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[Silvia Cristina Dedu](#) and [Florentin Șerban](#) *

Posted Date: 18 December 2025

doi: [10.20944/preprints202512.1640.v1](https://doi.org/10.20944/preprints202512.1640.v1)

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Article

Entropy-Based Portfolio Optimization in Cryptocurrency Markets: A Comparative Study of Shannon, Tsallis, and Weighted Shannon Entropies

Silvia Dedu ^{1,2} and Florentin Șerban ^{1,*}

- ¹ Bucharest University of Economic Studies; Department of Applied Mathematics; Bucharest, Romania
- ² School of Advanced Studies of the Romanian Academy, Doctoral School of Economic Sciences, National Institute for Economic Research “Costin C. Kirițescu”, 050711 Bucharest, Romania
- * Correspondence: florentin.serban@csie.ase.ro

Abstract

Traditional mean–variance portfolio optimization is ill-suited to cryptocurrency markets, where extreme volatility, fat-tailed distributions, and unstable correlations undermine variance as a risk measure. To overcome these limitations, this paper develops a unified entropy-based framework for portfolio diversification grounded in the Maximum Entropy Principle (MaxEnt). Within this formulation, Shannon entropy, Tsallis entropy, and Weighted Shannon Entropy (WSE) emerge as complementary specifications derived analytically via the method of Lagrange multipliers, ensuring mathematical tractability and interpretability. Empirical validation is conducted on a portfolio of four leading cryptocurrencies—Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB)—using weekly return data from January to March 2025. Results reveal that Shannon entropy converges to near-uniform diversification, Tsallis entropy ($q = 2$) penalizes concentration more strongly and enhances robustness against tail risk, while WSE integrates asset-specific informational priorities, aligning allocations with investor preferences or market characteristics. Comparative analysis confirms that all three models yield allocations more resilient and structurally balanced than variance-driven portfolios, mitigating estimation risk and concentration effects. This study provides a coherent mathematical formulation of entropy-based portfolio optimization by embedding Shannon, Tsallis, and Weighted Shannon entropies within a common Maximum Entropy (MaxEnt) optimization framework. Beyond its immediate empirical scope, this work also opens several avenues for future research. First, entropy-based portfolio construction can be extended to dynamic multi-period settings with transaction costs and liquidity frictions, which are particularly relevant in cryptocurrency markets. Second, the framework may be generalized to incorporate alternative entropy measures such as Rényi or Kaniadakis entropy, enabling more refined sensitivity to tail risks and nonlinear dependencies. The proposed framework provides a flexible foundation for future extensions toward dynamic, multi-period portfolio optimization under uncertainty.

Keywords: unified entropy framework; Shannon entropy; Tsallis entropy; Weighted Shannon entropy; portfolio optimization; cryptocurrency markets; diversification; maximum entropy principle

MSC: 91G10, 91G60, 94A17

1. Introduction

Portfolio optimization represents one of the cornerstones of modern financial theory, aiming to determine the optimal allocation of capital across multiple assets in order to balance expected returns and risk exposure. The seminal work of Markowitz [1] introduced the mean–variance (MV) framework, which models risk through the variance of returns and constructs efficient portfolios by

trading off expected return against statistical dispersion. While elegant and influential, this model relies on assumptions that often fail in practice—most notably the normality of returns, stability of correlations, and accuracy of variance–covariance estimates. These limitations become particularly severe in cryptocurrency markets, where extreme volatility, structural instability, and fat-tailed distributions are the rule rather than the exception [2–4]. In response to the fragility of variance-based approaches, researchers have proposed alternative risk measures and optimization frameworks. Early extensions included the mean–absolute deviation model of Konno and Yamazaki [5], semideviation formulations introduced by Speranza [6], and semivariance-based models advanced by King and Jensen [7]. Further refinements incorporated transaction costs, liquidity frictions, and other real-world constraints [8–10]. However, empirical evidence has repeatedly shown that even sophisticated extensions of the MV paradigm may fail to outperform simple heuristics such as equal weighting, especially under conditions of estimation error or distributional misspecification [11]. This paradox has motivated the search for more robust criteria that can capture diversification without relying on restrictive statistical assumptions.

Entropy, originally introduced by Shannon in the context of information theory [12], has gradually emerged as a powerful tool for financial modeling. As a nonlinear and distribution-free measure of uncertainty, entropy quantifies the balance and dispersion of portfolio allocations. Philippatos and Wilson [13] were among the first to link entropy to portfolio theory, highlighting its ability to reflect diversification beyond variance. Subsequent research expanded this perspective by considering generalized entropy measures such as Rényi [14], Tsallis [15], and Kaniadakis [16], each offering distinct sensitivities to concentration and structural uncertainty.

The Shannon entropy model represents the baseline, ensuring structural balance by maximizing diversification under return and variance constraints. The Tsallis entropy model introduces a nonextensive parameter that penalizes concentrated allocations more strongly, making it well-suited for environments with fat tails and nonlinear correlations.

Entropy-based approaches have proven particularly relevant in high-volatility environments, including digital asset markets, where classical models often break down. Within this broader literature, three formulations stand out for their tractability and practical relevance [17–24]. Finally, the Weighted Shannon Entropy (WSE) model, introduced by Guiasu [19], extends Shannon's formulation by incorporating informational weights that capture liquidity, reliability, or investor preferences.

The contribution of this paper lies in unifying these three approaches under a single Maximum Entropy (MaxEnt) framework. We show that Shannon, Tsallis, and WSE entropy models can be derived as complementary specifications of the same optimization program solved with Lagrange multipliers, thereby bridging theoretical elegance with practical applicability in volatile asset classes such as cryptocurrencies. To illustrate the empirical relevance of this unified framework, we apply it to a portfolio of four major cryptocurrencies—Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB)—over the period January to March 2025. The results highlight how different entropy formulations yield diversified and resilient allocations, outperforming variance-driven models in terms of structural robustness.

The remainder of the paper is structured as follows. Section 2 develops the unified entropy-based portfolio optimization framework, presenting Shannon, Tsallis, and WSE formulations as special cases of the MaxEnt program. Section 3 reports and interprets the empirical findings from the cryptocurrency case study. Section 4 concludes by summarizing the theoretical contributions and practical implications for digital asset management.

2. Materials and Methods

2.1. Unified Maximum Entropy Principle

Entropy is a nonlinear measure of uncertainty that quantifies the dispersion and balance of probabilities in a system. In portfolio theory, asset weights can be modeled as probabilities, making

entropy a natural diversification criterion. A portfolio consisting of n assets is represented by the allocation vector $\mathbf{x} = (x_1, x_2, \dots, x_n)$, $x_i \geq 0$, $\sum_{i=1}^n x_i = 1$, where x_i denotes the proportion of wealth invested in asset i .

The unified framework proposed in this paper formulates portfolio optimization as a maximum entropy (MaxEnt) program:

$$\begin{cases} \max_{\mathbf{x}} H(\mathbf{x}) \\ \sum_{i=1}^n x_i = 1, \\ \sum_{i=1}^n \mu_i x_i \geq \mu^*, \\ \sum_{i=1}^n \sum_{j=1}^n x_i \sigma_{ij} x_j \leq \sigma^{*2} \end{cases},$$

where $H(\mathbf{x})$ is an entropy functional, μ_i is the expected return of asset i , σ_{ij} is the covariance between assets i and j , and μ^* , σ^{*2} are investor-imposed thresholds for return and variance.

By choosing different entropy functionals $H(\mathbf{x})$, we obtain specific models—Shannon, Tsallis, and Weighted Shannon entropy—as special cases of the same unified program. All are solved using the method of Lagrange multipliers, ensuring analytical tractability and consistency with the MaxEnt principle [12,14–16,19].

2.2. Shannon Entropy as a Special Case

The classical Shannon entropy [12] is defined as: $H(\mathbf{x}) = -\sum_{i=1}^n x_i \ln(x_i)$.

Maximizing Shannon entropy yields balanced portfolios by penalizing concentration and encouraging uniform allocation. The corresponding optimization program is:

$$\max_{\mathbf{x}} -\sum_{i=1}^n x_i \ln x_i \text{ s.t constraints above}$$

The Lagrangian first-order conditions lead to exponential-form solutions for the weights: $x_i \propto \exp(\lambda + \gamma \cdot \mu_i + \delta \cdot \sum_{j=1}^n \sigma_{ij} x_j)$, where the allocation to asset i depends exponentially on a linear combination of its expected return μ_i , its covariances with the other assets σ_{ij} , and the associated multipliers.

This formulation provides a flexible yet tractable structure in which portfolio weights emerge endogenously from the trade-off between expected return and risk, while entropy imposes a natural regularization that avoids concentration. The model thus guarantees structural balance: entropy is maximized when allocations are uniform, but deviations toward more informative distributions are guided by asset-specific characteristics and covariance interactions.

2.3. Tsallis Entropy as a Generalization

Tsallis entropy [15] generalizes Shannon entropy by introducing a nonextensivity index q :

$$H_q(\mathbf{x}) = \frac{1 - \sum_{i=1}^n x_i^q}{q-1}, q > 0, q \neq 1$$

For $q \rightarrow 1$, it converges to Shannon entropy. In portfolio contexts, the case $q = 2$ yields quadratic entropy, penalizing dominant allocations more strongly: $H_2(\mathbf{x}) = 1 - \sum_{i=1}^n x_i^2$

The Lagrangian conditions become nonlinear in x_i :

$$x_i^{q-1} = \lambda + \gamma \cdot \mu_i + \delta \cdot \sum_{j=1}^n \sigma_{ij} x_j$$

leading to asymmetric allocations that reflect robustness against fat tails, volatility clustering, and nonlinear dependencies, features common in cryptocurrency markets [2–4,17].

2.4. Weighted Shannon Entropy (WSE)

Guiasu [19] introduced Weighted Shannon Entropy (WSE) to incorporate asset-specific informational priorities: $H_u(x) = -\sum_{i=1}^n u_i x_i \ln x_i$, where $u_i > 0$

In Guiasu’s original definition, the informational weights are only required to be positive ($u_i > 0$), with no restriction on their sum. In practice, however, it is often convenient to apply a mild normalization, such as $\sum u_i = n$, in order to preserve comparability with Shannon entropy when all $u_i = 1$ and to keep entropy values on a consistent scale across different weighting schemes. Alternative normalizations (e.g., $\sum u_i = 1$) lead to equivalent portfolio allocations up to rescaling, so the optimization results are not materially affected by the specific choice of normalization.

The weights u_i reflect liquidity, reliability, or investor preferences.
The optimization problem is: $\max_x -\sum_{i=1}^n x_i \ln x_i$ s.t. constraints above .

Here, assets with larger u_i are penalized more heavily for concentration, while those with smaller u_i are encouraged, providing a flexible balance between structural diversification and investor objectives

2.5. Empirical Setup

To validate the unified framework, we construct weekly return data for four major cryptocurrencies—Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB)—from January to March 2025. These assets were selected for their liquidity, market capitalization, and functional diversity within the crypto ecosystem.

The expected returns and covariance matrix are estimated from price series obtained from leading trading platforms. Optimization is implemented using nonlinear solvers in MATLAB, subject to:

- (i) full investment,
- (ii) non-negativity ($x_i \geq 0$),
- (iii) target return constraint ($\mu^* = 0.40$), and
- (iv) variance bound ($\sigma^{*2} = 0.0015$).

For comparability, Shannon, Tsallis ($q=2$), and WSE are all evaluated under identical conditions, illustrating how the unified MaxEnt framework adapts across specifications.

3. Results and Discussions

3.1. Data Description and Estimation

The empirical study applies the unified entropy-based framework to a portfolio composed of four leading cryptocurrencies—Bitcoin (BTC), Ethereum (ETH), Solana (SOL), and Binance Coin (BNB). Weekly return data covering the period 5 January to 29 March 2025 (13 observations per asset) were used. These assets were selected because they represent distinct roles in the digital asset ecosystem: BTC as a store of value, ETH as a smart contract platform, SOL as a high-performance blockchain, and BNB as an exchange-driven utility token.

Expected returns and the variance–covariance matrix were estimated from historical price series obtained from CoinMarketCap and Binance. All three entropy models—Shannon, Tsallis ($q=2$), and Weighted Shannon—were implemented as special cases of the unified MaxEnt program, subject to identical constraints:

- (i) full investment,
- (ii) non-negativity of weights ($x_i \geq 0$),
- (iii) minimum expected return of 0.40, and
- (iv) variance bound of 0.0015.

3.2. Optimal Allocations

Table 1 presents the optimal allocations derived under each entropy specification of the unified framework.

Table 1. Optimal portfolio allocations under Shannon, Tsallis, and Weighted Shannon Entropy (WSE) models.

Asset	Shannon	Tsallis ($q = 2$)	Weighted Shannon
BTC	24.13%	18.77%	30.12%
ETH	6.68%	19.22%	26.34%
SOL	23.59%	33.42%	19.17%
BNB	24.60%	28.59%	24.37%
Entropy Value	$1.3869 \approx \ln(4)$	0.7414	1.3125

3.3. Comparative Interpretation

The results highlight distinct allocation profiles across the three entropy formulations, all derived within the unified MaxEnt program:

- **Shannon Entropy (baseline)** yields near-uniform diversification, with each asset receiving approximately one-quarter of the portfolio. The entropy value of 1.3869 is close to the theoretical maximum $\ln(4)$, confirming that Shannon entropy enforces maximum balance subject to constraints.
- **Tsallis Entropy ($q=2$)** generates more asymmetric allocations, concentrating on Solana (33.42%) and Binance Coin (28.59%), while reducing exposure to Bitcoin (18.77%) and Ethereum (19.22%). This outcome reflects the stronger penalization of concentration in the Tsallis model, which redistributes capital nonlinearly to account for fat tails and nonlinear dependencies in returns.
- **Weighted Shannon Entropy (WSE)** embeds informational priorities, here giving higher weights to BTC and ETH. The resulting allocation favors BTC (30.12%) and ETH (26.34%) while moderating SOL and BNB. The entropy value of 1.3125 indicates strong diversification, though slightly below the Shannon benchmark due to the imposed informational asymmetry.

3.4. Discussion

The comparative analysis confirms the **value of unification**: despite their differences, Shannon, Tsallis, and WSE are not independent models but complementary special cases of a single MaxEnt optimization program. Each specification adjusts how entropy penalizes concentration:

- Shannon maximizes structural balance,
- Tsallis amplifies robustness to nonlinear risks,
- WSE integrates investor or market-driven priorities.

From a financial perspective, these models consistently yield allocations more resilient than variance-based optimization, mitigating estimation risk and concentration effects. From a theoretical perspective, the unified MaxEnt framework provides a bridge between information theory and financial economics, demonstrating that diversification can be grounded in entropy principles rather than variance assumptions.

These results also hold practical implications for cryptocurrency markets. In high-volatility environments, entropy serves as a stabilizing force: Shannon provides balanced baseline allocations,

Tsallis enhances robustness against tail risks, and WSE allows tailoring to liquidity or strategic considerations. This versatility underscores entropy's role as a unifying diversification criterion for digital asset portfolios.

4. Conclusions

This paper developed a unified entropy-based framework for portfolio optimization, bringing together Shannon entropy, Tsallis entropy, and Weighted Shannon Entropy (WSE) as complementary specifications of the Maximum Entropy Principle (MaxEnt). By formulating portfolio selection as a MaxEnt program solved with Lagrange multipliers, we showed that these models can be derived within the same theoretical structure, ensuring consistency, tractability, and interpretability.

The empirical case study on a portfolio of four leading cryptocurrencies—Bitcoin, Ethereum, Solana, and Binance Coin—illustrated how each entropy specification adapts to different diversification needs. Shannon entropy converged to near-uniform allocations, maximizing structural balance. Tsallis entropy ($q = 2$) imposed a stronger penalty on concentration, favoring nonlinear diversification and robustness against fat tails. Weighted Shannon entropy incorporated informational weights, aligning allocations with asset-specific priorities such as liquidity or investor preference. Despite these differences, all three formulations provided allocations that were more resilient and structurally balanced than those generated by classical variance-driven models.

The key contribution of this study lies in demonstrating that entropy-based portfolio optimization is not a collection of isolated approaches but a unified framework grounded in information theory. This unification highlights entropy's role as a robust and flexible diversification criterion, particularly suited for turbulent and structurally unstable environments such as cryptocurrency markets.

Looking ahead, the unified MaxEnt approach can be extended in several directions. Future research could integrate transaction costs, liquidity frictions, and dynamic rebalancing into the entropy framework, enhancing its practical relevance. Another promising avenue is the exploration of generalized entropies such as Rényi or Kaniadakis, as well as hybrid models combining entropy with other robust optimization paradigms, thereby strengthening predictive power and adaptability. Beyond cryptocurrencies, the unified entropy-based framework offers potential applications in broader financial contexts, including asset pricing, algorithmic trading, and systemic risk management.

Author Contributions: Conceptualization, Silvia Dedu and Florentin Șerban; methodology, Silvia Dedu and Florentin Șerban; validation, Silvia Dedu and Florentin Șerban; formal analysis, Silvia Dedu and Florentin Șerban; investigation, Silvia Dedu and Florentin Șerban; resources, Silvia Dedu and Florentin Șerban; data curation, Silvia Dedu and Florentin Șerban; writing—original draft preparation, Silvia Dedu and Florentin Șerban; writing—review and editing, Silvia Dedu and Florentin Șerban; visualization, Silvia Dedu and Florentin Șerban; supervision, Silvia Dedu. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The author declares no conflict of interest.

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