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Article

Temporal Dynamics as Time-First General Relativity: $k = 4G/c^2$ Linking Local Time Offsets, Curvature, and Cosmic Acceleration

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Abstract

We present *Temporal Dynamics* (TD) as a time-first reparameterization of General Relativity (GR). In TD, mass sources a local slow-time fraction $\Delta T(x)$; keeping the universal converter $UC = 1/c$ exact implies that the straight-path delay over a segment of length D corresponds to a curve-length increment $\Delta L = \overline{\Delta T} D$. Fixing the single TD mass constant to $k = 4G/c^2$ (m kg^{-1}) makes the $TD \leftrightarrow GR$ bridge exact: $\Delta L_{\text{rad}} = r_s$, $\Delta L_{\text{diam}} = 2r_s$, and the TD horizon rule $D_{\text{BH}} = kM$. The TD potential $\phi = -(c^2/2)\Delta T$ yields the general motion law $\mathbf{a} = (c^2/2)\nabla\Delta T$, reproducing the Newtonian limit $a_r = -GM/r^2$. Taking $\Delta T(r) = r_s/r$ reconstructs the Schwarzschild line element, so the PPN parameters match GR ($\beta = \gamma = 1$) along with classic tests (gravitational redshift, light bending, Shapiro delay, perihelion precession). A variational formulation identifies ΔT with the ADM lapse via $N^2 = 1 - \Delta T$; varying the action yields the Einstein equations with no extra propagating degrees of freedom (GR-equivalent dynamics, including gravitational waves with speed c and GR polarizations/flux). Cosmologically, a residual micro-push ϵ per clock unit defines $\kappa = \epsilon/c$ and recovers the Hubble law $v = \kappa D$; constant κ maps to $\Lambda = 3\kappa^2/c^2$, while a slow drift $\kappa(z)$ corresponds to $w(z) \neq -1$, reframing the H_0 tension as differing ϵ_0 estimates. TD thus organizes tested GR physics with a single constant k and provides a micro $\rightarrow$ macro interpretation of dark energy via ϵ , without modifying GR's empirical content.

Keywords: general relativity; Schwarzschild metric; gravitational time dilation; ADM formalism; post-Newtonian parameters; Shapiro delay; light deflection; dark energy; Hubble constant; cosmology

1. Introduction: The Cosmic Clock and the $TD \leftrightarrow GR$ Dictionary

Temporal Dynamics (TD) is a time-first description of gravity and expansion. Building on earlier TD work [1], its basic premise is that gravitational phenomena can be organized by a single bookkeeping of *time flow*: where the local flow of time along an otherwise straight path is fractionally slower, the corresponding null path acquires exactly the extra length that General Relativity (GR) encodes as curvature. In this sense TD and GR are two languages for the same physics.

We refer to the conversion between time and length as the *Universal Constant* $UC \equiv 1/c$ (s m^{-1}), which turns a geometric length D into the empty-space traversal time D/c . TD quantifies a path-averaged slow-time fraction ΔT (dimensionless) along a straight segment; the accumulated delay is

$$\Delta t_{\text{total}} = \Delta T \frac{D}{c}.$$

Converting back with $1/c$ gives the curve-length increment

$$\Delta L_{\text{diam}} = \Delta T D.$$

This is the $TD \rightarrow GR$ bridge: slow time along a straight path corresponds to extra geometric length along the actual path.

Fixing a single mass constant

$$k \equiv \frac{4G}{c^2} \quad (\text{units: m kg}^{-1}),$$

the bridge collapses to compact identities in the static, spherical sector:

$$\Delta L_{\text{diam}} = k M = 2 r_s, \quad \Delta L_{\text{rad}} = \frac{k M}{2} = r_s, \quad \Delta t_{\text{total}} = \frac{k M}{c} = \frac{4GM}{c^3},$$

where $r_s = 2GM/c^2$ is the Schwarzschild radius. Thus a single constant k ties the TD time–delay bookkeeping to the familiar GR lengths.

The equivalence to GR may be stated more geometrically. In the static sector we identify the GR lapse with the TD time–offset via

$$N^2 = 1 - \Delta T,$$

so that $d\tau = \sqrt{1 - \Delta T} dt$. With the radial profile $\Delta T(r) = r_s/r$, this immediately reproduces the Schwarzschild line element and, therefore, all classic tests in the solar system (light deflection, Shapiro delay, perihelion precession, gravitational redshift). Linearizing the theory around flat spacetime yields two tensor polarizations propagating at speed c with the standard quadrupole energy flux; the gravitational–wave sector thus coincides with GR.

For cosmology, TD introduces a microscopic residual rate ϵ (an extra “push” per light–second unit) whose macroscopic imprint is

$$\kappa \equiv \frac{\epsilon}{c} \quad (\text{s}^{-1}).$$

The late–time recession law is $v = \kappa D$, so today $H_0 = \kappa$ and a constant κ is exactly a cosmological constant,

$$\Lambda = \frac{3\kappa^2}{c^2}, \quad \rho_\Lambda = \frac{3\kappa^2}{8\pi G}, \quad p_\Lambda = -\rho_\Lambda c^2.$$

If desired, a gentle drift $\kappa(t)$ maps one–to–one to $w(z) \neq -1$ in the standard dark–energy language; distances, growth, and BAO/SNe/CMB inferences then proceed with the usual Friedmann machinery.

Scope and posture.

TD is presented here as a *reparameterization* of GR, not a modification: we write the Einstein–Hilbert action in TD variables and recover Einstein’s equations by variation. No new propagating degrees of freedom are introduced, and all tested predictions of GR are preserved. Conceptual pictures (e.g. viewing expansion via a global “cosmic clock”) are kept as interpretational aids; quantitative statements are anchored to the dictionary above.

Contributions.

(i) We establish the TD→GR bridge $\Delta L_{\text{diam}} = \Delta T D$ and, with $k = 4G/c^2$, obtain the compact identities $\Delta L_{\text{diam}} = 2r_s$, $\Delta L_{\text{rad}} = r_s$, and $\Delta t_{\text{total}} = 4GM/c^3$. (ii) We construct the metric from $\Delta T(r) = r_s/r$ and verify the PPN parameters $\gamma = \beta = 1$ (classic tests matched). (iii) We show that the linear gravitational–wave sector (polarizations, speed c , quadrupole flux) is identical to GR. (iv) We map the TD micro–parameter ϵ to the macroscopic cosmology via $\kappa = \epsilon/c$ and $\Lambda = 3\kappa^2/c^2$, provide observable formulas (distances and growth), and outline a data strategy consistent with BAO/SNe/CMB analyses.

Organization.

Section 2 fixes notation and units. Section 3 develops the TD→GR bridge and a solar example. Section 4 gives the variational principle and the lapse identification. Section 5 builds the metric from $\Delta T(r)$; Section 6 presents the PPN validation. Section 7 treats gravitational waves. Section 9 covers cosmology and observables. Conceptual notes and technical expansions appear in the Appendices.

2. The Mass Constant k : Definition, Units, and Equivalences

2.1. Purpose

To maintain continuity with prior TD papers [1] we retain the symbol k while fixing its definition and units so all TD results (gravitational acceleration, black-hole diameter, TD→GR bridge) are consistent and exactly aligned with GR.

2.2. Definition (Operational and Fundamental)

Operational (used throughout TD).

$$\Delta T = \frac{k M}{D}, \quad (1)$$

where ΔT is a *dimensionless* path-averaged slow-time fraction across a straight diameter D through (or past) mass M .

Fundamental equivalence (fixes the value by GR correspondence).

$$k \equiv \frac{4G}{c^2}. \quad (2)$$

2.3. Units and Numerical Value

$$[k] = \text{m kg}^{-1} \quad (\text{so } \Delta T \text{ is dimensionless; lengths inferred from TD are in meters}).$$

SI value (using standard G and exact c):

$$k = \frac{4G}{c^2} \approx 2.9704641076 \times 10^{-27} \text{ m kg}^{-1}.$$

Previously used in drafts: $k_{\text{old}} = 2.97033058787623 \times 10^{-27} \text{ m kg}^{-1}$; the difference is $\sim 0.0045\%$. Adopting $k = 4G/c^2$ makes the TD↔GR identities exact.

2.4. Unit Audit and Immediate Consequences

From Eq. (1), ΔT is dimensionless. The diameter and radius-style curve-length increments are

$$\Delta L_{\text{diam}} = \Delta T D = k M, \quad (3)$$

$$\Delta L_{\text{rad}} = \frac{1}{2} \Delta L_{\text{diam}} = \frac{k M}{2}. \quad (4)$$

2.5. Canonical Identities (Single-Constant Form)

Scope: the following hold in the static, spherically symmetric *exterior* (vacuum) solution. Let $r_s = 2GM/c^2$.

1. Schwarzschild match (any spherical mass).

$$r_s = \frac{2GM}{c^2} = \frac{kM}{2}, \quad \Delta L_{\text{diam}} = kM = 2r_s, \quad \Delta L_{\text{rad}} = r_s. \quad (5)$$

(See Sec. 5 for the line element derived from $\Delta T(r) = r_s/r$.)

2. Black-hole diameter (TD horizon rule). Horizon when $\Delta T = (kM)/D = 1$:

$$D_{\text{BH}} = kM = 2r_s. \quad (6)$$

3. Radial form (take $D = 2r$).

$$\Delta T(r) = \frac{kM}{2r} = \frac{r_s}{r}. \quad (7)$$

4. **Potential and surface gravity (Newtonian limit).** Define $\phi(r) = -(c^2/2) \Delta T(r)$, then

$$\phi(r) = -\frac{GM}{r}, \quad g(r) = \left| \frac{d\phi}{dr} \right| = \frac{GM}{r^2}. \quad (8)$$

(Consistency with PPN $\beta = \gamma = 1$ is shown in Sec. 6.)

5. **Total time delay across a diameter (mass-only invariant).**

$$\Delta t_{\text{total}} = \Delta T \frac{D}{c} = \frac{kM}{c} = \frac{4GM}{c^3}, \quad \Delta L_{\text{diam}} = c \Delta t_{\text{total}} = kM. \quad (9)$$

2.6. Backward-Compatibility

Symbol. We continue to write k to match earlier publications.

Units. Any occurrence of $[k] = \text{m s kg}^{-1}$ in older text should be corrected to $[k] = \text{m kg}^{-1}$. Prior formulas remain numerically consistent because ΔT was used as dimensionless and ΔL in meters.

Numerics. The prior value differs by $\sim 0.005\%$; fixing $k = 4G/c^2$ removes rounding and makes TD $\leftrightarrow$ GR identities exact at the Schwarzschild scale.

2.7. Interpretation (What k Means)

- **Mass $\rightarrow$ curve-length converter.** k (m kg^{-1}) tells how much effective path length space must add per unit mass to keep the universal clock exact: $\Delta L_{\text{diam}} = kM = \frac{4GM}{c^2}$ and $\Delta L_{\text{rad}} = (kM)/2 = \frac{2GM}{c^2}$.
- **Built-in GR alignment.** Since $r_s = 2GM/c^2$, one gets $\Delta L_{\text{diam}} = 2r_s$, $\Delta L_{\text{rad}} = r_s$ and the horizon rule $D_{\text{BH}} = kM = 2r_s$.
- **Time-slowness as horizon fraction.** With $D = 2r$, $\Delta T(r) = kM/(2r) = r_s/r$ —the local slow-time is literally the “how close to horizon” fraction.
- **Newtonian limit.** $\phi(r) = -(c^2/2)\Delta T(r)$ gives $\phi = -GM/r$ and $g = GM/r^2$ automatically (see Sec. 6).
- **Mass-energy view.** Using $E = Mc^2$, $kM = (4G/c^4)E$ —a length per energy, echoing GR’s coupling scale.
- **Null-path signature.** The factor “4” echoes familiar lightlike coefficients in GR (e.g. deflection $\sim 4GM/(c^2b)$), signaling that k encodes how light (or expansion at c) couples to mass in TD.

One-line meaning.

$k = 4G/c^2$ is TD’s mass-to-curve (mass-to-time-offset) proportionality that makes the TD time-delay and GR’s geometric curvature *two faces of the same calculation*, exactly.

Drop-in summary.

We retain the mass constant k for continuity and fix its value by the Schwarzschild correspondence: $k = 4G/c^2$ (m kg^{-1}). Then $\Delta T = (kM)/D$ is dimensionless and $\Delta L_{\text{diam}} = kM = 2r_s$, $\Delta L_{\text{rad}} = r_s$, $D_{\text{BH}} = kM$, $\Delta T(r) = kM/(2r) = r_s/r$, $\phi = -(c^2/2)\Delta T = -GM/r$, $g = GM/r^2$, and the invariant diameter delay $\Delta t_{\text{total}} = 4GM/c^3$.

Fixing k from the Newtonian limit (one line)

Linearizing the Schwarzschild lapse gives $g_{tt} \simeq -(1 - r_s/r)c^2$ so the potential is $\phi = \frac{1}{2}c^2(g_{tt}/(-c^2) + 1) = -GM/r$. TD defines $\phi(r) = -(c^2/2)\Delta T(r)$ with $\Delta T(r) = kM/(2r)$, hence $-(c^2/2)kM/(2r) = -GM/r \Rightarrow k = 4G/c^2$.

Fixing k from the diameter delay

TD gives the mass-only invariant $\Delta t_{\text{total}} = \Delta T(D/c) = (kM)/c$. Shapiro-type analyses isolate the same mass factor $4GM/c^3$ in the one-body limit; therefore $(kM)/c = 4GM/c^3 \Rightarrow k = 4G/c^2$.

3. TD→GR Bridge: Core Identity and Solar Example

This section establishes the algebraic bridge between TD's path-averaged slow-time fraction ΔT and the geometric excess length that GR encodes as curvature. The result is a pair of identities that will be used repeatedly: a time-delay identity and a length identity. With $k = 4G/c^2$ (Sec. 2), these immediately reproduce the Schwarzschild scale.

3.1. Core Identities

Consider a straight segment of geometric length D through (or past) a mass M .

Empty-space traversal time.

$$t_{\text{geom}} = \frac{D}{c}. \quad (10)$$

Accumulated slow-time (TD).

If the path-averaged slow-time fraction is ΔT (dimensionless), the total delay is

$$\Delta t_{\text{total}} = \Delta T t_{\text{geom}} = \Delta T \frac{D}{c}. \quad (11)$$

Convert back to length (GR).

Multiplying by c gives the curve-length increment across the diameter:

$$\Delta L_{\text{diam}} = c \Delta t_{\text{total}} = \Delta T D. \quad (12)$$

For "radius-style" quantities:

$$\Delta L_{\text{rad}} = \frac{1}{2} \Delta L_{\text{diam}} = \frac{1}{2} \Delta T D. \quad (13)$$

Insert the operational TD law.

From Eq. (1) ($\Delta T = kM/D$),

$$\Delta L_{\text{diam}} = \Delta T D = k M, \quad \Delta L_{\text{rad}} = \frac{k M}{2}. \quad (14)$$

With $k = 4G/c^2$,

$$\Delta L_{\text{diam}} = 2 r_s, \quad \Delta L_{\text{rad}} = r_s, \quad r_s = \frac{2GM}{c^2}. \quad (15)$$

Equations (11)–(15) constitute the TD→GR bridge used throughout. (Also see Figure 1).

3.2. Worked Example: The Sun

Constants (SI): $c = 299,792,458 \text{ m s}^{-1}$ (exact), $G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$. Solar values: $M_{\odot} = 1.98847 \times 10^{30} \text{ kg}$, $D_{\odot} = 1.3914 \times 10^9 \text{ m}$.

TD time-offset (diameter).

$$\Delta T = \frac{k M_{\odot}}{D_{\odot}} = \frac{(4G/c^2) M_{\odot}}{D_{\odot}} \approx 4.24104 \times 10^{-6}. \quad (16)$$

Curve-length increments.

$$\Delta L_{\text{diam}} = \Delta T D_{\odot} \approx 5.906 \times 10^3 \text{ m}, \quad (17)$$

$$\Delta L_{\text{rad}} = \frac{1}{2} \Delta L_{\text{diam}} \approx 2.953 \times 10^3 \text{ m}. \quad (18)$$



Figure 1. Two rulers describing the same physics. (a) TD: mass creates a local slow-time fraction ΔT along a straight diameter D ; the integrated delay is $\Delta t_{\text{total}} = \overline{\Delta T} D/c$. (b) GR: the same effect appears as an added curve length $\Delta L = \overline{\Delta T} D$. With $k = 4G/c^2$ one gets $\Delta L_{\text{rad}} = r_s$ and $\Delta L_{\text{diam}} = 2r_s$.

Schwarzschild check.

The Sun's Schwarzschild radius is

$$r_s = \frac{2GM_\odot}{c^2} \approx 2.953 \times 10^3 \text{ m}, \quad (19)$$

matching ΔL_{rad} within rounding, as predicted by Eq. (15).

3.3. Remark on Unit Handling (“No Double Conversion”)

A common pitfall is to multiply ΔT by a traversal time *and then* multiply by c twice, effectively performing two time $\leftrightarrow$ length conversions. The correct chain is

$$\Delta T \xrightarrow{\times D/c} \Delta t_{\text{total}} \xrightarrow{\times c} \Delta L_{\text{diam}},$$

which yields Eqs. (11)–(12) and the exact Schwarzschild match in Eq. (15).

3.4. Looking Ahead

Using $D = 2r$ in Eq. (1) gives the radial profile $\Delta T(r) = kM/(2r) = r_s/r$, which we use in Sec. 5 to reconstruct the Schwarzschild line element and in Sec. 6 to read off the PPN parameters ($\beta = \gamma = 1$).

4. Variational Principle and Field Equations for Temporal Dynamics (TD)

4.1. Aim and Philosophy

We present a variational formulation in which the TD “time–offset field” ΔT is not a new dynamical degree of freedom but a repackaging of the lapse function in the ADM form of General Relativity (GR). Varying the action yields the usual Einstein equations. In this sense, TD is a covariant reparameterization of GR: ΔT carries the causal interpretation (time slowdown), while the metric carries the geometry. This section makes that equivalence precise.

4.2. Minimal Setup and Units

Constants: c (speed of light), G (Newton’s constant), $k = 4G/c^2$ (units m kg^{-1}). TD time–offset: ΔT (dimensionless).

ADM decomposition (shift-free for clarity).

$$ds^2 = -N^2 c^2 dt^2 + h_{ij} dx^i dx^j, \quad (20)$$

where N is the lapse (dimensionless) and h_{ij} is the spatial 3-metric.

TD identification (core dictionary).

$$N^2 = 1 - \Delta T \iff \Delta T = 1 - N^2. \quad (21)$$

This matches the local redshift statement $d\tau = \sqrt{1 - \Delta T} dt$ and the static spherical result $\Delta T(r) = r_s/r$ used in Sec. 5.

4.3. Action

We take the standard Einstein–Hilbert action with matter, written in ADM variables, and express the lapse through ΔT :

$$S_{\text{total}}[h_{ij}, N, \text{matter}] = \frac{c^3}{16\pi G} \int dt d^3x N \sqrt{h} \left[{}^{(3)}R + K_{ij}K^{ij} - K^2 - 2\Lambda \right] + S_m[g, \Psi], \quad (22)$$

with the identifications $N = \sqrt{1 - \Delta T}$ and (for clarity here) $N^i = 0$. Here ${}^{(3)}R$ is the Ricci scalar of h_{ij} , K_{ij} is the extrinsic curvature,

$$K_{ij} = \frac{1}{2N} \partial_t h_{ij} \quad (\text{for } N^i = 0), \quad K = h^{ij} K_{ij}, \quad (23)$$

and Ψ denotes matter fields. A Gibbons–Hawking–York boundary term is understood to make the metric variation well-posed; it does not affect the bulk field equations and is omitted here for brevity.

4.4. Variations and Field Equations (Equivalence to GR)

Variation with respect to ΔT (Hamiltonian constraint)

Because $N = \sqrt{1 - \Delta T}$,

$$\frac{dN}{d(\Delta T)} = -\frac{1}{2\sqrt{1 - \Delta T}} = -\frac{1}{2N}.$$

By the chain rule,

$$\frac{\delta S_{\text{total}}}{\delta(\Delta T)} = \frac{\delta S_{\text{total}}}{\delta N} \frac{dN}{d(\Delta T)} \implies \frac{\delta S_{\text{total}}}{\delta(\Delta T)} = 0 \Leftrightarrow \frac{\delta S_{\text{total}}}{\delta N} = 0.$$

But $\delta S_{\text{total}}/\delta N = 0$ is precisely the ADM *Hamiltonian constraint*:

$${}^{(3)}R + K_{ij}K^{ij} - K^2 - 2\Lambda = \frac{16\pi G}{c^4} \rho, \quad (24)$$

where $\rho = T_{\mu\nu} n^\mu n^\nu$ is the energy density measured by the unit normal n^μ to the spatial slice. Hence the “ ΔT equation” is not extra dynamics; it is exactly the GR Hamiltonian constraint.

Variation with respect to h_{ij} (evolution equations)

Varying (22) with respect to h_{ij} yields the ADM evolution equations (trace and traceless parts), equivalent to the spatial components of Einstein’s equations. Since ΔT only enters via N , and N has already been fixed by (24), these equations are unchanged from GR.

Variation with respect to the shift N^i (momentum constraint)

If the shift is included, varying with respect to N^i gives the ADM momentum constraints (equivalent to the mixed $0i$ Einstein equations). In the present shift-free presentation they are trivially satisfied.

Takeaway.

The full GR field–equation content (Hamiltonian and momentum constraints, plus evolution equations) is recovered by varying the TD–augmented action. The “TD field equation” is exactly the Hamiltonian constraint; TD introduces no extra gravitating degree of freedom.

4.5. Static, Spherically Symmetric Sector (Schwarzschild)

In vacuum and static symmetry, $K_{ij} = 0$ and we may set $\Lambda = 0$ for the Schwarzschild case. The Hamiltonian constraint reduces to ${}^{(3)}R = 0$ on each slice, and the evolution equations fix the 3-metric h_{ij} together with the lapse N to the Schwarzschild solution. In TD language this is

$$g_{tt} = -\left(1 - \frac{r_s}{r}\right)c^2 \Rightarrow N^2 = 1 - \frac{r_s}{r} \Rightarrow \Delta T(r) = \frac{r_s}{r}, \quad (25)$$

recovering Sec. 5. All classic tests (light deflection, Shapiro delay, perihelion advance, gravitational redshift) follow because the line element is Schwarzschild.

4.6. Cosmology (Consistency with Λ and H)

In homogeneous, isotropic slicing (FLRW) with zero shift, the ADM action reduces to the standard cosmological action. Choosing $N = 1$ implies $\Delta T = 0$ for the background clock (as usual in comoving time). A cosmological constant Λ gives $H^2 = \Lambda c^2/3$. The TD dark-energy dictionary identifies

$$H \equiv \kappa = \frac{\epsilon}{c}, \quad \Lambda = \frac{3\kappa^2}{c^2},$$

so the GR Λ sector and the TD micro-rate ϵ are in one-to-one correspondence. Linear perturbations proceed in the usual way; TD does not add a propagating scalar beyond GR.

4.7. Covariant “Link” Formulation

To make the identification manifestly covariant, introduce a scalar “clock” field $\chi(x)$ whose level sets define the time foliation. Define the (future-pointing) unit normal to $\chi = \text{const}$ slices by

$$u_\mu = -A \partial_\mu \chi, \quad A \text{ chosen so that } g^{\mu\nu} u_\mu u_\nu = -1.$$

The lapse is $N = (-g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi)^{-1/2}$. Impose the TD link via a Lagrange multiplier:

$$S_{\text{link}} = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} \lambda(x) \left[\Delta T - (1 - N^2) \right]. \quad (26)$$

Variations give: $\delta S/\delta \lambda \Rightarrow \Delta T = 1 - N^2$ and $\delta S/\delta(\Delta T) \Rightarrow \lambda = 0$ on-shell; the Einstein–Hilbert dynamics are unchanged. This simply ties ΔT to the foliation without introducing new dynamics.

4.8. Interpretation and Novelty

- ΔT plays the role of the lapse via (21). Varying ΔT enforces the Hamiltonian constraint; there is no extra dynamical scalar.
- The action principle is GR’s action written in TD variables. Hence the field equations are exactly Einstein’s equations.
- The novelty is interpretational and organizational: the single constant $k = 4G/c^2$ and the TD time–offset bookkeeping yield compact, mass–only identities (e.g., $\Delta t_{\text{total}} = 4GM/c^3$) and a micro-to-macro path for cosmic acceleration ($H_0 = \epsilon/c$) without modifying GR’s tested predictions.

4.9. What Would Change TD into a New Theory (and Why We Do Not Do That Here)

If one added a genuine kinetic term for ΔT (e.g., gradients of ΔT in the Lagrangian), ΔT would become dynamical and generally produce deviations from GR (extra propagating modes, fifth forces, or modified wave speeds). We do not introduce such terms; TD is presented here as a reparameterization of GR with a causal interpretation, not as a modification.

Boxed field-equation summary (TD form)

Action (bulk):

$$S_{\text{total}} = \frac{c^3}{16\pi G} \int dt d^3x N \sqrt{h} \left[{}^{(3)}R + K_{ij}K^{ij} - K^2 - 2\Lambda \right] + S_m.$$

TD dictionary: $N^2 = 1 - \Delta T$ (so $d\tau = \sqrt{1 - \Delta T} dt$).

Variations: $\delta S / \delta(\Delta T) = 0 \Leftrightarrow$ Hamiltonian constraint (GR); $\delta S / \delta h_{ij} = 0 \Rightarrow$ evolution equations (GR); $\delta S / \delta N^i = 0 \Rightarrow$ momentum constraints (if shift included).

Static spherical vacuum: $\Delta T(r) = r_s/r$, $g_{tt} = -(1 - r_s/r)c^2$, $g_{rr} = (1 - r_s/r)^{-1}$.

Cosmology: $H = \kappa = \epsilon/c$, $\Lambda = 3\kappa^2/c^2$, $\rho_\Lambda = 3\kappa^2/(8\pi G)$.

5. Metric Construction from $\Delta T(r)$

This section builds the static, spherically symmetric line element from the TD radial profile

$$\Delta T(r) = \frac{kM}{2r} = \frac{r_s}{r}, \quad r_s \equiv \frac{2GM}{c^2},$$

and the dictionary $N^2 = 1 - \Delta T$ (so $d\tau = \sqrt{1 - \Delta T} dt$). Building on Sec. 4, we now recover the Schwarzschild metric.

5.1. Ansatz and TD Dictionary

For a static, spherically symmetric vacuum exterior we write

$$ds^2 = -N(r)^2 c^2 dt^2 + B(r) dr^2 + r^2 d\Omega^2, \quad (27)$$

with $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$. The TD dictionary fixes the lapse via

$$N(r)^2 = 1 - \Delta T(r) = 1 - \frac{r_s}{r}. \quad (28)$$

Thus $g_{tt} = -(1 - r_s/r)c^2$ is fixed by TD; it remains to determine $B(r)$.

5.2. Fixing the Radial Factor from Vacuum Einstein Equations (Sketch)

Imposing the vacuum equations $G_{\mu\nu} = 0$ for the ansatz (27) gives (see any GR text)

$$\frac{d}{dr} \left[r \left(1 - \frac{1}{B(r)} \right) \right] = 0 \quad \Rightarrow \quad B(r) = \frac{1}{1 - \frac{C}{r}}, \quad (29)$$

with C an integration constant. Asymptotic flatness requires $B(r) \rightarrow 1$ as $r \rightarrow \infty$, so C is finite. The $G^r_r = 0$ equation (or matching to the Newtonian potential via g_{tt}) fixes $C = r_s$, which also ensures consistency with (28). Therefore

$$B(r) = \frac{1}{1 - \frac{r_s}{r}}. \quad (30)$$

5.3. Result: Schwarzschild Line Element

Combining (28) and (30) yields

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 d\Omega^2, \quad (31)$$

the standard Schwarzschild metric in curvature (areal-radius) coordinates. In TD language, (31) is precisely the geometry that corresponds to the slow-time profile $\Delta T(r) = r_s/r$ fixed in Sec. 3.

5.4. Immediate Checks

(i) Gravitational redshift / proper time.

From (31), clocks at fixed r satisfy $d\tau = \sqrt{1 - r_s/r} dt$, which equals the TD rule $d\tau = \sqrt{1 - \Delta T} dt$ once $\Delta T = r_s/r$.

(ii) Radial null rays and the horizon.

Setting $ds^2 = 0 = d\Omega^2$ gives $\frac{dr}{dt} = \pm c(1 - \frac{r_s}{r})$, so $r = r_s$ is a null surface (the horizon). This matches the TD “horizon rule” $\Delta T = 1$ at $D = 2r = r_s \times 2$.

(iii) Weak-field expansion and PPN match.

For $r \gg r_s$,

$$g_{tt} = -c^2\left(1 - \frac{r_s}{r}\right) = -c^2\left(1 + \frac{2\phi}{c^2}\right), \quad \phi = -\frac{GM}{r},$$

and

$$g_{rr} = \left(1 - \frac{r_s}{r}\right)^{-1} = 1 + \frac{r_s}{r} + O\left(\frac{r_s^2}{r^2}\right).$$

Reading off the Parametrized Post-Newtonian coefficients gives $\gamma = \beta = 1$, which we collect and test in Sec. 6.

5.5. Summary of the Construction

TD fixes the lapse via the slow-time profile $N^2 = 1 - \Delta T(r)$; imposing the vacuum Einstein equations then determines the radial factor and returns the unique exterior solution (31). Thus the TD time-offset bookkeeping and GR geometry are exactly equivalent in the static, spherical sector.

6. PPN Validation of Temporal Dynamics (TD)

6.1. Goal and Strategy

We show that TD makes the same first-post-Newtonian (PPN) predictions as General Relativity (GR). Strategy: (i) build the line element from $\Delta T(r)$; (ii) expand to 1PN in isotropic coordinates and read off (β, γ) ; (iii) list classic tests and verify agreement.

Conventions.

$r_s = 2GM/c^2$ (Schwarzschild radius), $U(r) = GM/r$ (positive magnitude), $\Delta T(r) = r_s/r = 2U/c^2$, and the local TD clock relation $d\tau = \sqrt{1 - \Delta T(r)} dt$.

TD motion law (Newtonian limit).

With $\phi = -(c^2/2)\Delta T$ one has $\mathbf{a} = (c^2/2)\nabla\Delta T$; for $\Delta T(r) = 2GM/(rc^2)$ this gives $a_r = -GM/r^2$.

6.2. Line Element from $\Delta T(r)$ (Schwarzschild Sector)

From $d\tau = \sqrt{1 - \Delta T} dt$ with $\Delta T(r) = r_s/r$ we have

$$g_{tt} = -\left(1 - \frac{r_s}{r}\right)c^2. \quad (32)$$

Spherical vacuum fixes $g_{rr} = (1 - r_s/r)^{-1}$, with angular part $r^2 d\Omega^2$. Thus

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 d\Omega^2. \quad (33)$$

Because this is Schwarzschild, TD inherits all GR test results; the PPN check below makes this explicit.

6.3. PPN Extraction (β, γ)

The PPN template (isotropic spatial metric) to 1PN is

$$g_{tt} = -1 + \frac{2U}{c^2} - 2\beta \frac{U^2}{c^4} + \dots, \quad g_{ij} = \left(1 + 2\gamma \frac{U}{c^2} + \dots\right) \delta_{ij}. \quad (34)$$

Write Schwarzschild in isotropic radius ρ using

$$r = \rho \left(1 + \frac{m}{2\rho}\right)^2, \quad m \equiv \frac{GM}{c^2}, \quad (35)$$

which gives the exact form

$$ds^2 = -\left(\frac{1 - \frac{m}{2\rho}}{1 + \frac{m}{2\rho}}\right)^2 c^2 dt^2 + \left(1 + \frac{m}{2\rho}\right)^4 (d\rho^2 + \rho^2 d\Omega^2). \quad (36)$$

Define $x \equiv m/(2\rho)$ and note $U \simeq GM/\rho$ at this order. Expanding for $x \ll 1$:

$$\frac{g_{tt}}{c^2} = -\left(\frac{1-x}{1+x}\right)^2 = -(1-4x+8x^2+O(x^3)) = -1 + \frac{2U}{c^2} - 2\frac{U^2}{c^4} + O\left(\frac{U^3}{c^6}\right), \quad (37)$$

$$g_{ij} = (1+x)^4 \delta_{ij} = \left(1+4x+6x^2+O(x^3)\right) \delta_{ij} = \left(1+2\frac{U}{c^2} + \frac{3}{2}\frac{U^2}{c^4} + \dots\right) \delta_{ij}. \quad (38)$$

Comparing with the template yields

$$\gamma = 1, \quad \beta = 1,$$

and (in this static, spherically symmetric vacuum) vanishing preferred-frame parameters ($\alpha_1 = \alpha_2 = 0$), i.e. the GR point in PPN space.

6.4. Classic Tests (TD = GR)

Gravitational redshift / time dilation.

From TD: $d\tau/dt = \sqrt{1 - r_s/r}$. Weak field: $d\tau/dt \simeq 1 - U/c^2 - \frac{1}{2}(U/c^2)^2 + \dots$. Frequency shift between r_1 and r_2 : $\Delta\nu/\nu \simeq [U(r_2) - U(r_1)]/c^2$.

Deflection of light by mass M .

PPN: $\alpha = (1 + \gamma) 2GM/(c^2 b)$. With $\gamma = 1$: $\alpha = 4GM/(c^2 b)$ ($\approx 1.75''$ for a grazing solar ray).

Shapiro time delay (radar echo).

Near superior conjunction, one-way extra delay for impact parameter b and endpoints r_E, r_R :

$$\Delta t_{\text{Shapiro}} \simeq (1 + \gamma) \frac{GM}{c^3} \ln\left(\frac{4 r_E r_R}{b^2}\right).$$

With $\gamma = 1$: coefficient $2GM/c^3$ (one-way); round-trip doubles to $4GM/c^3$.

Perihelion precession (bound orbits).

PPN per-orbit advance:

$$\Delta\omega = \frac{2 - \beta + 2\gamma}{3} \frac{6\pi GM}{a(1 - e^2)c^2}.$$

With $\beta = \gamma = 1$: $\Delta\omega = 6\pi GM/[a(1 - e^2)c^2]$ (e.g. $\sim 43''/\text{century}$ for Mercury after Newtonian perturbations).

6.5. Scope and Coordinates

The extraction uses isotropic coordinates because the PPN template is defined there; the TD metric in standard Schwarzschild coordinates (33) is coordinate-equivalent at this order. For rotating sources (Kerr), additional frame-dragging terms enter the full PPN counting; TD as formulated here (static mapping with $\Delta T = r_s/r$) covers the non-rotating sector. Extending the dictionary to rotation is left for future work.

Boxed results (quick reference)

Metric (TD): $ds^2 = -(1 - r_s/r)c^2 dt^2 + (1 - r_s/r)^{-1} dr^2 + r^2 d\Omega^2$.

PPN: $\gamma = 1, \beta = 1, \alpha_1 = \alpha_2 = 0$ (static, spherical vacuum).

Light deflection: $\alpha = 4GM/(c^2 b)$.

Shapiro delay (one-way): $\Delta t = 2GM/c^3 \ln(4r_E r_R/b^2)$.

Perihelion precession: $\Delta\omega = 6\pi GM/[a(1 - e^2)c^2]$.

Redshift: $\Delta\nu/\nu \simeq \Delta U/c^2$ (weak field). (See Table 1).

Table 1. Classic tests in the TD/GR mapping (PPN with $\beta = \gamma = 1$).

Effect	Prediction
Gravitational redshift	$d\tau/dt = \sqrt{1 - r_s/r}$
Light deflection	$\alpha = 4GM/(c^2 b)$
Shapiro delay (one way)	$\Delta t = \frac{2GM}{c^3} \ln \frac{4r_E r_R}{b^2}$
Perihelion precession	$\Delta\omega = \frac{6\pi GM}{a(1 - e^2)c^2}$

7. Gravitational Waves: TD = GR in the Radiative Sector

7.1. Claim and Scope

Temporal Dynamics (TD) introduces no new propagating degree of freedom: ΔT is a repackaging of the lapse ($N^2 = 1 - \Delta T$; Sec. 4). Therefore the radiative sector coincides with GR: two transverse-traceless tensor polarizations, waves propagate at c , and the far-zone flux is the standard quadrupole formula. This section shows the equivalence by linearizing around flat spacetime.

7.2. Linearization and Gauge

Linearize the metric about Minkowski:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1. \quad (39)$$

In ADM language with vanishing background curvature, $N = 1$ and $\Delta T = 0$ at zeroth order. A first-order perturbation of the TD field $\delta(\Delta T)$ appears only through the lapse perturbation $N = 1 - \frac{1}{2}\delta(\Delta T) + O(h^2)$; since ΔT has *no kinetic term* (Sec. 4), its variation enforces a constraint rather than a wave equation. After imposing the linearized constraints (Hamiltonian and momentum), the two radiative tensor modes in h_{ij} remain, just as in GR.

Adopt the harmonic (Lorenz) gauge $\partial^\mu \bar{h}_{\mu\nu} = 0$ with

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h, \quad h \equiv \eta^{\alpha\beta}h_{\alpha\beta}. \quad (40)$$

The linearized Einstein equations are

$$\square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \quad \square \equiv -\frac{1}{c^2} \partial_t^2 + \nabla^2. \quad (41)$$

Because TD's action is the Einstein-Hilbert action written in TD variables (Sec. 4), Eq. (41) holds unchanged.

7.3. Propagation Speed and Polarizations

In vacuum ($T_{\mu\nu} = 0$), Eq. (41) reduces to

$$\square \bar{h}_{\mu\nu} = 0, \quad (42)$$

whose plane-wave solutions travel at speed c . In the transverse-traceless (TT) gauge, only the spatial components survive and are traceless and transverse:

$$h_{0\mu}^{\text{TT}} = 0, \quad \partial_i h_{ij}^{\text{TT}} = 0, \quad \delta^{ij} h_{ij}^{\text{TT}} = 0. \quad (43)$$

For a wave propagating along $\hat{z}$,

$$h_{ij}^{\text{TT}}(t, z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} (t - z/c), \quad (44)$$

revealing the two tensor polarizations $(+, \times)$, identical to GR.

7.4. Far-Zone Solution and Energy Flux

In the slow-motion, weak-field source regime, the far-zone TT strain at distance R is

$$h_{ij}^{\text{TT}}(t, \mathbf{x}) = \frac{2G}{c^4 R} \frac{d^2 Q_{ij}^{\text{TT}}(t - R/c)}{dt^2}, \quad (45)$$

where Q_{ij} is the (trace-free) mass quadrupole moment of the source. The averaged GW stress-energy (Isaacson) is

$$T_{\mu\nu}^{\text{GW}} = \frac{c^4}{32\pi G} \langle \partial_\mu h_{ij}^{\text{TT}} \partial_\nu h_{ij}^{\text{TT}} \rangle, \quad (46)$$

which yields the standard quadrupole luminosity

$$P = \frac{G}{5c^5} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle, \quad (47)$$

and the energy flux $\mathcal{F} = c T_{0r}^{\text{GW}}$ at large R . Equations (45)–(47) are the GR results and follow here because TD introduces no extra wave modes and no modified couplings.

7.5. Observational Mapping

Because the wave equation, polarizations, speed, and flux are all the GR ones, TD makes the same quantitative predictions for compact-binary inspirals: phase evolution (chirp), amplitude-distance relation (standard sirens), and polarization response in detector networks. In particular, the propagation speed equals c and no scalar/vector polarizations are present, aligning TD with the current interferometer constraints.

Boxed Results (Quick Reference)

Wave equation (vacuum): $\square \bar{h}_{\mu\nu} = 0$ (speed c).

Polarizations: two tensor modes $(+, \times)$ in TT gauge.

Far-zone strain: $h_{ij}^{\text{TT}} = \frac{2G}{c^4 R} \frac{d^2 Q_{ij}^{\text{TT}}}{dt^2}$.

Power: $P = \frac{G}{5c^5} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle$.

TD role: ΔT is non-propagating (lapse), so no extra GW modes; TD = GR for GWs.

8. Dark Energy from Temporal Dynamics (TD)

8.1. Postulates

(1) Cosmic clock (normal expansion).

Space expands at the universal clock rate set by light. Define the cosmic unit length

$$L^{\star} \equiv c \, \text{m} \tag{48}$$

(one light-second of length). Earlier sections used $\text{UC} = 1/c \, (\text{s m}^{-1})$; here we work in lengths, so there is no conflict.

(2) Residual field (dark energy).

Today’s space carries a faint residual “push” per unit per second, denoted ϵ . *Units:* ϵ has dimensions of m s^{-2} per cosmic unit $L^{\star}$ (i.e., extra velocity gained each second *per* light-second of separation).

(3) Normal expansion is shared.

Co-moving masses share the normal expansion and do not separate because of it. Only the residual ϵ produces relative drift.

8.2. Geometry of Separation

For two masses at physical distance D , the number of cosmic units (light-second lengths) between them is

$$N = \frac{D}{L^{\star}} = \frac{D}{c} \quad (\text{dimensionless}). \tag{49}$$

8.3. Residual Expansion Adds Linearly Over Units

Each unit contributes $\epsilon \, (\text{m s}^{-2})$ of extra velocity per second. Over one second, summing across N units gives the recession speed due to dark energy:

$$v_{\text{DE}} = N \epsilon \, (1 \, \text{s}) = \frac{D}{L^{\star}} \epsilon \, (1 \, \text{s}) = \frac{\epsilon}{c} D. \tag{50}$$

8.4. Macroscopic Residual Rate and Hubble Identification

Define the macroscopic residual rate

$$\kappa \equiv \frac{\epsilon}{c} \quad (\text{s}^{-1}), \tag{51}$$

so that the TD recession law is

$$v_{\text{DE}} = \kappa D. \tag{52}$$

Comparing with Hubble’s law $v_{\text{DE}} = H_0 D$ gives

$$H_0 = \kappa = \frac{\epsilon}{c}. \tag{53}$$

Thus, in TD, H_0 is not free; it is derived from the micro-rate ϵ .

8.5. Deriving ϵ and H_0 from Observations

Given a single galaxy (peculiar motion removed) with measured recession speed v_{obs} and proper distance D_{obs} :

Step 1 — solve for ϵ .

$$\epsilon = v_{\text{obs}} \frac{c}{D_{\text{obs}}} \quad (\text{m s}^{-2} \text{ per } L^{\star}). \tag{54}$$

Illustrative example.

$$v_{\text{obs}} = 7.50 \times 10^7 \text{ m s}^{-1}, \quad D_{\text{obs}} = 3.086 \times 10^{25} \text{ m}$$

$$\Rightarrow \epsilon \approx \left(7.50 \times 10^7\right) \frac{2.99792458 \times 10^8}{3.086 \times 10^{25}} \approx 7.29 \times 10^{-10} \text{ m s}^{-2} \text{ per } L^*.$$

Step 2 — from ϵ to H_0 .

$$H_0 = \frac{\epsilon}{c}. \quad (55)$$

Using the example ϵ :

$$H_0 \approx \frac{7.29 \times 10^{-10}}{2.99792458 \times 10^8} \approx 2.43 \times 10^{-18} \text{ s}^{-1} \approx 75 \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

Many-galaxy fit: Fitting the slope of v_{obs} vs. D gives H_0 directly; then $\epsilon = H_0 c$.

8.6. Time Evolution / Acceleration (Optional)

If ϵ (hence κ) varies with cosmic time,

$$\frac{d}{dt} v_{\text{DE}} = \kappa \dot{D} + D \dot{\kappa}. \quad (56)$$

With κ constant (de Sitter) and $\dot{D} = \kappa D$,

$$D(t) = D_0 e^{\kappa t}, \quad a_{\text{DE}} \equiv \frac{dv_{\text{DE}}}{dt} = \kappa v_{\text{DE}} = \kappa^2 D. \quad (57)$$

8.7. Units and Symbols (Quick Check)

$$L^* = c \text{ m}, \quad \epsilon : \text{m s}^{-2} \text{ per } L^*, \quad \kappa = \epsilon/c : \text{s}^{-1}, \quad v_{\text{DE}} : \text{m s}^{-1}, \quad D : \text{m}.$$

The dimensional chain in Eq. (50) is $[\text{m s}^{-1}] = ([\text{m s}^{-2}]/[\text{m s}^{-1}]) [\text{m}]$.

Summary box (key results)

Core law: $v_{\text{DE}} = (\epsilon/c) D$. Macroscopic rate: $\kappa = \epsilon/c$. Hubble identification: $H_0 = \kappa = \epsilon/c$.

Inference from data: $\epsilon = H_0 c$. Constant κ : $D(t) = D_0 e^{\kappa t}$ (de Sitter-like).

9. Cosmology from TD: Friedmann Equations, Λ Mapping, and Observational Constraints

9.1. Setup (TD $\rightarrow$ Macro)

Micro parameter: ϵ (per-unit residual push, m s^{-2} per L^*). Macro rate: $\kappa = \epsilon/c$ (s^{-1}). TD recession law: $v_{\text{DE}} = \kappa D$. In homogeneous, isotropic FLRW, treat κ as the dark-energy driver. (See Figure A2).

9.2. Friedmann Equations in TD Variables

A constant κ corresponds exactly to a cosmological constant:

$$\Lambda = \frac{3\kappa^2}{c^2}, \quad \rho_\Lambda = \frac{\Lambda c^2}{8\pi G} = \frac{3\kappa^2}{8\pi G}, \quad p_\Lambda = -\rho_\Lambda c^2. \quad (58)$$

Then the Friedmann pair (curvature $k_{\text{geom}} \in \{-1, 0, +1\}$) reads

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} (\rho_m + \rho_r + \rho_\Lambda) - k_{\text{geom}} \frac{c^2}{a^2}, \quad (59)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho_m + \rho_r + \frac{3p_r}{c^2} - 2\rho_\Lambda \right). \quad (60)$$

In a flat matter+ Λ case,

$$\frac{H^2(z)}{H_0^2} = \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{\Lambda0}, \quad \Omega_{\Lambda0} \equiv \frac{\rho_{\Lambda0}}{\rho_{\text{crit0}}} = \left(\frac{\kappa_0}{H_0}\right)^2. \quad (61)$$

9.3. If κ Varies with Time

Define $\rho_{\text{DE}}(t) \equiv 3\kappa(t)^2/(8\pi G)$. The continuity equation gives an effective equation of state

$$w(a) = -1 - \frac{1}{3} \frac{d \ln \rho_{\text{DE}}}{d \ln a} = -1 - \frac{2}{3} \frac{d \ln \kappa}{d \ln a} = -1 - \frac{2}{3} \frac{\dot{\kappa}}{\kappa H}. \quad (62)$$

In DE-domination ($H \simeq \kappa$) this reduces to $w \simeq -1 - \frac{2}{3} \dot{\kappa}/\kappa^2$.

9.4. Background in Terms of Redshift

With $E(z) \equiv H(z)/H_0$ and flat geometry,

$$E^2(z) = \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \left[\frac{\kappa(z)}{H_0}\right]^2. \quad (63)$$

Two common calibrations: (i) TD-dominant late time (heuristic): $\kappa_0 \approx H_0$. (ii) GR split (exact): $\kappa_0 = H_0 \sqrt{\Omega_{\Lambda0}}$ with $\Omega_{\Lambda0} = 1 - \Omega_{m0} - \Omega_{r0}$ in flat space. In either case, $\epsilon_0 = c \kappa_0$ is the micro residual rate today.

9.5. Distances and Observables

Given $E(z)$ from Eq. (63),

$$\chi(z) = c \int_0^z \frac{dz'}{H_0 E(z')} \quad (\text{comoving distance}), \quad (64)$$

$$D_L(z) = (1+z) \chi(z), \quad D_A(z) = \frac{\chi(z)}{1+z}. \quad (65)$$

Thus SNe (via D_L), BAO (via χ and D_A), and CMB (via the acoustic angle) are computed as usual once $\kappa(z)$ is specified.

9.6. Growth of Structure

TD does not modify gravity (Sec. 4); linear growth follows the GR equation in the chosen background:

$$\frac{d^2 D_+}{d \ln a^2} + \left[2 + \frac{d \ln H}{d \ln a}\right] \frac{d D_+}{d \ln a} - \frac{3}{2} \Omega_m(a) D_+ = 0, \quad \Omega_m(a) = \frac{\Omega_{m0} a^{-3} H_0^2}{H^2(a)}. \quad (66)$$

Predictions for $f\sigma_8$ and lensing therefore match GR given the same $H(z)$.

9.7. Numerical Mapping ($H_0 \leftrightarrow \epsilon_0$)

By Definition $H_0 = \kappa_0 = \epsilon_0/c$:

$$\begin{aligned} H_0 = 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1} &\Rightarrow \kappa_0 \approx 2.18 \times 10^{-18} \text{ s}^{-1} \Rightarrow \epsilon_0 \approx 6.55 \times 10^{-10} \text{ m s}^{-2} \text{ per } L^*, \\ H_0 = 73.0 \text{ km s}^{-1} \text{ Mpc}^{-1} &\Rightarrow \kappa_0 \approx 2.37 \times 10^{-18} \text{ s}^{-1} \Rightarrow \epsilon_0 \approx 7.11 \times 10^{-10} \text{ m s}^{-2} \text{ per } L^*, \\ H_0 = 68.5 \text{ km s}^{-1} \text{ Mpc}^{-1} &\Rightarrow \kappa_0 \approx 2.22 \times 10^{-18} \text{ s}^{-1} \Rightarrow \epsilon_0 \approx 6.66 \times 10^{-10} \text{ m s}^{-2} \text{ per } L^*. \end{aligned}$$

(Use $c = 2.99792458 \times 10^8 \text{ m s}^{-1}$ and $1 \text{ Mpc} = 3.0856776 \times 10^{22} \text{ m}$. Values shown are illustrative.)

9.8. Observational Constraints and the H_0 Tension (TD View)

In TD, the H_0 tension is simply two estimates of the same micro rate ϵ_0 : an early-time inference (CMB/BAO-anchored) versus a late-time distance-ladder inference. If κ is constant, TD reduces to Λ CDM ($w = -1$). Allowing a mild late drift $\kappa(z)$ corresponds to $w(z) \neq -1$ via Eq. (62), offering a controlled way to adjust local H_0 while preserving early-time observables.

Boxed identities (cosmology)

$$\begin{aligned} \kappa &= \epsilon/c; \quad \Lambda = 3\kappa^2/c^2; \quad \rho_\Lambda = 3\kappa^2/(8\pi G); \quad p_\Lambda = -\rho_\Lambda c^2. \\ H^2 &= \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_\Lambda) - k_{\text{geom}}c^2/a^2. \quad 1 + w = -\frac{2}{3}\dot{\kappa}/(\kappa H). \quad H_0 \leftrightarrow \epsilon_0 : \epsilon_0 = H_0 c. \end{aligned}$$

10. Observables, Parameterizations, and Data Strategy for TD Dark Energy

10.1. Distance–Redshift Relations (SNe and BAO)

Use $H(z)$ from Sec. 9 (Friedmann with $\rho_\Lambda(z) = 3\kappa(z)^2/(8\pi G)$):

$$\chi(z) = \int_0^z \frac{c \, dz'}{H(z')}, \quad (67)$$

$$D_A(z) = \frac{\chi(z)}{1+z}, \quad D_L(z) = (1+z)\chi(z), \quad (68)$$

$$\mu(z) = 5 \log_{10} \left(\frac{D_L(z)}{10 \text{ pc}} \right). \quad (69)$$

Flat baseline: $H^2(z) = H_0^2 [\Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_\Lambda(z)]$, with $\Omega_\Lambda(z) = \rho_\Lambda(z)/\rho_{\text{crit}0}$ and $\rho_\Lambda(z) = 3\kappa(z)^2/(8\pi G)$. For curvature, add $\Omega_{k0}(1+z)^2$ in H^2 and use the standard $S_k(\chi)$ prescription for distances.

10.2. Linear Growth of Structure (RSD and Weak Lensing)

TD (as presented) is GR-equivalent for background and linear perturbations, so

$$D''(a) + \left[2 + \frac{d \ln H}{d \ln a} \right] D'(a) - \frac{3}{2} \Omega_m(a) D(a) = 0, \quad (70)$$

where primes denote $d/d \ln a$. A useful summary is $f(a) \equiv d \ln D / d \ln a \simeq \Omega_m(a)^\gamma$ with $\gamma \simeq 0.55$ for Λ CDM; mild $w \neq -1$ shifts γ slightly.

10.3. Parameterizing $\kappa(z)$ (or $w(z)$)

Option A (direct TD).

$$\kappa(z) = \kappa_0 \left[1 + a_1 \frac{z}{1+z_*} + a_2 \left(\frac{z}{1+z_*} \right)^2 \right], \quad (71)$$

with a pivot z_* (e.g., 0.5 or 1.0) to reduce parameter correlation. Then $\rho_\Lambda(z) = 3\kappa(z)^2/(8\pi G)$.

Option B (CPL equation of state).

$$w(a) = w_0 + w_a(1 - a), \quad (72)$$

$$\rho_{\text{DE}}(a) = \rho_{\text{DE}0} a^{-3(1+w_0+w_a)} \exp[-3w_a(1-a)], \quad (73)$$

$$\kappa(a) = \sqrt{\frac{8\pi G}{3} \rho_{\text{DE}}(a)}, \quad \epsilon(a) = c \kappa(a). \quad (74)$$

Reporting: give both TD and GR faces for transparency: $\{H_0, \Omega_{\text{m}0}, \kappa_0$ (or $w_0, w_a\})$ and the micro value $\{\epsilon_0 = H_0 c\}$.

10.4. Curvature and the Sound Horizon (Degeneracies)

Allow Ω_{k0} if needed: $H^2(a) = H_0^2 [\Omega_{\text{m}0} a^{-3} + \Omega_{\text{r}0} a^{-4} + \Omega_{\Lambda}(a) + \Omega_{k0} a^{-2}]$. For BAO, specify whether r_d (sound horizon) is fixed by an early-universe prior (CMB-informed) or floated in late-time-only fits.

10.5. Minimal Fitting Workflow (Practical)

1. Choose geometry: flat by default ($\Omega_{k0} = 0$) unless testing curvature.
2. Choose parameterization: constant κ (Λ) or one of the drifts above.
3. Compute $H(z)$, then $\chi(z)$, $D_A(z)$, $D_L(z)$, $\mu(z)$.
4. Growth: integrate $D(a)$ or use $f \simeq \Omega_{\text{m}}(a)^\gamma$ for RSD/lensing likelihoods.
5. Likelihoods: combine SNe (distance moduli), BAO (D_M/r_d , $H r_d$), and CMB anchors (acoustic angle or r_d prior) as appropriate.
6. Report posteriors for $\{H_0, \Omega_{\text{m}0}, \kappa_0$ or $w_0, w_a\}$ and derived $\{\epsilon_0 = H_0 c\}$.
7. Model comparison: if you freed $\kappa(z)$, quote AIC/BIC against constant κ .

Boxed formulas (quick reference)

$\kappa = \epsilon/c$; $\epsilon = H_0 c$; $\Lambda = 3\kappa^2/c^2$; $\rho_{\Lambda} = 3\kappa^2/(8\pi G)$; $p_{\Lambda} = -\rho_{\Lambda} c^2$.
 $\chi(z) = \int_0^z c/H(z') dz'$; $D_A = \chi/(1+z)$; $D_L = (1+z)\chi$; $\mu = 5 \log_{10}(D_L/10 \text{ pc})$.
 Growth: $D'' + [2 + d \ln H/d \ln a] D' - (3/2)\Omega_{\text{m}}(a)D = 0$; $f \simeq \Omega_{\text{m}}^{0.55}$ (near Λ).

Worked numerical example (from H_0 to ϵ_0 and Λ)

Constants.

$$c = 2.99792458 \times 10^8 \text{ m s}^{-1}, \quad G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}, \quad 1 \text{ Mpc} = 3.0856776 \times 10^{22} \text{ m}.$$

Definitions.

$$H_0 \text{ in s}^{-1}: H_0 = (H_0/\text{km s}^{-1} \text{ Mpc}^{-1}) \times 10^3/(1 \text{ Mpc}).$$

$$\text{TD micro rate: } \epsilon_0 = c \kappa_0. \quad \text{Mapping: } \Lambda = 3\kappa_0^2/c^2, \quad \rho_{\Lambda} = 3\kappa_0^2/(8\pi G).$$

Calibration A (GR split; flat Λ CDM).

$$\text{Here } \kappa_0 = H_0 \sqrt{\Omega_{\Lambda 0}}.$$

$$\text{Example A1 (Planck-like): } H_0 = 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad \Omega_{\Lambda 0} = 0.685.$$

$$H_0 = 2.184285 \times 10^{-18} \text{ s}^{-1}, \quad \kappa_0 = 1.807818 \times 10^{-18} \text{ s}^{-1},$$

$$\epsilon_0 = c \kappa_0 = 5.423 \times 10^{-10} \text{ m s}^{-2} \text{ per } L^*,$$

$$\Lambda = \frac{3\kappa_0^2}{c^2} = 1.091 \times 10^{-52} \text{ m}^{-2}, \quad \rho_{\Lambda} = \frac{3\kappa_0^2}{8\pi G} = 5.85 \times 10^{-27} \text{ kg m}^{-3}.$$

$$\text{Example A2 (ladder-like): } H_0 = 73.0 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad \Omega_{\Lambda 0} = 0.70.$$

$$H_0 = 2.365769 \times 10^{-18} \text{ s}^{-1}, \quad \kappa_0 = 1.979344 \times 10^{-18} \text{ s}^{-1},$$

$$\epsilon_0 = 5.935 \times 10^{-10} \text{ m s}^{-2} \text{ per } L^*, \quad \Lambda = 1.308 \times 10^{-52} \text{ m}^{-2}, \quad \rho_\Lambda = 7.01 \times 10^{-27} \text{ kg m}^{-3}.$$

Calibration B (TD late-time heuristic; DE-dominated).

Here $\kappa_0 \approx H_0$ (pure de Sitter at late times).

Example B1: $H_0 = 67.4$.

$$\kappa_0 = H_0 = 2.184285 \times 10^{-18} \text{ s}^{-1}, \quad \epsilon_0 = 6.548 \times 10^{-10} \text{ m s}^{-2} \text{ per } L^*,$$

$$\Lambda = \frac{3H_0^2}{c^2} = 1.593 \times 10^{-52} \text{ m}^{-2}, \quad \rho_\Lambda = \frac{3H_0^2}{8\pi G} = 8.53 \times 10^{-27} \text{ kg m}^{-3}.$$

Example B2: $H_0 = 73.0$.

$$\kappa_0 = H_0 = 2.365769 \times 10^{-18} \text{ s}^{-1}, \quad \epsilon_0 = 7.092 \times 10^{-10} \text{ m s}^{-2} \text{ per } L^*,$$

$$\Lambda = 1.868 \times 10^{-52} \text{ m}^{-2}, \quad \rho_\Lambda = 1.001 \times 10^{-26} \text{ kg m}^{-3}.$$

Comment.

Calibration A (GR split) uses the measured $\Omega_{\Lambda 0}$ and reproduces the standard Λ values; Calibration B (TD heuristic) is appropriate only in a DE-dominated asymptotic view where $\kappa_0 \approx H_0$. In all cases the TD micro value follows transparently as $\epsilon_0 = c \kappa_0$.

11. Conclusions

We presented Temporal Dynamics (TD) as a time-first reparameterization of General Relativity (GR). The single mass constant $k = 4G/c^2$ makes TD's path-averaged slow-time ΔT quantitatively equivalent to GR curvature: $\Delta L_{\text{diam}} = kM = 2r_s$ and $\Delta T(r) = r_s/r$. From the variational principle (Sec. 4), varying ΔT enforces the Hamiltonian constraint, so TD introduces no new degrees of freedom. The metric built from $\Delta T(r)$ (Sec. 5) is Schwarzschild; the PPN parameters (Sec. 6) are $\beta = \gamma = 1$; gravitational waves (Sec. 7) have the GR tensor polarizations, speed c , and quadrupole flux.

In cosmology (Sec. 9), a uniform micro residual rate ϵ maps to a macroscopic rate $\kappa = \epsilon/c$ and to $\Lambda = 3\kappa^2/c^2$. Constant κ reproduces Λ CDM; mild late-time drift $\kappa(z)$ is equivalent to $w(z) \neq -1$ in the standard language and can be constrained by BAO/SNe/CMB and growth.

Outlook.

Natural next steps include: (i) extending the TD dictionary to rotation (Kerr and Lense–Thirring), (ii) a full cosmological perturbation treatment with $\kappa(z)$ fits, and (iii) exploring whether TD's book-keeping yields new compact analytic identities (e.g., lensing integrals, timing delays) useful for data analysis. TD, as formulated here, is observationally GR-equivalent in the tested regimes; any phenomenology beyond GR would arise only if ΔT were promoted to a dynamical field (which we do not do here). **Acknowledgments:** I

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Author contributions

Conceptualization, formal analysis, writing, and verification: O. O. Francis.

Competing interests

The author declares no competing interests.

Data and code availability

No new datasets or custom code were generated. Numerical values use standard physical constants; all reproducible numbers are given in the text and appendices.

Appendix L Conceptual Notes: The Cosmic Clock and TD Intuition

Appendix L.1 The Cosmic Clock and the Unit Length $L^* = c$

The starting picture is a universal clock: for every second of time, light marks out exactly one light-second of length. Two equivalent conversion constants are useful:

$$UC = \frac{1}{c} \text{ (s m}^{-1}\text{)}, \quad L^* = c \text{ (m per s)}.$$

Earlier sections sometimes use UC (time per length) while here we use L^* (length per second). They are reciprocals, so switching between “time-first” and “length-first” viewpoints is straightforward.

Intuition.

Think of the universe as a giant metronome: every tick of duration 1 s lays down c m of “clock length” in every direction. TD asks: what happens to this steady mapping between time and length when mass or a residual cosmic push is present?

Appendix L.2 Two Rulers: Straight Time vs. Curved Space

GR measures how paths bend: the geometry between two points becomes longer than the Euclidean straight line. TD measures how the *straight* path’s local time slows by a fraction ΔT ; to keep the global clock exact, space “adds” just enough extra length along the actual path.

Bridge identity.

If a straight segment has geometric length D and a path-averaged slow-time fraction ΔT ,

$$\Delta t_{\text{total}} = \Delta T \frac{D}{c}, \quad \Delta L_{\text{diam}} = c \Delta t_{\text{total}} = \Delta T D.$$

No double conversion: multiply by D/c *once* to get a time delay, then multiply by c *once* to get a length increment.

Sun thought experiment.

For a diameter through (or a limb-skimming chord past) the Sun, using $k = 4G/c^2$ and $\Delta T = (kM)/D$ gives $\Delta L_{\text{rad}} = \frac{1}{2}\Delta L_{\text{diam}} = r_s$, matching the Schwarzschild scale. GR’s curved surface length and TD’s straight-path slow-time are two readings of the *same* mass effect.

Appendix L.3 Mass as Concentrated Time; the Meaning of $k = 4G/c^2$

TD uses one constant

$$k = \frac{4G}{c^2} \text{ (m kg}^{-1}\text{)},$$

so that

$$\Delta T = \frac{kM}{D}, \quad \Delta L_{\text{diam}} = kM, \quad \Delta L_{\text{rad}} = \frac{kM}{2}.$$

Interpretations:

- **Length per mass.** k tells how many meters of effective path length are “added” per kilogram to preserve the universal clock.
- **Per energy.** Using $E = Mc^2$, $kM = (4G/c^4) E$ is the same meter-per-joule scale that appears in GR’s coupling (compare $8\pi G/c^4$).
- **Horizon fraction.** With $D = 2r$, $\Delta T(r) = \frac{kM}{2r} = \frac{r_s}{r}$.

- **Diameter invariant.** The mass-only time delay across a diameter is $\Delta t_{\text{total}} = \frac{kM}{c} = \frac{4GM}{c^3}$.

Appendix L.4 Where Gravity “Comes from” in TD Language

In weak fields, define a potential

$$\phi(r) = -\frac{c^2}{2} \Delta T(r).$$

With $\Delta T(r) = r_s/r = 2GM/(rc^2)$ this gives $\phi(r) = -GM/r$ and

$$\mathbf{g}(\mathbf{r}) = -\nabla\phi = -\frac{GM}{r^2} \hat{\mathbf{r}}.$$

TD says: matter drifts toward regions where local time runs slower (larger ΔT), which is equivalent to sliding down $-\phi$. GR says: free fall follows geodesics in the curved metric. Both statements describe the same motion.

Appendix L.5 Dark Energy as a Residual Micro-Push

TD posits a tiny per-unit residual push ϵ (m s⁻² per light-second). Over a separation D there are $N = D/L^* = D/c$ units, so in one second the extra recession is

$$v_{\text{DE}} = \frac{\epsilon}{c} D.$$

Define the macroscopic rate $\kappa = \epsilon/c$ (s⁻¹). Then

$$v_{\text{DE}} = \kappa D, \quad H_0 = \kappa = \frac{\epsilon}{c}, \quad \Lambda = \frac{3\kappa^2}{c^2}, \quad \rho_\Lambda = \frac{3\kappa^2}{8\pi G}.$$

Constant κ reproduces Λ CDM ($w = -1$). A slowly drifting $\kappa(z)$ is equivalent to $w(z) \neq -1$ via

$$w(a) = -1 - \frac{2}{3} \frac{d \ln \kappa}{d \ln a}.$$

What observations “measure.”

A direct slope of v vs. D at low z gives $H_0 = \kappa$; then $\epsilon_0 = c \kappa_0$. Early-time probes (CMB+BAO) infer κ indirectly through the background expansion; late-time distance ladders infer it locally. In TD language the H_0 tension is two estimates of the same micro rate ϵ_0 .

Appendix L.6 What TD Does Not Change

- **Field equations.** The variational principle with $N^2 = 1 - \Delta T$ reproduces Einstein’s equations; ΔT is not a new propagating field.
- **Solar-system tests.** PPN parameters are $\beta = \gamma = 1$; light bending, Shapiro delay, perihelion advance, and redshift match GR.
- **Gravitational waves.** Two tensor polarizations, speed c , and quadrupole flux; same wave equation as GR.
- **Early universe.** TD does not modify pre-recombination physics at the background or linear level; the sound horizon r_d and CMB acoustic physics are unchanged once $\kappa(z)$ (or $w(z)$) is specified.
- **Causality and c .** The speed of light is constant; TD never requires c to vary.

Appendix L.7 What Would Make TD a New Theory (and Why We Avoid It)

Promoting ΔT to a dynamical field by adding gradient or kinetic terms in the action would introduce an extra scalar mode (“fifth force”), alter wave speeds, and generically deviate from GR. The formulation in this paper deliberately *does not* add such terms: TD is GR in time-first coordinates.

Appendix L.8 Frequently Asked Questions

Why $\Delta T(r) = r_s/r$?

Because TD fixes the lapse via $N^2 = 1 - \Delta T(r)$ and matches the Newtonian limit $\phi = -GM/r$ through $\phi = -(c^2/2)\Delta T$. This pins $\Delta T(r) = 2GM/(rc^2) = r_s/r$, which then yields the Schwarzschild metric.

Does TD Change Coordinates?

No. TD gives a dictionary for the lapse; the resulting metric is Schwarzschild in standard coordinates. For PPN comparisons one passes to isotropic coordinates in the usual way.

Is the factor “4” in $k = 4G/c^2$ special?

Yes: it encodes null-path effects (e.g., $4GM/(c^2b)$ for light deflection) and ensures the diameter invariant $\Delta t_{\text{total}} = 4GM/c^3$.

Is the residual ϵ just $H_0 c$?

At $z \approx 0$, yes: $\epsilon_0 = H_0 c$. If $\kappa(z)$ drifts, $\epsilon(z) = c \kappa(z)$ maps one-to-one to $w(z)$.

What about rotation and frame dragging?

This paper treats the static spherical sector. Extending the TD dictionary to include off-diagonal $g_{t\varphi}$ pieces recovers Lense–Thirring and, in full, the Kerr metric (left for future work).

How should I think about gravitational waves in TD?

As tiny, transverse ripples in how the global clock synchronizes across space; mathematically they are the same tensor waves GR describes.

Appendix L.9 Minimal TD↔GR Dictionary (One Page)

- $k = \frac{4G}{c^2}$ (m kg⁻¹); $\Delta T = \frac{kM}{D}$; $\Delta L_{\text{diam}} = kM$; $\Delta L_{\text{rad}} = \frac{kM}{2}$.
- $r_s = \frac{2GM}{c^2}$; $\Delta T(r) = \frac{r_s}{r}$; $g_{tt} = -(1 - r_s/r)c^2$; $g_{rr} = (1 - r_s/r)^{-1}$.
- $\phi = -(c^2/2)\Delta T$; $\mathbf{g} = -\nabla\phi$.
- ϵ (m s⁻² per L^{*}); $\kappa = \epsilon/c$ (s⁻¹); $H_0 = \kappa$; $\Lambda = 3\kappa^2/c^2$; $\rho_\Lambda = 3\kappa^2/(8\pi G)$.

Appendix L.10 Common Pitfalls (and Quick Fixes)

- **Double converting time and length.** Use $\Delta t_{\text{total}} = \Delta T (D/c)$ then $\Delta L = c \Delta t_{\text{total}}$.
- **Mixing UC and L^{*}.** Work either in time-per-length (UC) or length-per-time (L^{*}) consistently; they are reciprocals.
- **Treating ΔT as clock time, not a fraction.** ΔT is dimensionless (a “seconds per second” fraction). The actual delay is Δt_{total} .
- **Forgetting ϵ is per unit L^{*}.** The macroscopic law is $v_{\text{DE}} = (\epsilon/c)D$, not $v_{\text{DE}} = \epsilon D$.

Appendix L.11 Roadmap Back to the Mathematics

For formal derivations and tests: mass constant and identities (Sec. 2); TD→GR bridge (Sec. 3); variational principle and field equations (Sec. 4); metric construction (Sec. 5); PPN validation (Sec. 6); gravitational waves (Sec. 7); cosmology and data strategy (Sec. 9, Sec. 10).

Appendix L.12 Normal Expansion (the Baseline Clock Field)

Definition.

The normal expansion is the global mapping between time and length set by light:

$$\text{UC} = \frac{1}{c} \text{ (s m}^{-1}\text{)}, \quad L^* = c \text{ (m per s)}.$$

By itself, this clock field acts *uniformly* everywhere. Two co-moving masses that share the same frame do not acquire relative motion from the normal expansion, because there is no differential between them.

Operational statement.

Let D be the physical separation between two co-moving masses. The normal expansion contributes the same D/c of coordinate time along either worldline; therefore it creates no separation. Relative motion arises only from (i) gravity (through ΔT gradients), (ii) peculiar velocities, or (iii) the residual “micro push” ϵ summed over $N = D/L^*$ units (Sec. 8).

Appendix L.13 TD Gravity Ratio and the General Motion Law

Local clock ratio (the right “gravity ratio”).

In TD (and GR), the *local* slow-time factor is

$$\chi(r) \equiv \frac{d\tau}{dt} = \sqrt{1 - \Delta T(r)}, \quad \Delta T(r) = \frac{r_s}{r} = \frac{2GM}{rc^2}, \quad r > r_s. \quad (\text{A75})$$

We call χ the *gravity ratio*. It is dimensionless, $0 < \chi \leq 1$, and exactly reproduces gravitational redshift/time-dilation.

Avoid a common pitfall (path fraction vs. local ratio).

If one forms the path fraction

$$\chi_{\text{path}} \equiv \frac{t_{\text{geom}}}{t_{\text{geom}} + \Delta t_{\text{total}}} = \frac{D/c}{(D/c) + \overline{\Delta T}(D/c)} = \frac{1}{1 + \overline{\Delta T}},$$

this is a *path-averaged* ratio, not a local clock factor. Here

$$\Delta t_{\text{total}} = \frac{1}{c} \int_{\text{path}} \Delta T(s) ds \approx \frac{\overline{\Delta T} D}{c}.$$

For small ΔT , $\chi_{\text{path}} \approx 1 - \Delta T$ while the correct local ratio is $\chi \approx 1 - \frac{1}{2}\Delta T$. Use $\chi = \sqrt{1 - \Delta T}$ for dynamics and clocks.

Flow speed and the “never crosses c ” statement.

Define the local flow speed of the clock field

$$v_{\text{flow}}(r) \equiv \chi(r) c = c \sqrt{1 - \Delta T(r)} \leq c.$$

As $\Delta T \uparrow$, v_{flow} decreases but never exceeds c . This captures the intuition that gravity “slows the flow” locally.

General motion law (weak field / Newtonian limit).

The acceleration of a slow test particle follows from the TD potential

$$\phi(r) = -\frac{c^2}{2} \Delta T(r), \quad \mathbf{a} = -\nabla \phi = \frac{c^2}{2} \nabla \Delta T. \quad (\text{A76})$$

For a point mass with $\Delta T(r) = 2GM/(rc^2)$,

$$\frac{d\Delta T}{dr} = -\frac{2GM}{c^2} \frac{1}{r^2} \Rightarrow a_r = -\frac{GM}{r^2},$$

i.e. the standard Newtonian acceleration. Thus TD’s *dynamics* are encoded in the *gradient* of ΔT , not in the difference $c - v_{\text{flow}}$.

Relation to the velocity-style heuristic.

The heuristic “ c minus gravity ratio times c ” is a good *intuition* for a local speed deficit:

$$\delta v(r) \equiv c - v_{\text{flow}}(r) = c \left[1 - \sqrt{1 - \Delta T(r)} \right] \approx \frac{c \Delta T(r)}{2} \quad (\Delta T \ll 1).$$

But δv has units of speed; acceleration comes from spatial variation:

$$a_r \sim -\frac{d}{dr} [\delta v(r)] \propto -\frac{d}{dr} \Delta T(r),$$

which reproduces Eq. (A76) exactly when you use the TD potential $\phi = -(c^2/2)\Delta T$.

Boxed: TD motion and ratios (quick reference)

Local gravity ratio: $\chi = \frac{d\tau}{dt} = \sqrt{1 - \Delta T}$. Clock flow speed: $v_{\text{flow}} = \chi c \leq c$. TD potential: $\phi = -(c^2/2)\Delta T$. General motion law: $\mathbf{a} = \frac{c^2}{2} \nabla \Delta T$. Point mass: $\Delta T(r) = 2GM/(rc^2) \Rightarrow a_r = -GM/r^2$.

Worked example: Earth surface gravity (numbers).

Constants. $G = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$, $c = 2.99792458 \times 10^8 \text{ m s}^{-1}$, $M_{\text{Earth}} = 5.972 \times 10^{24} \text{ kg}$, $R_{\text{Earth}} = 6.371 \times 10^6 \text{ m}$.

Schwarzschild scale.

$$r_s = \frac{2GM_{\text{Earth}}}{c^2} \approx 8.87 \times 10^{-3} \text{ m}.$$

Local slow-time at the surface.

$$\Delta T(R_E) = \frac{r_s}{R_E} \approx 1.392 \times 10^{-9}, \quad \chi(R_E) = \sqrt{1 - \Delta T} \approx 1 - 0.695 \times 10^{-9}.$$

Clock-flow “speed deficit” (intuition only).

$$\delta v \equiv c[1 - \chi] \approx \frac{c \Delta T}{2} \approx 0.209 \text{ m s}^{-1}.$$

Acceleration from the TD motion law. Using $\frac{d\Delta T}{dr} = -\frac{r_s}{r^2}$ and $\mathbf{a} = \frac{c^2}{2} \nabla \Delta T$,

$$\left. \frac{d\Delta T}{dr} \right|_{r=R_E} \approx -2.18 \times 10^{-16} \text{ m}^{-1}, \quad a_r = \frac{c^2}{2} \frac{d\Delta T}{dr} \approx -9.82 \text{ m s}^{-2},$$

which equals the Newtonian result $a_r = -GM_{\text{Earth}}/R_{\text{Earth}}^2$ to the stated precision.

Assumptions and domain (scope of formulas)

Unless stated otherwise, we assume: static, spherically symmetric, exterior region $r > r_s$; weak-field expansions use $\Delta T \ll 1$. Coordinates are Schwarzschild (standard) unless noted; isotropic coordinates are used only for PPN matching. Local slow-time is $\chi = \sqrt{1 - \Delta T}$; path-averaged quantities carry a bar, e.g. $\overline{\Delta T}$. **Units.** c exact (SI); $G = 6.67430(15) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ (CODATA 2018 [2]).

Notation (quick reference)

UC = $1/c$ (s m^{-1}): time-per-length converter. $L^* = c$ (m per s): length per tick (one light-second). $k = 4G/c^2$ (m kg^{-1}). $r_s = 2GM/c^2$. ΔT (local, dimensionless); $\overline{\Delta T}$ (path average). $\chi = d\tau/dt = \sqrt{1 - \Delta T}$. ϵ (m s^{-2} per L^*); $\kappa = \epsilon/c$ (s^{-1}).

Appendix L.14 Dark Energy as a Residual (“Twin”) Expansion: An Expanded Intuition

Dark energy has long been described operationally as a negative–pressure component that accelerates expansion. TD adds a causal picture: the universe carries a faint *residual* expansion imprint, a micro push per light–second ϵ (Sec. 8), left over from an earlier, faster epoch. Summing this imprint across many clock units reproduces the macroscopic Hubble law and, in GR variables, is exactly a cosmological constant (or a mild $w(z) \neq -1$ if the imprint drifts).

Normal expansion vs. relative motion.

- **Cosmic clock field.** The normal expansion is the global mapping $1 \text{ s} \leftrightarrow c \text{ m}$ (Sec. L). It acts *uniformly* and synchronously, so two co–moving masses sharing the same frame do not separate because of it.
- **No differential, no drift.** If gravity between m_1 and m_2 vanishes and both are co–moving, their *relative* motion from the clock alone is zero. TD and GR agree: relative motion requires a differential driver (gravity, peculiar velocity, or—in TD—the residual micro push).

Why dark energy is not the normal clock.

If the normal clock itself created relative motion, any two co–moving galaxies would drift apart purely from the background, contrary to how the FRW frame is defined. TD therefore separates:

- (i) normal expansion (shared, no differential) vs. (ii) residual expansion (not shared, differential).

The residual *does* create relative motion because its effect *adds* across the $N = D/L^*$ units between two points.

The “twin expansion” hypothesis (cartoon).

Think of two expansion signatures:

1. **Normal expansion:** defines the spacetime ruler (the clock) and keeps c exact.
2. **Dark expansion:** a faint twin with a slightly different temporal rate.

The mismatch leaves a residual “memory” per unit, ϵ . Across a separation D there are $N = D/L^*$ units, so in one second the extra recession is

$$v_{\text{DE}} = \frac{\epsilon}{c} D = \kappa D, \quad \kappa \equiv \frac{\epsilon}{c}.$$

Thus the farther the separation, the larger the accumulated residual and the larger the recession speed—exactly the observed linear law. In GR variables,

$$\Lambda = \frac{3\kappa^2}{c^2}, \quad \rho_\Lambda = \frac{3\kappa^2}{8\pi G},$$

so a constant κ is indistinguishable from Λ CDM at the background level (Sec. 9).

Light and temporal signatures.

- **No second metric.** TD remains single–metric GR (Sec. 4). The normal clock sets the null cone; locally measured light speed is c , everywhere.
- **Operational view.** Light is *bound* to the normal clock: it rides the same lapse that defines time flow. The residual expansion never introduces a new propagation channel; it only changes the *background rate* κ that distances accumulate against (hence $D_L(z)$, $D_A(z)$, etc.).

Putting the pieces together.

- **Cause.** A past, faster phase left a uniform micro residual per unit, ϵ .
- **Effect.** Summed over $N = D/L^*$ units, it yields $v_{\text{DE}} = (\epsilon/c)D$.

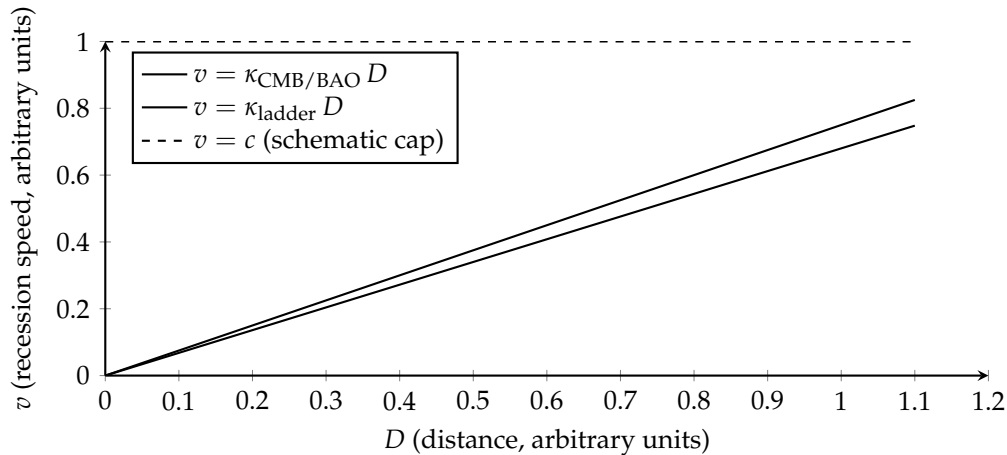


Figure A2. TD dark energy as a residual rate $\kappa = \epsilon/c$: recession is $v = \kappa D$. Different data combinations infer slightly different slopes (the H_0 tension recast as κ or ϵ).

- **GR face.** $\kappa = \epsilon/c$ maps to $\Lambda = 3\kappa^2/c^2$ (constant case) or to $w(a) = -1 - \frac{2}{3} d \ln \kappa / d \ln a$ (drifting case).
- **Observables.** Distances and growth are computed exactly as in GR given $\kappa(z)$ (Secs. 9–10); local c and solar–system physics are unchanged.

Referee-facing clarifications.

- *Not bimetric:* There is no second metric or extra mode; ΔT is an auxiliary lapse variable, not a propagating field.
- *Energy–momentum:* The residual maps to an effective $T_{\mu\nu}^{(\Lambda)} = -\rho_\Lambda g_{\mu\nu}$ with $\rho_\Lambda = 3\kappa^2/(8\pi G)$, the standard GR vacuum form.
- *Predictions:* Constant $\kappa \Rightarrow \Lambda$ CDM. Mild late drift in $\kappa(z) \Rightarrow$ standard w_0 – w_a phenomenology. Discriminants are BAO/SNe distances, CMB acoustic angle, and growth $f\sigma_8(z)$ (Sec. 10).

One–paragraph summary.

In TD, the universe’s acceleration is not a new force but an echo of its own past: a tiny, uniform residual micro push per light–second, ϵ , accumulated across many units into a macroscopic rate $\kappa = \epsilon/c$. This reproduces Hubble’s law and maps one–to–one to GR’s Λ (or to $w(z)$ if κ drifts), while preserving a single metric, local c , and all tested GR phenomenology.

Appendix L.15 What “One Unit of Space per Second” Really Means

The clock unit is a *conversion*, not a wind.

TD uses two reciprocal constants to translate between time and length:

$$\text{UC} = \frac{1}{c} \text{ (s m}^{-1}\text{)}, \quad L^* = c \text{ (m per s)}.$$

Saying “one unit of space per second” means *one light–second of length corresponds to one second of time* by definition of c . It does *not* mean space streams past objects adding c meters every second.

Replication vs. addition (the cartoon vs. the math).

The “each unit replicates every second” image is a cartoon for counting. The mathematics is continuous:

- **Normal clock (shared):** contributes equally to all co–moving worldlines. Two co–moving masses separated by D remain at fixed comoving separation from the clock alone: no differential $\Rightarrow$ no relative drift.

- **Residual push (dark energy):** a tiny per-unit imprint ϵ (m s^{-2} per L^*) *does* sum across the $N = D/L^*$ units, giving in one second

$$v_{\text{DE}} = \frac{\epsilon}{c} D \equiv \kappa D, \quad \kappa = \frac{\epsilon}{c}.$$

This is Hubble's law in TD variables. If κ is (approximately) constant, the separation solves $\dot{D} = \kappa D \Rightarrow D(t) = D_0 e^{\kappa t}$ (de Sitter).

Mapping to FRW (no new physics).

In an FLRW background with scale factor $a(t)$, TD's $\kappa(t)$ is just the vacuum component in the Friedmann equations:

$$\Lambda = \frac{3\kappa^2}{c^2}, \quad \rho_\Lambda = \frac{3\kappa^2}{8\pi G}, \quad H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_\Lambda) - \frac{k_{\text{geom}} c^2}{a^2}.$$

Constant κ reproduces Λ CDM ($w = -1$); a slow drift $\kappa(z)$ is a standard $w(z) \neq -1$ model via $w(a) = -1 - \frac{2}{3} d \ln \kappa / d \ln a$.

No "space wind," no second metric.

Local physics uses the single GR metric; light speed is c everywhere. The clock unit L^* is a bookkeeping device (a ruler synchronized to c), not a fluid that flows through matter.

A 30-second thought experiment.

Place two free test masses that are co-moving (no peculiar velocity, no mutual gravity) at separation D .

1. *Normal clock only:* both accumulate the same coordinate time D/c per light-second; separation remains D .
2. *Add residual ϵ :* across $N = D/L^*$ units, one second later their relative velocity gained is $v_{\text{DE}} = \kappa D$. This is the observed late-time acceleration.

Units sanity check.

$[\epsilon] = \text{m s}^{-2}$ per L^* , $[\kappa] = \text{s}^{-1}$, $[v_{\text{DE}}] = \text{m s}^{-1}$, $[\Lambda] = \text{m}^{-2}$. Dimensional chain: $v_{\text{DE}} = (\epsilon/c) D$ gives $[\text{m s}^{-2}]/[\text{m s}^{-1}] \times [\text{m}] = [\text{m s}^{-1}]$.

Edge cases and scope.

All statements here refer to the exterior region $r > r_s$ and to background expansion at sub-horizon scales where the FLRW picture applies. Normal expansion produces no local relative motion between co-moving masses; relative drift arises from $\nabla \Delta T$ (gravity) or from κ (the residual).

Glossary (mini-card)

c Speed of light; sets the clock length $L^* = c$.

UC Universal converter = $1/c$ (s m^{-1}), time $\leftrightarrow$ length.

G Newton's constant.

k TD mass constant = $4G/c^2$ (m kg^{-1}); mass $\rightarrow$ curve converter.

r_s Schwarzschild radius = $2GM/c^2$.

$\Delta T(\mathbf{x})$ **Local** slow-time fraction (dimensionless).

$\overline{\Delta T}$ **Path-averaged** slow-time fraction along a segment.

χ Local gravity ratio = $d\tau/dt = \sqrt{1 - \Delta T}$.

ϕ TD potential = $-(c^2/2) \Delta T$.

a Motion law (Newtonian limit) $\mathbf{a} = (c^2/2)\nabla\Delta T$.

ΔL Curve-length increment for a segment: $\Delta L = \overline{\Delta T} D$.

ϵ Residual micro push per clock unit (m s^{-2} per L^*).

κ Macroscopic residual rate $= \epsilon/c$ (s^{-1}).

Λ Cosmological constant $= 3\kappa^2/c^2$ (m^{-2}) for constant κ .

ρ_Λ, p_Λ Vacuum density/pressure: $\rho_\Lambda = 3\kappa^2/(8\pi G)$, $p_\Lambda = -\rho_\Lambda c^2$.

H, H_0 Hubble rate, present value.

$a(t), z$ Scale factor, redshift ($1+z = 1/a$ with $a_0 = 1$).

$\Omega_m, \Omega_r, \Omega_\Lambda, \Omega_k$ Density parameters (today).

$w(a), w_0, w_a$ Dark-energy equation-of-state and CPL params.

$D_C(z)$ Comoving distance $D_C(z) = \int_0^z \frac{c dz'}{H(z')}$.

D_A, D_L, μ Angular-diameter, luminosity distance; distance modulus $\mu = 5 \log_{10}(D_L/10 \text{ pc})$.

R_H, R_e, R_p Hubble radius c/H ; event horizon $\rightarrow c/\kappa$ (de Sitter); particle horizon (observable radius).

$D, \mathcal{N}$ Proper separation; number of clock units $\mathcal{N} = D/L^*$.

$g_{\mu\nu}, h_{ij}, N, N^i$ 4-metric; spatial 3-metric; lapse; shift (ADM).

$K_{ij}, R^{(3)}$ Extrinsic curvature; 3-Ricci scalar (ADM).

β, γ PPN parameters (TD/GR give $\beta = \gamma = 1$ in tested regimes).

Notation hygiene.

(1) We reserve χ for the TD gravity ratio; to avoid collision with cosmology, we use $D_C(z)$ for comoving distance (not $\chi(z)$). (2) We use $\mathcal{N} = D/L^*$ for “number of clock units”, reserving N for the ADM lapse. (3) Curvature index is written $k_{\text{geom}} \in \{-1, 0, +1\}$ to avoid confusion with the TD constant $k = 4G/c^2$. (4) Context disambiguates κ (TD residual rate) from BH “surface gravity” κ_{sg} ; we never use κ_{sg} here.

References

- Francis, O.O. Temporal Dynamics: For Space-Time and Gravity. *Preprints* **2025**. preprint, version 2 (posted 2025-05-02), <https://doi.org/10.20944/preprints202503.0453.v2>.
- Tiesinga, E.; Mohr, P.J.; Newell, D.B.; Taylor, B.N. The 2018 CODATA recommended values of the fundamental physical constants. *Reviews of Modern Physics* **2021**, *93*, 025010. <https://doi.org/10.1103/RevModPhys.93.025010>.
- Will, C.M. The Confrontation between General Relativity and Experiment. *Living Reviews in Relativity* **2018**, *21*. <https://doi.org/10.1007/s41114-018-0017-5>.
- Arnowitt, R.; Deser, S.; Misner, C.W. Republication of: The Dynamics of General Relativity. *General Relativity and Gravitation* **2008**, *40*, 1997–2027. <https://doi.org/10.1007/s10714-008-0661-1>.
- Wald, R.M. *General Relativity*; University of Chicago Press, 1984. <https://doi.org/10.7208/chicago/9780226870373.001.0001>.
- Misner, C.W.; Thorne, K.S.; Wheeler, J.A. *Gravitation*; W. H. Freeman (1973); reprint Princeton Univ. Press (2017), 1973.
- Schwarzschild, K. On the Gravitational Field of a Point-Mass, According to Einstein's Theory. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften* **1916**, pp. 189–196, [[arXiv:physics.hist-ph/physics/9905030](https://arxiv.org/abs/physics/9905030)]. English translation: [arXiv:physics/9905030](https://arxiv.org/abs/physics/9905030), <https://doi.org/10.48550/arXiv.physics/9905030>.
- Dyson, F.W.; Eddington, A.S.; Davidson, C. A Determination of the Deflection of Light by the Sun's Gravitational Field. *Philosophical Transactions of the Royal Society A* **1920**, *220*, 291–333. <https://doi.org/10.1098/rsta.1920.0009>.
- Pound, R.V.; Rebka, G.A. Apparent Weight of Photons. *Physical Review Letters* **1960**, *4*, 337–341. <https://doi.org/10.1103/PhysRevLett.4.337>.

10. Shapiro, I.I. Fourth Test of General Relativity. *Physical Review Letters* **1964**, *13*, 789–791. <https://doi.org/10.1103/PhysRevLett.13.789>.
11. Abbott, B.P.; et al. Observation of Gravitational Waves from a Binary Black Hole Merger. *Physical Review Letters* **2016**, *116*, 061102. <https://doi.org/10.1103/PhysRevLett.116.061102>.
12. Aghanim, N.; et al. Planck 2018 Results. VI. Cosmological Parameters. *Astronomy & Astrophysics* **2020**, *641*, A6, [arXiv:astro-ph.CO/1807.06209]. <https://doi.org/10.1051/0004-6361/201833910>.
13. Scolnic, D.; et al. The Pantheon+ Analysis: Cosmological Constraints. *The Astrophysical Journal* **2022**, *938*, 113, [arXiv:astro-ph.CO/2112.03863]. <https://doi.org/10.3847/1538-4357/ac8b1d>.
14. Francis, O.O. Temporal Conundrums: Navigating the Gravitational Time of Flow. *Preprints* **2024**. preprint, version 1 (posted 2024-10-24), <https://doi.org/10.20944/preprints202410.1755.v1>.
15. Adame, A.G.; et al. DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations. *arXiv e-prints* **2024**, [arXiv:astro-ph.CO/2404.03002].
16. Adame, A.G.; et al. DESI 2024 III: Baryon Acoustic Oscillations from Galaxies and Quasars. *arXiv e-prints* **2024**, [arXiv:astro-ph.CO/2404.03000].
17. Chevallier, M.; Polarski, D. Accelerating Universes with Scaling Dark Matter. *International Journal of Modern Physics D* **2001**, *10*, 213–224, [gr-qc/0009008]. <https://doi.org/10.1142/S0218271801000822>.
18. Linder, E.V. Exploring the Expansion History of the Universe. *Physical Review Letters* **2003**, *90*, 091301, [astro-ph/0208512]. <https://doi.org/10.1103/PhysRevLett.90.091301>.

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