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Article

Non-Equipartition as a Signature of Structural Memory in Self-Gravitating Systems

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Abstract

Self-gravitating, collisionless systems such as dark-matter halos, stellar halos, and satellite galaxies consistently exhibit long-lived violations of equipartition. Standard Λ CDM relaxation theory attributes these deviations to extremely long relaxation times, predicting slow asymptotic decay toward the partial-equipartition curves derived from the Fokker–Planck equation. This work introduces an alternative explanation, the *Structural Memory Framework* (SMF), based on a history-dependent constraint. A dimensionless structural information budget $\mathcal{I}_0$ is defined, computed directly from a halo’s merger tree, together with a corresponding structural information functional $\mathcal{I}[f]$ defined as the Kullback–Leibler divergence of a phase-space distribution relative to the Markovian baseline f_M . The stationary state is obtained by maximising the entropy $S[f]$ subject to conservation of energy, particle number, and the constraint $\mathcal{I}[f] = \mathcal{I}_0$. This produces a history-tilted Boltzmann distribution $f_{\text{stat}} \propto f_M^\alpha \exp[-\beta_{\text{eff}} H]$, in which the deformation exponent α is set by $\mathcal{I}_0$. The Structural Memory Framework predicts a qualitatively distinct evolution of the equipartition deficit Δ_{eq} . Whereas Λ CDM relaxation theory requires monotonic decay $d\Delta_{\text{eq}}/dt < 0$, SMF predicts saturation $d\Delta_{\text{eq}}/dt \rightarrow 0$ to a non-zero floor determined by $\mathcal{I}_0$. Three binary falsification tests are formalised, including a corrected “Twins Test” using the *History–Kinematic Correlation Ratio* $\mathcal{R}_{\mathcal{IK}} = \frac{\text{Corr}(\Delta_{\text{eq}}, \mathcal{I}_0)}{\max[\text{Corr}(\Delta_{\text{eq}}, \Lambda\text{CDM proxies})]}$, and outer stellar halos and tidally processed satellites are identified as the primary observational targets. The framework provides a mathematically rigorous and observationally testable alternative to the Markovian assumption underlying Λ CDM relaxation theory, and establishes the structural parameter α as a unifying quantity linking non-equipartition, the S_8 suppression, and rotation-curve stabilisation within the broader CIOU paradigm.

Keywords: structural memory; equipartition; Λ CDM; collisionless dynamics; dark matter halos

1. Introduction

Non-equipartition is widely observed in self-gravitating systems: multi-temperature components, anisotropy, mass segregation plateaus, and persistent substructure. Standard Λ CDM gravitational physics (collisionless Boltzmann plus slow two-body relaxation) predicts incomplete mixing but with asymptotic decay of non-equipartition toward partial-equipartition solutions.

This work proposes a different mechanism: non-equipartition as a stationary consequence of structural information accumulated through a system’s dynamical history.

Thesis. The stationary distribution of a real halo is not the maximum-entropy, memoryless distribution. It is the maximum-entropy distribution subject to a history-derived structural information constraint. This predicts an asymptotically stable, non-zero equipartition deficit whose magnitude correlates with the integrated dynamical history, not just with instantaneous halo properties. The prediction differs from Λ CDM quantitatively and measurably.

2. Markovian Λ CDM Baseline

2.1. Evolution Equation

The collisionless Boltzmann equation reads

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f - \nabla_{\mathbf{x}} \Phi \cdot \nabla_{\mathbf{v}} f = \mathcal{C}(f), \quad (1)$$

where $f(\mathbf{x}, \mathbf{v}, t)$ is the phase-space distribution, Φ is the potential, and $\mathcal{C}(f)$ is an effective collision or relaxation term.

Markov closure assumes

$$\mathcal{C}(f(t)) = \mathcal{C}(f), \quad (2)$$

with no explicit dependence on past structural states.

2.2. Equipartition Under Markovian Closure

For a multi-mass system with species a and b , standard relaxation theory leads, in the full equipartition limit, to

$$m_a \sigma_a^2 = m_b \sigma_b^2, \quad (3)$$

where m_a and σ_a are the mass and velocity dispersion of component a .

In Λ CDM, non-equipartition may persist for long periods, but the amplitude is expected to decrease, asymptotically converging toward a partial-equipartition Fokker–Planck solution. In a schematic representation,

$$\frac{d\Delta_{\text{eq}}}{dt} < 0, \quad \lim_{t \rightarrow \infty} \Delta_{\text{eq}} = 0 \quad (4)$$

for the idealised Markovian baseline, with realistic systems approaching known partial-equipartition curves.

2.3. Operational Definition: “Sufficiently Relaxed”

To avoid straw-man arguments, a system is defined as *sufficiently relaxed* when:

- it has experienced no major merger in the last 5–8 dynamical times;
- its mass accretion history is stable over the same interval;
- it exhibits no large-amplitude, rapid potential fluctuations.

This regime is precisely where Λ CDM predicts monotonic decay of Δ_{eq} under Markovian relaxation theory.

3. Structural Information Budget $\mathcal{I}_0$: A History-Derived Constraint

The structural information budget $\mathcal{I}_0$ must be defined independently of the present-day distribution $f(\mathbf{z})$ (with $\mathbf{z}$ denoting the full phase-space coordinate). It encodes the irreversible dynamical history of a halo and acts as an external constraint on the stationary state.

To make the definition operational and simulation-ready, a cumulative, merger-tree-based functional is adopted:

$$\mathcal{I}_0 \equiv \int_0^{t_0} \left(\frac{\dot{M}_{\text{acc}}(t)}{M(t)} \right)^2 dt, \quad (5)$$

where:

- $M(t)$ is the halo mass;
- $\dot{M}_{\text{acc}}(t)$ is the instantaneous (smooth plus clumpy) mass accretion rate.

The quantity $\mathcal{I}_0$ is:

- dimensionless;
- monotonic under irreversible events;
- directly computable from standard merger trees;

- independent of the present-day phase-space state.

An alternative discrete definition for comparison is

$$\mathcal{I}_0 = \sum_{i \in \text{mergers}} \left(\frac{M_i}{M_{\text{final}}} \right)^2 \left(\frac{2T_i}{|U_i|} \right), \quad (6)$$

where each merger contributes according to its mass ratio M_i/M_{final} and virial impulse $2T_i/|U_i|$. Both forms satisfy the requirement that $\mathcal{I}_0$ is a causal history variable, not a kinematic diagnostic.

4. Structural Information Functional $\mathcal{I}[f]$

Given a candidate phase-space distribution $f(\mathbf{z})$, the structural information functional is defined as the Kullback–Leibler divergence relative to the Markovian baseline $f_M(\mathbf{z})$:

$$\mathcal{I}[f] = \int f(\mathbf{z}) \ln \left(\frac{f(\mathbf{z})}{f_M(\mathbf{z})} \right) d^6z. \quad (7)$$

Key properties are:

- $\mathcal{I}[f]$ is dimensionless;
- $\mathcal{I}[f] = 0$ if and only if $f = f_M$;
- $\mathcal{I}[f]$ is monotonic under coarse-graining.

The central constraint of the Structural Memory Framework is

$$\mathcal{I}[f_{\text{stat}}] = \mathcal{I}_0. \quad (8)$$

This enforces historical memory on the stationary state without ever defining memory from f itself, avoiding circularity.

5. Constrained Variational Principle

The goal is to determine the stationary distribution f_{stat} that:

1. conserves particle number;
2. conserves energy;
3. maximises entropy;
4. satisfies the structural memory constraint (8).

Define the dimensionless entropy

$$S[f] = - \int f \ln f d^6z. \quad (9)$$

Define the total particle number:

$$N[f] = \int f d^6z. \quad (10)$$

Define the total energy:

$$E[f] = \int f(\mathbf{z}) H(\mathbf{z}) d^6z, \quad (11)$$

where H has physical units of energy.

The functional to be maximised is

$$\mathcal{F}[f] = S[f] - \beta E[f] - \mu N[f] - \eta \mathcal{I}[f], \quad (12)$$

where:

- β has units of 1/energy;
- μ and η are dimensionless multipliers.

This construction is dimensionally consistent: $S[f]$, $N[f]$, and $\mathcal{I}[f]$ are dimensionless, and only $E[f]$ couples to a dimensional quantity β .

5.1. Euler–Lagrange Condition

Varying $\mathcal{F}$ with respect to f yields

$$\delta\mathcal{F} = \int \delta f \left(-\ln f - 1 - \beta H - \mu - \eta \left[1 + \ln \left(\frac{f}{f_M} \right) \right] \right) d^6z = 0. \quad (13)$$

Hence the stationary condition is

$$(1 + \eta) \ln f - \eta \ln f_M = -(1 + \eta + \mu) - \beta H. \quad (14)$$

Solving for $\ln f_{\text{stat}}$ gives

$$\ln f_{\text{stat}} = \frac{\eta}{1 + \eta} \ln f_M - \frac{\beta}{1 + \eta} H + \text{const.} \quad (15)$$

Exponentiating,

$$f_{\text{stat}}(\mathbf{z}) \propto f_M(\mathbf{z})^\alpha \exp[-\beta_{\text{eff}} H(\mathbf{z})], \quad (16)$$

with

$$\alpha = \frac{\eta}{1 + \eta}, \quad \beta_{\text{eff}} = \frac{\beta}{1 + \eta}. \quad (17)$$

5.2. Interpretation

If $\eta = 0$, one recovers the Markovian baseline

$$f_{\text{stat}} = f_M \exp(-\beta H), \quad (18)$$

up to normalisation. If $\eta > 0$, the stationary state is tilted away from f_M , with the deformation amplitude fixed by the historical memory budget $\mathcal{I}_0$ via the constraint $\mathcal{I}[f_{\text{stat}}] = \mathcal{I}_0$.

The result is a generalised family of stationary states that differ not only in amplitude but also in mass and spatial dependence, enabling sharp contrasts with Λ CDM predictions. The derivation is dimensionally correct, non-circular, and reproducible.

6. Predictions for Equipartition Deficits

6.1. Defining the Equipartition Deficit

For a multicomponent system, the equipartition deficit is defined as

$$\Delta_{\text{eq}} = \frac{1}{N} \sum_a \left(\frac{T_a - \bar{T}}{\bar{T}} \right)^2, \quad (19)$$

where T_a is the kinetic temperature of component a , $\bar{T}$ is the mean temperature, and N is the number of components.

Λ CDM standard relaxation theory predicts known scaling behaviours: partial equipartition for multi-mass populations, Spitzer criteria determining stability of segregation, and characteristic radial dependence in relaxed systems (e.g. [1,3,4]). Thus Λ CDM does not, in general, predict $\Delta_{\text{eq}} \rightarrow 0$, but rather

$$\Delta_{\text{eq}}^{\Lambda\text{CDM}}(m, r) = f_{\text{FP}}(m, r), \quad (20)$$

where f_{FP} denotes the Fokker–Planck solution curve.

6.2. SMF Prediction: A Distinct Asymptotic Form

The SMF stationary distribution (16) introduces an additional, history-dependent deformation characterised by the parameter $\alpha(\eta)$:

$$f_{\text{stat}} \propto f_M^\alpha \exp(-\beta_{\text{eff}} H). \quad (21)$$

This produces a non-equipartition profile of the form

$$\Delta_{\text{eq}}^{\text{SMF}}(m, r) = f_{\text{FP}}(m, r) + g_{\text{hist}}(m, r; \mathcal{I}_0), \quad (22)$$

with $g_{\text{hist}} \neq 0$.

Here, ΛCDM determines the Fokker–Planck baseline f_{FP} , and SMF adds an irreducible deformation tied to $\mathcal{I}_0$. The SMF prediction is therefore an entirely different functional form, not merely a higher amplitude of the same curve.

6.3. Time-Derivative Test (Binary Falsifier)

For sufficiently relaxed halos (Section 2), the asymptotic behaviour of $\Delta_{\text{eq}}(t)$ distinguishes the models:

$$\Lambda\text{CDM}: \quad \frac{d}{dt} \Delta_{\text{eq}}^{\Lambda\text{CDM}} < 0 \quad (\text{slow decay}) \quad (23)$$

$$\text{SMF}: \quad \frac{d}{dt} \Delta_{\text{eq}}^{\text{SMF}} \rightarrow 0 \quad (\text{saturation at a floor tied to } \mathcal{I}_0). \quad (24)$$

This slope-based discriminator is binary and falsifiable in simulations.

6.4. Mass- and Radius-Dependent Predictions

Because f_M is isotropic and the deformation enters multiplicatively, lower-mass components receive stronger tilting from f_M^α , and outer regions with weaker binding amplify the SMF tilt. Thus

$$\Delta_{\text{eq}}^{\text{SMF}}(m, r) \quad \text{departs most strongly from} \quad f_{\text{FP}}(m, r) \quad \text{at large } r \text{ and small } m. \quad (25)$$

This provides clear observational targets.

7. Falsification Tests

7.1. Test 1: Asymptotic Slope Test (Primary Kill-Switch)

Objective.

Measure the time evolution of the equipartition deficit $\Delta_{\text{eq}}(t)$ in halos with no major merger for at least 5–8 dynamical times.

ΛCDM Prediction.

Standard Markovian relaxation theory implies

$$\frac{d}{dt} \Delta_{\text{eq}}^{\Lambda\text{CDM}} < 0, \quad (26)$$

i.e. monotonic, though slow, decay toward the partial-equilibrium Fokker–Planck curve $f_{\text{FP}}(m, r)$.

SMF Prediction.

The SMF prediction is

$$\frac{d}{dt} \Delta_{\text{eq}}^{\text{SMF}} \rightarrow 0, \quad (27)$$

i.e. saturation onto a history-constrained floor

$$\Delta_{\text{eq}}^{\text{SMF}}(t) \rightarrow \Delta_{\text{eq}}^{\text{floor}}(\mathcal{I}_0). \quad (28)$$

This difference in time derivative is binary and cannot be reconciled by parameter tuning. If Δ_{eq} continues to decay indefinitely in sufficiently relaxed halos, SMF is falsified. If Δ_{eq} flattens to a constant that correlates with $\mathcal{I}_0$, the Markovian baseline of Λ CDM is falsified.

7.2. Test 2: Corrected Twins Test with Quantitative Benchmarking

Objective.

Identify pairs of halos (“twins”) with nearly identical present-day mass, concentration, and virial state, but radically different merger histories.

Λ CDM Prediction.

Λ CDM already predicts some history dependence in internal kinematics through:

- concentration c ,
- splashback radius R_{sp} ,
- formation redshift z_{form} ,
- $V_{\text{max}}/V_{\text{circ}}$.

These standard Λ CDM correlations must serve as the null benchmark.

SMF Prediction.

Under SMF, the history-derived structural information budget $\mathcal{I}_0$ is expected to be a stronger predictor of present-day non-equipartition than any Λ CDM proxy. A quantitative discriminator is defined as

$$r(\Delta_{\text{eq}}, \mathcal{I}_0) \gg r(\Delta_{\text{eq}}, c), \quad r(\Delta_{\text{eq}}, V_{\text{max}}/V_{\text{circ}}), \quad r(\Delta_{\text{eq}}, z_{\text{form}}), \quad (29)$$

where $r(\cdot, \cdot)$ is the correlation coefficient across the halo sample.

7.2.1. History–Kinematic Correlation Ratio (HKCR)

To quantify predictive power more sharply, define the *History–Kinematic Correlation Ratio*

$$\mathcal{R}_{\mathcal{IK}} \equiv \frac{\text{Corr}(\Delta_{\text{eq}}, \mathcal{I}_{\text{hist}})}{\max[\text{Corr}(\Delta_{\text{eq}}, \Lambda\text{CDM proxies})]}, \quad (30)$$

where $\mathcal{I}_{\text{hist}}$ is the chosen realisation of $\mathcal{I}_0$ (e.g. from Equation (5) or (6)), and Λ CDM proxies include c , z_{form} , R_{sp} , and $V_{\text{max}}/V_{\text{circ}}$.

Binary Criterion.

$$\mathcal{R}_{\mathcal{IK}} > 1 \Rightarrow \text{SMF supported}, \quad (31)$$

$$\mathcal{R}_{\mathcal{IK}} \leq 1 \Rightarrow \Lambda\text{CDM Markovianity supported}. \quad (32)$$

This transforms the Twins Test into a high-precision model discriminator rather than a qualitative comparison. The ratio $\mathcal{R}_{\mathcal{IK}}$ plays a role analogous to a Bayes factor: a single scalar discriminator unifying multiple competing predictors.

7.3. Test 3: Controlled Merger Experiment (Dynamical Impulse Test)

Objective

Perform controlled N -body mergers with tunable initial conditions to isolate the dynamical effects of major and minor mergers on $\Delta_{\text{eq}}(t)$ and $\mathcal{I}[f(t)]$.

Λ CDM Prediction

Mergers drive temporary spikes in anisotropy and temperature differences, but after sufficient relaxation time:

$$\Delta_{\text{eq}}(t) \downarrow \quad \text{towards} \quad f_{\text{FP}}(m, r). \quad (33)$$

SMF Prediction

Under SMF, a merger irreversibly increases $\mathcal{I}_0$, and the system settles onto a new plateau:

$$\Delta_{\text{eq}}(t) \rightarrow \Delta_{\text{eq}}^{\text{floor}}(\mathcal{I}_0^{\text{new}}), \quad (34)$$

with $\mathcal{I}_0^{\text{new}} > \mathcal{I}_0^{\text{old}}$.

Binary Criterion.

A permanent increase in the floor supports SMF; decay of non-equipartition back to the baseline falsifies SMF. This test isolates the mechanism directly.

8. Observational Signatures

A rigorous theory must make principled predictions for empirical data, independent of simulation environments. SMF predicts the strongest deviations from Λ CDM relaxation theory in environments where:

- external tidal history is strong (large $\mathcal{I}_0$);
- internal relaxation is weak (long dynamical times).

These regions minimise Λ CDM partial-relaxation noise and amplify structural memory signals.

8.1. Primary Observational Targets

8.1.1. Outer Stellar Halos (50–150 kpc)

Outer stellar halos feature long dynamical times, so internal mixing is extremely weak, and they have strong tidal histories, implying large $\mathcal{I}_0$. Λ CDM predicts mild decay of anisotropies and multi-temperature structures, whereas SMF predicts persistent, radius-dependent non-equipartition that correlates with known orbital passages and accretion events.

Measurable quantities include:

- radial dispersion gradients for different tracer populations;
- kinematic misalignments between metal-rich and metal-poor components;
- long-lived non-Gaussian velocity distributions.

8.1.2. Satellite Galaxies with Reconstructed Orbits

Satellite galaxies of similar mass and size but with different pericentric passages and tidal shock histories provide a natural test. The SMF prediction is that

$$\Delta_{\text{eq}} \quad \text{should correlate with reconstructed tidal history,} \quad (35)$$

whereas the Λ CDM baseline predicts only a weak dependence on tides.

8.1.3. Cold Streams and Shells

Under Λ CDM, streams are gradually phase-mixed. In SMF, mixing stalls once the structural memory budget saturates. Observationally, the persistence of multi-temperature or anisotropic signatures in very old streams would support SMF.

8.2. Secondary Observational Anchors

These systems are still valuable but noisier.

8.2.1. Globular Clusters

Partial equipartition is already known in Λ CDM. SMF predicts an additional, non-Fokker–Planck deformation that may be detectable in sufficiently collisionless regimes.

8.2.2. Dwarf Spheroidals

Age-dependent dispersions, if controlled for tides, can test history-linked structural information and provide an additional check of the SMF predictions.

8.2.3. Brightest Cluster Galaxies (BCGs)

Velocity-dispersion offsets between the BCG and intra-cluster light provide a complementary test of persistent non-equipartition in cluster cores.

These secondary probes are supportive but less decisive than the outer-halo tests.

9. Discussion

The Structural Memory Framework proposes that the stationary state of a self-gravitating, collisionless system is not determined solely by its present conditions but also by a history-derived structural information budget $\mathcal{I}_0$. This yields:

- a family of stationary solutions,

$$f_{\text{stat}} \propto f_M^\alpha \exp(-\beta_{\text{eff}} H), \quad (36)$$

distinct from traditional Fokker–Planck partial-equipartition curves;

- a natural explanation for persistent non-equipartition, not as a transient due to long relaxation times but as a structural consequence of irreversible dynamical history;
- binary testability via slopes, with $d\Delta_{\text{eq}}/dt$ serving as the decisive discriminator;
- predictive correlations between present-day dynamics and historical accretion, expressible in terms of $\mathcal{I}_0$.

Crucially, SMF does not dispute Λ CDM gravitational physics; it disputes the Markovian assumption that present-day dynamics contain no explicit memory of the system's past.

If any of the following holds, SMF is falsified:

- the equipartition deficit decays monotonically in relaxed halos;
- merger-induced increases in Δ_{eq} are erased over time;
- Λ CDM proxies outperform $\mathcal{I}_0$ in predicting internal kinematics;
- outer halos show no history-correlated deformation from Fokker–Planck predictions.

The theory is therefore testable, falsifiable, and non-adjustable, satisfying the standards of a physical model.

9.1. Connection to CIOU Unification: The Shared Structural Parameter α

A central implication of the Structural Memory Framework is that the same dimensionless structural parameter α —which governs:

- the deviation of the stationary distribution

$$f_{\text{stat}} \propto f_M^\alpha \exp(-\beta_{\text{eff}} H), \quad (37)$$

- the stabilisation amplitude in galactic rotation curves;
- the suppression term in the S_8 growth discrepancy,

must also determine the saturation amplitude of the equipartition deficit:

$$\Delta_{\text{eq}}^{\text{sat}} = \Delta_{\text{eq}}(\mathcal{I}_0) \propto \alpha(\mathcal{I}_0). \quad (38)$$

Thus:

- rotation curves (galactic stability),
- large-scale clustering (S_8 tension),
- halo non-equipartition (this work)

all emerge from the same structural memory parameter, constrained by the history-derived $\mathcal{I}_0$. This transforms SMF from a local dynamical hypothesis into a unified description of structural deviations from Λ CDM across multiple cosmic scales.

10. Limitations and Scope

The Structural Memory Framework developed here establishes a variational foundation for history-dependent stationary states, but several limitations must be noted.

1. **No explicit dynamical evolution equation is provided.** A closed-form time-dependent evolution law for $f(t)$ is not derived. The predictions concern the asymptotic behaviour of $\Delta_{\text{eq}}(t)$ in sufficiently relaxed halos.
2. **The deformation function $g_{\text{hist}}(m, r; \mathcal{I}_0)$ is not analytically specified.** Its existence is demonstrated from first principles, but its explicit form is left to future numerical and empirical calibration.
3. **The framework does not modify gravity.** SMF operates entirely within standard Newtonian/ Λ CDM gravitational potentials. Any deviations arise from the entropy–information constraints, not from new forces or particle species.
4. **Applicability is limited to collisionless, long-relaxation systems.** Systems dominated by short collisional relaxation (e.g. dense star clusters) fall outside the scope of the present model.
5. **Dependence on merger-tree reconstruction.** The history metric $\mathcal{I}_0$ requires accurate merger trees; observational proxies may introduce noise and bias.
6. **Equifinality caveat.** A confirmed correlation between Δ_{eq} and $\mathcal{I}_0$, while falsifying the strict Markovian assumption, does not automatically validate the specific mechanism of entropy maximisation under a structural constraint. An alternative, non-Markovian dynamical process not captured by the variational principle could in principle produce a similar saturation effect. A positive result confirms the failure of memoryless equilibration and the role of history, but the precise thermodynamic mechanism proposed here would require further validation through the specific functional form of f_{stat} .

Despite these limitations, the core prediction—saturation of Δ_{eq} governed by the history parameter $\mathcal{I}_0$ —remains binary, falsifiable, and independent of the unresolved details.

11. Conclusion

This work has established the mathematical and physical basis for a history-dependent stationary state in self-gravitating, collisionless systems. By defining a structural information budget $\mathcal{I}_0$ directly from the merger tree and enforcing the constraint $\mathcal{I}[f] = \mathcal{I}_0$ in a dimensionally consistent variational principle, a generalised stationary distribution

$$f_{\text{stat}} \propto f_M^\alpha \exp(-\beta_{\text{eff}} H) \quad (39)$$

is derived, which recovers the Markovian baseline when $\alpha = 0$ and introduces a history-induced deformation when $\alpha > 0$.

The resulting Structural Memory Framework predicts that the equipartition deficit Δ_{eq} —a well-known feature of halos in Λ CDM simulations—should saturate rather than decay in sufficiently

relaxed systems. This prediction is orthogonal to the traditional “slow relaxation” explanation and is falsifiable through the asymptotic time derivative:

$$\frac{d\Delta_{\text{eq}}}{dt} < 0 \quad (\Lambda\text{CDM}) \quad \text{vs.} \quad \frac{d\Delta_{\text{eq}}}{dt} \rightarrow 0 \quad (\text{SMF}). \quad (40)$$

A set of binary tests has been formalised, including (i) the Asymptotic Slope Test, (ii) a corrected Twins Test quantified through the History–Kinematic Correlation Ratio $\mathcal{R}_{\mathcal{IK}}$, and (iii) controlled merger experiments. These tests enable direct separation of Markovian and non-Markovian relaxation dynamics in cosmological simulations.

Observationally, SMF predicts history-correlated non-equipartition signatures in outer stellar halos, tidally processed satellites, and long-lived phase-space structures such as streams and shells, where long dynamical times suppress the standard ΛCDM relaxation pathways.

The structural parameter α linking f_{stat} to $\mathcal{I}_0$ is the same parameter that appears in CIOU analyses of rotation-curve stability and the S_8 growth suppression. This positions SMF not as an isolated hypothesis but as a foundational component of a broader unifying framework for non-Markovian cosmological dynamics.

The theory presented here is testable, falsifiable, and compatible with standard gravitational dynamics. The next generation of simulations and observations can determine its success or failure.

References

1. J. Binney and S. Tremaine. *Galactic Dynamics* (2nd ed.). Princeton University Press, 2008.
2. D. Lynden-Bell. Statistical mechanics of violent relaxation in stellar systems. *MNRAS*, 136:101–121, 1967.
3. L. Spitzer and M. H. Hart. Random gravitational encounters and the evolution of spherical systems. *ApJ*, 164:399, 1971.
4. S. Tremaine et al. The Fokker–Planck evolution of star clusters. *ApJ*, 300:154, 1986.
5. L. Hernquist. An analytical model for spherical galaxies and bulges. *ApJ*, 356:359, 1990.
6. V. Springel et al. The Aquarius Project: the subhalos of galactic halos. *MNRAS*, 391:1685–1711, 2008.
7. P. Behroozi et al. A unified analysis of star formation histories from $z = 0$ –8. *ApJ*, 770:57, 2013.
8. A. Helmi. Streams, substructures and the early formation history of the Milky Way. *ARA&A*, 58:205–256, 2020.
9. C. E. Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27:379–423, 623–656, 1948.
10. T. M. Cover and J. A. Thomas. *Elements of Information Theory*. Wiley, 2006.

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