

# 1 MRST-Shale: An Open-Source Framework for Generic Numerical Modeling of 2 Unconventional Shale and Tight Gas Reservoirs

3 Bin Wang<sup>1</sup>

4 1. Craft and Hawkins Department of Petroleum Engineering, Louisiana State University; [bwang31@lsu.edu](mailto:bwang31@lsu.edu)

5

## 6 Highlights

- 7 • A generic numerical model for shale gas flow in tight reservoir is proposed
- 8 • A flexible open-source framework OpenShale is developed with EDFM
- 9 • EDFM can lead to large error for shale gas flow without help of grid refinement
- 10 • A new geomechanics model for hydraulic and natural fractures is proposed and evaluated
- 11 • OpenShale successfully applied in field history matching and new model evaluation

12

## 13 Abstract

14 We present a generic and open-source framework for the numerical modeling of the expected  
15 transport and storage mechanisms in unconventional gas reservoirs. These unconventional reservoirs  
16 typically contain natural fractures at multiple scales. Considering the importance of these fractures in  
17 shale gas production, we perform a rigorous study on the accuracy of different fracture models. The  
18 framework is validated against an industrial simulator and is used to perform a history-matching  
19 study on the Barnett shale. This work presents an open-source code that leverages cutting-edge  
20 numerical modeling capabilities like automatic differentiation, stochastic fracture modeling,  
21 multi-continuum modeling and other explicit and discrete fracture models. We modified the  
22 conventional mass balance equation to account for the physical mechanisms that are unique to  
23 organic-rich source rocks. Some of these include the use of an adsorption isotherm, a dynamic  
24 permeability-correction function, and an embedded discrete fracture model (EDFM) with  
25 fracture-well connectivity. We explore the accuracy of the EDFM for modeling  
26 hydraulically-fractured shale-gas wells, which could be connected to natural fractures of finite or  
27 infinite conductivity, and could deform during production. Simulation results indicates that although  
28 the EDFM provides a computationally efficient model for describing flow in natural and hydraulic  
29 fractures, it could be inaccurate under these three conditions: 1. when the fracture conductivity is  
30 very low. 2. when the fractures are not orthogonal to the underlying Cartesian grid blocks, and 3.  
31 when sharp pressure drops occur in large grid blocks with insufficient mesh refinement. Each of  
32 these results are very significant considering that most of the fluids in these ultra-low matrix  
33 permeability reservoirs get produced through the interconnected natural fractures, which are  
34 expected to have very low fracture conductivities. We also expect sharp pressure drops near the  
35 fractures in these shale gas reservoirs, and it is very unrealistic to expect the hydraulic fractures or  
36 complex fracture networks to be orthogonal to any structured grid. In conclusion, this paper presents

1 an open-source numerical framework to facilitate the modeling of the expected physical mechanisms  
2 in shale-gas reservoirs. The code was validated against published results and a commercial simulator.  
3 We also performed a history-matching study on a naturally-fractured Barnett shale-gas well  
4 considering adsorption, gas slippage & diffusion and fracture closure as well as proppant embedment,  
5 using the framework presented. This work provides the first open-source code that can be used to  
6 facilitate the modeling and optimization of fractured shale-gas reservoirs. To provide the numerical  
7 flexibility to accurately model stochastic natural fractures that are connected to  
8 hydraulically-fractured wells, it is built atop other related open-source codes. We also present the  
9 first rigorous study on the accuracy of using EDFM to model both hydraulic fractures and natural  
10 fractures that may or may not be interconnected.

11 Source code is available at [https://github.com/BinWang0213/MRST\\_Shale](https://github.com/BinWang0213/MRST_Shale)

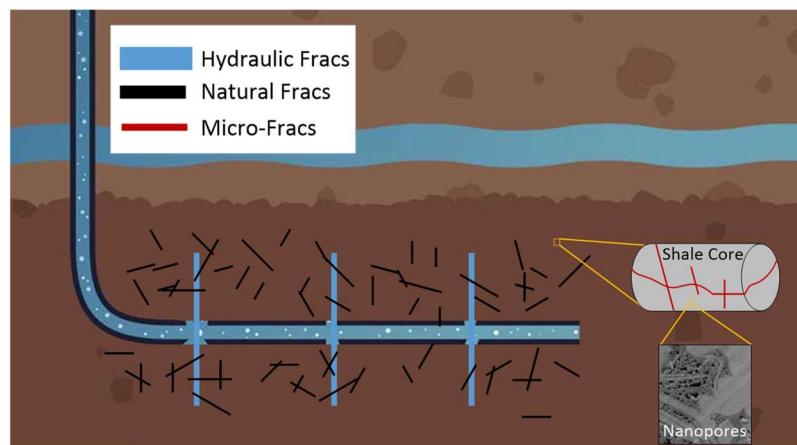
12 **Key words:** shale gas; MRST; embedded discrete fracture model; open-source implementation

## 13 1 Introduction

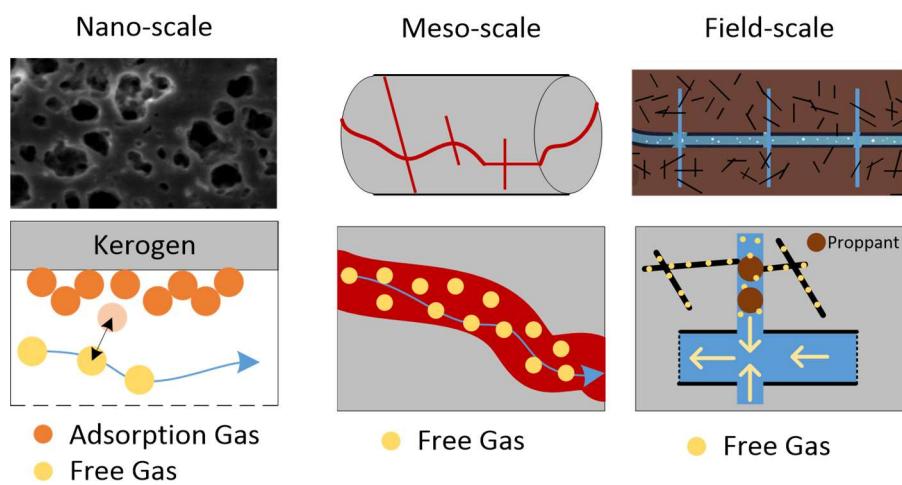
14 Unconventional gas resources gain great interest recently due to successful economic development  
15 and strong energy supply around the world. Advancement of horizontal well drilling and hydraulic  
16 fracturing technology as well as better understanding unconventional reservoirs drives substantial  
17 growth of shale gas production (Bowker, 2007). Unlike conventional reservoirs, unconventional  
18 shale gas reservoirs can be characterized by ultra-low permeability, low porosity, complex transport  
19 mechanism and multi-scale fractures (Akkutlu et al, 2018). Development of unconventional  
20 resources is more technology-demanding and expensive. Thus, accurate modeling and numerical  
21 simulation of shale gas flow is critical for evaluating, designing and managing stimulation and  
22 production processes.

23 Well-established flow and transport theory for conventional reservoir rocks are not directly  
24 applicable to unconventional porous media (Gensterblum et al, 2015). For decades, researchers have  
25 been investigating the storage and transport mechanisms for unconventional reservoirs, which  
26 includes gas desorption, adsorbed gas porosity, gas slippage, and Knudsen diffusion, etc (Javadpour  
27 et al, 2007; Wang and Reed, 2009; Civan et al, 2010,2011; Sakhaei and Bryant, 2012; Akkutlu and  
28 Fathi, 2012, Yu et al, 2016 and Tan et al, 2018). In addition, the fractured shale matrix is comprised  
29 of a hierarchical network of pores down to a few nanometers, cracks and micro-fractures, which  
30 makes the formation a multi-scale porous medium with large heterogeneity and anisotropy (Akkutlu  
31 et al, 2018). Hence, the complex gas transport mechanisms and multiscale fracture system (**Figs. 1-2**)

1 pose a great challenge to accurately and efficiently evaluate and simulate well performance in shale  
 2 gas reservoirs.



3  
 4 **Fig 1 – Multi-scale nature of shale gas production**  
 5



6  
 7 **Fig 2 – Multi-scale shale gas storage and transport**

8 In recent years, significant efforts have been made to model gas flow in unconventional reservoirs.  
 9 These methods can be categorized into analytical model, semi-analytical model and numerical  
 10 simulations. The analytical method dates back to 1970s, where the line-source fundamental solution is  
 11 derived for simple fracture geometry such as single bi-wing hydraulic fractures and pseudo-pressure is  
 12 applied to linearized the non-linear real gas equation (Gingarten et al, 1974, Cinco et al, 1978 and  
 13 Agarwal 1979). Recently, the analytical method is extended into semi-analytical method to consider  
 14 complex fracture networks and shale gas storage mechanism based on the boundary element method  
 15 (Zuo et al, 2016, Chen et al, 2015, 2016, 2017, 2018; Yang et al, 2016a, 2016b, 2017, Yu et al, 2016b,  
 16 2017 and Li et al, 2018). Although analytical-based method is fast and accurate, it is difficult to handle  
 17 rock heterogeneity, multi-phase, multi-compositional and strong non-linear transport mechanisms in

1 shale gas flow problems (Houze et al, 2010 and Olorode et al, 2013). On the other hand, numerical  
 2 simulation has been proven to be is the most general and rigorous method to account for arbitrary  
 3 non-linear physics and fracture geometry for unconventional reservoirs (Olorode et al, 2013,2017 and  
 4 Cipolla et al, 2012). Highly coupled non-linear physics and treatment of multi-scale fractured system  
 5 are two key issues in shale gas flow simulation. Fully implicit scheme with Automatic Differentiation  
 6 (AD) is a robust and generic method to solve the highly coupled non-linear problem accurately and  
 7 efficiently (Zhou et al, 2011 and Krogstad et al, 2015). In terms of multi-scale fractured system, dual  
 8 continuum method (Warren and Root, 1963) and discrete fracture method (Karimi-fard et al, 2004,  
 9 Hoteit and Firoozabadi, 2005, Hajibeygi et al, 2011 and Moinfar et al, 2014) are generally used to model  
 10 highly connected fractures and long, disconnected hydraulic/natural fractures (Fig. 1), respectively. A  
 11 hierarchical method is also proposed by integrating continuum method and discrete fracture method for  
 12 multi-scale fractured system where the micro-fractures are upscaled into matrix permeability tensor and  
 13 hydraulic/natural fractures are modeled explicitly (Lee et al, 2001 and Karimi-Fard et al, 2006).  
 14 Unstructured gridding with local grid refinement (LGR) is generally used to capture the irregular  
 15 fracture geometry and sharp pressure gradient near the fractures. However, it is still challenging to  
 16 generate conforming mesh efficiently for complex fracture networks (Karimi-Fard, Durlofsky, 2016).  
 17 Recently, an embedded discrete fracture model is developed to resolve the complex gridding issue.  
 18 Using EDFM, the complex fractures are embedded in conventional matrix grids without conforming the  
 19 matrix grids with fracture plane, thus it is more efficient for complex fracture networks. In addition, it  
 20 can be easily integrated into well-established reservoir simulator without accessing the code (Xu, 2015  
 21 and Olorode et al, 2017). **Table 1** shows the advantages and disadvantages of these method where  
 22 unstructured grid and EDFM are the two most promising methods for generic shale gas simulation with  
 23 multi-scale fractures.

24 **Table 1. Comparison of shale gas flow simulation methods**

	Analytical	Semi-analytical	Structured grid	Unstructured grid	EDFM
Accuracy	++	++	+++	+++	++
Nonlinear mechanisms*	+	+	+++	+++	+++
Rock heterogeneity	+	+	+++	+++	+++
Fracture gridding	+++	+++	+	+	+++
Preprocessing** efficiency	+++	+++	+++	+++	++
Computational*** efficiency	+++	+++	+	++	++

---

\* Nonlinear gas transport & storage model, multi-phase flow, compositional flow

\*\* 2D/3D geometry calculations, such plane-plane intersection, point-plane distance

\*\*\* linear algebra and Newton's calculations

1 Flow and transport theory and models for unconventional reservoir is a rapid evolving area of  
 2 research, many of the existing and newly discovered phenomenon have not been completely understood.  
 3 Also, the effect of these mechanism on practical well performance is not clear. To the best of our  
 4 knowledge, almost all existing numerical models for shale gas reservoir are implemented in in-house  
 5 simulators or commercial simulators (Jiang and Younis, 2015, Cao et al, 2016, Xu et al, 2017, Wang et  
 6 al, 2017 and Akkutlu et al, 2018). Hence, it is necessary to develop a flexible and generic open-source  
 7 framework to fill this gap.

8 In this paper, a generic numerical model is developed to simulate shale gas flow in unconventional  
 9 reservoirs with multi-scaled fractures, which can be used to integrate any shale gas transport and  
 10 storage mechanism for unconventional reservoirs as well as the geomechanics effect for fracture  
 11 system. An efficient and flexible framework (OpenShale) is also developed using an open-source  
 12 reservoir simulation toolkit (MRST) and EDFM. OpenShale can handle deterministic hydraulic  
 13 fractures and stochastic natural fractures with arbitrary geometry and distribution. The framework is  
 14 firstly verified against a commercial simulator and an in-house reservoir simulator that employs  
 15 unstructured grid to simulate shale gas transport with non-planar hydraulic fracture, gas desorption,  
 16 gas slippage & diffusion. The advantages and limitation of EDFM for shale gas flow problem is also  
 17 discussed. Finally, field application of history matching and new geomechanics model evaluation are  
 18 studied.

19 **2 Mathematical equations**

20 Considering the isothermal single-component single-phase gas flow in 2D fractured porous media  
 21 with 1D fracture line without gravity effect. The general governing equation for shale gas flow in matrix  
 22 ( $\Omega_m$ ), considering storage ( $m_{ad}$ ) and transport mechanisms ( $F_{app}$ ), can be expressed as follows:

$$23 \quad \frac{\partial}{\partial t} \left( \rho_g \phi + (1-\phi)m_{ad} \right) + \nabla \cdot \left( -\rho_g \frac{\prod_i F_{app,i} k_0}{\mu_g} \nabla p \right) = \rho_g q_w \quad \text{in } \Omega_m \quad (1)$$

24 Similarly, the governing equation for fracture ( $\Omega_f$ ), only considering transport mechanisms, can  
 25 be expressed as follows:

$$\frac{\partial}{\partial t}(\rho_g \phi) + \nabla \cdot (-\rho_g \frac{\prod_i F_{app,i} k_0}{\mu_g} \nabla p) = \rho_g q_w \quad \text{in } \Omega_f \quad (2)$$

Introducing inverse formation volume factor  $b_g = \rho_g / \rho_{gsc}$  ( $\rho_g = b_g \rho_{gsc}$ ), the above equation can be rewritten as follows:

$$\begin{aligned} \frac{\partial}{\partial t} \left( b_g \phi + \frac{(1-\phi)}{\rho_{gsc}} m_{ad} \right) + \nabla \cdot \left( -b_g \frac{\prod_i F_{app,i} k_0}{\mu_g} \nabla p \right) &= b_g q_w \quad \text{in } \Omega_m \\ \frac{\partial}{\partial t} (b_g \phi) + \nabla \cdot \left( -b_g \frac{\prod_i F_{app,i} k_0}{\mu_g} \nabla p \right) &= b_g q_w \quad \text{in } \Omega_f \end{aligned} \quad (3)$$

where  $\rho_g$  is the mass density of gas,  $\text{M/L}^3$ ;  $\mu_g$  is the dynamic viscosity of natural gas,  $\text{N.T/L}^2$   $m_{ad}$  is the accumulation term due to adsorption,  $\text{M/L}^3$ ;  $\phi$  is the matrix porosity, dimensionless;  $k_0$  is the absolute Darcy permeability of the reservoir rock,  $\text{L}^2$ .  $F_{app,i}$  is the  $i$ -th permeability correction factor for a specific shale gas transport mechanism;  $q_w$  is the volumetric sink/source term,  $\text{M/L}^3/\text{T}$ .  $k_0$  is the absolute Darcy permeability of the reservoir rock,  $\text{L}^2$ .

## 2.1 Gas properties

*Density:* The pressure-dependent density of natural gas can be calculated by the real gas law:

$$\rho_g = \frac{pM}{Z(p,T)RT} \quad (4)$$

where  $M$  is the molecule weight of the natural gas,  $\text{M/Mol}$ ;  $R$  is the Boltzmann constant,  $8.314 \text{ J/K.mol}$ ;  $T$  is the reservoir temperature,  $\text{K}$ ;

The compressibility factor  $Z$  can be calculated using either implicit Peng-Robinson equation-of-state (PR-EOS) equation or empirical explicit equation. Using the empirical equation, the complex natural gas mixture can be considered as a single component with pseudo-temperature and pseudo-pressure. Mahmoud (2014) developed an explicit empirical equation for natural gas mixture as follows:

$$Z(p,T) = 0.702e^{-2.5T_{pr}} \cdot p_{pr}^2 - 5.524e^{-2.5T_{pr}} \cdot p_{pr} + (0.044T_{pr}^2 - 0.164T_{pr} + 1.15) \quad (5)$$

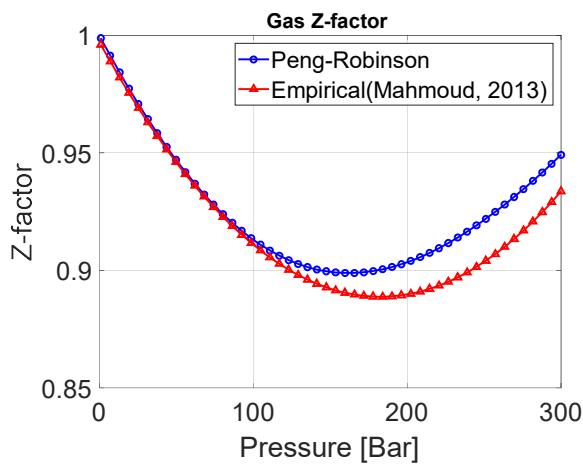
where the reduced-temperature and reduced-pressure can be expressed as  $T_{pr} = T / T_c$  and  $p_{pr} = p / p_c$ , respectively.  $T_{pc}$  and  $p_{pc}$  are the pseudo-critical pressure and pseudo-critical temperature for the shale gas mixture, respectively.

Also, for single component gas simulation, such as methane, the  $Z$  factor can be accurately

1 estimated by solving a cubic function of PR-EOS as follows (Lira and Elliott, 2012):

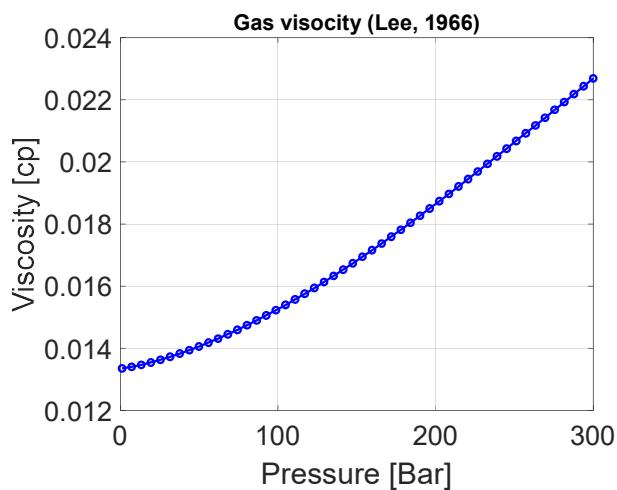
$$\begin{aligned}
 Z^3 + a_2 Z^2 + a_1 Z + a_0 &= 0 \\
 a_0(p, T) &= (AB - B^2 - B^3), \quad a_1(p, T) = A - 3B^2 - 2B, \quad a_2(p, T) = B - 1 \\
 2 \quad A &= ap / (RT)^2, \quad B = bp / (RT) \\
 a &= \frac{0.457235R^2T_c^2}{p_c}, \quad b = \frac{0.0777961RT_c}{p_c}
 \end{aligned} \tag{6}$$

3 In this paper, an analytical solution (see details in appendix B of Lira and Elliott, 2012) is used  
 4 for solving the cubic equation. For more complex natural gas mixture, it requires complex flash  
 5 calculation and belongs multi-component compositional simulation which will be investigated in our  
 6 future work. **Fig. 3** shows an estimation of Z-factor for methane using Eq.5 and Eq. 6, respectively.



7

8 **Fig. 3** Evaluated natural gas Z-factor for empirical and PR-EOS models with T=352 K,  
 9  $T_c=191$  K,  $p_c=4.64$  MPa,  $R=8.314$  J/(K.mol)



10

11 **Fig. 4** Evaluated natural gas viscosity using Lee Lee-Gonzalez-Eakin empirical correlation  
 12 with M=16.04 g/mol and T=633.6 Rankine

1        *Viscosity*: The density-dependent viscosity of natural gas can be estimated by  
 2        Lee-Gonzalez-Eakin empirical correlation (Lee et al, 1966) as follows:

$$\mu_g = 10^{-7} K \exp(X \rho_g^Y) \quad (7)$$

$$K = \frac{(9.379 + 0.01607M)T^{1.5}}{209.2 + 19.26M + T}, \quad X = 3.448 + \frac{986.4}{T} + 0.01009M, \quad Y = 2.447 - 0.2224X$$

4        where the unit of  $M$ ,  $T$  are g/mol and Rankine, respectively. **Fig. 4** shows an estimation of  
 5        viscosity for methane using Eq.7.

6        Noted that although the usage of pseudo-pressure equation can eliminate the nonlinearity issue  
 7        introduced by pressure-dependent gas viscosity and compressibility (Eqs. 5-6), it leads to even  
 8        larger errors especially for tight shale reservoirs (Houze et al, 2010). Thus, in this paper, the real-gas  
 9        equation is used.

## 10        2.2 Transport and storage mechanism

11        Since rapid commercial development of unconventional tight reservoirs in recent years, many  
 12        researchers spend enormous effort to understand the transport and storage mechanism of shale gas in  
 13        such complex multi-scale systems (Figs. 1-2). Several key physical mechanisms (Yu et al, 2016;  
 14        Klinkenberg, 1941; Florence et al, 2007; Javadpour, 2007; Civan, 2010) can be summarized as in **Table**  
 15        2.

16        In the presented open-source code, *OpenShale*, any storage and transport mechanisms models can be  
 17        easily implemented via defining nonlinear gas storage function ( $m_{ad}$ ) and permeability correction  
 18        function ( $F_{app}$ ). Demonstrative storage and transport models implemented in OpenShale this study are  
 19        shown as follows:

20        **Table 2. Key transport and storage mechanism for shale gas flow**

Mechanism	Models	Type	Continuum
Adsorption	Langmuir, BET	S*	Matrix
Slip flow & Diffusion	Klinkenberg, Florence, Javadpour, Civan	T*	Matrix
Non-Darcy flow	Darcy-Forchheimer	T	Fracture

21        \*S-Storage mechanism, T-Transport mechanism

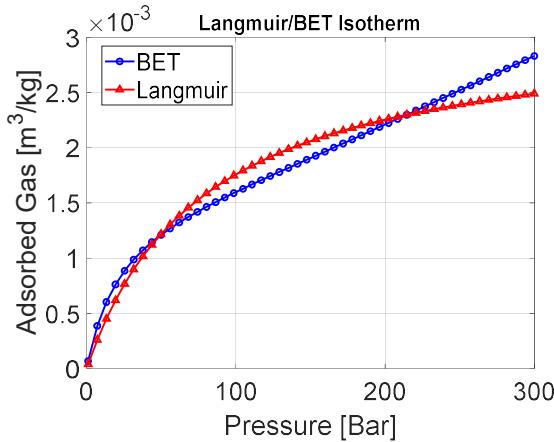
22        *Adsorption*: The gas molecules adsorbed in the pore wall of Kerogen in shale reservoir can be  
 23        modeled using monolayer Langmuir isotherm and multiple layer BET isotherm as follows (Yu et al,  
 2016a):

$$1 \quad \text{Langmuir: } m_{ad} = \rho_s \rho_{gsc} \frac{p V_L}{p + P_L} \quad (8)$$

$$2 \quad \text{BET: } m_{ad} = \rho_s \rho_{gsc} \frac{V_m C p_r}{1 - p_r} \left[ \frac{1 - (n+1)p_r^n + np_r^{n+1}}{1 + (C-1)p_r - Cp_r^{n+1}} \right] \quad (9)$$

$$p_r = \frac{p}{P_s}, \quad P_s = \exp(7.7437 - \frac{1306.5485}{19.4362 + T})$$

3 where  $V_L$  is the Langmuir volume,  $\text{L}^3/\text{M}$ .  $P_L$  is the Langmuir pressure,  $\text{M/L/T}^2$ .  $\rho_s$  is the density  
 4 of rock bulk matrix  $\text{M/L}^3$ ,  $V_L$  is the Langmuir volume (the maximum adsorption capacity at a given  
 5 temperature),  $\text{L}^3/\text{M}$ .  $P_L$  is the Langmuir pressure (the pressure at which the adsorbed gas volume is  
 6 equal to  $V_L/2$ ),  $\text{M/L/T}^2$ .  $V_m$  is the BET adsorption volume,  $\text{L}^3/\text{M}$ .  $C$  is the BET adsorption constant,  
 7 dimensionless.  $n$  is the BET adsorption molecular layers, dimensionless.  $p_s$  is the pseudo-saturation  
 8 pressure,  $\text{M/L/T}^2$ . Noted that, the unit of  $P_s$  is MPa. **Fig. 5** shows an estimation of adsorption  
 9 isotherm using Eq.8 and Eq. 9, respectively.



10  
 11 **Fig. 5 Langmuir and BET isotherms curve with  $V_L=0.0031 \text{ m}^3/\text{kg}$  and  $P_L=7.89 \text{ MPa}$ ,**  
 12  **$T=327.59 \text{ K}$ ,  $P_s=53.45 \text{ MPa}$ ,  $V_m=0.0015 \text{ m}^3/\text{kg}$ ,  $C=24.56$  and  $n=4.46$**

13 *Slippage flow & Diffusion:* Considering slippage and diffusion effect of shale gas flow in the  
 14 matrix, the apparent permeability in the low-pressure region around the fracture will be increased. In  
 15 the OpenShale, the Florence's (2007) permeability correction factor (**Fig. 4**) is implemented as  
 16 follows:

$$17 \quad F_{app} = (1 + \alpha K_n) \left( 1 + \frac{4K_n}{1 + K_n} \right) \quad (10)$$

$$1 \quad K_n = \frac{\mu_g}{2.8284 p_g} \sqrt{\frac{\pi R T}{2 M} \frac{\phi}{k_0}} \quad (11)$$

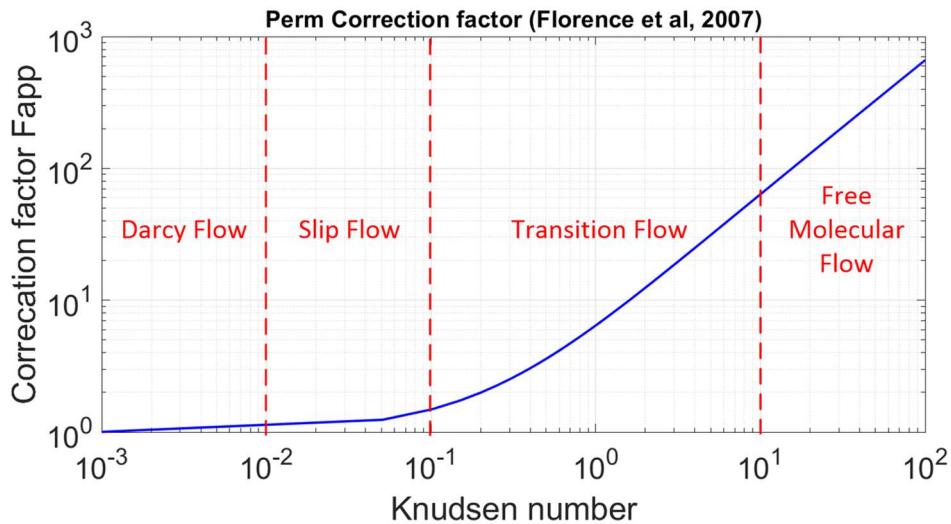
$$\alpha = \frac{128}{15\pi^2} \tan^{-1}(4K_n^{0.4})$$

2 where  $Kn$  is the Knudsen number, dimensionless.  $\alpha$  is the rarefaction parameter, dimensionless.

3 **Fig. 6** shows an estimation of gas slippage and diffusion permeability correction factor for methane  
4 using Eqs. 10-11.

5 *Non-Darcy Flow*: In case of high Forchheimer number ( $F_{oc} > 0.11$ ) in the hydraulic fractures, the  
6 linear Darcy flow is no longer applicable (Zeng and Grigg, 2006). The permeability correction factor  
7 (Barree and Conway, 2004) for Darcy-Forchheimer flow can be expressed as follows:

$$8 \quad F_{app} = \frac{2}{1 + \sqrt{1 + 4\rho_g \beta \left( \frac{k_0}{\mu_g} \right)^2 |\nabla p|}} \quad (12)$$



9  
10 **Fig. 6 – Permeability correction factor  $F_{app}$  versus Knudsen number for all flow regions with  
11 methane properties in Table 2,  $T=191$  K,  $k_0=1e-10$  and  $\phi=0.1$**

12 where  $\beta$  is the empirical Forchheimer coefficient, for propped hydraulic fractures, which can be  
13 evaluated as follows (Rubin, 2010):

$$14 \quad \beta = 3.2808 \frac{1.485 \times 10^9}{(k_0 \times 10^{-15})^{1.021}} \quad (13)$$

### 15 **2.3 Geomechanics effect**

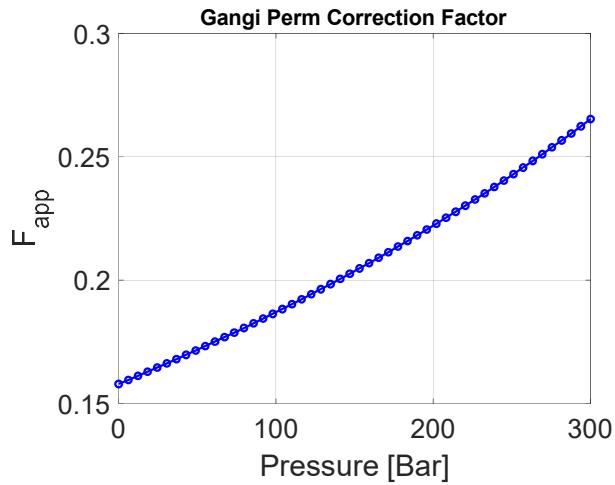
16 As shown in Fig. 2, shale reservoir has multi-scale fractures. The fracture conductivity will be  
17 decreased with increasing of production time due to the proppant embedment and fracture closure  
18 under high stress concentration near the fracture (Akkutlu et al, 2018, Hu et al, 2018a, 2018b). In this

1 paper, three types of fractures are defined based on their various length scales, including hydraulic  
 2 fracture (half-length 50-100 meters, aperture 1mm), natural fracture (half-length 1-20 m, aperture  
 3 0.1mm), and micro-fracture (half-length < 1m, aperture <0.1 mm). A new geomechanics model is  
 4 proposed herein by considering closure of micro-fracture, unpropped natural fracture and propped  
 5 fractures.

6 To consider the micro-fracture closure, Gangi's (1978) empirical pressure-dependent  
 7 permeability reduction model can be applied as follows:

$$8 \quad k = k_0 F_{app} = k_0 \left[ 1 - \left( \frac{P_c - \alpha_B p}{P_1} \right)^m \right]^3 \quad (14)$$

9 Where  $\alpha_B$  is the Biot's constant,  $P_c$  is the confining overburden pressure,  $P_1$  is the effective  
 10 stress when micro-fracture completely closed.  $m$  is a constant related to surface roughness. **Fig. 7**  
 11 shows an estimation of Gangi permeability correction factor for methane using Eqs. 14.



12  
 13 **Fig. 7 – Permeability correction factor  $F_{frac}$  versus pore pressure with  $m=0.5$ ,  $p_1=180$  MPa,  
 14  $p_c=38$  MPa and  $\alpha=0.5$**

15 To consider the closure of hydraulic and natural fractures, Alramahi and Sundberg (2012)  
 16 performed experiment to measure the effect of closure pressure on propped fracture conductivity for  
 17 different shale samples from stiff shale to soft shale. An empirical model of normalized fracture  
 18 conductivity for propped fractures,  $F_{cd,N}$ , can be fitted as follows:

$$19 \quad \begin{aligned} \text{Stiff Shale: } F_{cd,N}(p) &= 10^{-0.00011\sigma - 0.0971}, \quad R^2 = 0.961 \\ \text{Meidum Shale: } F_{cd,N}(p) &= 10^{-0.00035\sigma + 0.2396}, \quad R^2 = 0.996 \\ \text{Soft Shale: } F_{cd,N}(p) &= 10^{-0.00064\sigma - 0.4585}, \quad R^2 = 0.987 \end{aligned} \quad (15)$$

1 Wu et al (2018) performed similar experiment to investigate the effect of closure pressure on  
 2 unpropped fracture conductivity. An empirical model of normalized fracture conductivity for  
 3 unpropped fractures can be fitted as follows:

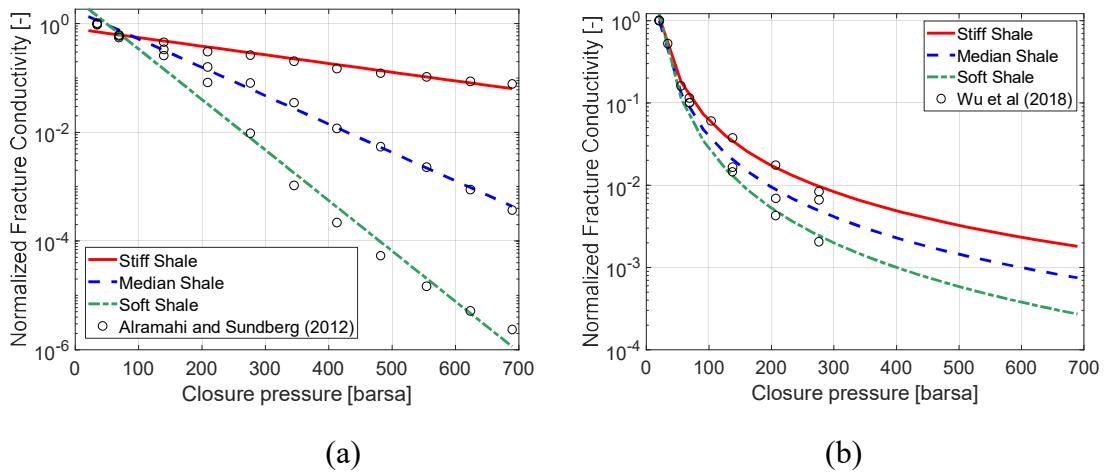
4

$$\text{Stiff Shale: } F_{cd,N}(p) = 10^{-0.793\ln(\sigma_c)+4.5618}, R^2 = 0.995$$

$$\text{Medium Shale: } F_{cd,N}(p) = 10^{-0.89\ln(\sigma_c)+5.0725}, R^2 = 0.988$$

$$\text{Soft Shale: } F_{cd,N}(p) = 10^{-1.041\ln(\sigma_c)+6.0216}, R^2 = 0.989$$

(16)



7 **Fig. 8 – Empirical correlation between normalized fracture conductivity and closure**  
 8 **pressure for propped fractures (a) and unpropped fractures (b)**

9 Where effective closure stress  $\sigma_c$  can be calculated by reservoir horizontal stress and in-situ

10 fracture pore pressure,  $\sigma_c(p) = \sigma_h - p$ . Plane direction of hydraulic fracture is normally orthogonal  
 11 to the minimum horizontal stress and it support by rigid proppant, while the plane of natural fracture  
 12 has stochastic orientation and lacking support from proppant. Thus, the closure stress for hydraulic  
 13 fracture and natural fracture can be expressed as follows:

14

$$\text{HydraulicFrac: } \sigma_{HF} = \sigma_{h\min} - p$$

$$\text{NaturalFrac: } \sigma_{NF} = \frac{\sigma_{h\min} + \sigma_{h\max}}{2} - p$$

(17)

15 The empirical correlation between fracture conductivity and closure pressure are shown in **Fig.**  
 16 **8**. In the OpenShale, the fracture permeability can be reduced by a dynamic permeability correction  
 17 factor as follows:

18

$$k_f = k_0 F_{app} = k_0 \frac{F_{cd}(p)}{F_{cd}(p_0)}$$

(18)

19 Based on proposed empirical correlation model in Eqs. 15-16, a typical permeability correction  
 20 factors for fracture closure can be shown as follows (**Fig. 9**):

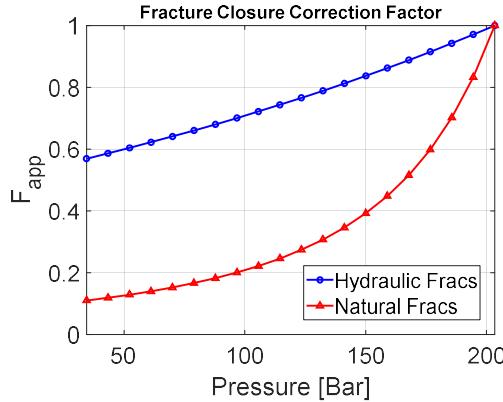


Fig 9 – Permeability correction factor  $F_{\text{frac}}$  for hydraulic fractures and natural fractures with  $p_o = 20.34 \text{ MPa}$  and  $p_{wf} = 34.5 \text{ MPa}$   $\sigma_{h\min} = 29 \text{ MPa}$  and  $\sigma_{h\max} = 34 \text{ MPa}$

### 3 Numerical Model

In this paper, a new shale gas simulation framework, OpenShale, is developed using the automatic differentiation module (ad-core, ad-props), black-oil module (ad-blackoil) and hierarchical fracture model (hfm) module in open-source MATLAB Reservoir Simulation Toolbox (Lie, 2012). Two-point flux approximated finite volume method (TPFA-FVM) is applied for discretizing the governing equations (Eq. 3). Time discretization is implemented using a fully implicit first-order backward scheme, where the Jacobian matrix of the nonlinear system is calculated by Automatic Differentiation. All nonlinear functions for shale gas transport and storage mechanisms as well as geomechanics effect are defined as separate function. For multi-scale fracture system, the larger fracture, such as the hydraulic fracture and natural fracture are explicitly modeled using EDFM. The micro-fractures are assumed highly connected and thus upscaled into the matrix permeability.

#### 3.1 Numerical discretization

The discretized governing equation of Eq. 3 can be expressed as follows:

$$\begin{aligned}
 & \frac{\phi V}{\Delta t} (b_g(p^{n+1}) - b_g(p^n)) + \frac{(1-\phi)V}{\Delta t} \frac{1}{\rho_{gsc}} (m_{ad}(p^{n+1}) - m_{ad}(p^n)) \\
 & - \text{div} \left( b_g(p^{n+1}) \frac{\prod_i F_{app,i}(p^{n+1})}{\mu_g(p^{n+1})} T \cdot \mathbf{grad}(p^{n+1}) \right) \\
 & - V b_g(p^{n+1}) q_w(p^{n+1}) - V b_g(p^{n+1}) \psi_{f-m}(p^{n+1}) = 0
 \end{aligned} \tag{19}$$

The discretized governing equation for each 1D fracture system can be expressed as follows:

$$\begin{aligned}
& \frac{\phi V}{\Delta t} (b_g(p^{n+1}) - b_g(p^n)) \\
& - \mathbf{div} \left( b_g(p^{n+1}) \frac{\prod_i F_{app,i}(p^{n+1})}{\mu_g(p^{n+1})} T \cdot \mathbf{grad}(p^{n+1}) \right) \\
& - V b_g(p^{n+1}) q_w(p^{n+1}) - V b_g(p^{n+1}) \psi_{m-f}(p^{n+1}) = 0
\end{aligned} \tag{20}$$

where  $V$  is the bulk volume of a grid cell.  $\psi_{f-m/m-f}$  is the flow coupling term between fracture and matrix. To simplify the implementation of governing equations (Eqs. 18-19), three discrete domain delta  $\delta$  functions for matrix ( $\Omega_m$ ), hydraulic fractures ( $\Omega_{HF}$ ) and natural fractures ( $\Omega_{NF}$ ) can be defined as follows:

$$\delta_m(x) = \begin{cases} 1 & x \in \Omega_m \\ 0 & x \notin \Omega_m \end{cases}, \quad \delta_{HF}(x) = \begin{cases} 1 & x \in \Omega_{HF} \\ 0 & x \notin \Omega_{HF} \end{cases}, \quad \delta_{NF}(x) = \begin{cases} 1 & x \in \Omega_{NF} \\ 0 & x \notin \Omega_{NF} \end{cases} \tag{21}$$

A generic numerical model for fractured reservoir considering shale gas transport and storage mechanism can be expressed as follows:

$$\begin{aligned}
& \frac{\phi V_{ijk}}{\Delta t} (b_g(p^{n+1}) - b_g(p^n)) + \delta_m \frac{(1-\phi)V_{ijk}}{\Delta t} \frac{1}{\rho_{gsc}} (m_{ad}(p^{n+1}) - m_{ad}(p^n)) \\
& - \mathbf{div} \left( b_g(p^{n+1}) \frac{\prod_i [1 + \delta_{HF/NF,i} F_{app,i}(p^{n+1})]}{\mu_g(p^{n+1})} T \cdot \mathbf{grad}(p^{n+1}) \right) \\
& - V_{ijk} b_g(p^{n+1}) q_w(p^{n+1}) - V_{ijk} b_g(p^{n+1}) \psi_{f-m/m-f}(p^{n+1}) = 0
\end{aligned} \tag{22}$$

Assuming vertical well fully penetrate the reservoir thickness, a semi-analytical well model (Peaceman, 1983) for a vertical well can be expressed as follows:

$$q_w = WI / \mu_g (p_{bh} - p) \tag{23}$$

where  $p_{bh}$  is the bottom hole pressure of a wellbore, M/L/T<sup>2</sup>. WI is the wellbore flow index.

The solution matrix from Eqs. 21 can be expressed as follows:

$$\begin{bmatrix} \mathbf{A}_{mm} & \mathbf{A}_{mf} & \mathbf{A}_{mw} \\ \mathbf{A}_{fm} & \mathbf{A}_{ff} & \mathbf{A}_{fw} \\ \mathbf{A}_{wm} & \mathbf{A}_{wf} & \mathbf{A}_{ww} \end{bmatrix} \begin{Bmatrix} \mathbf{p}_m \\ \mathbf{p}_f \\ \mathbf{p}_w \end{Bmatrix} = \begin{Bmatrix} \mathbf{Q}_m \\ \mathbf{Q}_f \\ \mathbf{Q}_w \end{Bmatrix} \tag{24}$$

#  $\mathbf{p}_m$  = # MatrixEles, #  $\mathbf{p}_f$  = # FractureEles, #  $\mathbf{p}_w$  = #Eles has well

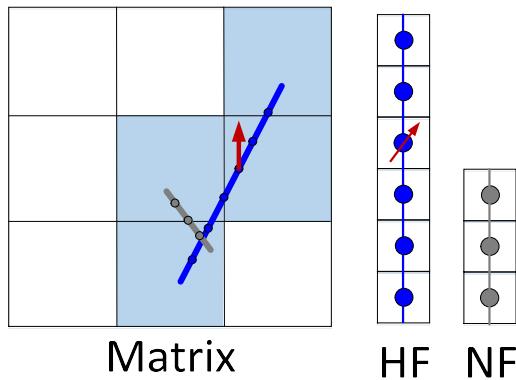
1 Noted that the shale gas viscosity, density and permeability corrections terms are all depends on  
 2 solution variables. To solve non-linear system of Eq. 23, the residual form of Newton's iterations can  
 3 be expressed as follows:

4

$$\mathbf{J}(\mathbf{x}^i)(\mathbf{x}^{i+1} - \mathbf{x}^i) = \frac{d\mathbf{R}}{d\mathbf{x}}(\mathbf{x}^i)(\mathbf{x}^{i+1} - \mathbf{x}^i) = -\mathbf{R}(\mathbf{x}^i) \quad (25)$$

5 The Jacobian matrix  $\mathbf{J}$  is calculated by automatic differentiation in MRST.

6 **3.2 EDFM**



**Fig. 10 – Grid system in EDFM for matrix, natural fracture and hydraulic fracture**

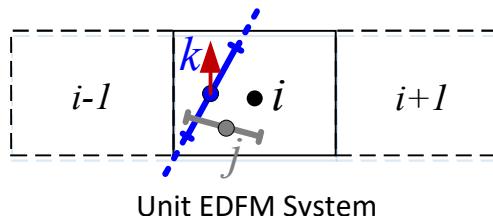
As shown in **Fig. 10**, EDFM adopted the concept of dual-continuum fracture modeling method,

the flow coupling term  $\psi_{f-m/m-f}$  is introduced to couple the solution among matrix and fractures.

Thus, the matrix grid is not necessary conforming with the fracture plane. As shown in **Fig. 11**, there are three kinds of non-neighbor connection (NNC) in EDFM formulation: 1) fracture-matrix connectivity, 2) fracture-fracture connectivity and 3) fracture-wellbore connectivity. The general NNC model can be expressed as follows (Xu, 2015):

15

$$\begin{aligned} \psi_{f-m}^{NNC} &= T_{f-m}^{NNC}(p_f^{n+1} - p_m^{n+1}) \\ \psi_{f-m}^{NNC} &= -\psi_{m-f}^{NNC} \end{aligned} \quad (26)$$



**Fig. 11 – Unit EDFM NNCs of 1) fracture-matrix (*i*-*k* pair) connectivity 2) fracture-fracture (*j*-*k* pair) connectivity and 3) fracture-wellbore (well-*k* pair) connectivity**

*Fracture-matrix NNC:* The fracture-matrix transmissibility ( $T_{f-m}$ ) can be expressed as follows:

$$T_{ik}^{NNC} = \frac{k_{0,ik}}{\mu_{g,ik}} \frac{A_{ik}}{\langle d \rangle_{ik}} \quad (27)$$

where  $A_{i,k}$  is the intersection area fraction between a fracture plane and a gridblock. For 2D grid, the area is the product of intersected fracture cell length within the matrix cell and uniform formation thickness,  $DZ$ . Noted that the harmonic average and upwind scheme are used for the permeability and viscosity, respectively.  $\langle d \rangle_{i,k}$  is the average normal distance between matrix cell and fracture plane, which can be calculated as follows:

$$\langle d \rangle_{ik} = \frac{\int d_{ik} dv}{V_i} \quad (28)$$

For 2D structured grid, an analytical solution is available for the average normal distance (see Tene et al, 2016).

*Fracture-fracture NNC:* the star-delta transformation can be used to calculate the transmissibility between intersected fractures as follows (Hajibeygi et al, 2011):

$$T_{jk}^{NNC} = \frac{t_j t_k}{\sum_{m=1}^{N_{ints}} t_m}, \quad t_m = \frac{A_{f,m}}{0.5h_{f,m}} \frac{k_{0,m}}{\mu_{g,m}} \quad (29)$$

where  $A_f$  is the cross-section area of a fracture plane, for 2D cell, which can be calculated by product of fracture aperture,  $w_f$ , and formation thickness.  $h_f$  is the fracture cell length.

*Fracture-well NNC:* If a well intersected with a fracture cell, the effective wellbore index (WI) and equivalent radius ( $r_e$ ) can be expressed as follows (Xu, 2015):

$$WI_f = \frac{2\pi k_f w_f}{\ln(r_e / r_w) + s}, \quad r_e = 0.14\sqrt{h_f^2 + DZ^2} \quad (30)$$

where  $s$  is the skin factor, dimensionless, which will be used as a correction factor to correct the error introduced by EDFM when model low-permeability fractures.  $DZ$  is the formation thickness, L.

## 4 Verification

To verify the presented general shale gas model (Eq. 21), two numerical simulations are performed against a commercial simulator (CMG, 2015) and an in-house simulator with unstructured mesh (Jiang and Younis, 2015). The base model and simulation parameters for all cases as shown in **Table 3**:

**Table 3—Base model and simulation parameters for all cases**

Property	Unit	Value
----------	------	-------

Rock density	kg/m <sup>3</sup>	2500
Molecular weight, CH <sub>4</sub>	kg/mol	0.01604
Critical pressure, CH <sub>4</sub>	MPa	4.60
Critical temperature, CH <sub>4</sub>	K	190.6
Acentric factor, CH <sub>4</sub>	-	0.01142
Well radius	m	0.1

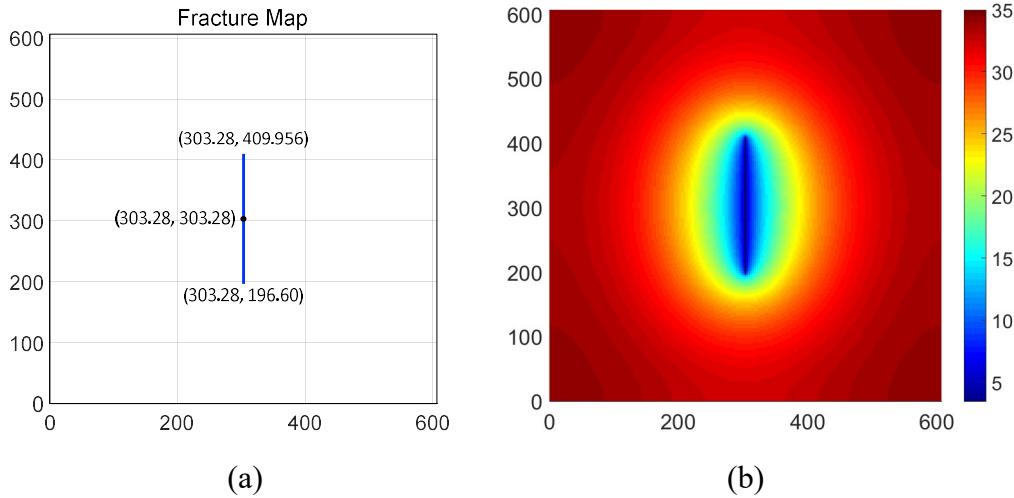
1

2 **4.1 Case 1 – Verification against commercial simulator**

3 OpenShale is firstly verified in a simple methane production case against a commercial  
 4 simulator (CMG) with a single vertical hydraulic fracture (Fig. 12). By changing the hydraulic  
 5 conductivity, grid schemes and natural fractures, three subcases (Case1a, Case1b and Case1c) are  
 6 investigated. The accuracy of OpenShale with explicit fracture modeling (EFM) and EDFM are  
 7 systematically studied. In this simulation, only Langmuir adsorption (Eq. 8) is considered. All fluid  
 8 properties and simulation parameters are the same with the commercial simulator. The  
 9 compressibility factor Z and natural gas viscosity are directly interpolated from the properties table  
 10 of the commercial simulator. Detailed simulation properties are shown in **Table 4**.

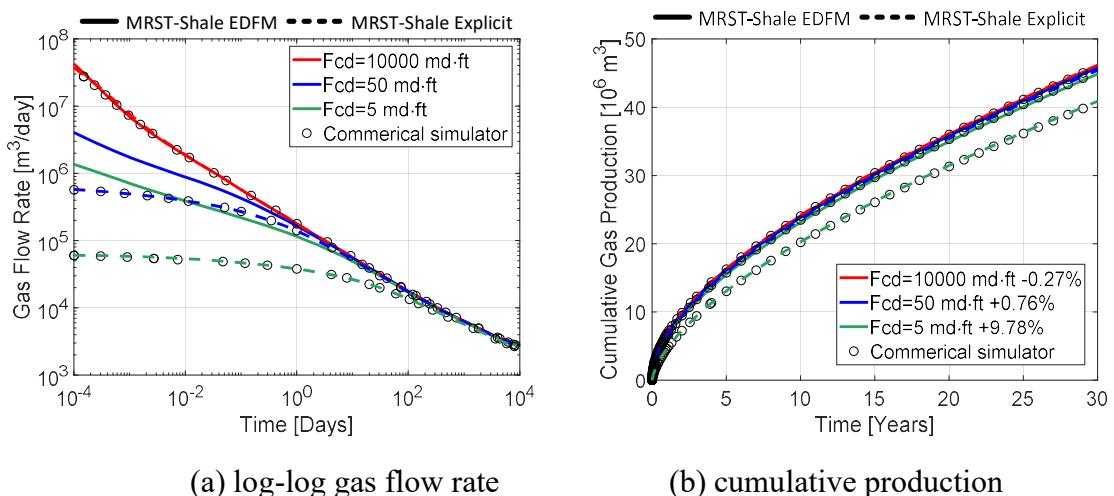
11 **Table 4. Key reservoir and simulation parameters of Case 1**

Property	Unit	Value
Domain dimensions (x,y)	m	606.6,606.6
Grid (nx,ny)	-	201,65
Formation thickness	m	45.72
Initial reservoir pressure	MPa	34.47
Temperature	K	327.60
Langmuir pressure	MPa	8.96
Langmuir volume	m <sup>3</sup> /kg	0.0041
Matrix porosity		0.07
Matrix compressibility	1/Pa	1.45e-10
Matrix permeability	nD	500
Fracture permeability	mD	0.5-1000
Fracture width	m	0.003
Fracture half-length	m	106.68
Fracture conductivity	md-ft	5-10000
Well BHP	MPa	3.45
Production time	years	30



**Fig. 12 Fracture map (a) and pressure contour after 30 years production (b) of Case 1a and Case1b**

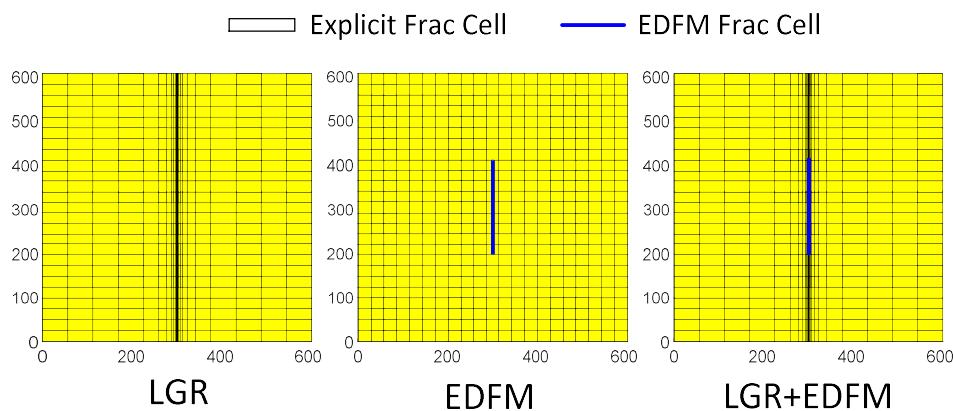
**Case1a:** In the first subcase, three fracture conductivities (10000 md-ft, 50 md-ft, 5 md-ft) are used to verify the accuracy of OpenShale with EFM and EDFM. **Fig. 13** shows a good agreement of both gas flow rate and cumulative production between OpenShale and commercial simulator. Results show that OpenShale with EFM (dash line) always gives consistent results against commercial simulator. But OpenShale with EDFM (solid) has significant error (up to 10.92%) when fracture conductivity is low (5 md-ft). Fig. 12a shows that OpenShale EDFM only converges to reference solution under infinite fracture conductivity (10000 md-ft). This is observation matches Tene (2017)'s conclusion that EDFM can not handle the fracture with low permeability.



**Fig. 13 Comparison of gas flow rate (a) and cumulative production (b) for Case 1a between OpenShale EDFM (solid line), OpenShale EFM (dash line) and a commercial**

1 simulator (dots) with respect to fracture conductivities of 5 md-ft (green lines), 50 md-ft (blue  
2 lines) and 10000 md-ft (red lines)

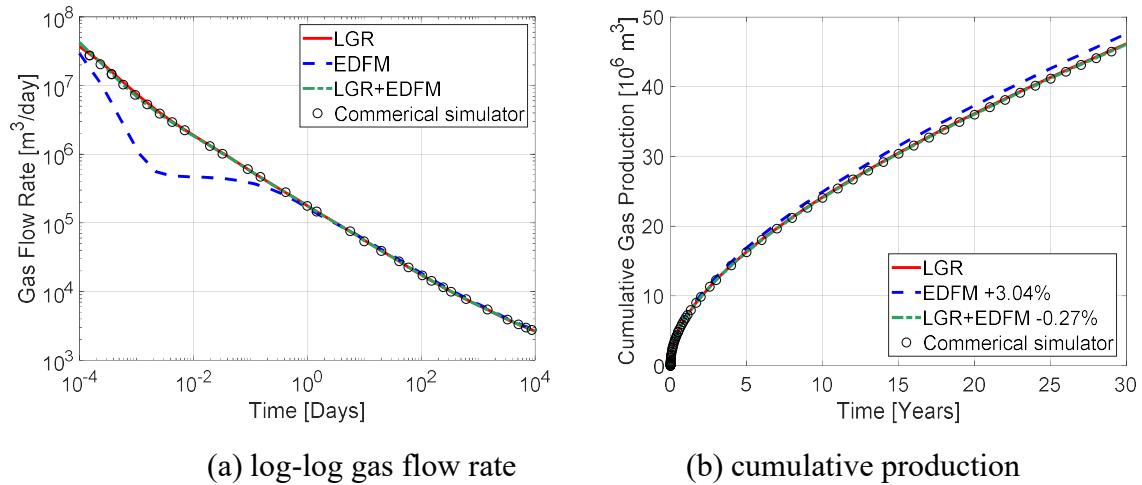
3        *Case1b*: For unconventional tight reservoir, LGR is usually required to capture the transient  
 4 flow behavior and sharp pressure gradient near the hydraulic fractures. In the second subcase, the  
 5 effect of grid schemes on accuracy of OpenShale with EFM and EDFM are investigated. In this case,  
 6 the fracture conductivity is set as 10000 md-ft to eliminate the EDFM error mentioned in Case1a. All  
 7 other parameter is the same with Case1a. As shown in **Fig. 14**, three grid schemes are investigated,  
 8 where LGR scheme with logarithmic refinement that is solved by OpenShale EFM; EDFM scheme is  
 9 the standard EDFM grid scheme (Xu et al, 2017 and Tene et al, 2017) with uniform grid that is  
 10 solved by EDFM; EDFM+LGR scheme is the same grid scheme as LGR scheme that an additional  
 11 EDFM fracture cell is added and that is solved by EDFM. Noted that all grid scheme has the same  
 12 grid dimension (nx,ny) of 499x61.



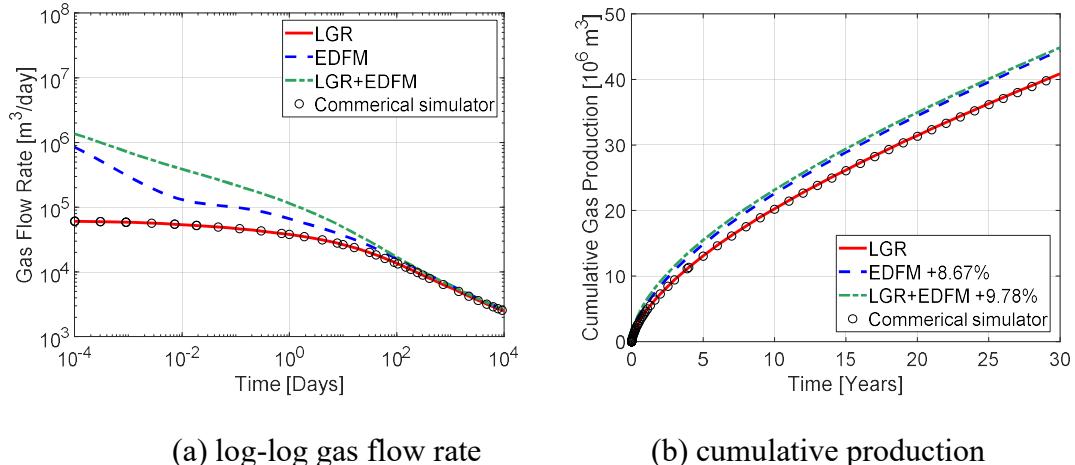
**Fig. 14 EFM and EDFM Grid schemes for Case1b, fracture cell is shown 10 times larger the real size, where logarithmic refinement and uniform used in LGR and EDFM scheme, respectively**

17 Figs. 15-16 shows a good agreement of gas flow rate and cumulative production between  
18 OpenShale and commercial simulator with respect to high fracture conductivity and low fracture  
19 conductivity. OpenShale with EFM again gives consistent results against commercial simulator for  
20 all grid schemes. However, the standard EDFM grid scheme can introduce an error of 3.31% for high  
21 fracture conductivity and 1.11% for low fracture conductivity. The error is measured by the  
22 difference of cumulative production between grid schemes of LGR+EDFM and EDFM. This

1 benchmark case demonstrates that EDFM cannot capture transient flow behavior and sharp pressure  
 2 gradient near the hydraulic fracture without helping of LGR.



5 **Fig. 15 Comparison of gas flow rate (a) and cumulative production (b) for Case 1b with**  
 6 **high fracture conductivity of 10000 md-ft between OpenShale and a commercial simulator**  
 7 **(dots)**

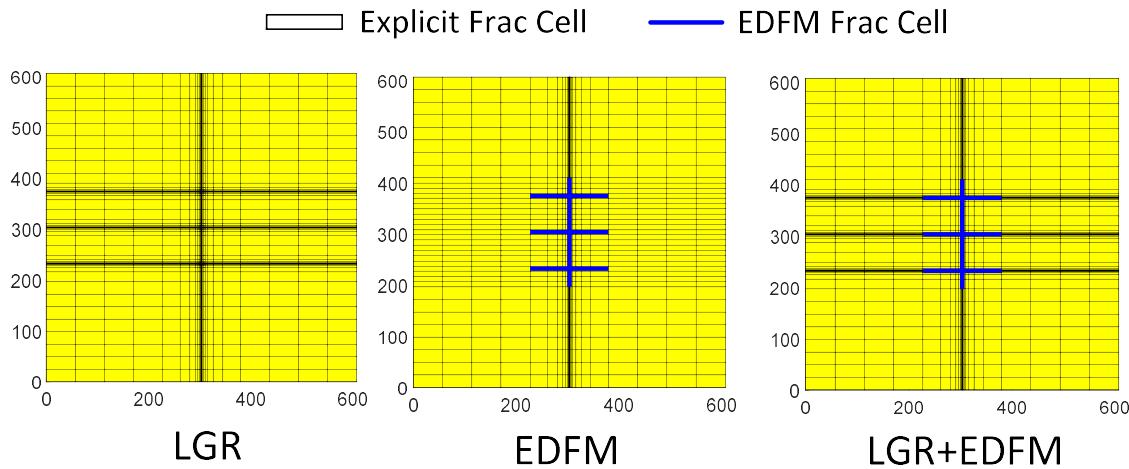


8 **Fig. 16 Comparison of gas flow rate (a) and cumulative production (b) for Case 1b with**  
 9 **low fracture conductivity of 5 md-ft between OpenShale and a commercial simulator (dots)**

10 *Case 1c:* As mentioned in Case1a and Case1b, EDFM cannot handle low-permeability fracture

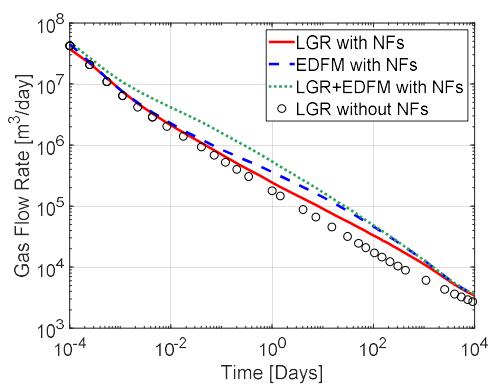
11 and hydraulic fractures with sharp pressure gradient. But modeling of natural fracture network is  
 12 quite challenge for the EFM. In this subcase, the effect of different grid schemes of natural fractures  
 13 on accuracy for OpenShale with EDFM is also investigated. As shown in **Fig. 17**, six natural  
 14 fractures with the same length of 116.74 m are added based on the Case1a. The well performance of  
 15 two grid schemes with and without LGR for natural fractures are studied. Fracture conductivity for  
 16 two grid schemes with and without LGR for natural fractures are studied. Fracture conductivity for  
 17

1 hydraulic fracture and natural fractures are set as 10000 md-ft and 5 md-ft, respectively. All other  
 2 parameters are the same with Table 3.

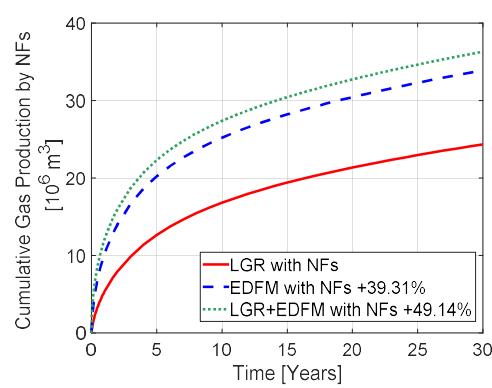


3  
 4 **Fig. 17 Grid schemes of Case 1c, number of grids are shown 20 times coarser than the real**  
 5 **scheme. LGR scheme with LGR for natural fractures, EDFM scheme without LGR for natural**  
 6 **fractures**

7 **Fig. 18** demonstrates that OpenShale with EDFM can lead to a significant error (up to 16.99%)  
 8 for the case where low-permeability natural fractures connected with high-permeability hydraulic  
 9 fractures. Also, EDFM without LGR for natural fractures tends to underestimate the well  
 10 performance (error of 3.4% for six natural fractures). This benchmark case indicates that EDFM is  
 11 not capable to accurately model well performance of shale gas flow in ultra-tight reservoir due to the  
 12 errors introduced by low-permeability fracture and grid refinement.



13  
 14 (a) log-log gas flow rate



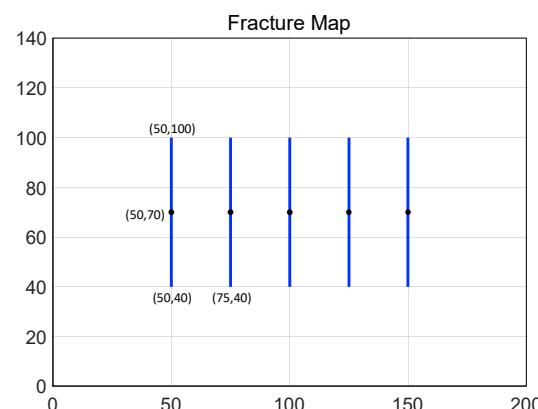
(b) normalized cumulative production

15 **Fig. 18 Comparison of gas flow rate (a) and normalized cumulative production by NFs (b)**  
 16 **for Case 1c between different grid scheme for natural fractures**

1 In sum, this case study shows that OpenShale with EFM always give consistence results against  
 2 commercial simulator, while OpenShale with EDFM only converge to the reference solution at  
 3 infinite fracture conductivity (Case1a). Also, OpenShale with EDFM cannot handle low-permeability  
 4 fracture (Case1b) and cannot capture transient behavior and sharp gradient without LGR (Case1c).  
 5 OpenShale with EDFM can model complex and irregular natural fractures accurately and efficiently.  
 6 Thus, in the following simulations, an empirical skin-factor and uniform grid refinement are adopted  
 7 to relieve the limitations of EDFM. More advanced projected EDFM (Tene et al, 2017) and  
 8 adaptively grid refinement will be implemented in our future work.

#### 9 **4.2 Case 2 – Verification against in-house simulator**

10 OpenShale is further verified against an in-house simulator (Jiang and Younis, 2015) by  
 11 considering more comprehensive state-of-art transport mechanisms and fracture geometries. For the  
 12 reference solution, it used fully unstructured mesh with LGR to capture the complex fracture  
 13 geometries as well as the sharp pressure gradient near the fracture. In this case, the gas rate solution  
 14 of two sub-case are investigated. In the first sub-case (Case2a), the well performance with and  
 15 without storage (Eq. 8) and transport mechanism (Eq. 10) is considered. In the second sub-case  
 16 (Case2b), the irregular fracture geometry is considered. The fracture map of Case2a is shown in **Fig.**  
 17 **19**. Detailed simulation parameters for Case 2 are elaborated in **Table 4**.



18 **Fig. 19 Fracture map and EDFM grid of Case 2**

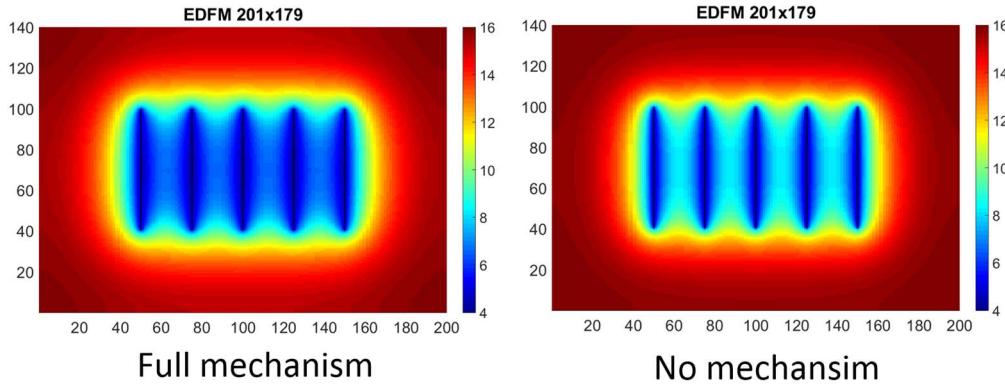
19 **Table 4. Key reservoir and simulation parameters of Case 2**

Property	Unit	Value
Domain dimensions (x,y)	m	200,140
Formation thickness,	m	10
Initial reservoir pressure	MPa	16
Temperature	K	343.15
Langmuir pressure	MPa	4

Langmuir volume	$\text{m}^3/\text{kg}$	0.018
Matrix porosity		0.1
Matrix compressibility	1/Pa	1.0e-9
Fracture porosity		1.0
Matrix permeability	nD	100
Fracture permeability	D	1
Fracture width	m	1e-3
Well BHP	MPa	4
Correction skin factor	-	43
Production time	days	10000

Other parameters are the same as in Table 2

1 **Fig. 20** shows pressure contour after 2500 days of production for Case 2 with and without  
 2 transport mechanisms. It can be observed that the sub-case with full mechanism has better pressure  
 3 depletion (dark blue region) than one without any mechanism. **Fig. 21** shows a good agreement  
 4 between gas flow rate between OpenShale and an in-house simulator, where demonstrates that the  
 5 both adsorption and gas slippage and diffusion effect increase the gas production significantly. In  
 6 tight unconventional reservoirs, smaller pore-throat and lower bottom-hole pressure can lead to  
 7 higher production due to gas slippage flow and releasing adsorbed gas.



8  
 9 **Fig. 20 Pressure contour with and without full shale gas transport mechanism @ 2500  
 10 days of Case 2**

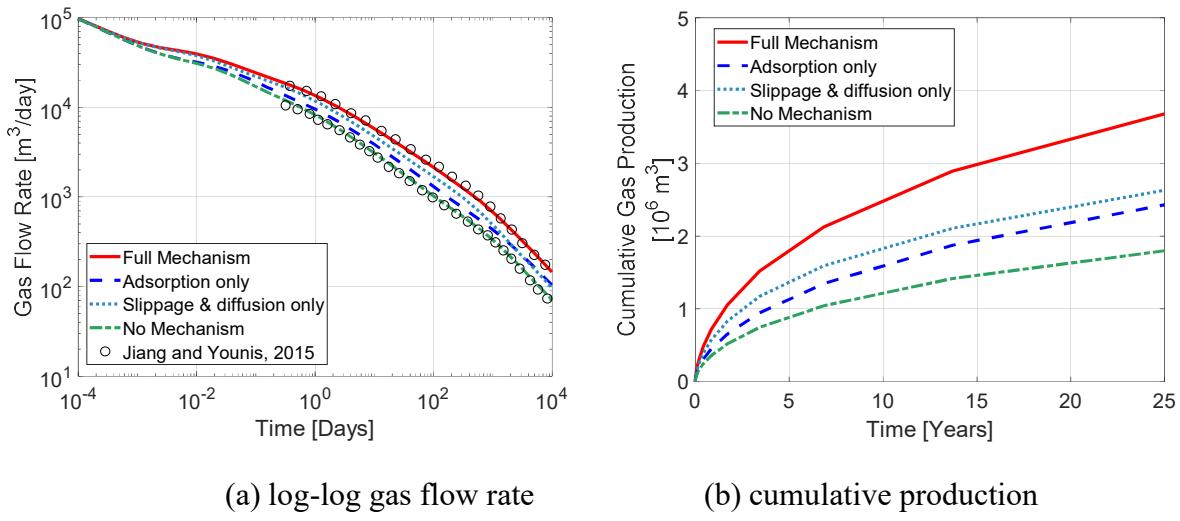


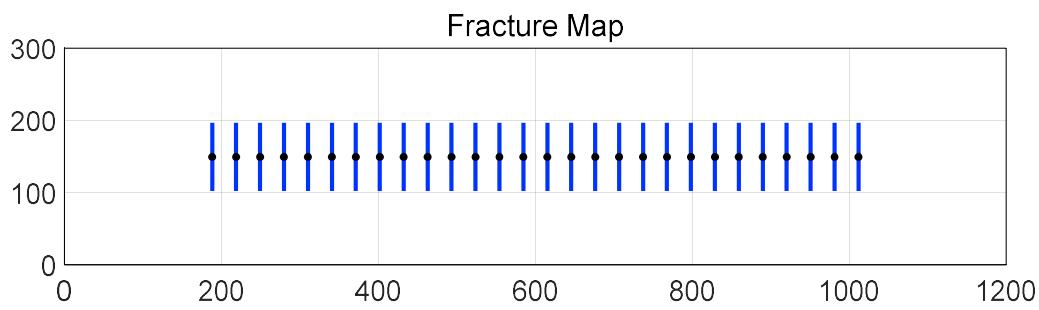
Fig. 21 Comparison of gas flow rate (a) and cumulative production (b) for Case 2a between OpenShale and an in-house simulator

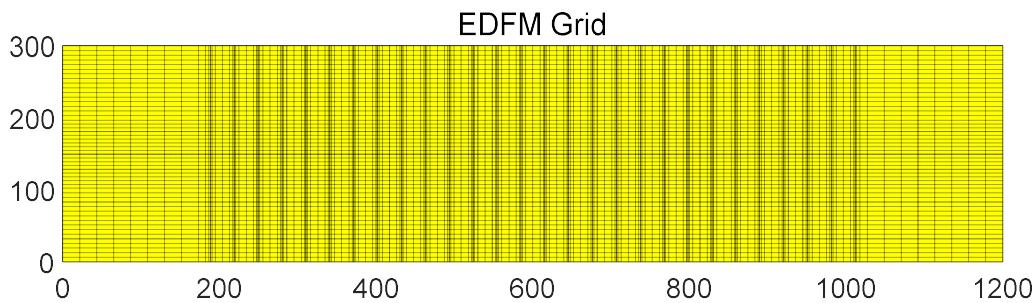
## 5 Application

In the previous sections, OpenShale shows its capability to handle arbitrary transport and storage mechanism and fracture geometries. To further illustrate the applicability of OpenShale in practical problems, two case studies of OpenShale in realistic unconventional reservoirs with complex fracture network are presented.

### 5.1 Case 3: History matching and production forecast

To further verify the applicability of the OpenShale. A history matching with field production data on a Barnett shale has performed. The field production and simulation data are adopted from literature (Cao, Liu and Leong, 2016; Yu and Kamy Sepehrnoori, 2014). The detailed reservoir and fluid parameters are shown as in **Fig. 22** and **Table 5**.





**Fig. 22 Fracture map and EDFM LGR grid with 28 planar hydraulic fractures of Case 3**

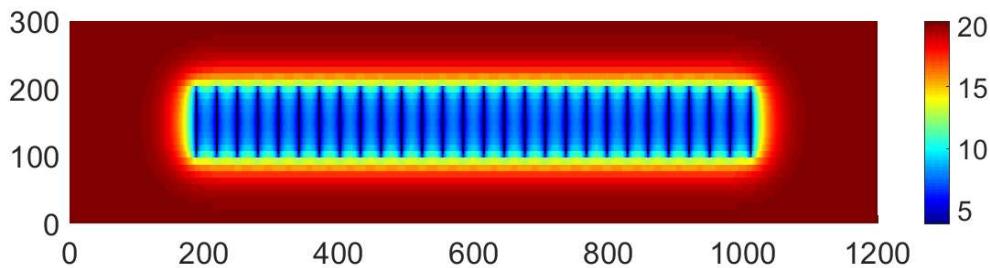
**Table 5. Key reservoir and simulation parameters of Barnett shale for Case 3 (Cao, 2016)**

Property	Unit	Value
Domain dimensions (x,y)	m	1200,300
Depth	m	5463
Formation thickness,	m	90
Initial reservoir pressure	MPa	20.34
Temperature	K	352
Rock density	kg/m <sup>3</sup>	2500
Langmuir pressure	MPa	4.47
Langmuir volume	m <sup>3</sup> /kg	0.00272
Matrix porosity		0.03
Matrix compressibility	1/Pa	1.5e-10
Fracture compressibility	1/Pa	1.0e-8
Matrix permeability	nD	200
Fracture permeability	mD	100
Fracture width	m	0.003
Fracture spacing	m	30.5
Fracture half-length	m	47.2
Fracture conductivity	md-ft	1
Well BHP	MPa	3.69
Correction skin factor	-	19
Production time	days	1600

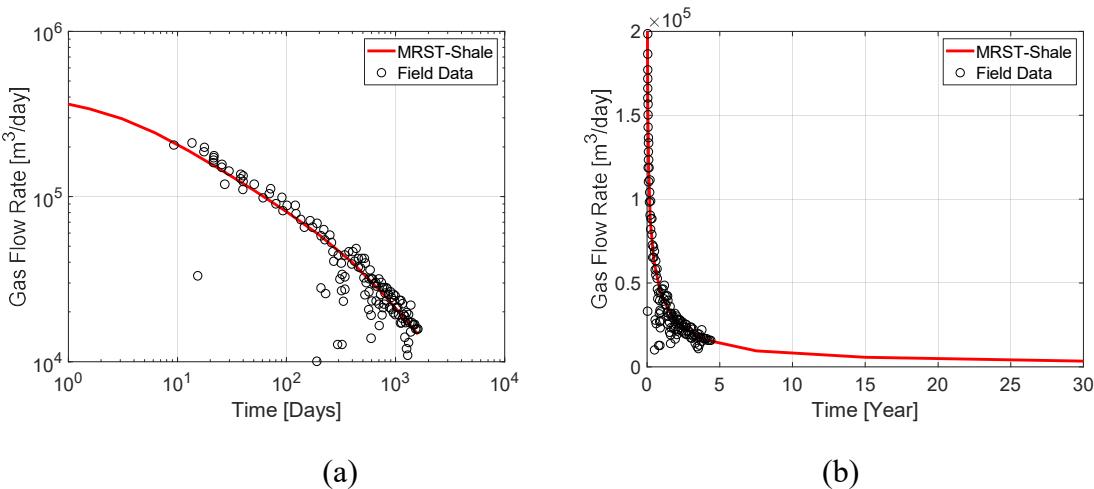
Other parameters are the same as in Table 2

In this simulation, a rectangle reservoir with dimension of  $1100 \times 290 \times 90$  m was discretized by  $148 \times 39 \times 1$  grids. 28 stages hydraulic fractures in the center of domain with the half-length of 47.2 m and the fracture spacing of 30.5 m. The fractures are assumed have constant aperture of 0.003 m and permeability of 100 md. Only shale gas storage mechanism of Langmuir adsorption (Eq. 8) is considered. **Fig. 23** shows the pressure contour at different production time (400 days and 1600 days). **Fig. 24** shows the comparison of production rate between OpenShale and field data which

1 shows good agreements with the field production data. Based on matched simulation parameters, the  
 2 production forecast can be easily performed as in Fig. 21.



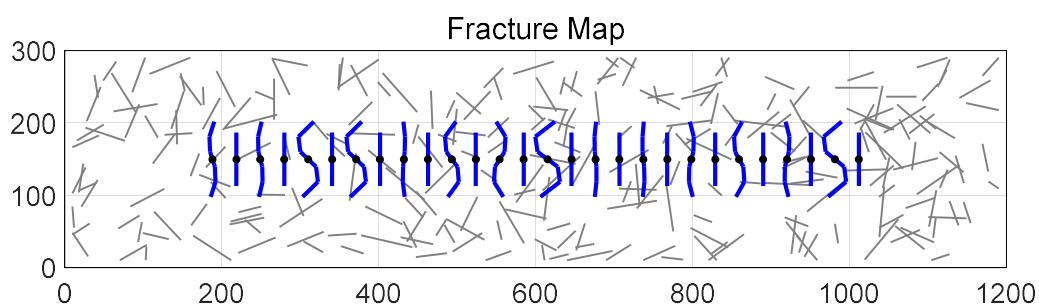
3  
 4 **Figure 23 Pressure contour after 1600 days production for Barnett shale reservoir (Case 3)**



5  
 6 **Fig. 24 History matching (a) and production forecast (b) of a Barnett shale well (Case 3)**

## 7 **5.2 Case 4: New model evaluation**

8 To illustrate the capability of modular design and rapid prototyping of OpenShale, a new shale  
 9 gas model considering geomechanics effect (Eqs. 15-17) for multi-scale fractured network is  
 10 implemented and evaluated using OpenShale. In this section, the influence of multi-scale fracture  
 11 network and geomechanics effect on shale gas production performance will be investigated.  
 12



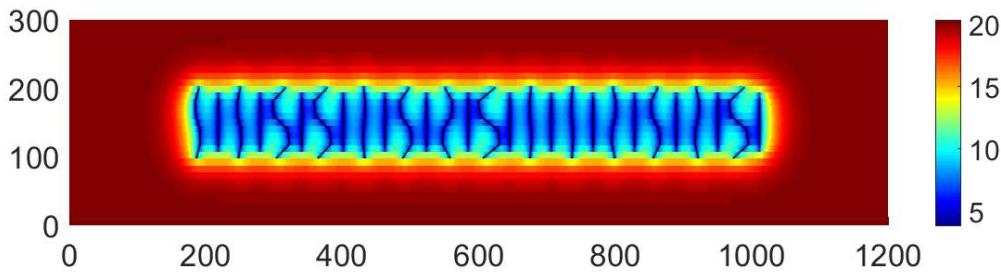
13  
 14 **Fig. 25 Fracture map with 28 non-planar hydraulic fractures and 248 natural fractures of**  
 15 **Case 4**

1 In this case, all the simulation parameters are the same with Case 3 of Barnett shale reservoir.  
 2 The total length of non-planar hydraulic fractures (blue lines in **Fig. 25**) is the same as planar  
 3 fractures used in Case 3 (blue lines in Fig. 14). Natural fractures are stochastically generated by an  
 4 open-source fracture generator ADFNE (Alghalandis, 2017). The geomechanics parameters for shale  
 5 reservoir are assumed (Wasaki and Akkutlu, 2015) as follows (**Table 5**):

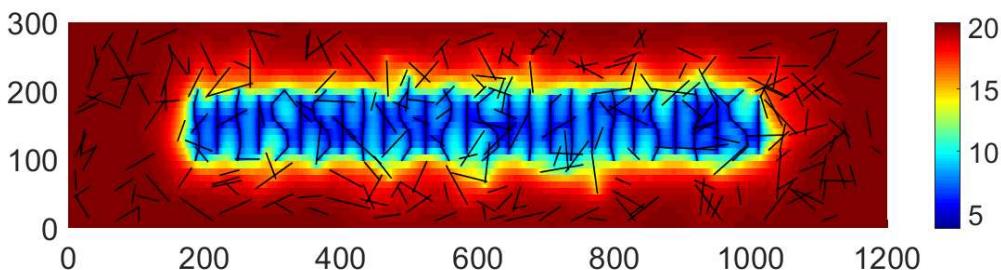
6 **Table 5. Geomechanics parameters of Barnett shale for Case 4**

Property	Unit	Value
Biot constant, $\alpha$	-	0.5
Overburden confining stress, $p_c$	MPa	38
Maximum horizontal stress, $s_{hmax}$	MPa	34
Minimum horizontal stress, $s_{hmin}$	MPa	29
Maximum closure stress for micro-fracture, $p_1$	MPa	180
Gangi exponential constant, $m$	-	0.5
Natural fractures permeability	md	10

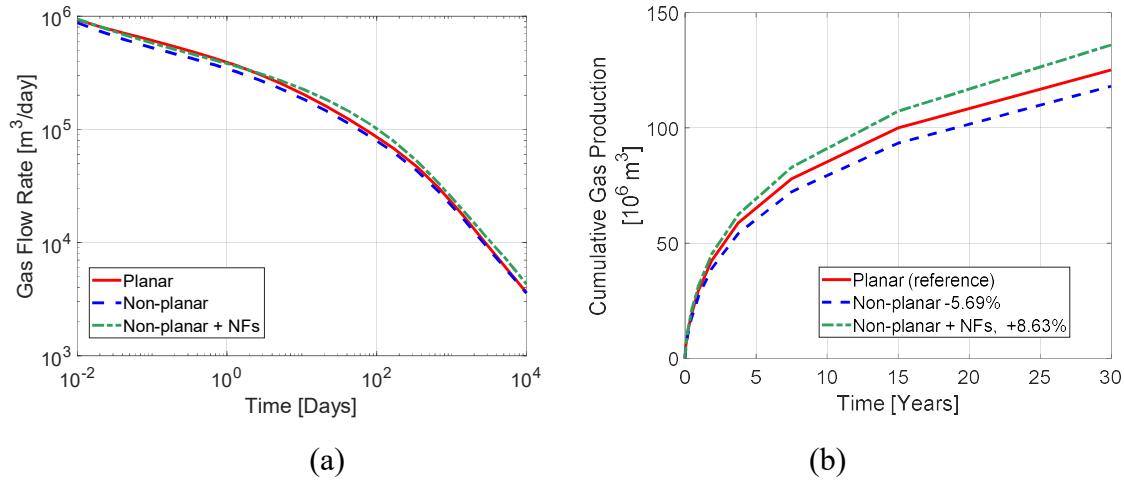
Other parameters are the same as in Tables 2-3



7 (a) Non-planar hydraulic fracture (Same total fracture length with Case 3)  
 8  
 9

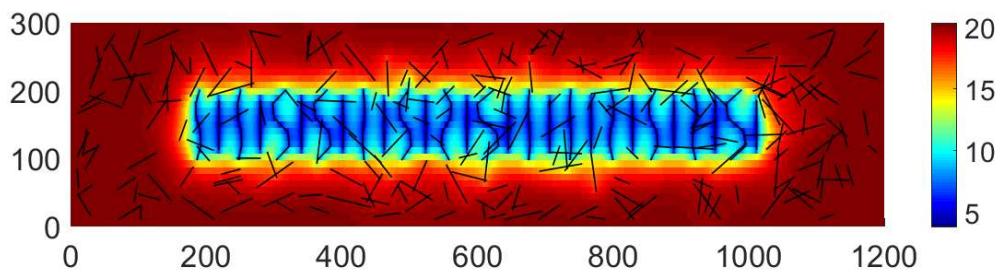


10 (b) Non-planar hydraulic fracture + natural fractures  
 11  
 12 **Fig. 26 Pressure contour at the 3.75 years for Barnett shale reservoir with Non-planar fracture**  
 13 **geometries and natural fractures (case 4)**



**Fig. 27 Comparison of gas flow rate (a) and cumulative production (b) between Planar, Non-planar and Non-planar & natural fractures cases**

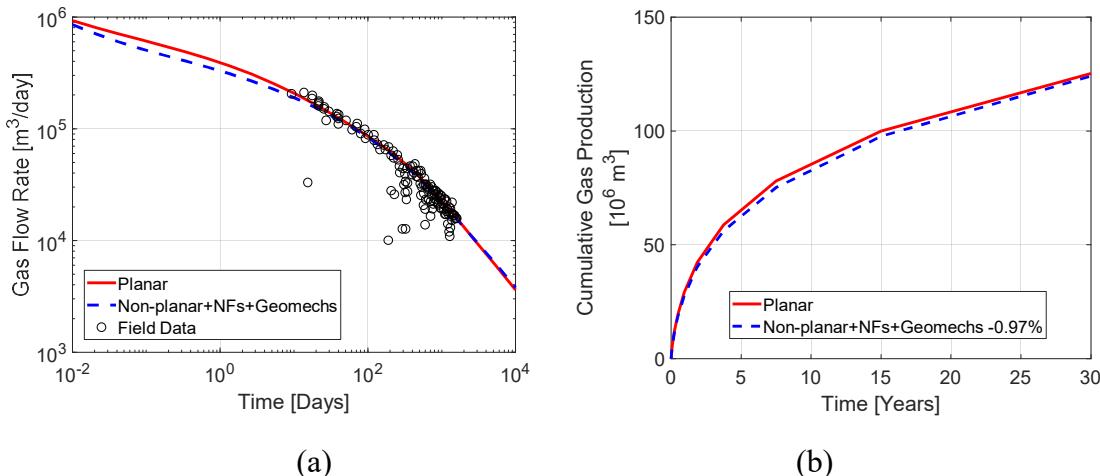
Firstly, the effect of complex fracture network on well performance is studied. **Fig 26** shows the pressure contour for the non-planar fractures with and without the natural fractures. Obviously, the case of natural fractures has larger and better stimulated reservoir volume (SRV). Thus, as shown in **Fig. 27**, the cumulative gas production of non-planar case with natural fracture has much higher value (14.56% improvements) than the planar case in the Case 3. While in the case of same total length, the non-planar fracture geometries will slightly degenerate the well performance (-5.69% reduction).



**Fig. 28 Pressure contour at 3.75 years for Barnett shale reservoir with planar hydraulic fractures case and realistic case**

The influence of geomechanics effect with fracture closure on well performance is further investigated by implementing Eqs. 15-17. **Fig. 28** shows the pressure contour at the 3.75 years for the planar case (Fig. 20a) and realistic case (Fig. 20b) with non-planar hydraulic fracture, natural fractures and geomechanics effect. As shown in **Fig. 29**, at the earlier production period, even realistic case has lower production than simple planar case due to geomechanics effect. But in the

1 later production time, the contribution of natural fractures makes identical well performance between  
 2 realistic case and simple planar case. Thus, the modeling of natural fractures and geomechanics  
 3 effect is important for long-term production evaluation.



5 **Fig. 29 Comparison of gas flow rate (a) and cumulative production (b) between planar**

6 **hydraulic fracture case and realistic case with non-planar, natural fractures and geomechanics**  
 7 **effect**

## 9 **6 Conclusion**

10 In this work, A generic numerical model and an open-source framework OpenShale are  
 11 developed for shale gas simulation with state-of-art flow and storage mechanisms. It is verified  
 12 against commercial and in-house reservoir simulators. The limitation of EDFM are also investigated  
 13 quantity. Also, a field application of history matching and new model evaluation of geomechanics  
 14 effect are successful performed. Several conclusions can be drawn as follows:

15 (1) A generic shale gas numerical model is developed which can be used to model any state-of-art  
 16 storage and transport mechanisms, including gas adsorption, gas slippage & diffusion,  
 17 non-Darcy flow as well as geomechanics effect by considering complex multi-scale fracture  
 18 geometries.

19 (2) A general and open-source framework, OpenShale is developed and verified. With the help of  
 20 the EDFM, Automatic Differentiation, and object-designed framework of OpenShale, one can  
 21 easily use and extend OpenShale to simulate practical shale gas problem with arbitrary fracture  
 22 geometries and new storage and transport mechanisms.

23 (3) EDFM can efficiently and accurately model irregular fracture geometry and complex fracture

1 networks. However, it cannot accurately model low-permeability fracture (error of 12.22%) and  
2 hydraulic fractures without help of LGR where have strong transient behavior and sharp  
3 gradient (error of 2.84%). Thus, projected EDFM and adaptively grid refinement will be  
4 implemented and tested in our future work.

5 (4) Shale gas transport and storage mechanisms, such as gas desorption and gas slippage & diffusion  
6 flow gas, are the most significant impact on well performance, follow by natural fractures,  
7 geomechanics effect and fracture geometry.

8 (5) OpenShale is capable of serving as an efficient, flexible research tool to evaluate new models  
9 with arbitrary non-linearity and fracture complexity. It can serve as a bridge between mechanism  
10 study and field scale engineering application.

## 12 Nomenclature

13  $\rho_g$  = mass density of natural gas, kg/m<sup>3</sup>

14  $\phi$  = absolute rock porosity, dimensionless

15  $\Omega_m$  = matrix domain

16  $\Omega_f$  = fracture domain

17  $m_{ad}$  = storage mechanism term, kg/m<sup>3</sup>

18  $F_{app}$  = transport mechanism term, dimensionless

19  $k_0$  = absolute Darcy rock permeability, m<sup>2</sup>

20  $\mu_g$  = viscosity of natural gas, Pa·s

21  $p$  = pore pressure, Pa

22  $q_w$  = volumetric sink/source term, m<sup>3</sup>/day

23  $b_g$  = inverse formation volume factor, dimensionless

24  $M$  = molecular weight of natural gas, kg/mol

25  $Z$  = compressibility factor of natural gas, dimensionless

26  $R$  = ideal gas constant, 8.314 J/(mol·K)

27  $T$  = reservoir temperature, K

28  $T_{pr}$  = pseudo-temperature for natural gas, dimensionless

29  $T_c$  = critical-temperature for natural gas, K

30  $p_{pr}$  = pseudo-pressure for natural gas, dimensionless

31  $p_c$  = critical-pressure for natural gas, Pa

32  $a_{0,1,2}$  = constants for Peng-Robinson equation of state, dimensionless

33  $a, b$  = constants for Peng-Robinson equation of state, dimensionless

34  $A, B$  = constants for Peng-Robinson equation of state, dimensionless

35  $K, X, Y$  = constants for Lee-Conzalez-Eakin natural gas viscosity, dimensionless

36  $\rho_s$  = mass density of bulk matrix, kg/m<sup>3</sup>

37  $\rho_{gsc}$  = mass density of natural gas at the standard condition, kg/m<sup>3</sup>

38  $V_L$  = Langmuir volume, m<sup>3</sup>/kg

1  $P_L$  = Langmuir pressure, Pa  
2  $V_m$  = BET volume,  $\text{m}^3/\text{kg}$   
3  $P_s$  = BET pseudo-saturation pressure, Pa  
4  $p_r$  = psueo-pressure for BET isotherm, dimensionless  
5  $C$  = constant for BET isotherm, dimensionless  
6  $n$  = constant for BET isotherm, dimensionless  
7  $\alpha$  = rarefaction coefficient for gas slippage flow, dimensionless  
8  $K_n$  = Knudsen number, dimensionless  
9  $\beta$  = Darcy-Forchheimer coefficient, dimensionless  
10  $\alpha_B$  = Biot's coefficient, dimensionless  
11  $P_c$  = reservoir confining overburden pressure, Pa  
12  $P_I$  = reservoir effective stress when micro-fracture completely closed, Pa  
13  $m$  = constant for the Gangi's model, Pa  
14  $F_{cd}$  = fracture conductivity,  $\text{md}\cdot\text{ft}$   
15  $p_0$  = initial reservoir pressure, m  
16  $\sigma$  = effective fracture closure stress, Pa  
17  $\sigma_{hf}$  = effective closure stress for hydraulic fracture, Pa  
18  $\sigma_{nf}$  = effective closure stress for natural fracture, Pa  
19  $\sigma_h$  = reservoir horizontal principle stress, Pa  
20  $\sigma_{hmin}$  = minimum reservoir horizontal principle stress, Pa  
21  $\sigma_{hmax}$  = maximum reservoir horizontal principle stress, Pa  
22  $k_f$  = absolute Darcy permeability of fracture,  $\text{m}^2$   
23  $w_f$  = fracture width, m  
24  $V$  = bulk volume of a grid cell, m  
25  $\delta$  = discrete domain delta function, dimensionless  
26  $\Delta t$  = solution time-step, day  
27  $\psi_{f-m}$  = mass coupling term for matrix, dimensionless  
28  $\psi_{m-f}$  = mass coupling term for fracture, dimensionless  
29  $p_{bh}$  = wellbore bottom hole pressure, Pa  
30  $k_{11}$  = absolute Darcy rock permeability in x-direction,  $\text{m}^2$   
31  $k_{22}$  = absolute Darcy rock permeability in y-direction,  $\text{m}^2$   
32  $r_e$  = equivalent radius for wellbore model, m  
33  $r_w$  = wellbore radius, m  
34  $s$  = wellbore skin factor, dimensionless  
35  $\Delta x$  = grid cell size in x-direction, m  
36  $\Delta y$  = grid cell size in y-direction, m  
37  $\Delta z$  = grid cell size in z-direction, m  
38  $WI$  = wellbore index, dimensionless  
39  $\mathbf{x}$  = Unknown vector, -  
40  $\mathbf{J}$  = Jacobian matrix, -  
41  $\mathbf{R}$  = Residual vector, -  
42  $WI$  = wellbore index, dimensionless  
43  $p_f$  = pore pressure at the fracture domain, Pa  
44  $p_m$  = pore pressure at matrix domain, Pa

1  $T$  = transmissibility, dimensionless  
2  $A$  = intersection area among fracture and matrix,  $\text{m}^2$   
3  $d$  = average normal distance among fracture and matrix, m  
4  $h_f$  = length of a fracture cell, m  
5  $t$  = fracture transmissibility for fracture-fracture NNC, dimensionless  
6

7 Subscripts:

8  $NF$  = natural fracture  
9  $HF$  = hydraulic fracture  
10  $m$  = matrix  
11  $f$  = fracture  
12  $g$  = gas  
13  $w$  = well  
14

## 15 References

16 Akkutlu, I.Y., Efendiev, Y., Vasilyeva, M. and Wang, Y., 2018. Multiscale model reduction for  
17 shale gas transport in poroelastic fractured media. *Journal of Computational Physics*, 353,  
18 pp.356-376.

19 Akkutlu IY, Fathi E. Multi-scale gas transport in shales with local kerogen heterogeneities. *SPE*  
20 *J.* 2012;17(4):1002–1011.

21 Alghalandis, Y.F., 2017. ADFNE: Open source software for discrete fracture network  
22 engineering, two and three dimensional applications. *Computers & Geosciences*, 102, pp.1-11.

23 Alramahi, B. and Sundberg, M.I., 2012, January. Proppant embedment and conductivity of  
24 hydraulic fractures in shales. In 46th US Rock Mechanics/Geomechanics Symposium. American  
25 Rock Mechanics Association.

26 Agarwal, R.G., 1979, January. " Real Gas Pseudo-Time"-A New Function For Pressure Buildup  
27 Analysis Of MHF Gas Wells. In *SPE Annual Technical Conference and Exhibition*. Society of  
28 Petroleum Engineers.

29 Bowker, K.A., 2007. Barnett shale gas production, Fort Worth Basin: Issues and discussion.  
30 *AAPG bulletin*, 91(4), pp.523-533.

31 Cipolla, C.L., Lolom, E.P., Erdle, J.C. and Rubin, B., 2010. Reservoir modeling in shale-gas  
32 reservoirs. *SPE reservoir evaluation & engineering*, 13(04), pp.638-653.

33 Cao, P., Liu, J. and Leong, Y.K., 2016. A fully coupled multiscale shale deformation-gas  
34 transport model for the evaluation of shale gas extraction. *Fuel*, 178, pp.103-117.

35 Civan F. Effective correlation of apparent gas permeability in tight porous media. *Transp Porous  
36 Media*. 2010;82(2):375–384.

37 Chen, Z., Liao, X., Zhao, X., Dou, X. and Zhu, L., 2015. Performance of horizontal wells with  
38 fracture networks in shale gas formation. *Journal of Petroleum Science and Engineering*, 133,  
39 pp.646-664.

40 Chen, Z., Liao, X., Zhao, X., Dou, X. and Zhu, L., 2016. A semi-analytical mathematical model  
41 for transient pressure behavior of multiple fractured vertical well in coal reservoirs incorporating  
42 with diffusion, adsorption, and stress-sensitivity. *Journal of Natural Gas Science and  
43 Engineering*, 29, pp.570-582.

1 Chen, Z., Liao, X., Sepehrnoori, K. and Yu, W., 2018. A Semianalytical Model for  
2 Pressure-Transient Analysis of Fractured Wells in Unconventional Plays With Arbitrarily  
3 Distributed Discrete Fractures. *SPE Journal*.

4 Chen Z, Liao X, Zhao X, Lyu S, Zhu L. A comprehensive productivity equation for multiple  
5 fractured vertical wells with non-linear effects under steady-state flow. *J Pet Sci Eng.* 2017;  
6 149:9–24.

7 Civan F, Rai CS, Sondergeld CH. Shale-gas permeability and diffusivity inferred by improved  
8 formulation of relevant retention and transport mechanisms. *Transp Porous Media.*  
9 2011;86(3):925–944.

10 Cinco, L., Samaniego, V. and Dominguez, A., 1978. Transient pressure behavior for a well with  
11 a finite-conductivity vertical fracture. *Society of Petroleum Engineers Journal*, 18(04),  
12 pp.253-264.

13 Elliott, J.R. and Lira, C.T., 2011. *Introductory chemical engineering thermodynamics* (Vol. 184).  
14 Upper Saddle River, NJ: Prentice Hall PTR.

15 Florence, F.A., Rushing, J., Newsham, K.E. and Blasingame, T.A., 2007, January. Improved  
16 permeability prediction relations for low permeability sands. In *Rocky Mountain Oil & Gas*  
17 *Technology Symposium*. Society of Petroleum Engineers.

18 Hoteit, H. and Firoozabadi, A., 2005. Multicomponent fluid flow by discontinuous Galerkin and  
19 mixed methods in unfractured and fractured media. *Water Resources Research*, 41(11).

20 Houze, O., Tauzin, E., Artus, V. and Larsen, L., 2010. *The Analysis of Dynamic Data in Shale*  
21 *Gas Reservoirs—Part 1*. company report, Kappa engineering, Houston, Texas, USA.

22 Gringarten, A.C., Ramey Jr, H.J. and Raghavan, R., 1974. Unsteady-state pressure distributions  
23 created by a well with a single infinite-conductivity vertical fracture. *Society of Petroleum*  
24 *Engineers Journal*, 14(04), pp.347-360.

25 Karimi-Fard, M., Durlofsky, L.J., Aziz, K., 2004. An efficient discrete-fracture model applicable  
26 for general purpose reservoir simulators. *SPE J.* 9 (2), 227e236.

27 Karimi-Fard, M., Gong, B. and Durlofsky, L.J., 2006. Generation of coarse - scale continuum  
28 flow models from detailed fracture characterizations. *Water resources research*, 42(10).

29 Karimi-Fard, M. and Durlofsky, L.J., 2016. A general gridding, discretization, and coarsening  
30 methodology for modeling flow in porous formations with discrete geological features.  
31 *Advances in water resources*, 96, pp.354-372.

32 Krogstad, S., Lie, K.A., Møyner, O., Nilsen, H.M., Raynaud, X. and Skaflestad, B., 2015,  
33 February. MRST-AD—an open-source framework for rapid prototyping and evaluation of  
34 reservoir simulation problems. In *SPE reservoir simulation symposium*. Society of Petroleum  
35 Engineers.

36 Gangi, A.F., 1978, October. Variation of whole and fractured porous rock permeability with  
37 confining pressure. In *International Journal of Rock Mechanics and Mining Sciences &*  
38 *Geomechanics Abstracts* (Vol. 15, No. 5, pp. 249-257). Pergamon.

39 Gensterblum, Y., Ghanizadeh, A., Cuss, R.J., Amann-Hildenbrand, A., Krooss, B.M., Clarkson,  
40 C.R., Harrington, J.F. and Zoback, M.D., 2015. Gas transport and storage capacity in shale gas  
41 reservoirs—A review. Part A: Transport processes. *Journal of Unconventional Oil and Gas*  
42 *Resources*, 12, pp.87-122.

43 Hajibeygi, H., Karvounis, D. and Jenny, P., 2011. A hierarchical fracture model for the iterative  
44 multiscale finite volume method. *Journal of Computational Physics*, 230(24), pp.8729-8743.

1 Hu, X., Wu, K., Li, G., Tang, J. and Shen, Z., 2018a. Effect of proppant addition schedule on the  
2 proppant distribution in a straight fracture for slickwater treatment. *Journal of Petroleum Science  
3 and Engineering*, 167, pp.110-119.

4 Hu, X., Wu, K., Song, X., Yu, W., Tang, J., Li, G. and Shen, Z., 2018b. A new model for  
5 simulating particle transport in a low - viscosity fluid for fluid - driven fracturing. *AIChE  
6 Journal*.

7 Javadpour F, Fisher D, Unsworth M. Nanoscale gas flow in shale gas sediments. *J Can Pet  
8 Technol.* 2007;46(10):55–61.

9 Jiang, J. and Younis, R.M., 2015. Numerical study of complex fracture geometries for  
10 unconventional gas reservoirs using a discrete fracture-matrix model. *Journal of Natural Gas  
11 Science and Engineering*, 26, pp.1174-1186.

12 Klinkenberg, L.J., 1941, January. The permeability of porous media to liquids and gases. In  
13 Drilling and production practice. American Petroleum Institute.

14 Lie, K.A., Krogstad, S., Ligaarden, I.S., Natvig, J.R., Nilsen, H.M. and Skaflestad, B., 2012.  
15 Open-source MATLAB implementation of consistent discretisations on complex grids.  
16 *Computational Geosciences*, 16(2), pp.297-322.

17 Li, Y., Zuo, L., Yu, W. and Chen, Y., 2018. A Fully Three Dimensional Semianalytical Model for  
18 Shale Gas Reservoirs with Hydraulic Fractures. *Energies*, 11(2), p.436.

19 Lee, S.H., Lough, M.F. and Jensen, C.L., 2001. Hierarchical modeling of flow in naturally  
20 fractured formations with multiple length scales. *Water resources research*, 37(3), pp.443-455.

21 Moinfar, A., Varavei, A., Sepehrnoori, K. et al. 2014. Development of an Efficient Embedded  
22 Discrete Fracture Model for 3D Compositional Reservoir Simulation in Fractured Reservoirs.  
23 *SPE J.* 19 (2): 289-303. *SPE-154246-PA*. <http://dx.doi.org/10.2118/154246-PA>.

24 Mahmoud, M., 2014. Development of a new correlation of gas compressibility factor (Z-factor)  
25 for high pressure gas reservoirs. *Journal of Energy Resources Technology*, 136(1), p.012903.

26 Olorode, O., Freeman, C.M., Mordini, G. and Blasingame, T.A., 2013. High-resolution  
27 numerical modeling of complex and irregular fracture patterns in shale-gas reservoirs and tight  
28 gas reservoirs. *SPE Reservoir Evaluation & Engineering*, 16(04), pp.443-455.

29 Olorode, O., Akkutlu, I.Y. and Efendiev, Y., 2017. Compositional Reservoir-Flow Simulation for  
30 Organic-Rich Gas Shale. *SPE Journal*.

31 Peaceman, D.W., 1983. Interpretation of well-block pressures in numerical reservoir simulation  
32 with nonsquare grid blocks and anisotropic permeability. *Society of Petroleum Engineers  
33 Journal*, 23(03), pp.531-543.

34 Rubin B. Accurate simulation of non-darcy flow in stimulated fractured shale reservoirs. In: *SPE  
35 132093*, presented at the SPE western regional meeting, Anaheim, CA; May 27-29, 2010

36 Sakhaei-Pour A, Bryant SL. Gas permeability of shale. *SPE Reservoir Eval Eng*.  
37 2012;15(4):401–409.

38 Sandve TH, Berre I, Nordbotten JM. An efficient multi-point flux approximation method for  
39 discrete fracture–matrix simulations. *J Comput Phys.* 2012;231(9):3784–3800

40 Tene, M., Al Kobaisi, M.S. and Hajibeygi, H., 2016. Algebraic multiscale method for flow in  
41 heterogeneous porous media with embedded discrete fractures (F-AMS). *Journal of  
42 Computational Physics*, 321, pp.819-845.

43 Tan, L., Zuo, L. and Wang, B., 2018. Methods of decline curve analysis for shale gas reservoirs.  
44 *Energies*, 11(3), p.552.

1 Tene, M., Bosma, S. B., Al Kobaisi, M. S., & Hajibeygi, H. (2017). Projection-based embedded  
2 discrete fracture model (pEDFM). *Advances in Water Resources*, 105, 205-216.

3 Wang FP, Reed RM. Pore networks and fluid flow in gas shales. In: SPE Annual Technical  
4 Conference and Exhibition. New Orleans, Louisiana, 2009.

5 Wang, K., Liu, H., Luo, J., Wu, K. and Chen, Z., 2017. A comprehensive model coupling  
6 embedded discrete fractures, multiple interacting continua, and geomechanics in shale gas  
7 reservoirs with multiscale fractures. *Energy & Fuels*, 31(8), pp.7758-7776.

8 Wasaki, A. and Akkutlu, I.Y., 2015. Permeability of organic-rich shale. *SPE Journal*, 20(06),  
9 pp.1-384.

10 Wu, W., Zhou, J., Kakkar, P., Russell, R. and Sharma, M.M., 2018. An Experimental Study on  
11 Conductivity of Unpropped Fractures in Preserved Shales. *SPE Production & Operations*.

12 Xu Y. Implementation and application of the Embedded Discrete Fracture Model (EDFM) for  
13 reservoir simulation in fractured reservoirs. Master Thesis, The University of Texas at Austin,  
14 2015.

15 Xu, Y., Cavalcante Filho, J.S., Yu, W. and Sepehrnoori, K., 2017. Discrete-fracture modeling of  
16 complex hydraulic-fracture geometries in reservoir simulators. *SPE Reservoir Evaluation &*  
17 *Engineering*, 20(02), pp.403-422.

18 Yu, W., Xu, Y., Liu, M., Wu, K. and Sepehrnoori, K., 2018. Simulation of shale gas transport and  
19 production with complex fractures using embedded discrete fracture model. *AIChE Journal*,  
20 64(6), pp.2251-2264.

21 Yu, W. and Sepehrnoori, K., 2014. Simulation of gas desorption and geomechanics effects for  
22 unconventional gas reservoirs. *Fuel*, 116, pp.455-464.

23 Yu, W., Sepehrnoori, K. and Patzek, T.W., 2016a. Modeling gas adsorption in Marcellus shale  
24 with Langmuir and bet isotherms. *SPE Journal*, 21(02), pp.589-600.

25 Yu, W., Wu, K., Zuo, L., Tan, X. and Weijermars, R., 2016b, August. Physical models for  
26 inter-well interference in shale reservoirs: relative impacts of fracture hits and matrix  
27 permeability. In *Unconventional Resources Technology Conference*, San Antonio, Texas, 1-3  
28 August 2016 (pp. 1535-1558). Society of Exploration Geophysicists, American Association of  
29 Petroleum Geologists, Society of Petroleum Engineers.

30 Yu, W., Wu, K., Sepehrnoori, K. and Xu, W., 2017. A comprehensive model for simulation of  
31 gas transport in shale formation with complex hydraulic-fracture geometry. *SPE Reservoir  
32 Evaluation & Engineering*, 20(03), pp.547-561.

33 Yang\*, R., Huang, Z., Li, G., Yu, W., Lashgari, H.R., Sepehrnoori, K. and Shen, Z., 2016a,  
34 August. A semianalytical method for modeling two-phase flow in coalbed methane reservoirs  
35 with complex fracture networks. In *Unconventional Resources Technology Conference*, San  
36 Antonio, Texas, 1-3 August 2016 (pp. 2387-2404). Society of Exploration Geophysicists,  
37 American Association of Petroleum Geologists, Society of Petroleum Engineers.

38 Yang R, Huang Z, Yu W, Li G, Ren W, Zuo L, Tan X, Sepehrnoori K, Tian S, Sheng M. A  
39 comprehensive model for real gas transport in shale formations with complex non-planar  
40 fracture networks. *Sci Rep*. 2016b; 6:36673.

41 Yang, R., Huang, Z., Li, G., Yu, W., Sepehrnoori, K., Lashgari, H.R., Tian, S., Song, X. and  
42 Sheng, M., 2017. A Semianalytical Approach to Model Two-Phase Flowback of Shale-Gas Wells  
43 with Complex-Fracture-Network Geometries. *SPE Journal*.

44 Zhang Y, Yu W, Sepehrnoori K, Di Y. A comprehensive numerical model for simulating fluid

1 transport in nanopores. *Sci Rep.* 2017;7: 40507.

2 Zhou, Y., Tchelepi, H.A. and Mallison, B.T., 2011, January. Automatic differentiation framework  
3 for compositional simulation on unstructured grids with multi-point discretization schemes. In  
4 SPE Reservoir Simulation Symposium. Society of Petroleum Engineers.

5 Zuo, L., Yu, W. and Wu, K., 2016. A fractional decline curve analysis model for shale gas  
6 reservoirs. *International Journal of Coal Geology*, 163, pp.140-148.

7 Zeng, Z. and Grigg, R., 2006. A criterion for non-Darcy flow in porous media. *Transport in  
8 porous media*, 63(1), pp.57-69.

9