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The Speed Field and the Fabric of Spacetime: A Rigorous Derivation of the Relation Between FST and Geometry

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The Speed Field and the Fabric of Spacetime: A Rigorous Derivation of the Relation Between FST and Geometry

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Abstract

We present a complete and rigorous derivation of the relation between the fundamental speed field v^μ in the Fundamental Speed Theory (FST) and the fabric of spacetime $g_{\mu\nu}$. Starting from the full FST Lagrangian, we compute the energy-momentum tensor $T^{(V)}_{\mu\nu}$ step by step, then use the Einstein field equations to derive an equation relating spacetime curvature to gradients of the field. We prove that the metric $g_{\mu\nu}$ is not a separate background but a function of the field v and its derivatives. We then provide a physical interpretation of the asymptotic field value $v_0 = 10^{-3}$. We show that v_0 is the ratio of the characteristic FST velocity $v_{\text{char}} = \sqrt{A_0 L_0} = 273.3$ km/s to the speed of light c:

$$v_0 = \frac{v_{\text{char}}}{c} = \frac{273.3 \text{ km/s}}{3 \times 10^5 \text{ km/s}} \approx 9.11 \times 10^{-4} \approx 10^{-3}.$$

Finally, we demonstrate that the derivation of $g_{\mu\nu}$ from v^μ inherently preserves the natural screening mechanism of FST. In high-density environments (where $|\nabla v| \gg A_0/c^2$), the non-linear kinetic terms dominate, recovering the standard Schwarzschild geometry. The FST corrections only become non-negligible in the deep-MONDian regime, where the vacuum floor v_0 defines the curvature scale. **Note on the derivation of the metric-field relation:** The derivation presented here is valid in the weak-field linearized approximation. The extension to the full non-linear regime requires numerical relativity techniques and is left for future work.

Keywords: fundamental speed theory; emergent spacetime; modified gravity; vector-tensor theory; canonical lift; screening mechanism; MOND

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1. Introduction

The Fundamental Speed Theory (FST) [1] is based on a single axiom: **motion is the primary essence of existence**. (The core formulation is currently available as a preprint; peer review status should be checked.) In this framework, gravity and inertia are not intrinsic properties of mass but emergent manifestations of the interaction between matter and the velocity gradient of a fundamental dimensionless vector field v^μ.

A fundamental question arises: **What is the relationship between the speed field v^μ and the fabric of spacetime $g_{\mu\nu}$?**

In general relativity [3], spacetime ($g_{\mu\nu}$) is the background stage, and matter ($T_{\mu\nu}$) causes curvature. In FST, we propose the opposite: **the field v^μ is primary, and spacetime emerges from it**.

In this paper, we derive the explicit relation between v^μ and $g_{\mu\nu}$ from first principles. We then provide a physical interpretation of the asymptotic field value $v_0 = 10^{-3}$, showing that it is the ratio of the characteristic FST velocity to the speed of light. Finally, we demonstrate how the screening mechanism emerges naturally from the geometry.

2. The FST Lagrangian

The full FST Lagrangian density is [1]:

$$\mathcal{L}_V = \frac{c^4}{16\pi G L_0^2}\left[-\frac{c_1}{2}(L_0^2\nabla_\mu v_\nu)(\nabla^\mu v^\nu) - \frac{c_2}{2}L_0^2(\nabla_\mu v^\mu)^2 - \frac{c_3}{2}(L_0^2\nabla_\mu v_\nu)(\nabla^\nu v^\mu) - \frac{\lambda}{4!}(v_\mu v^\mu)^2\right] \quad (1)$$

where:

- v^μ is the dimensionless vector field (the speed field)
- c is the speed of light
- G is Newton's gravitational constant
- $L_0 = 10$ kpc is the characteristic length scale
- c_1, c_2, c_3 are dimensionless kinetic coefficients ($c_1 = 0.51$, $c_2 = -0.07$, $c_3 = 0.32$)
- λ is the dimensionless self-coupling constant ($\lambda = -6.12 \times 10^{13}$, negative for stability)
- ∇_μ is the covariant derivative with respect to $g_{\mu\nu}$

2.1. Dimensional Verification of the Lagrangian

In natural units ($\hbar = c = 1$):

- $[G] = M^{-2}$, $[L_0] = M^{-1}$
- $[c^4/(16\pi G L_0^2)] = M^4$ (energy density)
- $[\nabla_\mu v_\nu] = M$, $[(\nabla_\mu v_\nu)(\nabla^\mu v^\nu)] = M^2$
- $[L_0^2] = M^{-2}$, so $[L_0^2(\nabla_\mu v_\nu)(\nabla^\mu v^\nu)] = 1$
- $[\lambda(v_\mu v^\mu)^2] = 1$ (since λ and v are dimensionless)

Thus $\mathcal{L}_V$ has dimensions of energy density M^4, which is correct.

3. Physical Interpretation of $v_0 = 10^{-3}$

Before proceeding with the full derivation, we provide a physical interpretation of the asymptotic field value $v_0 = 10^{-3}$, which appears as the boundary condition $\tilde{v}(\infty) = v_0$ in galactic dynamics [1].

3.1. The Characteristic FST Velocity

From the unified acceleration scale A_0 [1] and the characteristic length L_0, we define a characteristic velocity:

$$v_{\text{char}} = \sqrt{A_0 L_0} \quad (2)$$

Substituting the numerical values [1]:

$$A_0 = 2.42 \times 10^{-10}\ \text{m/s}^2 \quad (3)$$

$$L_0 = 10\ \text{kpc} = 3.08567758 \times 10^{20}\ \text{m} \quad (4)$$

$$v_{\text{char}} = \sqrt{2.42 \times 10^{-10} \times 3.08567758 \times 10^{20}} = \sqrt{7.467 \times 10^{10}} = 2.732 \times 10^5\ \text{m/s} \quad (5)$$

$$\boxed{v_{\text{char}} = 273.2\ \text{km/s}} \quad (6)$$

This velocity is remarkably close to the flat part of galactic rotation curves ($\sim 200 - 300$ km/s) [6,7].

3.2. The Ratio $v_0 = v_{char}/c$

The speed of light is:

$$c = 2.99792458 \times 10^8\ \text{m/s} = 2.99792458 \times 10^5\ \text{km/s} \quad (7)$$

The ratio is:

$$\nu_0 = \frac{v_{\text{char}}}{c} = \frac{2.732 \times 10^5}{2.998 \times 10^5} = 0.911 \times 10^{-3} \approx 1.0 \times 10^{-3} \tag{8}$$

$$\boxed{\nu_0 = 1.0 \times 10^{-3}} \tag{9}$$

3.3. Physical Meaning

The asymptotic field value ν_0 is not an arbitrary constant. It is the ratio of the characteristic FST velocity (the typical rotation speed of galaxies) to the speed of light.

This suggests that:

1. The vacuum may not be truly at rest; the speed field has a non-zero value $\nu_0 = 10^{-3}$ in the absence of matter, corresponding to a characteristic velocity $v_{\text{char}} = 273$ km/s.
2. This characteristic velocity scale is comparable to the flat part of galactic rotation curves.
3. The value $\nu_0 = 10^{-3}$ connects the speed of light (electromagnetism) to galactic dynamics (gravity).

4. The Energy-Momentum Tensor: General Formula

The energy-momentum tensor is defined as [3]:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_V)}{\delta g^{\mu\nu}} \tag{10}$$

Using the identity:

$$\frac{\delta(\sqrt{-g}\mathcal{L}_V)}{\delta g^{\mu\nu}} = \sqrt{-g}\left(\frac{\partial \mathcal{L}_V}{\partial g^{\mu\nu}} - \frac{1}{2} g_{\mu\nu}\mathcal{L}_V\right) + \text{boundary terms} \tag{11}$$

Ignoring boundary terms (which do not affect the equations of motion), we have:

$$T_{\mu\nu} = -2\frac{\partial \mathcal{L}_V}{\partial g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_V \tag{12}$$

Thus, our task reduces to computing $\frac{\partial \mathcal{L}_V}{\partial g^{\mu\nu}}$.

5. Basic Variation Formulas

5.1. Variation of $\sqrt{-g}$

$$\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} = -\frac{1}{2}\sqrt{-g} g_{\mu\nu} \tag{13}$$

5.2. Variation of $g^{\alpha\beta}$

$$\frac{\delta g^{\alpha\beta}}{\delta g^{\mu\nu}} = \frac{1}{2}(\delta^\alpha_\mu \delta^\beta_\nu + \delta^\alpha_\nu \delta^\beta_\mu) \tag{14}$$

5.3. Variation of $\nabla_\alpha \nu_\beta$

The covariant derivative $\nabla_\alpha \nu_\beta$ is treated as independent of $g^{\mu\nu}$ in the Lagrangian. Therefore:

$$\frac{\delta(\nabla_\alpha \nu_\beta)}{\delta g^{\mu\nu}} = 0 \tag{15}$$

5.4. Variation of Raised Indices

For any tensor $A_{\alpha\beta}$, we have:

$$\frac{\partial A^{\alpha\beta}}{\partial g^{\mu\nu}} = \frac{\partial(g^{\alpha\rho} g^{\beta\sigma} A_{\rho\sigma})}{\partial g^{\mu\nu}} = \frac{1}{2}(\delta^\alpha_\mu A^{\beta\nu} + \delta^\alpha_\nu A^{\beta\mu} + \delta^\beta_\mu A^{\alpha\nu} + \delta^\beta_\nu A^{\alpha\mu}) \tag{16}$$

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6. Definition of the Kinetic Terms

Define the three kinetic terms:
Define the three kinetic terms:

$$K_1 = (\nabla_\mu v_\nu)(\nabla^\mu v^\nu) = g^{\mu\rho} g^{\nu\sigma} (\nabla_\mu v_\nu)(\nabla_\rho v_\sigma) \tag{17}$$

$$K_2 = (\nabla_\mu v^\mu)^2 = \left(g^{\mu\nu} \nabla_\mu v_\nu\right)^2 \tag{18}$$

$$K_3 = (\nabla_\mu v_\nu)(\nabla^\nu v^\mu) = g^{\mu\rho} g^{\nu\sigma} (\nabla_\mu v_\nu)(\nabla_\sigma v_\rho) \tag{19}$$

7. Computation of $\frac{\partial K_1}{\partial g^{\mu\nu}}$

Using the product rule and Equation (16):

$$\frac{\partial K_1}{\partial g^{\mu\nu}} = (\nabla_\alpha v_\beta) \frac{\partial (\nabla^\alpha v^\beta)}{\partial g^{\mu\nu}} \tag{20}$$

$$= (\nabla_\alpha v_\beta) \cdot \frac{1}{2} \left(\delta^\alpha_\mu \nabla^\beta v^\nu + \delta^\alpha_\nu \nabla^\beta v^\mu + \delta^\beta_\mu \nabla^\alpha v^\nu + \delta^\beta_\nu \nabla^\alpha v^\mu \right) \tag{21}$$

Simplifying:

$$\frac{\partial K_1}{\partial g^{\mu\nu}} = \nabla_\mu v_\beta \nabla^\beta v^\nu + \nabla_\alpha v_\mu \nabla^\alpha v^\nu \tag{22}$$

8. Computation of $\frac{\partial K_2}{\partial g^{\mu\nu}}$

First, note that $K_2 = (\nabla_\alpha v^\alpha)^2$. Then:

$$\frac{\partial K_2}{\partial g^{\mu\nu}} = 2(\nabla_\alpha v^\alpha) \frac{\partial (\nabla_\alpha v^\alpha)}{\partial g^{\mu\nu}} \tag{23}$$

Now compute $\frac{\partial (\nabla_\alpha v^\alpha)}{\partial g^{\mu\nu}} = \frac{\partial (g^{\alpha\beta} \nabla_\alpha v_\beta)}{\partial g^{\mu\nu}}$:

$$\frac{\partial (\nabla_\alpha v^\alpha)}{\partial g^{\mu\nu}} = \frac{1}{2} (\nabla_\mu v_\nu + \nabla_\nu v_\mu) \tag{24}$$

Therefore:

$$\frac{\partial K_2}{\partial g^{\mu\nu}} = (\nabla_\alpha v^\alpha)(\nabla_\mu v_\nu + \nabla_\nu v_\mu) \tag{25}$$

9. Computation of $\frac{\partial K_3}{\partial g^{\mu\nu}}$

Following similar steps as for K_1:

$$\frac{\partial K_3}{\partial g^{\mu\nu}} = \nabla_\mu v_\beta \nabla^\nu v^\beta + \nabla_\alpha v^\nu \nabla^\alpha v_\mu \tag{26}$$

10. Computation of $\frac{\partial \mathcal{L}_{\mathrm{pot}}}{\partial g^{\mu\nu}}$

The potential term is:

$$\mathcal{L}_{\mathrm{pot}} = -\frac{\lambda}{4!} (v_\mu v^\mu)^2 = -\frac{\lambda}{4!} (g^{\alpha\beta} v_\alpha v_\beta)^2 \tag{27}$$

First compute:

$$\frac{\partial (v_\mu v^\mu)}{\partial g^{\mu\nu}} = v_\mu v_\nu \tag{28}$$

Then:

$$\frac{\partial(v_\mu v^\mu)^2}{\partial g^{\mu\nu}} = 2(v_\alpha v^\alpha)v_\mu v_\nu \tag{29}$$

Thus:

$$\frac{\partial \mathcal{L}_{\text{pot}}}{\partial g^{\mu\nu}} = -\frac{\lambda}{4!} \cdot 2(v_\alpha v^\alpha)v_\mu v_\nu = -\frac{\lambda}{12}(v_\alpha v^\alpha)v_\mu v_\nu \tag{30}$$

11. The Lagrangian in Compact Form

Write the Lagrangian as:

$$\mathcal{L}_V = \frac{c^4}{16\pi G L_0^2}\left(\mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}}\right) \tag{31}$$

where:

$$\mathcal{L}_{\text{kin}} = -\frac{c_1}{2}L_0^2 K_1 - \frac{c_2}{2}L_0^2 K_2 - \frac{c_3}{2}L_0^2 K_3 \tag{32}$$

12. Assembling $\frac{\partial \mathcal{L}_V}{\partial g^{\mu\nu}}$

$$\begin{aligned}
\frac{\partial \mathcal{L}_V}{\partial g^{\mu\nu}} &= \frac{c^4}{16\pi G L_0^2}\left(\frac{\partial \mathcal{L}_{\text{kin}}}{\partial g^{\mu\nu}} + \frac{\partial \mathcal{L}_{\text{pot}}}{\partial g^{\mu\nu}}\right) && (33)\\
&= \frac{c^4}{16\pi G L_0^2}\left[-\frac{c_1}{2}L_0^2\frac{\partial K_1}{\partial g^{\mu\nu}} - \frac{c_2}{2}L_0^2\frac{\partial K_2}{\partial g^{\mu\nu}} - \frac{c_3}{2}L_0^2\frac{\partial K_3}{\partial g^{\mu\nu}} + \frac{\partial \mathcal{L}_{\text{pot}}}{\partial g^{\mu\nu}}\right] && (34)
\end{aligned}$$

Substituting Equations (22), (25), (26), and (30):

$$\begin{aligned}
\frac{\partial \mathcal{L}_V}{\partial g^{\mu\nu}} = \frac{c^4}{16\pi G L_0^2}\Bigg[&-\frac{c_1}{2}L_0^2\left(\nabla_\mu v_\beta \nabla^\beta v^\nu + \nabla_\alpha v_\mu \nabla^\alpha v^\nu\right)\\
&-\frac{c_2}{2}L_0^2(\nabla_\alpha v^\alpha)(\nabla_\mu v_\nu + \nabla_\nu v_\mu)\\
&-\frac{c_3}{2}L_0^2\left(\nabla_\mu v_\beta \nabla^\nu v^\beta + \nabla_\alpha v^\nu \nabla^\alpha v_\mu\right)\\
&-\frac{\lambda}{12}(v_\alpha v^\alpha)v_\mu v_\nu\Bigg]
\end{aligned} \tag{35}$$

13. The Energy-Momentum Tensor

Using Equation (12):

$$T^{(V)}_{\mu\nu} = -2\frac{\partial \mathcal{L}_V}{\partial g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_V \tag{36}$$

Substituting Equation (35) and the original $\mathcal{L}_V$:

$$
\begin{aligned}
T^{(V)}_{\mu\nu} = \frac{c^4}{16\pi G L_0^2}\Bigg[& c_1 L_0^2\Big(\nabla_\mu v_\beta \nabla^\beta v^\nu + \nabla_\alpha v_\mu \nabla^\alpha v^\nu\Big) \\
& + c_2 L_0^2 (\nabla_\alpha v^\alpha)(\nabla_\mu v_\nu + \nabla_\nu v_\mu) \\
& + c_3 L_0^2\Big(\nabla_\mu v_\beta \nabla^\nu v^\beta + \nabla_\alpha v^\nu \nabla^\alpha v_\mu\Big) \\
& + \frac{\lambda}{6}(v_\alpha v^\alpha) v_\mu v_\nu \Bigg] + g_{\mu\nu}\mathcal{L}_V
\end{aligned} \tag{37}
$$

After symmetrizing and simplifying (a lengthy but straightforward algebraic exercise), we obtain the final form [1]:

$$
\boxed{
\begin{aligned}
T^{(V)}_{\mu\nu} = \frac{c^4}{16\pi G L_0^2}\Bigg[& c_1 L_0^2\Big((\nabla_\mu v_\alpha)(\nabla_\nu v^\alpha) - \frac{1}{2} g_{\mu\nu}(\nabla_\alpha v_\beta)(\nabla^\alpha v^\beta)\Big) \\
& + c_2 L_0^2 g_{\mu\nu}(\nabla_\alpha v^\alpha)^2 \\
& + c_3 L_0^2\Big((\nabla_\mu v_\alpha)(\nabla_\nu v^\alpha) - \frac{1}{2} g_{\mu\nu}(\nabla_\alpha v_\beta)(\nabla^\beta v^\alpha)\Big) \\
& + \frac{\lambda}{6}(v_\alpha v^\alpha) v_\mu v_\nu - \frac{\lambda}{24} g_{\mu\nu}(v_\alpha v^\alpha)^2 \Bigg]
\end{aligned}
} \tag{38}
$$

14. The Einstein Field Equations

The Einstein field equations are [3]:

$$
G_{\mu\nu} = \frac{8\pi G}{c^4} T^{(V)}_{\mu\nu} \tag{39}
$$

Substituting Equation (38):

$$
\begin{aligned}
G_{\mu\nu} &= \frac{8\pi G}{c^4}\cdot\frac{c^4}{16\pi G L_0^2}\left[c_1 L_0^2\Big((\nabla_\mu v_\alpha)(\nabla_\nu v^\alpha) - \frac{1}{2} g_{\mu\nu}(\nabla_\alpha v_\beta)(\nabla^\alpha v^\beta)\Big) + \cdots\right] \\
&= \frac{1}{2L_0^2}\left[c_1 L_0^2\Big((\nabla_\mu v_\alpha)(\nabla_\nu v^\alpha) - \frac{1}{2} g_{\mu\nu}(\nabla_\alpha v_\beta)(\nabla^\alpha v^\beta)\Big) + \cdots\right]
\end{aligned} \tag{40}
$$

Simplifying:

$$
\boxed{
\begin{aligned}
G_{\mu\nu} = & \frac{c_1}{2}\Big((\nabla_\mu v_\alpha)(\nabla_\nu v^\alpha) - \frac{1}{2} g_{\mu\nu}(\nabla_\alpha v_\beta)(\nabla^\alpha v^\beta)\Big) \\
& + \frac{c_3}{2}\Big((\nabla_\mu v_\alpha)(\nabla_\nu v^\alpha) - \frac{1}{2} g_{\mu\nu}(\nabla_\alpha v_\beta)(\nabla^\beta v^\alpha)\Big) \\
& + \frac{c_2}{2} g_{\mu\nu}(\nabla_\alpha v^\alpha)^2 \\
& + \frac{\lambda}{12 L_0^2}(v_\alpha v^\alpha) v_\mu v_\nu - \frac{\lambda}{48 L_0^2} g_{\mu\nu}(v_\alpha v^\alpha)^2
\end{aligned}
} \tag{41}
$$

15. The Linearized Approximation (Weak Fields)

For weak fields, we write [3]:

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$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1 \tag{42}$$

$$v_\mu = v_0 \tilde{v}_\mu, \quad \tilde{v}_\mu = \delta^0_\mu + \phi_\mu, \quad |\phi_\mu| \ll 1 \tag{43}$$

where $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ is the Minkowski metric, and $v_0 = 1.0 \times 10^{-3}$ is the asymptotic field value [1].

15.1. Choice of Gauge

To simplify the linearized Einstein equations, we adopt the Lorenz gauge condition [3]:

$$\partial_\mu h^{\mu\nu} = \frac{1}{2}\partial^\nu h \tag{44}$$

where $h = h^\alpha_\alpha$. In this gauge, the linearized Einstein tensor reduces to:

$$G_{\mu\nu} \approx -\frac{1}{2}\Box h_{\mu\nu} \tag{45}$$

This is a standard result in general relativity [3] and significantly simplifies the analysis.

To first order in perturbations, the left-hand side of Equation (41) becomes:

$$G_{\mu\nu} \approx -\frac{1}{2}\Box h_{\mu\nu} \tag{46}$$

The right-hand side, to first order in ϕ_μ, becomes:

$$RHS \approx \frac{c_1}{2} v_0^2 (\partial_\mu \phi)(\partial_\nu \phi) + \cdots \tag{47}$$

where we have kept only the dominant term from the kinetic part. Thus, in the weak-field limit and Lorenz gauge, we obtain:

$$\boxed{\Box h_{\mu\nu} \approx -c_1 v_0^2 (\partial_\mu \phi)(\partial_\nu \phi)} \tag{48}$$

15.2. Note on the Sign

The negative sign in Equation (48) indicates that the source term for the metric perturbation is negative definite, which will lead to an attractive gravitational force in the Newtonian limit.

16. Solving for $h_{\mu\nu}$

Equation (48) is a wave equation with a source term. Using the retarded Green's function for the d'Alembertian in Minkowski spacetime [3]:

$$G_{\text{ret}}(x - x') = \frac{\delta(t - t' - |\mathbf{x} - \mathbf{x}'|)}{4\pi|\mathbf{x} - \mathbf{x}'|}\Theta(t - t') \tag{49}$$

which satisfies:

$$\Box G_{\text{ret}}(x - x') = \delta^{(4)}(x - x') \tag{50}$$

The solution for $h_{\mu\nu}$ is:

$$h_{\mu\nu}(x) = -c_1 v_0^2 \int G_{\text{ret}}(x - x')(\partial'_\mu \phi)(\partial'_\nu \phi)\, d^4x' \tag{51}$$

where $\partial'_\mu = \partial/\partial x'^\mu$ denotes differentiation with respect to the source coordinates.

17. The Metric-Field Relation

Since $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, we obtain:

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$$\boxed{g_{\mu\nu}(x) = \eta_{\mu\nu} - c_1 v_0^2 \int G_{\text{ret}}(x - x')(\partial'_\mu \phi)(\partial'_\nu \phi)\, d^4x' + \mathcal{O}(h^2)} \tag{52}$$

The term $\mathcal{O}(h^2)$ denotes higher-order contributions from the non-linear parts of the Einstein equations and from the potential term in the Lagrangian. These are negligible in the weak-field limit but become important in strong-field regimes.

17.1. Important Clarifications

1. **Green's function**: We use the retarded Green's function G_{ret} to maintain causality.
2. **Derivatives**: The derivatives $\partial'_\mu \phi$ are taken with respect to the source coordinates x'^μ.
3. **Linear approximation**: This relation is valid only in the weak-field linearized regime ($|h_{\mu\nu}| \ll 1$, $|\phi_\mu| \ll 1$).
4. **Gauge dependence**: The expression depends on the choice of Lorenz gauge. Physical observables are gauge-independent.

18. Physical Interpretation of the Metric-Field Relation

Equation (52) shows explicitly that:

1. **The metric $g_{\mu\nu}$ is a function of the field ϕ and its gradients.**
2. **In the absence of field gradients ($\partial_\mu \phi = 0$), $g_{\mu\nu} = \eta_{\mu\nu}$ (Minkowski spacetime).**
3. **In the presence of field gradients, curvature emerges ($g_{\mu\nu} \neq \eta_{\mu\nu}$).**
4. **The fabric of spacetime is not a separate background; it emerges from the speed field.**
5. **Causality is preserved through the use of the retarded Green's function.**

19. The Inherent Screening Mechanism

The derivation of $g_{\mu\nu}$ from the speed field v^μ inherently preserves the natural screening mechanism of FST [1].

19.1. High-Density Environments

In high-density environments (where $|\nabla v| \gg A_0/c^2$), the non-linear kinetic terms in Equation (41) dominate. In this regime, the field equation reduces to:

$$G_{\mu\nu} \approx \frac{c_1 + c_3}{2} (\nabla_\mu v_\alpha)(\nabla_\nu v^\alpha) \tag{53}$$

For a static, spherically symmetric source, solving this equation in conjunction with the vector field equation [1] recovers the standard Schwarzschild geometry [3] to high precision:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2 + \mathcal{O}\left(\frac{r}{\lambda_{\text{screen}}}\right) \tag{54}$$

Thus, in high-density environments (stars, planets, the Solar System), FST reproduces general relativity to within current observational limits.

19.2. Low-Density Environments

In low-density environments (where $|\nabla v| \lesssim A_0/c^2$), the quartic term becomes comparable to the kinetic terms. Using $v_\mu v^\mu = v_0^2 = 10^{-6}$, we have:

$$\frac{|\lambda|}{12L_0^2} v_0^4 \sim \frac{6.12 \times 10^{13}}{12L_0^2} \times 10^{-12} \sim \frac{5.1 \times 10^1}{L_0^2} \tag{55}$$

This scale is comparable to $A_0 L_0 / c^4$ and defines the MONDian transition [4,5,7]. The FST corrections only become non-negligible in this deep-MONDian regime, where the vacuum floor v_0 defines the curvature scale.

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19.3. The Screening Length

From the linearized vector field equation [1], the screening length emerges naturally:

$$\lambda_{\text{screen}} = \frac{L_0}{\sqrt{\frac{3\beta_{\text{eff}}c_1}{c_1+c_3}}} = 1.65 \text{ pc} \tag{56}$$

At distances $r \gg \lambda_{\text{screen}}$, the field perturbation ϕ decays exponentially, and the metric approaches Minkowski spacetime. At distances $r \ll \lambda_{\text{screen}}$, the field behaves as $1/r$, and the Schwarzschild geometry is recovered.

Table 1. Regimes of the FST field and corresponding geometry

Regime	$\lvert\nabla v\rvert$	Dominant terms	Geometry
High-density (Solar System)	$\gg A_0/c^2$	Kinetic terms (c_1, c_3)	Schwarzschild [3]
Transition (galactic outskirts)	$\sim A_0/c^2$	All terms	MOND-like [4,5,7]
Low-density (voids)	$\ll A_0/c^2$	Quartic term (λ)	Minkowski

20. Limitations and Future Work

The present work has several limitations that should be addressed in future publications:

1. **Weak-field approximation**: The derivation of the metric-field relation is valid only in the linearized regime. The full non-linear theory requires numerical relativity techniques.
2. **Full source term**: The linearized equation retained only the dominant term $(\partial_\mu\phi)(\partial_\nu\phi)$. A complete analysis including all terms from the energy-momentum tensor is left for future work.
3. **Quantum effects**: The theory is purely classical; quantization is left for future work.
4. **Observational tests**: The predicted screening length $\lambda_{\text{screen}} = 1.65$ pc is currently beyond observational reach but may be tested by future missions probing the outer Solar System.

21. Conclusions

We have derived the explicit relation between the speed field v^μ and the metric $g_{\mu\nu}$ from first principles in the weak-field linearized approximation [1,3]:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} - c_1 v_0^2 \int G_{\text{ret}}(x-x')(\partial'_\mu\phi)(\partial'_\nu\phi)\, d^4x' + \mathcal{O}(h^2)$$

Key clarifications regarding this derivation:

- The retarded Green's function G_{ret} ensures causality.
- The derivatives $\partial'_\mu\phi$ are taken with respect to the source coordinates.
- The Lorenz gauge condition $\partial_\mu h^{\mu\nu} = \frac{1}{2}\partial^\nu h$ was used to simplify the linearized Einstein equations [3].
- The result is valid only in the weak-field regime ($|h_{\mu\nu}| \ll 1$, $|\phi_\mu| \ll 1$).

We have also provided a physical interpretation of the asymptotic field value $v_0 = 10^{-3}$ as the ratio of the characteristic FST velocity to the speed of light [1]:

$$v_0 = \frac{v_{\text{char}}}{c} = \frac{\sqrt{A_0 L_0}}{c} = \frac{273 \text{ km/s}}{3 \times 10^5 \text{ km/s}} \approx 10^{-3}$$

Finally, we have demonstrated that the derivation inherently preserves the natural screening mechanism of FST [1]:

- In high-density environments, non-linear kinetic terms dominate, recovering standard Schwarzschild geometry [3].
- The screening length $\lambda_{\text{screen}} = 1.65$ pc emerges naturally.

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- FST corrections only become non-negligible in the deep-MONDian regime [4,5,7].

This derivation proves that:

1. **The speed field v^μ is primary.**
2. **The metric $g_{\mu\nu}$ (spacetime fabric) emerges from the speed field in the weak-field limit.**
3. **Gravity arises from gradients of the speed field.**
4. **The screening mechanism is inherent to the geometry, not an external assumption.**

Appendix A. Dimensional Analysis Summary

Table A1. Dimensional analysis of key quantities (natural units, $\hbar = c = 1$)

Quantity	Symbol	Dimensions
Speed field	v^μ	1
Metric	$g_{\mu\nu}$	1
Covariant derivative	∇_μ	M
Field strength	$F_{\mu\nu} = \nabla_\mu v_\nu - \nabla_\nu v_\mu$	M
Kinetic terms	K_1, K_2, K_3	M^2
Lagrangian	$\mathcal{L}_V$	M^4
Energy-momentum tensor	$T^{(V)}_{\mu\nu}$	M^4
Einstein tensor	$G_{\mu\nu}$	M^2
Characteristic velocity	$v_{\text{char}} = \sqrt{A_0 L_0}$	1 (in natural units)

Appendix B. Numerical Calculation of v_0

$$A_0 = 2.42 \times 10^{-10}\ \text{m/s}^2 \tag{A1}$$
$$L_0 = 10\ \text{kpc} = 3.08567758 \times 10^{20}\ \text{m} \tag{A2}$$
$$v_{\text{char}} = \sqrt{A_0 L_0} = \sqrt{2.42 \times 10^{-10} \times 3.08567758 \times 10^{20}} \tag{A3}$$
$$= \sqrt{7.467 \times 10^{10}} = 2.732 \times 10^5\ \text{m/s} = 273.2\ \text{km/s} \tag{A4}$$
$$c = 2.99792458 \times 10^8\ \text{m/s} = 2.99792458 \times 10^5\ \text{km/s} \tag{A5}$$
$$v_0 = \frac{v_{\text{char}}}{c} = \frac{2.732 \times 10^5}{2.998 \times 10^5} = 0.911 \times 10^{-3} \approx 1.0 \times 10^{-3} \tag{A6}$$

Appendix C. Appendix C: Green's Function for the d'Alembertian

The Green's function for the d'Alembertian $\Box = \partial_\alpha \partial^\alpha$ satisfies:

$$\Box G(x - x') = \delta^{(4)}(x - x') \tag{A7}$$

In Minkowski spacetime, the retarded Green's function is [3]:

$$G_{\text{ret}}(x - x') = \frac{\delta(t - t' - |\mathbf{x} - \mathbf{x}'|)}{4\pi|\mathbf{x} - \mathbf{x}'|}\Theta(t - t') \tag{A8}$$

where Θ is the Heaviside step function. This Green's function is used in Equation (51) to compute $h_{\mu\nu}$ while ensuring causality.

Appendix D. Step-by-Step Derivation Checklist

1. Start with FST Lagrangian (Equation 1) [1]
2. Define kinetic terms K_1, K_2, K_3 (Equation A1)
3. Compute $\frac{\partial K_1}{\partial g^{\mu\nu}}$ (Equation 22)
4. Compute $\frac{\partial K_2}{\partial g^{\mu\nu}}$ (Equation 25)

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5. Compute $\frac{\partial K_3}{\partial g^{\mu\nu}}$ (Equation 26)
6. Compute $\frac{\partial \mathcal{L}_{\text{pot}}}{\partial g^{\mu\nu}}$ (Equation 30)
7. Assemble $\frac{\partial \mathcal{L}_V}{\partial g^{\mu\nu}}$ (Equation 35)
8. Compute $T^{(V)}_{\mu\nu} = -2\frac{\partial \mathcal{L}_V}{\partial g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_V$
9. Simplify to final form (Equation 38) [1]
10. Substitute into Einstein equations (Equation 39) [3]
11. Derive FST-Einstein equation (Equation 41)
12. Choose Lorenz gauge: $\partial_\mu h^{\mu\nu} = \frac{1}{2}\partial^\nu h$ [3]
13. Linearize for weak fields
14. Obtain $\Box h_{\mu\nu} \approx -c_1 v_0^2 (\partial_\mu \phi)(\partial_\nu \phi)$
15. Solve for $h_{\mu\nu}$ using retarded Green's function (Equation 51) [3]
16. Obtain relation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ (Equation 52)
17. Compute $v_{\text{char}} = \sqrt{A_0 L_0} = 273$ km/s [1]
18. Compute $v_0 = v_{\text{char}}/c = 10^{-3}$
19. Derive screening mechanism from geometry [1]
20. Show recovery of Schwarzschild in high-density regime [3]
21. Identify $\lambda_{\text{screen}} = 1.65$ pc [1]
22. State limitations and future work

Appendix E. Numerical Constants

Table A2. Numerical constants used in the derivation

Constant	Symbol	Value
Speed of light	c	2.99792458×10^8 m/s
Gravitational constant	G	6.67430×10^{-11} m³/kg/s² [2]
Characteristic length	L_0	$3.08567758 \times 10^{20}$ m (10 kpc) [1]
FST acceleration	A_0	2.42×10^{-10} m/s² [1]
Characteristic velocity	$v_{\text{char}} = \sqrt{A_0 L_0}$	2.732×10^5 m/s (273.2 km/s)
Asymptotic field value	$v_0 = v_{\text{char}}/c$	1.0×10^{-3} [1]
Screening length	λ_{screen}	1.65 pc [1]
Kinetic coefficient sum	$c_1 + c_3$	0.83 [1]
Kinetic coefficient	c_1	0.51 [1]
Self-coupling magnitude	$\|\lambda\|$	6.12×10^{13} [1]
MOND acceleration scale	a_0	1.2×10^{-10} m/s² [4,5,7]

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