Article

Mathematical Modeling of Seepage Velocity Influence on Transport of Dissolved Heavy Metals from Waste-Dumps to River Channel

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Abstract: Increasing rate of seepage velocity from several formation characteristics, such as permeability and porosity, in water aquifer environment greatly prompt pollution of water reservoirs within a short period of time. Considerably, migration rate of dissolved heavy metals from Solid Waste Dumpsites (SWD), such as municipal dumpsites and landfills, through heterogeneous aquifer environment, and finally into nearby water reservoirs are mainly influenced by variation of seepage velocity within the soil and water environment. This presents a dynamic system for water pollution that was studied using a formulated mathematical model to describe the transport process of dissolved heavy metals, mainly characterized by seepage velocities, within the water aquiferous environment. Permeability, porosity, fluid pressure and concentration of heavy metals in aquiferous environment were used as principal parameters that influence seepage velocity of the metals, in dissolved state, through the structural formation of water aquifers. The derived mathematical equations that constitute the model of this study were generated through Darcy's law and the equation of continuity. The model was validated on structural river aquifer sediments, and it was solved using graphical method through matlab open-source software. The initial and boundary conditions were obtained by discretizing the geological setting of flow region so as to transform the gradient of the head, $\frac{\partial h}{\partial x}$ into the time domain.

Keywords: seepage velocity; heavy metals; reservoirs pollution

1. Introduction

Industrial activities and natural environment conditions have led to the introduction of heavy metals into soil and aquatic environment as a result of anthropogenic and gynogenic sources, respectively (Van W. *et al*., 2018). Omoko *et al.,* (2015) provides that common dissolved heavy metals in municipal wastes include cadmium, copper, lead, nickel and zinc. These metals are hazardous to human and aquatic lives. (Van Wezel, A. P, *et al.* 2018)

The movement of heavy metals in water aquiferous environment is restricted by partitioning reactions to aquifer sediments. Probable techniques that influence this partitioning to sub-surface aquiferous water include direct adsorption to clay minerals, adsorption and co-precipitation with metal oxides, combinations with natural organic compounds, ion exchange with charged surfaces, and direct precipitation as carbonates, hydroxides or sulfides. The extent of chemical speciation of heavy metals in solution exerts a significant influence on the extent and mechanism of partitioning to aquifer sediments, which may be influenced by oxidation-reduction reactions and acid-base reactions that influence the speciation of competing organic solution species (for example, aqueous sulfate verses sulfide), and interactions with dissolved organic compounds. In general, organic or inorganic species that form dissolved complexes (for example, dissolved carbon) with heavy metals tend to enhance their transport in soil profiles to sub-surface water (Zhang, Z.Z *et al*., 2019).

Field studies on transport in sub-surface water systems illustrate several general conditions that are anticipated to result in expanding heavy metal plumes, including: first, acidic conditions (Kloller, 2004); second, Manganese-reducing and ion-reducing conditions (Blowes *et al.*, 2000); and third, the presence of mobile organic compounds that form soluble heavy metal complexes (Lekula, M. and Lubczynski, M.W., 2019).).

Possible production concept that can be used for remediation of groundwater plume containing most heavy metals include physical removal of polluted soils or sediments that serve as a long-term source of heavy metal leached into groundwater, extraction of the dissolved plume with some method of treatment above groundwater, physical isolation of the dissolved plume, or in-site treatment of a dissolved plume resulting in mobilization of dissolved heavy metals within the aquifer. Of these technologies, the use of permeable reactive barriers (PRBs) for the capture and immobilization of heavy metal plumes has been investigated and applied in field settings due to favorable performance and cost characteristics (Naranjo-Fernández, N. *et al.*, 2018).

2. Background

Besides the chemical studies, the core aspects in the physical studies leverage on migration rates of dissolved heavy metals from potential solid waste dumpsites such as municipal dumpsites, through heterogeneous aquifer environment, and finally into nearby water reservoirs. In this study these migration rates have been attributed mainly to variation of seepage velocity within the soil and water environment. In municipal set-ups, these migration rates of dissolved pollutants from solid dumpsites into nearby water reservoirs remain a serious concern to environmental experts. (Stuyfzand, P. J. *et al.*, 2018).

Thus, the rate of seepage velocity greatly influences the transport of hazardous heavy metals such as nickel, copper, cadmium, lead and zinc which readily dissolve in water rendering them more vulnerable in water flow systems, both groundwater and surface water. Moreover, dissolved heavy metals regenerate from solid waste dumpsites due to continuous dumping of solid and aqueous wastes from municipal dumpsites and landfills. (Hameed, M.R. *et al.*, 2019).

In recognition of potential for heavy metal pollution on water reservoirs, both groundwater and surface water, it is essential that predictive analyzes are made routinely to monitor the rate of transport of ionic content of these metals into water reservoirs. Common sources of heavy metals in water pathways include majorly from solid waste dumpsites and landfills, agricultural fertilizers and pesticides, organic manures, urban and industrial wastes, metallurgical industries, mining and smelting of non-ferrous metals. (Nonterah, C., 2020).

Every day, local, state and international agencies (such as Environmental Protection Agency EPA in USA, World Health Organization (WHO), Kenya ministry of water and irrigation, NEMA) are taking steps to increase water-quality monitoring both for groundwater and surface water. The main objective of these agencies is to promote hydrologic practices that continuously refine analytical techniques on water sources so that early warning can be given, and plans can be implemented to mitigate or prevent water quality hazards. Three important tasks are worthy of brief mention among the principal aims of modern-day hydrologic practices in achieving proper utilization of water systems. First, to continuously evaluate water-quality conditions so as to curb water pollution; second, to predict the capacity of the water resource for long-term supply; third, for prudent management of water resource aimed at minimizing wastage and promoting recycling (Omoko B, 2015).

To monitor the rate of transport of dissolved heavy metals from municipal dumpsites into groundwater aquifers the concept of seepage velocity is imperative because knowing the rate of metal deposition in the environment where wastes are deposited will make the experts ensure that risk assessments are carried out (Kar D. *et al.*, 2008). This predictive concept used along with the existing groundwater models will integrate factors of safety in design to prevent water pollution from municipal dump-sites and other possible potential sources of heavy-metal pollution. The course of this mathematical modeling will widen knowledge on techniques always ventured in the direction of reducing water-related diseases. (Karimi, L., 2019).

So as to focus on the influence of seepage velocity on migration rates of heavy metal pollutants in water aquifer systems, the following theoretical background is necessary.

The volume flux is defined as

$$q = \frac{Q}{A} \tag{1}$$

Where, $q =$ Darcy flux (m/s), $A =$ Cross-sectional area (m^2), $Q =$ Volumetric flow rate $(m^3 s$ and. The volume flux is the volumetric flow rate per unit area. Then, Darcy law is

$$q = -k\frac{dh}{dl} \tag{2}$$

Where $h =$ head constant (or hydraulic head), $l =$ length of the sample (or medium). The hydraulic conductivity, K is related to permeability, k by equation:

$$K = \frac{\rho g}{\mu}k \tag{3}$$

where, $k =$ permeability (m^2), $\mu =$ viscosity of the fluid (Nsm^{-2}), $K =$ hydraulic conductivity (m/s), and

$\rho =$ density of the fluid, $(kgm^{-3}$). (Karimi, L., 2019).

The seepage velocity v is equal to the volume flux divided by porosity (n) and is given as:

$$v = \frac{Q}{An} = \frac{q}{n} \tag{4}$$

3. Governing Equations

Combining equations (2), (3) and (4), the following equation (5) is obtained in terms of aquifers sediments variables.

$$v_j\frac{\partial m}{\partial x_j} = k_j\frac{Q_j}{n} = k_j\left(\frac{h}{n}\frac{\partial m}{\partial x_j}\right) \tag{5}$$

Where,v_j is the seepage velocity; m is the heavy metal concentration; Q_j is the discharge velocity; k_j is the hydraulic conductivity; h is pressure head; n is the porosity of aquiferous soil; and x_j is the horizontal flow displacement.

3.1. Model Development

Equation (5) expresses the concentration of heavy metal content in aquiferous zone under the influence of seepage velocity and permeability in river bank systems near landfills and municipal waste dumpsites (Hameed, M.R , 2019). In order to predict the heavy metal concentration in river bank system, the task was set to modify the governing equation (5) to include variables that influence the flow-rate of water through the aquifer. These variables are mainly slope (s) of bank, seepage velocity(v), and intrinsic permeability (k_i).

The following initial and boundary conditions were set by discretizing the aquiferous zone on the study locations.

$$m(0) = m_{(o)} \tag{6}$$

$$\frac{\partial m}{\partial x_j} = sm_{(x)} - m_{(0)} \tag{7}$$

Laplace transformations were taken so as to express the variables in terms relating themselves to each other in the aquiferous system and also to introduce the time domain (Jaxa-Rozen, *et al.*, 2019) . Substituting equations (5) and (7) into equation (5) yields

$$v_j s m_{(t)} - v_j s m'_{(t)} - m_{(0)} = \frac{Q_j}{\varepsilon} k_j \left(\frac{h}{n} s m_{(0)}\right) - m_{(0)} \tag{8}$$

Considering the initial condition at $t = 0$

$$m'_{(0)} = m_{(0)} = 0 \tag{9}$$

Inserting equation (9) into equation (8) yields

$$m_{(x)} \left[s v_j - v_s - \frac{Q_j}{n} k_j \left(\frac{h}{n} s\right)\right] = 0 \tag{10}$$

$$m_{(t)} \neq 0 \tag{11}$$

Considering boundary condition, at $t > 0$

$$m'_{(0)} = m_{(0)} = m_0 = 0 \tag{12}$$

Equation (8) is the established boundary condition in the aquiferous system. The equation provides a condition of constant concentration for the period of flow in the aquifer. Thus, there is constant regeneration of heavy metal pollutant from the source, then through the soil aquifer and finally into water reservoir. The most probable parameter that influences this condition is low permeability of soil aquiferous environment.

$$s m_{(x)} - \frac{Q_j}{n} k_j \left(\frac{h}{n} s_{(x)}\right) - v_j s m_0 + v_j m_0 + \frac{Q_j}{n} k_j \left(\frac{h}{n} m\right) \tag{13}$$

$$[v_j s - \frac{Q_j}{n} k_j \left(\frac{h}{n} s\right)] m_{(x)} = [v_j s + v_j \frac{Q_j}{n} k_j \left(\frac{h}{n} s\right)] m_0 \tag{14}$$

$$m_{(x)} = \frac{v_j + \ v_j \frac{Q_j}{n} k_j \left(\frac{h}{n}\right)}{v_j - \ \frac{Q_j}{n} k_j \left(\frac{h}{n}\right)} m_0 \tag{15}$$

Equation (15) proportionately expresses two distinct roots of a symmetric quadratic function. To propagate the quadratic function, let

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{16}$$

Where, $a = v_j, b = \frac{Q_j}{n}$, c=$k_j \left(\frac{h}{n}\right)$. Then the roots λ_1 and λ_2 are given as

$$\lambda_{1,2} = \frac{\frac{Q_j}{n} \pm \sqrt{(\frac{Q_j}{n})^2 - \ 4 v_j k_j \left(\frac{h}{n}\right)}}{2 v_j} \tag{17}$$

For a physical system of flow, $\sqrt{(\frac{Q_j}{n})^2 - \ 4 v_j k_j \left(\frac{h}{n}\right)} \geq 0$ so that λ_1, λ_2 are real and distinct roots of equations (5) through boundary conditions in equations (6) and (7). Again, by symmetry of quadratic roots $\lambda_1 = \alpha \lambda_2$ for $\alpha \in \mathbb{R}$. Then, solution to (5) is

$$m_{(x)} = \beta_1 exp\left(\frac{\frac{Q_j}{n} + \left[\sqrt{(\frac{Q_j}{n})^2 - 4v_j k_j\left(\frac{h}{n}\right)}\right]^x}{2v_j}\right) + \beta_2 exp\left(\frac{\frac{Q_j}{n} - \left[\sqrt{(\frac{Q_j}{n})^2 - 4v_j k_j\left(\frac{h}{n}\right)}\right]^x}{2v_j}\right) \tag{18}$$

The initial condition at the onset of pollutant flow at dumpsite is set as,

$$t = 0, \quad m_0' = 0, \quad m_0 = 0 \tag{19}$$

Applying equations (6), (7), (12) and (19) in equation (18) and taking the inverse transform gives

$$m_{(t)} = (\frac{v_j}{t/_v} + v_j + \frac{Q_j}{n})m_0 \left\{exp\left(\frac{\frac{Q_j}{n} + \left[\sqrt{(\frac{Q_j}{n})^2 - 4v_j k_j\left(\frac{h}{n}\right)}\right]^{t/_v}}{2v_j}\right) + exp\left(\frac{\frac{Q_j}{n} - \left[\sqrt{(\frac{Q_j}{n})^2 - 4v_j k_j\left(\frac{h}{n}\right)}\right]^{t/_v}}{2v_j}\right)\right\} \tag{20}$$

where $x = {}^{t}/_{v}$.
At $m_0' = 0,\ t \neq 0$. Again, $m_0 = m_{(0)}$. Then,

$$m_0 = \left[v_j + \frac{Q_j}{n}\right] m_{(0)}[1 + 1] \quad \Rightarrow 0 = \left[0 + \frac{Q_j}{n}\right] \tag{21}$$

$$\frac{Q_j}{n} + \frac{Q_j}{n} = 0 \tag{22}$$

Leading to

$$m_{(t)} = \left[2\frac{v_j}{t/v}\right] m_0 \left\{exp\left(\frac{\frac{Q_j}{n} + \left[\sqrt{(\frac{Q_j}{n})^2 - 4v_j k_j\left(\frac{h}{n}\right)}\right]^{t/_v}}{2v_j}\right) + exp\left(\frac{\frac{Q_j}{n} - \left[\sqrt{(\frac{Q_j}{n})^2 - 4v_j k_j\left(\frac{h}{n}\right)}\right]^{t/_v}}{2v_j}\right)\right\} \tag{23}$$

Pollutant deposition in water reservoirs reflects inverse transformation of the aquiferous system. Sinusoidal expressions are known to offer these transformations. Using the identity: $e^y + e^{-y} = 2\cosh x$, $e^y + e^{-y} = 2\sinh y$ and $\tanh y = \frac{\cosh y}{\sinh y}$ equation (23) yields

$$m_{(t)} = \left[2\frac{v_j}{t/v}\right] m_0\, tanh\left(\frac{\frac{Q_j}{n} + \left[\sqrt{(\frac{Q_j}{n})^2 - 4v_j k_j\left(\frac{h}{n}\right)}\right]^{t/_v}}{2v_j}\right) \tag{24}$$

Equation (24) is the model equation that expresses concentration of heavy metal pollutant $m_{(t)}$ at any point in the aquifer in terms of seepage velocity(v_j), discharge velocity (Q_j), and initial concentration (m_0).

4. Validating Model Equation

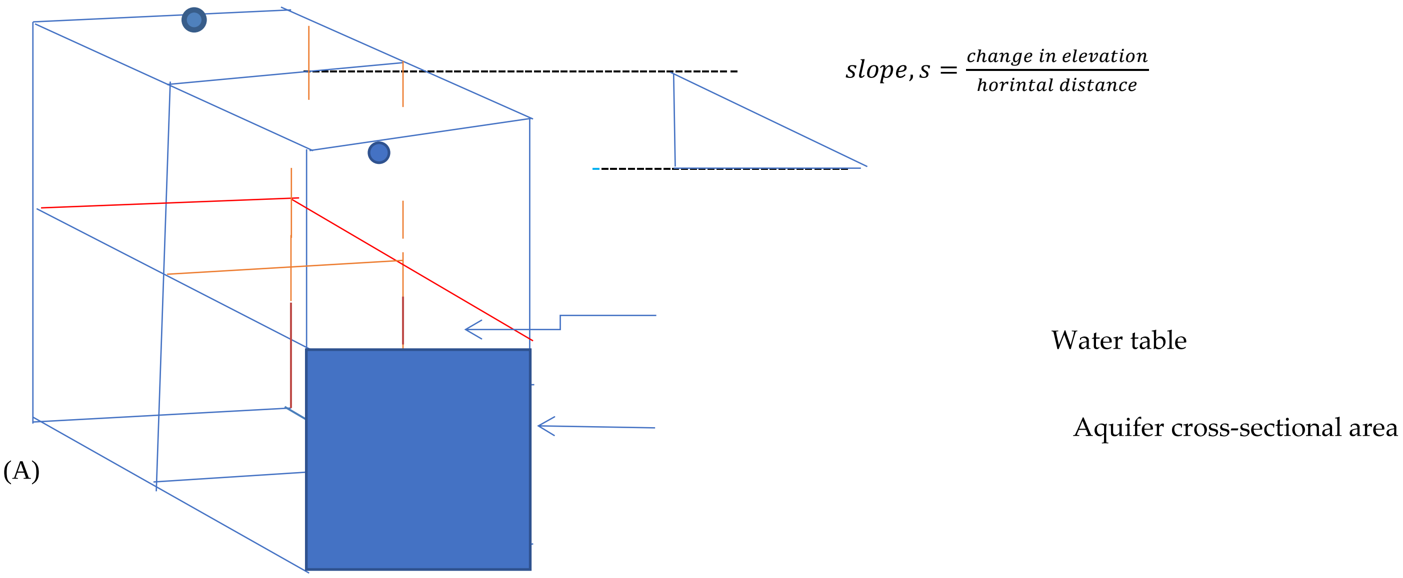

Figure 1. Discretizing the aquifer sediment.

$$Water\ Flux, Q = Aks \tag{25}$$

In an aquiferous system (river bank) identified with potential pollutant source such as a municipal waste-dumpsite of particular heavy metal, the following measurements were done.

Table 1. Data collected from municipal dumpsites: River Nyakomisaro and River Gucha in Kisii County, Kenya.

Quantity	River Nyakomisaro	River Gucha	**Units**
Slope (s)	0.48	0.60	–
Initial concentration (m_0)	0.46	0.77	ppm
Discharge velocity (Q_j)	32	44	–
Hydraulic conductivity (k)	0.12	0.25	m/s
Porosity (n)	0.62	0.45	–
Cross-sections (A)	0.04	0.25	m^2

5. Results

So as to observe the influence of seepage velocity on migration of pollutant in the soil (river bank) aquiferous environment, a graph of v_j against concentration $m(t)$ is desirable. However, it is difficult to directly make v_j in equation (24) the subject of the equation. Thus, by introducing an arbitrary function $f(v_j)$, model equation (24) is modified to the following equation (25).

$$f(v_j) = \left[2\frac{v_j}{t/v_j}\right] m_0\ tanh\left(\frac{\frac{Q_j}{n} + \left[\sqrt{\left(\frac{Q_j}{n}\right)^2 - 4v_j k_j \left(\frac{h}{n}\right)}\right]^{t/v_j}}{2v_j}\right) - m_{(t)} \tag{26}$$

A graph of $f(v_j)$ against v_j is then used to approximate the value of v_j when $f(v_j) = 0$.

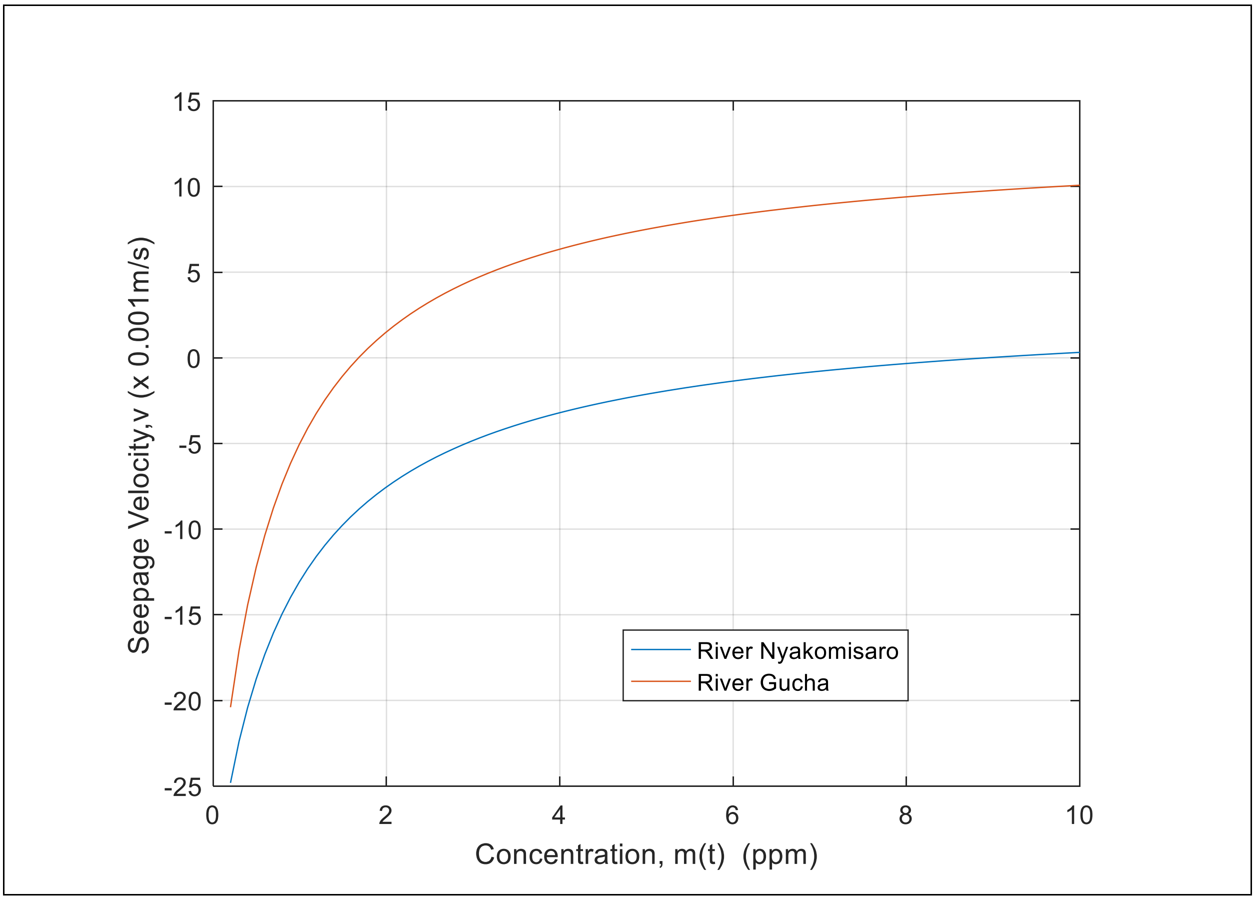

Figure 2. Plot of Seepage Velocity ($\boldsymbol{v}$) versus Concentration $\boldsymbol{m(t)}$ of Pollutant

The peak of seepage velocity on graph (4.1) with data from river Nyakomisaro is 9.8 m/s while that from river Gucha is $1.2m/s$. This comparison provides that seepage velocity is greater at aquifer sediments with high porosity, steep slopes, and high hydraulic conductivity (see Table 3.1 for collected and compared data). These properties greatly affect seepage velocity which in return influences particle transport in the aquifer media.

Recommendation: More aquifer locations (sediments) are recommended for model validation. It is also important to note that the on-site parameter values were obtained through physical measurements while some were retrieved from relevant existing literature. Experimental approach is recommended to obtain more certain and reliable measurements from validation aquifer sediments.

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