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Article

Decision-Making and Contract Coordination of Closed-Loop Supply Chain with Dual-Competitive Retail and Recycling Markets

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Abstract

The closed-loop supply chain (CLSC) plays an important role in reducing carbon emissions and effectively utilizing the residual value of used products. Sales competition and recycling rivalry are critical factors affecting the operation of CLSC. In this paper, we consider a CLSC system consisting of a single manufacturer, n retailers (where $n \geq 2$) and m third-party recyclers (where $m \geq 2$). The study constructs a Stackelberg game model with the manufacturer as a leader, and analyzes the impacts of retail competition and recycling rivalry on the decision-making and profits of both node enterprises and the supply chain system, and proposes a linear transfer-payment contract to coordinate the CLSC. Numerical analyses are conducted to visualize the effects of competition on the decisions and profits. The key findings are as follows: (1) In the centralized system, inter-retailer competition reduces optimal order quantities but does not affect optimal retail prices. In the decentralized system, however, it decreases both optimal order quantities and retail prices. (2) Rivalry among recyclers reduces their optimal recycling volumes but does not affect their optimal recycling prices in the centralized system. In the decentralized system, however, such rivalry not only decreases recycling volumes but also increases optimal recycling prices. (3) The manufacturer's product wholesale price and used product recycling price remain independent of competitive interactions among retailers and recyclers in the decentralized system. (4) Competition among retailers and recyclers positively affects the profits of the CLSC and the manufacturer, but negatively impacts those of retailers and recyclers. (5) When the reward-penalty factors for product order and used product recycling fall within specific range, the linear transfer-payment contract can coordinate the CLSC in the presence of competition in both retail and recycling.

Keywords: CLSC; Stackelberg game; competition; linear transfer-payment contract

1. Introduction

The rapid development of industrialization and the accelerated pace of product replacement have led to the generation of numerous used products [1]. Directly discarding them will not only cause environmental pollution but also result in significant waste of recoverable resources. As a result, the recycling and remanufacturing of used products have become a prevalent option in today's world [2]. As a cleaner production method, remanufacturing can not only provide appropriate revenues, economic growth, carbon emission reduction, and many new job opportunities, but also create environmental incentives for manufacturers. Many countries have therefore enacted legislation requiring manufacturers to adopt integrated green management systems and remanufacturing practices throughout the product lifecycle, from material sourcing to end-of-life recycling.

Research in the management of the CLSC has analyzed the impacts of various factors on decisions and profits. These factors include information sharing [3,4], environmental awareness [5,6],

government subsidies [7,8], return effort or return rate [9,10], and contract coordination [11,12], among others. However, the influence of dual competition in both forward and reverse channels on the CLSC has rarely been explored. In practice, manufacturers typically sell products through multiple retailers and outsource the collection of used products to numerous recyclers.

The profitability of a manufacturer implementing recycling and remanufacturing of used products exhibits dual dependencies. It depends on both the manufacturer's endogenous decisions and the exogenous operational decisions of retailers and third-party recyclers. However, traditional investment return theory is limited in analyzing such strategic dependence issues. Game theory, particularly Stackelberg game models, offers an effective approach to analyzing these problems. This method has been widely applied in research on the CLSC management for remanufacturing.

Therefore, this paper employs Stackelberg game theory to investigate management decisions, contract design, and the impact of competition within a CLSC comprising one manufacturer, n retailers ($n \geq 2$), and m recyclers ($m \geq 2$). The proposed model and analysis contribute to understanding three key aspects: (1) Interactions among supply chain members in a CLSC under dual competitive markets; (2) The optimal operational decisions made by these members; and (3) The optimal profits achieved through decisions on ordering quantities and recycling quantities. It examines the effectiveness of a linear transfer-payment contract in retail competition and recycling competition.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. The problem description, notations and assumptions are provided in Section 3 followed by the model formulation in Section 4. The numerical analysis is shown in Section 5 and the last section concludes the paper with main findings, management insights, limitations and directions for future research.

2. Literature Review

In this study, the literature relevant to this text is reviewed in three ways: competition in sales channels, competition in recycling channels, and dual competition in sales channels and recycling channels in closed-loop supply chains.

2.1. Competition in Sales Channels

The presence of multiple sales entities distributing products from the same manufacturer has intensified market competition. Consequently, scholars have extensively investigated decision-making in CLSC management involving sales competition within forward supply chain. Zheng et al. [13] and Pal and Sana [14] examined the effects of forward-channel sales competition on dual-channel CLSCs, which comprise a manufacturer, a retailer and a collector. The manufacturer may wholesale products to the retailer or sell directly to consumers, while the collector manages used product collection. Xie et al. [15] developed a revenue and cost sharing contract to coordinate a CLSC with dual sales channels. Zheng et al. [16] examined manufacturers' reverse channel choices and coordination mechanism in CLSCs with competitive dual sales channels. Wei and Zhao [17] incorporated fuzziness into collection and remanufacturing costs, using game theory and fuzzy set theory to study optimal pricing decisions for wholesale/retail prices and remanufacturing rates in a fuzzy CLSC with retail competition. Similarly, Ke et al. [18] addressed pricing and remanufacturing decisions in a fuzzy CLSC with one manufacturer, two competitive retailers, and one third-party collector.

2.2. Competition in Recycling Channels

In light of the competitive landscape in used product collection, scholars have explored decision-making challenges in CLSC management, building on Savaskan's foundational models. These studies typically address recycling competition within reverse supply chains. Ranjbar et al. [2] evaluated optimal pricing and collection decisions in a three-level CLSC under channel leadership with competitive dual-recycling channels: retailer collection and third-party collection. Shu et al. [19]

analyzed the impact of two collectors' fairness concerns on pricing decisions and coordination. Suvadarshini et al. [20] analyzed three return-channel structures under simultaneous influences of competition, collection efficiencies, individual rationality, and information asymmetry. Gaula and Jha [21] studied pricing and quality improvement strategies in a CLSC with dual collection channels. To explore consumer behavior effects on competitive dual-collection supply chains, Wang et al. [22] developed two CLSC models: (1) retailer and third-party collection, and (2) manufacturer and third-party collection. He et al. [23] proposed a hybrid game-theoretic CLSC model with a manufacturer, a retailer, and a third-party collector, devising collection functions accommodating varying competition levels-from monopoly to duopoly and hybrid scenarios.

2.2.3. Dual Competition in Sales Channels and Recycling Channels

To better align research with real-world complexities, a limited number of scholars have investigated optimization problems in CLSCs involving both retail competition and recycling competition. Giri et al. [24] analyzed a CLSC with two dual channels: a forward dual channel where a manufacturer sells products through traditional retail and e-tail channels, and a reverse dual channel where it collects used products for remanufacturing through third-party logistics and e-tail channels. Similarly, Zhang et al. [1] examined the impact of sales competition and recycling competition between manufacturer and retailer on CLSC management, while considering product quality and returns. They also designed an effective revenue-sharing contract to motivate retailers' recycling efforts. Hosseini-Motlagh et al. [25] studied a CLSC comprising a manufacturer investing in remanufacturing and energy-saving efforts, two retailers competing in sales, and two collectors competing in used product collection, proposing an energy-saving effort and cost-tariff contract for system coordination.

In summary, previous research has enriched the theoretical foundations of CLSC management, but most studies assume bilateral or trilateral monopolies within the supply chain, overlooking the influence of competitive factors. Some literature has considered competitions in sales and recycling, but only from the perspective of competition between two same entities. In reality, however, oligopolistic markets formed by multiple retailers are common within regional context, such as the dominance of Walmart, Carrefour, CR Vanguard, and Hualian in retail sectors. Additionally, a manufacturer typically outsources used product collection to multiple professional third-party collectors. Building on existing research and drawing on Cachon's revenue function, this paper intends to explore decisions and contract coordination in a CLSC with competition in both retail and recycling, with one manufacturer, n retailers ($n \geq 2$), and m recyclers ($m \geq 2$), to enhance the conclusion generalizability. The contribution of this study is twofold: (1) analyzing how retail and recycling competition separately impact decisions and profits of node enterprises and the supply chain, and (2) designing a linear transfer-payment contract to perfectly coordinate such a CLSC.

3. Problem Description and Notations

We consider the CLSC consisting of one manufacturer, n retailers, and m third-party collectors. The manufacturer engages in dual production modes: traditional manufacturing (using new materials) and remanufacturing (using used products). The n retailers submit orders to the manufacturer and sell products to consumers. The m third-party collectors collect used products from consumers and supply them to the manufacturer.

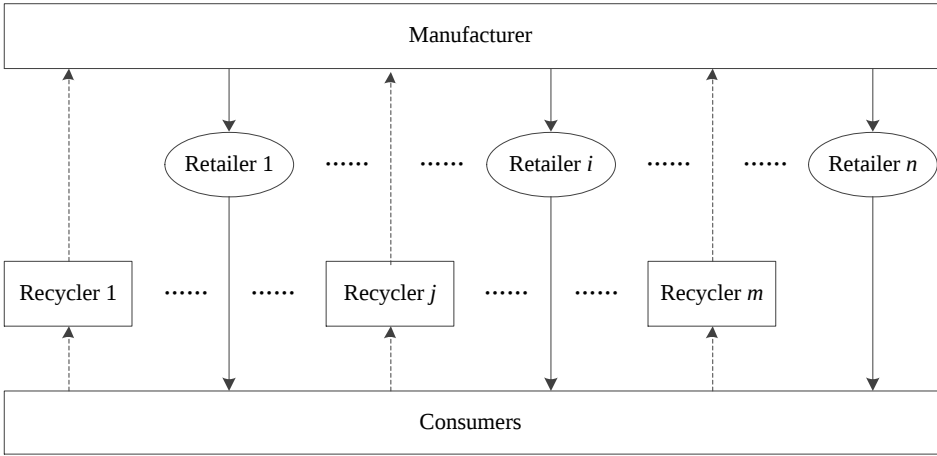


Figure 1. The proposed CLSC model.

The notations and their definitions used in this paper are listed in Table 1.

Table 1. Notations and definitions.

Notations	Definitions
c	Unit cost of manufacturing new products using new materials
c_r	Unit cost of remanufacturing used products
Δ	Unit remanufacturing cost savings, $\Delta = c - c_r > 0$
b	Unit recycling price of used products paid by the manufacturer to the third-party recycler
w	Wholesale price per unit set by the manufacturer
q_i	Retailer i 's order quantity from the manufacturer, $i = 1, 2, \dots, n$
l_j	Collection quantity of used products by the third-party collector j from consumers, $j = 1, 2, \dots, m$
γ	Reward-penalty factor for product-ordering within a linear transfer-payment contract
δ	Reward-penalty factor for recycling of used products within a linear transfer-payment contract
q_0	Minimum order quantity required by the manufacturer within a linear transfer-payment contract
l_0	Minimum recycling quantity required by the manufacturer within a linear transfer-payment contract
π_s	Profit of the manufacturer
π_{ri}	Profit of the retailer i , $i = 1, 2, \dots, n$
π_{rcj}	Profit of the recycler j , $j = 1, 2, \dots, m$
π_{clsc}	Profit of the CLSC
*	Decentralized decision-making
**	Centralized decision-making
tr	Coordinated decision-making

The mathematical models in this analysis rest on the following assumptions:

- Assumption 1.** Remanufactured products are functionally and qualitatively equivalent to new products. Consequently, the manufacturer sells both to retailers at a uniform wholesale price [26].
- Assumption 2.** The manufacturer acquires used products from third-party recyclers at a uniform price. All collected products meet remanufacturing quality standards.
- Assumption 3.** Retailer competition adheres to a Cournot model, with the retail price facing retailer i expressed as:

$$p_i = h - q_i - \alpha \sum_{e \neq i} q_e$$

Assumption 4. Analogously, the acquisition price by recycler j takes the form:

$$b_{rcj} = k + l_j + \beta \sum_{f \neq j} l_f$$

Assumption 5. The manufacturer serves as the Stackelberg leader in the CLSC, with retailers and recyclers acting as followers.

Assumption 6. All node enterprises in the CLSC are rational profit-maximizers with complete market information.

Assumption 7. An increase in order quantity by retailer e ($e \neq i$) may negatively impact retailer i 's profit (i.e., $\frac{\partial \pi_{ri}}{\partial q_e} < 0$). An increase in recycling quantity by recycler f ($f \neq j$) may negatively impact recycler j 's profit (i.e., $\frac{\partial \pi_{rcj}}{\partial l_f} < 0$).

Assumption 8. The unsold product inventories of retailer e and retailer i are interchangeable (i.e., $\frac{\partial^2 \pi_{ri}}{\partial q_i \partial q_e} \leq 0$). The recycling volumes of recycler f and recycler j are interchangeable (i.e., $\frac{\partial^2 \pi_{rcj}}{\partial l_j \partial l_f} \leq 0$).

The profit functions of the manufacturer, retailer i , recycler j , and the CLSC can be expressed as follows:

$$\pi_s = \sum_{i=1}^n (w - c) q_i + \sum_{j=1}^m (\Delta - b) l_j \quad (1)$$

$$\pi_{ri} = \left(h - q_i - \alpha \sum_{e \neq i} q_e \right) q_i - w q_i \quad (2)$$

$$\pi_{rcj} = b l_j - \left(k + l_j + \beta \sum_{f \neq j} l_f \right) l_j \quad (3)$$

$$\pi_{clsc} = \sum_{i=1}^n q_i \left(h - q_i - \alpha \sum_{e \neq i} q_e - c \right) + \sum_{j=1}^m l_j \left(\Delta - k - l_j - \beta \sum_{f \neq j} l_f \right) \quad (4)$$

4. The Model

4.1. Centralized Decision-Making

Centralized decision-making is modeled for a single selling period under a Cournot competition involving both sales and recycling. In the centralized system, all supply chain members are vertically integrated, and an unbiased decision-maker decides order quantities of retailers and recycling quantities of recyclers to maximize the total profit (π_{clsc}). The decision vectors are $\vec{q} = (q_1, q_2, \dots, q_n)$ and $\vec{l} = (l_1, l_2, \dots, l_m)$. The corresponding decision-making problem is:

$$\max_{\vec{q}, \vec{l}} \pi_{clsc} = \sum_{i=1}^n q_i \left(h - q_i - \alpha \sum_{e \neq i} q_e - c \right) + \sum_{j=1}^m l_j \left(\Delta - k - l_j - \beta \sum_{f \neq j} l_f \right) \quad (5)$$

By solving the equations $\frac{\partial \pi_{clsc}}{\partial q_i} = 0$ and $\frac{\partial \pi_{clsc}}{\partial l_j} = 0$, Proposition 1 is obtained.

Proposition 1. Under the centralized decision-making, the optimal order quantities and recycling quantities are:

$$q_i^{**} = \frac{h-c}{2\alpha(n-1)+2} \quad i = 1, 2, \dots, n$$

$$l_j^{**} = \frac{\Delta-k}{2\beta(m-1)+2} \quad j = 1, 2, \dots, m$$

Substituting q_i^{**} and l_j^{**} into Equation (4) yields the optimal system profit (π_{clsc}^{**}).

Proposition 2. Under centralized decision-making, the optimal order quantities decrease as the number of retailers increases, and the optimal recycling quantities decrease as the number of recyclers increases.

4.2. Decentralized Decision-Making

Under decentralized decision-making, the manufacturer, as the leader of the CLSC, first determines the wholesale price w of the products and the recycling price b of used products to maximize its profit. Subsequently, competitive retailers determine their order quantities based on w , and competitive recyclers determine their recycling quantities based on b , to maximize their respective profits. This decision-making problem is formulated as follows:

$$\begin{cases} \max_{w,b} \pi_s = \sum_{i=1}^n q_i(w-c) + \sum_{j=1}^m l_j(\Delta-b) \\ s.t. \max_{q_i} \pi_{ri} = (h-q_i-\alpha \sum_{e \neq i} q_e)q_i - wq_i \quad i = 1, 2, \dots, n \quad (6) \\ \max_{l_j} \pi_{rcj} = bl_j - (k+l_j+\beta \sum_{f \neq j} l_f)l_j \quad j = 1, 2, \dots, m \end{cases}$$

Using backward induction, solve $\frac{\partial \pi_{ri}}{\partial q_i} = 0$ and $\frac{\partial \pi_{rcj}}{\partial l_j} = 0$ to obtain: $q_i^* = \frac{h-w}{\alpha(n-1)+2}$ and $l_j^* = \frac{b-k}{\beta(m-1)+2}$. Substituting q_i^* and l_j^* into Equation (1) and solving $\frac{\partial \pi_s}{\partial w} = 0$ and $\frac{\partial \pi_s}{\partial b} = 0$, the manufacturer's optimal decentralized wholesale price w^* and recycling price b^* are derived. Substituting w^* into q_i^* and b^* into l_j^* yields the optimal decentralized order quantities for retailers and recycling quantities for recyclers.

Proposition 3. Under decentralized decision-making, the optimal wholesale price, recycling price, order quantities, and recycling quantities are:

$$w^* = \frac{h+c}{2}$$

$$b^* = \frac{k+\Delta}{2}$$

$$q_i^* = \frac{h-c}{2\alpha(n-1)+4} \quad i = 1, 2, \dots, n$$

$$l_j^* = \frac{\Delta-k}{2\beta(m-1)+4} \quad j = 1, 2, \dots, m$$

Proposition 4. Under decentralized decision-making, the manufacturer's optimal wholesale price and recycling price are independent of the numbers of retailers and recyclers. The optimal order quantities decrease as the number of retailers increases. The optimal recycling quantities decrease as the number of recyclers increases.

Using w^* , b^* , q_i^* , and l_j^* , the optimal profits π_s^* for the manufacturer, π_{ri}^* for retailers, and π_{rcj}^* for recyclers are derived under decentralized decision-making.

4.3. Coordinated Decision-Making

Proposition 5 can be obtained by comparing q_i^* with q_i^{**} and l_j^* with l_j^{**} .

Proposition 5. The optimal order quantities and recycling quantities in decentralized decision-making are less than those in centralized decision-making.

Proposition 5 indicates that decentralized decision-making can result in a loss of profit for the CLSC, implying that there is still room for improvement in the equilibrium profit of decentralized supply chain compared to the optimal profit of the integrated supply chain. Therefore, as the leader of the CLSC, the manufacturer can provide a cooperation contract to induce retailers and recyclers to make centralized decisions. This will maximize the profit of supply chain while ensuring that the profits of supply chain members are no less than their corresponding profits under decentralized decision-making.

Here, a linear transfer-payment contract is introduced to coordinate the CLSC. The manufacturer provides a linear transfer payment contingent on the retailer's order quantity, defined as $\theta_i(q_i) = \gamma(q_i - q_0)$ for $i = 1, 2, \dots, n$, aiming to incentivize retailers to increase their order quantities. Similarly, the manufacturer provides a linear transfer payment based on the recyclers' recycling quantities, formulated as $\varphi_j(l_j) = \delta(l_j - l_0)$ for $j = 1, 2, \dots, m$, aiming to incentivize recyclers to increase their recycling volumes. Furthermore, the manufacturer strategically sets the minimum order quantity q_0 and the minimum recycling amount l_0 , with q_0 depending on competing retailers' sales capacity and l_0 determined by recyclers' recycling capacity. To achieve coordination under this contract, the manufacturer determines the reward-penalty factors for product ordering and used product collection.

Under the linear transfer-payment contract, the profit functions of the manufacturer, competitive retailers, and recyclers are as follows:

$$\pi_s^{tr} = \sum_{i=1}^n q_i(w - c) + \sum_{j=1}^m l_j(\Delta - b) - \sum_{i=1}^n \theta_i(q_i) - \sum_{j=1}^m \varphi_j(l_j) \quad (7)$$

$$\pi_{ri}^{tr} = \left(h - q_i - \alpha \sum_{e \neq i} q_e \right) q_i - wq_i + \theta_i(q_i) \quad (8)$$

$$\pi_{rcj}^{tr} = bl_j - \left(k + l_j + \beta \sum_{f \neq j} l_f \right) l_j + \varphi_j(l_j) \quad (9)$$

Proposition 6. Under the linear transfer-payment contract $(w^{tr}, b^{tr}, \gamma, \delta)$, the CLSC can achieve perfect coordination when $w^{tr} = \gamma + \frac{\alpha(n-1)(h+c)+2c}{2\alpha(n-1)+2}$, $b^{tr} = -\delta + \frac{\beta(m-1)(\Delta+k)+2\Delta}{2\beta(m-1)+2}$, provided that the reward-penalty factors γ and δ satisfy the participation constraints of the manufacturer, retailers, and recyclers.

Proof 1. To achieve perfect coordination in the CLSC, decentralized decisions by competing retailers, on order quantities, and by recyclers, on used product recovery volumes, under a linear transfer-payment contract must align with centralized decision-making outcomes. By taking the partial derivatives of π_{ri}^{tr} with respect to q_i and of π_{rcj}^{tr} with respect to l_j , setting them equal to zero, and solving the resulting system of $n + m$ equations simultaneously, we obtain:

$$q_i^{tr} = \frac{h - w + \gamma}{\alpha(n-1) + 2}$$

$$l_j^{tr} = \frac{b - k + \delta}{\beta(m-1) + 2}$$

By comparing q_i^{tr} with q_i^{**} and l_j^{tr} with l_j^{**} , we can conclude that:

$$w^{tr} = \gamma + \frac{\alpha(n-1)(h+c)+2c}{2\alpha(n-1)+2}$$

$$b^{tr} = -\delta + \frac{\beta(m-1)(\Delta+k)+2\Delta}{2\beta(m-1)+2}$$

Furthermore, to coordinate the CLSC, the manufacturer's linear transfer-payment contract must satisfy the participation constraints of all supply chain members. This can be achieved by selecting appropriate values for the reward-penalty factors γ and δ .

Therefore, when the reward-penalty factors γ and δ satisfy the participation constraints of all supply chain members, the linear transfer-payment contract achieves perfect coordination of the CLSC with competition in both retailing and recycling. $\square$

5. Numerical Analysis

To validate the findings in Section 4, we will perform some numerical simulations on a CLSC consisting of one manufacturer, n retailers (where $n = 4,5,6,7$), and m recyclers (where $m = 2,3,4,5$). The values of key parameters are set as: $c = 255$, $c_r = 130$, $h = 1245$, $k = 5$, $\alpha = 0.4$ and $\beta = 0.1$. Using these parameters, we will analyze the changing trends in the optimal values of decision variables and in the maximum profits under decentralized and centralized decision-making scenarios as the numbers of retailers and recyclers vary.

Figure 2 illustrates the variation of order quantities as n increases from 4 to 7. Three conclusions are drawn:

- (1) Regardless of whether decision-making is decentralized or centralized, the optimal order quantities decrease as the number of retailers increases.
- (2) Under decentralized decision-making, the optimal order quantities are consistently lower than those under centralized scenario.
- (3) The gap between q_i^{**} and q_i^* gradually decreases as n increases.

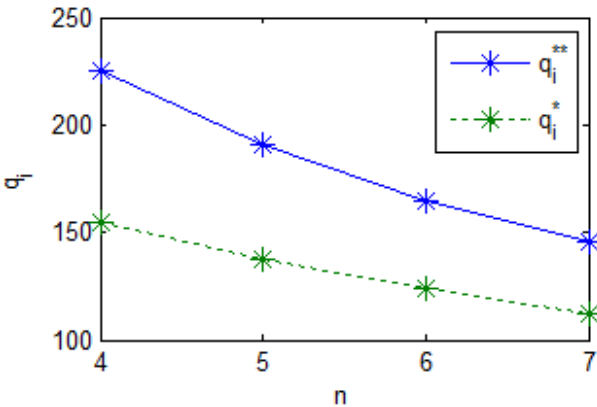


Figure 2. Dynamic variation of q_i with respect to n .

Figure 3 shows how the recycling quantities change as m increases from 2 to 5. From this figure, three key observations emerge:

- (1) Under both decentralized and centralized decision-making, the optimal recycling quantities decline as m increases.
- (2) The optimal recycling quantities under decentralized decision-making are consistently lower than those under centralized scenario.
- (3) As m increases, the difference between l_j^{**} and l_j^* gradually decreases.

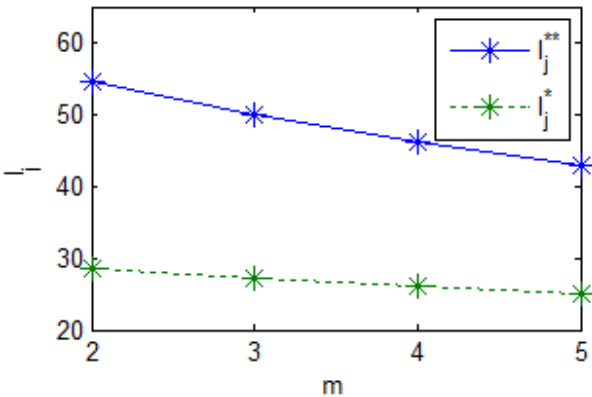


Figure 3. Dynamic variation of l_j with respect to m .

Figure 4 illustrates the variation in retail price as n increases from 4 to 7. Under centralized decision-making, the optimal retail price remains constant regardless of the number of retailers. In contrast, under the decentralized scenario, the optimal retail price decreases with an increasing number of retailers. As n increases, the discrepancy between p_i^{**} and p_i^* gradually decreases.

Figure 5 shows how the recycling price changes as m increases from 2 to 5. Under centralized decision-making, the optimal recycling price remains constant regardless of the number of recyclers. However, under decentralized decision-making, the optimal recycling price increases as the number of recyclers increases. As m increases, the difference between b_{rcj}^{**} and b_{rcj}^* gradually decreases.

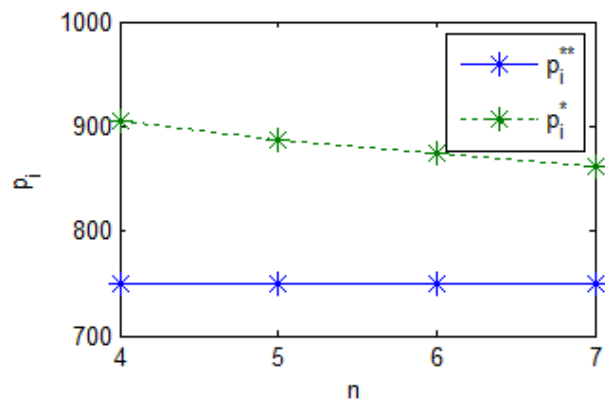


Figure 4. Dynamic variation of p_i with respect to n .

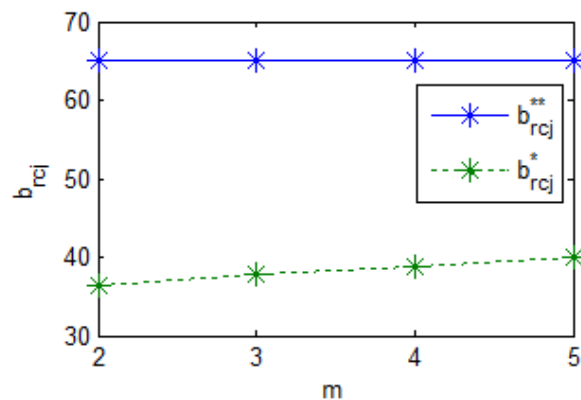


Figure 5. Dynamic variation of b_{rcj} with respect to m .

Figures 6 and 7 show how the profits of the CLSC change as n varies from 4 to 7 and m from 2 to 5. From these figures, two conclusions are drawn:

- (1) Under both decentralized and centralized decision-making, the optimal profits of the supply chain increase with the number of retailers or recyclers.
- (2) The optimal profits of the supply chain under decentralized decision-making are always lower than those under centralized scenario. These lower profits indicate that the supply chains are in a state of discoordination.

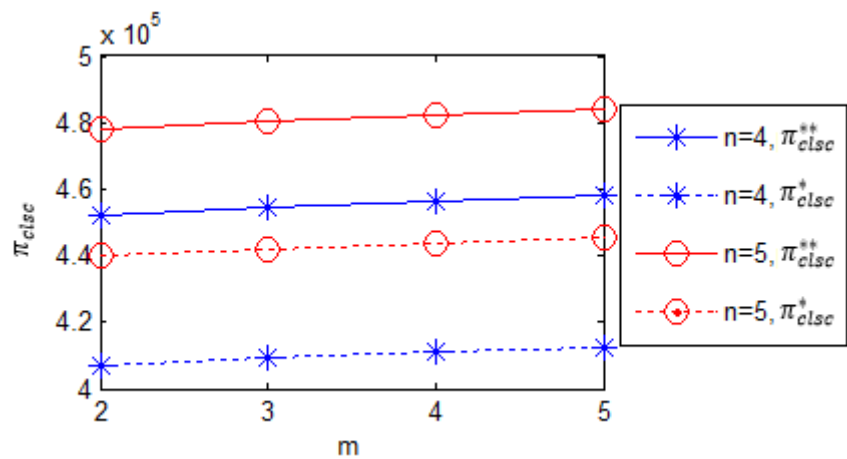


Figure 6. Dynamic variation in π_{clsc} for different n ($n = 4, 5$) and m ($m = 2, 3, 4, 5$).

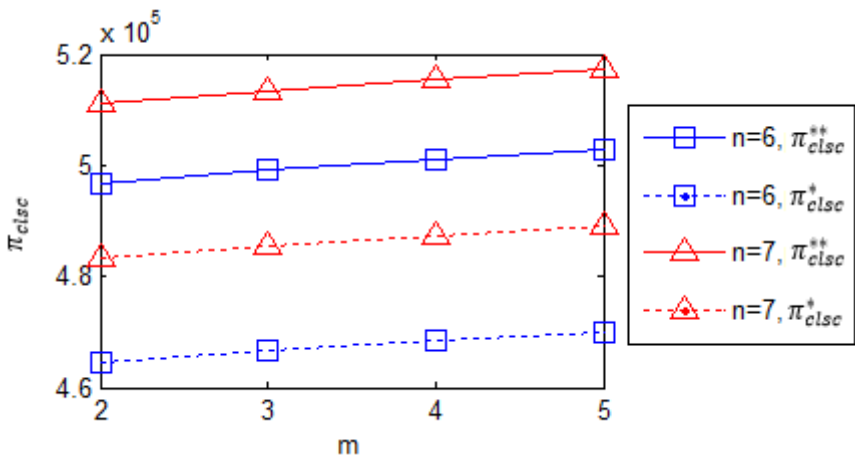


Figure 7. Dynamic variation in π_{clsc} for different n ($n = 6, 7$) and m ($m = 2, 3, 4, 5$).

Figure 8 shows the variation in the manufacturer's optimal profits as n ranges from 4 to 7 and m ranges from 2 to 5. Profits exhibit a positive correlation with retailer count and recycler count.

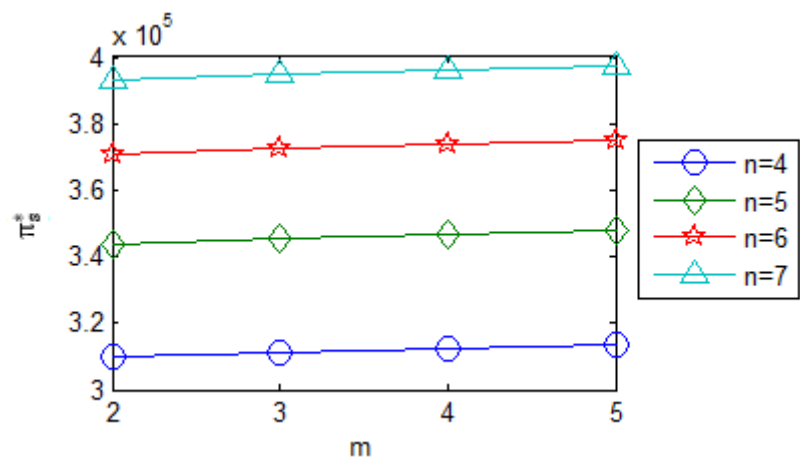


Figure 8. Dynamic variation in π_s^* for different n ($n = 4, 5, 6, 7$) and m ($m = 2, 3, 4, 5$).

Figure 9 demonstrates a negative correlation between the number of retailers and their optimal profits, while Figure 10 illustrates a similar inverse relationship between the number of recyclers and their optimal profits.

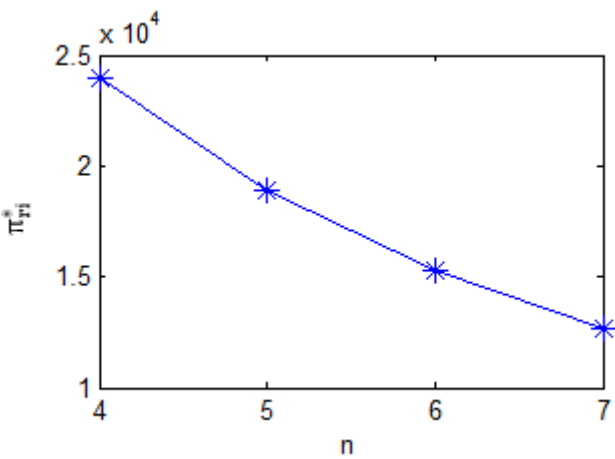


Figure 9. Dynamic variation in π_{ri}^* for different n ($n = 4,5,6,7$).

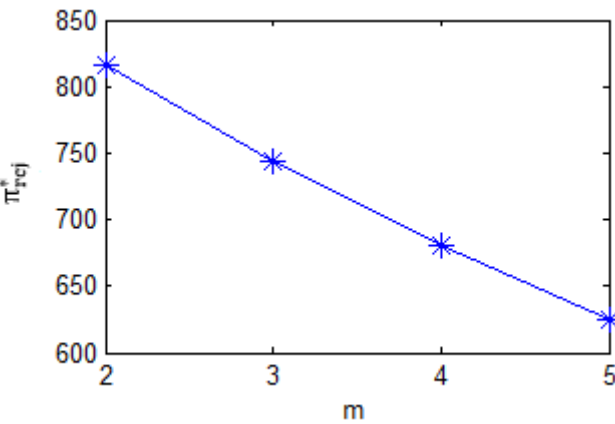


Figure 10. Dynamic variation in π_{rcj}^* for different m ($m = 2,3,4,5$).

The minimum order quantity and recycling quantity required by the manufacturer within a linear transfer-payment contract are set as: $q_0 = 140$ and $l_0 = 37$. Based on the participation constraints of the node enterprises and the condition $b^{tr} > 0$, the reward-penalty factors of the contract must satisfy the following derived conditions for each pair (n,m) in $\{(5,2), (6,3), (7,4)\}$.

$$\begin{cases} 700\gamma + 74\delta + 290565.4287 \geq 343740 \\ 36246.3073 - 140\gamma \geq 18906 \\ 2975.2061 - 37\delta \geq 816.3266 \\ 0 < \delta < 119.5455 \\ \gamma > 0 \end{cases}$$

$$\begin{cases} 1140\gamma + 111\delta + 328200 \geq 372450 \\ 27225 - 140\gamma \geq 15314 \\ 2500 - 37\delta \geq 743.8018 \\ 0 < \delta < 115 \\ \gamma > 0 \end{cases}$$

$$\begin{cases} 980\gamma + 148\delta + 358647.8641 \geq 396070 \\ 21195.9240 - 140\gamma \geq 12656 \\ 2130.1760 - 37\delta \geq 680.5290 \\ 0 < \delta < 111.1538 \\ \gamma > 0 \end{cases}$$

Using the above system of inequalities and related parameters, we can obtain the following:

Table 2. Values of coordination variables and evaluation of coordination effects.

(n,m)	γ	δ	w^{tr}	b^{tr}	$\pi_{clsc}^{tr}(\pi_{clsc}^{**})$	π_s^{tr}	π_{ri}^{tr}	π_{rcj}^{tr}
(5,2)	100	50	659.6154	69.54545	477750	364270	22246	1125
					π_{clsc}^*	π_s^*	π_{ri}^*	π_{rcj}^*

					407050	309710	23928	816.3266
(6,3)	50	40	635	75	$\pi_{clsc}^{tr}(\pi_{clsc}^{**})$	π_s^{tr}	π_{ri}^{tr}	π_{rcj}^{tr}
					499050	374640	20225	1020
					π_{clsc}^*	π_s^*	π_{ri}^*	π_{rcj}^*
					466570	372450	15314	743.8018
(7,4)	35	30	639.4118	81.1538	$\pi_{clsc}^{tr}(\pi_{clsc}^{**})$	π_s^{tr}	π_{ri}^{tr}	π_{rcj}^{tr}
					515542	397390	16296	1020
					π_{clsc}^*	π_s^*	π_{ri}^*	π_{rcj}^*
					487380	396070	12656	680.5290

where: $\pi_{clsc}^{tr} = \pi_s^{tr} + n\pi_{ri}^{tr} + m\pi_{rcj}^{tr}$.

From Table 2, two key conclusions can be drawn: (1) Under the linear transfer-payment contract, the profits of all supply chain members consistently exceed those under the decentralized scenario. (2) The total profit under this contract matches the maximum profit achievable by an integrated supply chain. These results demonstrate that the linear transfer-payment contract can achieve perfect coordination of the CLSC.

6. Conclusions

This paper investigates various aspects of product sales and used product recycling management, focusing on the competition effect and contract design in the CLSC. The study examines a CLSC consisting of a manufacturer, n retailers, and m recyclers, highlighting the impact of retailers’ and recyclers’ competition on order quantities, recycling volumes, and profits. A linear transfer-payment contract model is developed to maximize the total profit of the CLSC based on the Cachon profit function. The numerical results validate the theoretical analysis.

The main results of this study are summarized as follows.

Under decentralized decision-making, an increase in the number of retailers reduces their optimal order quantities and retail prices. In contrast, under centralized decision-making, more retailers lower optimal order quantities but leave the optimal retail price unaffected. For recyclers, decentralized decision-making results in decreased recycling volumes yet higher optimal recycling prices as their numbers grow. Under centralized decision-making, however, more recyclers reduce recycling volumes, while the optimal recycling price remains stable.

Competition among retailers or recyclers enhances profits for the supply chain and the manufacturer, whereas retailer competition erodes retailers’ profits, and recycler competition similarly diminishes recyclers’ profits.

When the order quantity reward-penalty factor γ and the recycling volume reward-penalty factor δ fall within specified ranges, the linear transfer-payment contract achieves coordination in a competitive CLSC comprising one manufacturer, n retailers, and m recyclers.

The following management insights can be drawn:

(1) The dominant manufacturer should collaborate with as many retailers and recyclers as possible, provided they are willing, as this benefits the manufacturer, the supply chain, and consumers.

(2) The manufacturer should coordinate the CLSC by establishing a linear transfer-payment contract with a larger pool of reliable retailers and recyclers, and setting the reward-penalty factors according to their participation constraints.

Further research should address the limitations of this study. First, this study assumes that new and remanufactured products are priced equally. Exploring scenarios where remanufactured products are sold at a discount relative to new ones could be a valuable extension. Second, the model in this paper does not account for the non-remanufacturing rate of used products. Future work could integrate the fuzzy set theory to quantify the non-remanufacturing rate. Third, manufacturer competition is excluded from the current framework. Given that product homogeneity intensifies

market competition, extending the model to include multiple manufacturers would better reflect the real world dynamics.

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