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ON MAHLER EXPANSION OF p-ADIC GAMMA FUNCTION AFFILIATED WITH THE q-BOOLE POLYNOMIALS

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Abstract. In this paper, we investigate several relations for p-adic gamma function by means of their Mahler expansion and fermionic p-adic q-integral on $\mathbb{Z}_p$. We also derive two fermionic p-adic q-integrals of p-adic gamma function in terms of q-Boole polynomials and numbers. Moreover, we discover fermionic p-adic q-integral of the derivative of p-adic gamma function. We acquire a representation for the p-adic Euler constant by means of the q-Boole polynomials. We finally develop a novel, explicit and interesting representation for the p-adic Euler constant including Stirling numbers of the first kind.

1. Introduction

Let $\mathbb{N} := \{1, 2, 3, \cdots\}$ and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Throughout this paper, $\mathbb{Z}$ denotes the set of integers, $\mathbb{R}$ denotes the set of real numbers and $\mathbb{C}$ denotes the set of complex numbers. Let p be chosen as an odd fixed prime number. The symbols $\mathbb{Z}_p$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ denote the ring of p-adic integers, the field of p-adic numbers and the completion of an algebraic closure of $\mathbb{Q}_p$, respectively. The normalized absolute value according to the theory of p-adic analysis is given by $|p|_p = p^{-1}$. The parameter q can be considered as an indeterminate, a complex number $q \in \mathbb{C}$ with $|q| < 1$, or a p-adic number $q \in \mathbb{C}_p$ with $|q-1|_p < p^{-\frac{1}{p-1}}$ and $q^x = \exp(x \log q)$ for $|x|_p \leq 1$. The q-analogue of x is defined by $[x]_q = (1-q^x)/(1-q)$. It is easy to show that $\lim_{q \to 1} [x]_q = x$ for any x with $|x|_p \leq 1$ in the p-adic case (for details, *cf.* [1-10]; see also the related references cited therein).

Let f be uniformly differentiable function at a point $a \in \mathbb{Z}_p$, denoted by $f \in UD(\mathbb{Z}_p)$. Kim [7] originally introduced the fermionic q-Volkenborn integral (or fermionic p-adic q-integral on $\mathbb{Z}_p$) of a function $f \in UD(\mathbb{Z}_p)$, as follows:

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)\, d\mu_{-q}(x) = \lim_{N \to \infty} \frac{1}{[p^N]_{-q}} \sum_{k=0}^{p^N-1} (-1)^k f(k)\, q^k. \tag{1.1}$$

Note that taking $q \to 1$ yields the familiar p-adic fermionic integral given by $\int_{\mathbb{Z}_p} f(x)\, d\mu_{-1}(x) = \lim_{N\to\infty} \frac{1}{p^N} \sum_{k=0}^{p^N-1} (-1)^k f(k)$, see [3,7-9].

Let $f_1(x) = f(x+1)$. By (1.1), the following integral equation holds true, see [7]:

$$qI_{-q}(f_1) + I_{-q}(f) = [2]_q f(0), \tag{1.2}$$

which intensely preserves usability in introducing diverse generalizations of several special polynomials such as Euler polynomials, Genocchi polynomials and Changhee polynomials. As a general case of (1.2), Kim [7], Korean mathematician, gave the following integral equality for $f_n(x) = f(x+n)$:

$$q^n I_{-q}(f_n) + (-1)^{n-1} I_{-q}(f) = [2]_q \sum_{r=0}^{n-1} (-1)^{n-r-1} q^r f(r).$$

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The usual Boole polynomials $Bl_n(x)$ are defined by means of the following generating function (*cf.* [9]):

$$\sum_{n=0}^{\infty} Bl_n\left(x\left|\omega\right.\right)\frac{t^n}{n!}=\frac{1}{1+(1+t)^{\omega}}(1+t)^x=\int_{\mathbb{Z}_p}(1+t)^{x+\omega y}\,d\mu_{-1}(y).$$

When $\omega=1$, we have $Bl_n(x\left|1\right.):=2^{-1}Ch_n(x)$ which are the familiar Changhee polynomials defined by the following generating function to be (*cf.* [8])

$$\sum_{n=0}^{\infty} Ch_n(x)\frac{t^n}{n!}=\frac{2}{2+t}(1+t)^x. \tag{1.3}$$

In the case $x=0$ in the (1.3), one can get $Ch_n(0):=Ch_n$ standing for n-th Changhee number (*cf.* [3,8]).

The q-Boole polynomials of the first kind are defined by means of the following fermionic q-Volkenborn integral (*cf.* [8,9]; see also the references cited in each of these earlier works):

$$\sum_{n=0}^{\infty} Bl_{n,q}\left(x\left|\omega\right.\right)\frac{t^n}{n!}=\frac{1}{1+q}\int_{\mathbb{Z}_p}(1+t)^{x+\omega y}\,d\mu_{-q}(y)=\frac{1}{1+q(1+t)^{\omega}}(1+t)^x. \tag{1.4}$$

The q-Boole polynomials of the first kind can be represented by

$$Bl_{n,q}\left(x\left|\omega\right.\right)=[2]_q^{-1}\int_{\mathbb{Z}_p}(x+\omega y)_n\,d\mu_{-q}(y), \tag{1.5}$$

where $(x)_n$ be falling factorial given by (*cf.* [1-3,8,9])

$$(x)_n=x(x-1)(x-2)\cdots(x-n+1). \tag{1.6}$$

In the special case, $Bl_{n,q}\left(0\left|\omega\right.\right):=Bl_{n,q}(\omega)$ is called n-th q-Boole number.

Upon setting $\omega=1$, we have $Bl_{n,q}(x\left|1\right.):=[2]_q^{-1}Ch_{n,q}(x)$ which are defined by

$$Ch_{n,q}(x)=\int_{\mathbb{Z}_p}(x+y)_n\,d\mu_{-q}(y).$$

Taking $x=0$ into the (1.4) gives $Ch_{n,q}(0):=Ch_{n,q}$ being called n-th q-Changhee number. It is obvious that $\lim_{q\to1}Ch_{n,q}(x):=Ch_n(x)$, see [3].

The q-Boole polynomials of the second kind are defined by means of the following fermionic q-Volkenborn integral, see [8]:

$$\sum_{n=0}^{\infty}\widehat{Bl}_{n,q}\left(x\left|\omega\right.\right)\frac{t^n}{n!}=\frac{1}{1+q}\int_{\mathbb{Z}_p}(1+t)^{x-\omega y}\,d\mu_{-q}(y)=\frac{(1+t)^{\omega}}{q+(1+t)^{\omega}}(1+t)^x. \tag{1.7}$$

The q-Boole polynomials of the second kind can be represented by

$$\widehat{Bl}_{n,q}\left(x\left|\omega\right.\right)=[2]_q^{-1}\int_{\mathbb{Z}_p}(x-\omega y)_n\,d\mu_{-q}(y). \tag{1.8}$$

When $x=0$, we have $\widehat{Bl}_{n,q}\left(0\left|\omega\right.\right):=\widehat{Bl}_{n,q}(\omega)$ which is called the q-Boole numbers of the second kind.

In recent years, the Boole and the Changhee polynomials in conjunction with their many generalizations studied and investigated by diverse mathematicians possess multifarious applications in p-adic analysis and q-analysis, *cf.* [3,8,9] and references cited therein.

The formula (1.6) satisfies the following identity:

$$(x)_n=\sum_{k=0}^{n}S_1(n,k)\,x^k, \tag{1.9}$$

where $S_1(n,k)$ is Stirling number of the first kind (*cf.* [1-3,8,9]).

The following relation holds true for $n\geq0$:

$$\int_{\mathbb{Z}_p}\binom{x+\omega y}{n}d\mu_{-q}(y)=\sum_{m=0}^{n}\omega^m S_1(n,m)\,E_{m,q}\left(\frac{x}{\omega}\right), \tag{1.10}$$

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where $E_{m,q}(x/\omega)$ is m-th q-Euler polynomials with the value x/ω defined by (*cf.* [8])

$$\sum_{n=0}^{\infty} E_{n,q}(y)\frac{t^n}{n!} = \int_{\mathbb{Z}_p}(x+y)^n d\mu_{-q}(x) = \frac{1+q}{1+qe^t}e^{yt}.$$

Note that when $y=0$, we have $E_{n,q}(0) := E_{n,q}$ called n-th q-Euler number (see [8]).

In this paper, we investigate several relations for p-adic gamma function by means of their Mahler expansion and fermionic p-adic q-integral on $\mathbb{Z}_p$. We also derived two fermionic p-adic q-integrals of p-adic gamma function in terms of q-Boole polynomials and numbers. Moreover, we discover fermionic p-adic q-integral of the derivative of p-adic gamma function. We acquire a representation for the p-adic Euler constant by means of the q-Boole polynomials. We finally develop a novel, explicit and interesting representation for the p-adic Euler constant including Stirling numbers of the first kind.

2. The q-Boole Polynomials Associated with p-Adic Gamma Function

Throughout this paper, we suppose that $t, q \in \mathbb{C}_p$ with $|q|_p < p^{-\frac{1}{1-p}}$ and $|t|_p < p^{-\frac{1}{1-p}}$. In this chapter, we perform to derive some relationships among the two types of q-Boole polynomials, p-adic gamma function and p-adic Euler constant by making use of the Mahler expansion of the p-adic gamma function.

The p-adic gamma function (see [1-3, 5, 6, 10]) is defined as follows

$$\Gamma_p(x) = \lim_{n\to x}(-1)^n \prod_{\substack{j<n \\ (p,j)=1}} j \qquad (x\in\mathbb{Z}_p), \tag{2.1}$$

where n approaches x through positive integers.

The p-adic Euler constant γ_p is defined by the following formula

$$\gamma_p := -\frac{\Gamma_p'(1)}{\Gamma_p(0)} = \Gamma_p'(1) = -\Gamma_p'(0). \tag{2.2}$$

The p-adic gamma function in conjunction with its a great deal of extensions and p-adic Euler constant have been developed by many physicists and mathematicians, *cf.* [1-3, 6, 10]; see also the references cited in each of these earlier works.

For $x \in \mathbb{Z}_p$, the symbol $\binom{x}{n}$ is given by $\binom{x}{0} = 1$ and $\binom{x}{n} = \frac{x(x-1)\cdots(x-n+1)}{n!}$ $(n\in\mathbb{N})$.

Let $x\in\mathbb{Z}_p$ and $n\in\mathbb{N}$. The functions $x\to\binom{x}{n}$ form an orthonormal base of the space $C(\mathbb{Z}_p\to\mathbb{C}_p)$ with respect to the Euclidean norm $\|\cdot\|_\infty$. The mentioned orthonormal base satisfy the following equality:

$$\binom{x}{n}' = \sum_{j=0}^{n-1}\frac{(-1)^{n-j-1}}{n-j}\binom{x}{j} \qquad (\text{see [4] and [6]}). \tag{2.3}$$

Mahler investigated a generalization for continuous maps of a p-adic variable utilizing the special polynomials as binomial coefficient polynomial [4] in 1958 as follows.

Theorem 1. [4]*Every continuous function $f:\mathbb{Z}_p\to\mathbb{C}_p$ can be written in the form*

$$f(x) = \sum_{n=0}^{\infty} a_n\binom{x}{n} \tag{2.4}$$

for all $x\in\mathbb{Z}_p$, where $a_n\in\mathbb{C}_p$ and $a_n\to 0$ as $n\to\infty$.

The base $\left\{\binom{*}{n} : n\in\mathbb{N}\right\}$ is named as Mahler base of the space $C(\mathbb{Z}_p\to\mathbb{C}_p)$, and the components $\{a_n : n\in\mathbb{N}\}$ in $f(x) = \sum_{n=0}^{\infty} a_n\binom{x}{n}$ are called Mahler coefficients of $f\in C(\mathbb{Z}_p\to\mathbb{C}_p)$. The Mahler expansion of the p-adic gamma function Γ_p and its Mahler coefficients are discovered in [6] as follows.

Proposition 1. *For $x\in\mathbb{Z}_p$, let $\Gamma_p(x+1) = \sum_{n=0}^{\infty} a_n\binom{x}{n}$ be Mahler series of Γ_p. Then its coefficients satisfy the following identity:*

$$\sum_{n\geqq 0}(-1)^{n+1}a_n\frac{x^n}{n!} = \frac{1-x^p}{1-x}\exp\left(x+\frac{x^p}{p}\right). \tag{2.5}$$

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The fermionic q-Volkenborn integral on $\mathbb{Z}_p$ of the p-adic gamma function via Eq. (1.5) and Proposition 1 is as follows.

Theorem 2. *The following identity holds true for $n \in \mathbb{N}$:*

$$\int_{\mathbb{Z}_p} \Gamma_p (\omega x + 1) \, d\mu_{-q} (x) = \sum_{n=0}^{\infty} \frac{a_n [2]_q}{n!} Bl_{n,q} (\omega) ,$$

where a_n is given by Proposition 1.

Proof. For $x, \omega \in \mathbb{Z}_p$, by Proposition 1, we get

$$\int_{\mathbb{Z}_p} \Gamma_p (\omega x + 1) \, d\mu_{-q} (x) = \sum_{n=0}^{\infty} a_n \int_{\mathbb{Z}_p} \binom{\omega x}{n} d\mu_{-q} (x)$$

and using the formula (1.5), we acquire

$$\int_{\mathbb{Z}_p} \Gamma_p (\omega x + 1) \, d\mu_{-q} (x) = \sum_{n=0}^{\infty} \frac{a_n [2]_q}{n!} Bl_{n,q} (\omega) ,$$

which gives the asserted result. □

We here present one other fermionic p-adic q-integral of the p-adic gamma function related to the q-Boole polynomials as follows.

Theorem 3. *Let $x, y, \omega \in \mathbb{Z}_p$. We have*

$$\int_{\mathbb{Z}_p} \Gamma_p (x + \omega y + 1) \, d\mu_{-q} (y) = \sum_{n=0}^{\infty} \frac{a_n [2]_q}{n!} Bl_{n,q} (x \,|\omega) , \tag{2.6}$$

where a_n is given by Proposition 1.

Proof. For $x, y, \omega \in \mathbb{Z}_p$, by the relation $\binom{x+\omega y}{n} = \frac{(x+\omega y)_n}{n!}$ and Proposition 1, we get

$$\int_{\mathbb{Z}_p} \Gamma_p (x + \omega y + 1) \, d\mu_{-q} (y) = \int_{\mathbb{Z}_p} \sum_{n=0}^{\infty} a_n \frac{(x + \omega y)_n}{n!} d\mu_{-q} (y) = \sum_{n=0}^{\infty} a_n \frac{1}{n!} \int_{\mathbb{Z}_p} (x + \omega y)_n \, d\mu_{-q} (y) ,$$

which is the desired result (2.6) via (1.4). □

We state the following theorem including a relation between $\Gamma_p (x)$ and $\widehat{Bl}_{n,q} (x \,|\omega)$.

Theorem 4. *For $x, y, \omega \in \mathbb{Z}_p$,we have*

$$\int_{\mathbb{Z}_p} \Gamma_p (x - \omega y + 1) \, d\mu_{-q} (y) = \sum_{n=0}^{\infty} a_n [2]_q \frac{\widehat{Bl}_{n,q} (x \,|\omega)}{n!},$$

where a_n is given by Proposition 1.

Proof. For $x, y, \omega \in \mathbb{Z}_p$, by the relation $\binom{-x-\omega y}{n} = \frac{(-x-\omega y)_n}{n!}$ and Proposition 1, we get

$$\begin{aligned} \int_{\mathbb{Z}_p} \Gamma_p (x - \omega y + 1) \, d\mu_{-q} (y) &= \int_{\mathbb{Z}_p} \sum_{n=0}^{\infty} a_n \frac{(x - \omega y)_n}{n!} d\mu_{-q} (y) \\ &= \sum_{n=0}^{\infty} a_n \frac{1}{n!} \int_{\mathbb{Z}_p} (x - \omega y)_n \, d\mu_{-q} (y) , \end{aligned}$$

which is the desired result thanks to (1.8). □

A consequence of Theorem 4 is given by the following corollary.

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Corollary 1. *Upon setting $x=0$ in Theorem 4 gives the following relation for Γ_p and $\widehat{Bl}_{n,q}(\omega)$:*

$$\int_{\mathbb{Z}_p} \Gamma_p(x-\omega y+1)\,d\mu_{-q}(y) = \sum_{n=0}^{\infty} a_n [2]_q \frac{\widehat{Bl}_{n,q}(\omega)}{n!},$$

where a_n is given by Proposition 1.

Here is the fermionic p-adic q-integral of the derivative of the p-adic gamma function.

Theorem 5. *For $x, y, \omega \in \mathbb{Z}_p$, we have*

$$\int_{\mathbb{Z}_p} \Gamma_p'(x+\omega y+1)\,d\mu_{-q}(y) = \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n [2]_q \frac{(-1)^{n-j-1} Bl_{j,q}(x\,|\omega)}{(n-j)\,j!}.$$

Proof. In view of Proposition 1, we obtain

$$\int_{\mathbb{Z}_p} \Gamma_p'(x+\omega y+1)\,d\mu_{-q}(y) = \int_{\mathbb{Z}_p} \sum_{n=0}^{\infty} a_n \binom{x+\omega y}{n}' d\mu_{-q}(y) = \sum_{n=0}^{\infty} a_n \int_{\mathbb{Z}_p} \binom{x+\omega y}{n}' d\mu_{-q}(y)$$

and using (2.3), we derive

$$\begin{aligned} \int_{\mathbb{Z}_p} \Gamma_p'(x+\omega y+1)\,d\mu_{-q}(y) &= \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n \frac{(-1)^{n-j-1}}{n-j} \int_{\mathbb{Z}_p} \binom{x+\omega y}{j} d\mu_{-q}(y) \\ &= \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n [2]_q \frac{(-1)^{n-j-1}}{n-j} \frac{Bl_{j,q}(x\,|\omega)}{j!}. \end{aligned}$$

□

The immediate result of Theorem 5 is given as follows.

Corollary 2. *For $y \in \mathbb{Z}_p$, we have*

$$\int_{\mathbb{Z}_p} \Gamma_p'(\omega y+1)\,d\mu_{-q}(y) = \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n [2]_q \frac{(-1)^{n-j-1} Bl_{j,q}(\omega)}{(n-j)\,j!}. \tag{2.7}$$

We now provide a new and interesting representation of the p-adic Euler constant by means of q-Boole polynomials of the second kind.

Theorem 6. *We have*

$$\gamma_p = \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n (-1)^{n-j} \frac{qBl_{j,q}(\omega-1\,|\omega) - Bl_{j,q}(-1\,|\omega)}{(n-j)\,j!}. \tag{2.8}$$

Proof. Taking $f(y) = \Gamma_p'(\omega y)$ in (1.2) yields the following result

$$q\int_{\mathbb{Z}_p} \Gamma_p'(\omega y+\omega-1+1)\,d\mu_{-q}(y) + \int_{\mathbb{Z}_p} \Gamma_p'(\omega y)\,d\mu_{-q}(y) = [2]_q \Gamma_p'(0).$$

Using (2.2), (2.7) and Theorem 5 along with some basic calculations, we have

$$q\sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n [2]_q \frac{(-1)^{n-j-1} Bl_{j,q}(\omega-1\,|\omega)}{(n-j)\,j!} + \sum_{n=0}^{\infty}\sum_{j=0}^{n-1} a_n [2]_q \frac{(-1)^{n-j-1} Bl_{j,q}(-1\,|\omega)}{(n-j)\,j!} = -\gamma_p [2]_q,$$

which implies the asserted result. □

We give the following explicit formula for the p-adic Euler constant.

Theorem 7. *The following explicit formula is valid:*

$$\gamma_p=\sum_{n=0}^{\infty}\sum_{j=0}^{n-1}\frac{a_n}{(n-j)\,j!}\sum_{m=0}^{\infty}(-1)^{m+n-j}\,q^{-m-1}\sum_{k=0}^{n}S_1(n,k)\left(q(-1-\omega m)^k-(-1-\omega-\omega m)^k\right).$$

Proof. By (1.4), we get

$$\begin{aligned}\sum_{n=0}^{\infty}\widehat{Bl}_{n,q}(x\,|\omega)\frac{t^n}{n!} &= \frac{q^{-1}}{1+\frac{(1+t)^{\omega}}{q}}(1+t)^{x+\omega}=\sum_{m=0}^{\infty}(-1)^m\,q^{-m-1}(1+t)^{x+\omega+\omega m}\\ &= \sum_{m=0}^{\infty}(-1)^m\,q^{-m-1}(1+t)^{x+\omega+\omega m}=\sum_{m=0}^{\infty}(-1)^m\,q^{-m-1}\sum_{n=0}^{\infty}\binom{x+\omega+\omega m}{n}t^n\\ &= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{\infty}(-1)^m\,q^{-m-1}(x+\omega+\omega m)_n\right)\frac{t^n}{n!},\end{aligned}$$

which gives, from (1.9), that

$$\widehat{Bl}_{n,q}(x\,|\omega)=\sum_{m=0}^{\infty}(-1)^m\,q^{-m-1}\sum_{k=0}^{n}S_1(n,k)(x+\omega+\omega m)^k.$$

In view of (1.4) and (1.7), we easily obtain that

$$\widehat{Bl}_{n,q}(x\,|\omega)=Bl_{n,q}(x\,|{-\omega}).$$

So, we derive that

$$Bl_{n,q}(x\,|\omega)=\sum_{m=0}^{\infty}(-1)^m\,q^{-m-1}\sum_{k=0}^{n}S_1(n,k)(x-\omega-\omega m)^k. \tag{2.9}$$

Thus, we have

$$Bl_{n,q}(-1\,|\omega)=\sum_{m=0}^{\infty}(-1)^m\,q^{-m-1}\sum_{k=0}^{n}S_1(n,k)(-1-\omega-\omega m)^k \tag{2.10}$$

and

$$Bl_{n,q}(\omega-1\,|\omega)=\sum_{m=0}^{\infty}(-1)^m\,q^{-m-1}\sum_{k=0}^{n}S_1(n,k)(-1-\omega m)^k. \tag{2.11}$$

By combining (2.8), (2.10) and (2.11), we arrive at the desired result. □

3. Conclusions and Observations

In the present work, we first have considered multifarious relationships among the two types of q-Boole polynomials and p-adic gamma function. Also, we have computed the fermionic p-adic q-integral of the derivative of p-adic gamma function. Moreover, we have given a novel representation for the p-adic Euler constant by means of the q-Boole polynomials of both sides. We have finally provided a quirky explicit formula for p-adic Euler constant.

References

[1] U. Duran, M. Acikgoz, Some relationships including p-adic gamma function and q-Daehee polynomials and numbers, Preprints 2018, 2018020118, doi: 10.20944/preprints201802.0118.v1.

[2] Ö. Ç. Havare, H. Menken, The Volkenborn integral of the p-adic gamma function, *Int. J. Adv. Appl. Funct.*, 5 (2), 2018, 56-59.

[3] Ö. Ç. Havare, H. Menken, A note on p-adic gamma function and q-Changhee polynomials, *J. Math. Comput. Sci.*, 18, 2018, 11-17.

[4] K. Mahler, An interpolation series for continuous functions of a p-adic variable, *J. Reine Angew. Math.*, 199, 1958, 23-34.

[5] Y. Morita, A p-adic analogue of the Γ-function, *J. Fac. Sci.*, 22 (2), 1975, 255-266.

[6] A. M. Robert, *A course in p-adic analysis*, Springer-Verlag New York, Inc., 2000.

[7] T. Kim, A note on q-Volkenborn integration, *Proc. Jangjeon Math. Soc.*, 8, 2005, 13-17.

[8] D. S. Kim, T. Kim, J. J. Seo, A note on q-analogue of Boole polynomials, *Appl. Math. Inf. Sci.,* 9 (6), 2015, 3153-3158.
[9] D. S. Kim, T. Kim, A note on Boole polynomials, *Integral Transforms Spec. Funct.*, 25 (8), 2014, 627-633.
[10] Y. S. Kim, q-analogues of p-adic gamma functions and p-adic Euler constants, *Comm. Korean Math. Soc.*, 13 (4), 1998, 735–741.

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