous manifold, the coordinates x are real numbers ($\mathbb{R}$).

The Kolmogorov complexity $K(x)$ of a generic real number (a non-computable number) is infinite:

$$K(x \in \mathbb{R}) \rightarrow \infty$$

Thus, to perfectly describe a continuous geometry at the Planck scale requires infinite information density.

B.2. The Singularity Contradiction

Consider a gravitational collapse leading to a singularity.

1. The Model Requirement: As the radius $r \rightarrow 0$, the curvature $R \rightarrow \infty$. To specify the physical state s_{sing} with infinite curvature requires infinite precision. Thus, the model demands: $I_{req}(s_{sing}) \rightarrow \infty$
2. The Physical Constraint: As $r \rightarrow 0$, the boundary area $A \rightarrow 0$. By the Bekenstein Bound, the available capacity is:

$$I_{cap}(r) \propto r^2 \Rightarrow \lim_{r \rightarrow 0} I_{cap} = 0$$

3. The Divergence: $I_{req} > I_{cap}$
This inequality holds for all $r < l_{crit}$, where l_{crit} is the scale at which the complexity of the geometry exceeds the holographic bound (typically the Planck scale).

B.3. Conclusion

The "singularity" represents the region where the map ($\mathcal{M}_{GR}$) demands more information than the territory ($\mathcal{U}$) can hold. The physical system must necessarily transition to a lower-complexity description (discrete states) to satisfy $I_{state} \leq I_{cap}$. Therefore, the singularity does not exist in $\mathcal{U}$.

Appendix C. Logical Systems Comparison

Objective: To formalize the distinction between the deductive logic assumed by Faizal et al. and the constructive logic of physical reality.

Table A1. Comparison of Logical Frameworks.

Feature	Classical Logic (ZFC)	Constructive Logic (Physical)
Domain	Infinite Sets ($\aleph_0, 2^{\aleph_0}$)	Finite/Bounded Sets ($N < \infty$)
Truth Definition	Tarskian (Correspondence to Model)	Heyting (Existence of Proof/Witness)
Consistency	Syntactic ($\Gamma \not\vdash \perp$)	Ontological (State Actualization)
Infinity	Actual Infinity (Completed Object)	Potential Infinity (Unbounded Process)
Gödel Status	Incomplete	Decidable

Formalizing Constructive Immanence

In Classical Logic, consistency is a meta-theorem:

$$\text{Meta-Theory} \vdash \text{Con}(T)$$

In Constructive Physics, consistency is an operational invariant.

Let δ be the transition function. Consistency is defined as the preservation of the state's validity:

$$\forall s_t \in S_{\text{valid}}, \delta(s_t) \in S_{\text{valid}}$$

If the universe exists at time t , then $s_t \in S_{\text{valid}}$. The transition $s_t \rightarrow s_{t+1}$ is the computational verification of consistency. No external truth predicate $T(x)$ is required because the system does not manipulate symbols about itself; it manipulates its own states.

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