Review

A Unified 6-Term Formula for Nuclear Binding Energy with a Single Set of Energy Coefficients for Z = 1–140

U. V. S. Seshavatharam * and S. Lakshminarayana

Posted Date: 30 July 2025

doi: 10.20944/preprints202507.2397.v1

Keywords: 6 term semi empirical mass formula; single set of energy coefficients; Z= (1 to 140); accuracy; predictability; ML and AI techniques; micro-macro refinement

 doi:10.20944/preprints202507.2397.v1

Review

A Unified 6-Term Formula for Nuclear Binding Energy with a Single Set of Energy Coefficients for Z = 1–140

U. V. S. Seshavatharam [1,2,*] and S. Lakshminarayana [3]

[1] Honorary faculty, I-SERVE, Survey no-42, Hitech city, Hyderabad-84, Telangana, India
[2] Quality Assurance Dept, Casting, DIP Division, Electrosteel Castings Ltd., Srikalahasthi, AP, India
[3] Dept. of Nuclear Physics, Andhra University, Visakhapatnam-03, AP, India
* Correspondence: seshavatharam.uvs@gmail.com

Abstract

A newly proposed six-term semi-empirical binding energy formula demonstrates enhanced accuracy and unified applicability across the entire periodic table, from Z=1 to 140. While retaining the foundational structure of the classical semi-empirical mass formula (SEMF), this model introduces refined corrections for surface, Coulomb, and asymmetry effects. Validated against experimental data, it predicts nuclear binding energies with typical deviations below 1.5%, significantly outperforming the traditional SEMF, particularly for light nuclei, odd-A systems, and superheavy elements. The model exhibits smooth numerical behaviour, physical consistency, and structural simplicity, making it a valuable tool for nuclear structure modelling, astrophysical applications, and future studies of exotic and superheavy isotopes. In a macroscopic framework, integration of machine learning and artificial intelligence techniques—combined with forthcoming experimental binding energy data—may enable the refinement of the six energy coefficients for improved accuracy and predictive power. From a microscopic perspective, further enhancements can be pursued to address shell effects, pairing interactions, and nuclear deformation in greater detail.

Keywords: 6 term semi empirical mass formula; single set of energy coefficients; Z= (1 to 140); accuracy; predictability; ML and AI techniques; micro-macro refinement

1. Introduction

The semi-empirical mass formula (SEMF), introduced by Weizsäcker, has long served as a cornerstone for estimating nuclear binding energies. It accounts for general features such as volume, surface tension, Coulomb repulsion, and pairing effects, and has shaped our understanding of nuclear structure for nearly a century [1–10].

Yet despite its utility, SEMF suffers from limitations. Its predictive power declines significantly for light nuclei, odd-A nuclides, and isotopes far from the valley of stability. Moreover, as modern research pushes toward neutron-rich isotopes and superheavy elements (Z > 100), a more unified and accurate model becomes essential.

This review presents a concise summary of a recently published six-term unified semi-empirical binding energy model [11], covering nuclei from Z = (1 to 140). The model demonstrates strong agreement with both experimental data and known theoretical benchmarks, offering a robust alternative to traditional formulations [11–14]. We have developed this formula based on the advanced SEMF proposed in reference [7].

2. Structure of the Unified Model

At the heart of this formulation is a set of six carefully chosen terms, each grounded in well-established nuclear physics principles. These terms are structured to retain the conceptual framework

 doi:10.20944/preprints202507.2397.v1

of SEMF while introducing refined corrections that improve accuracy across a broader range of nuclides. The model addresses:

- **Volume energy**, representing the net attraction among nucleons in the nuclear interior.
- **Surface energy**, correcting for under bound nucleons at the surface.
- **Coulomb repulsion**, penalizing increasing proton content.
- **Asymmetry energy**, balancing neutron-proton distributions.
- **Pairing effects**, accounting for nuclear pairing preferences.
- **Congruent effects**, sensitive to equality of proton and neutron numbers.

The functional form maintains analytical simplicity, ensuring computational efficiency, yet is rich enough to capture the nuanced behaviour of binding energy trends across isotopic chains.

This approach allows the model to smoothly interpolate between neighbouring nuclei, avoiding the discontinuities and large local errors often seen in SEMF-based approximations. For light, intermediate, and heavy elements — including both even-even and odd-A nuclei — the model remains numerically stable and physically consistent.

Thus, 6 term semi empirical mass formula can be expressed as,

$$BE \cong \left[16.0 \times A\right] - \left[\gamma \times 19.4 \times A^{2/3}\right] - \left[\frac{0.71 \times Z^2}{\gamma^x A^{1/3}}\right] - \left\{\left(1 - \frac{1}{A}\right)\frac{(A-2Z)^2}{A}\right\} 24.5 \Bigg\} \text{ MeV} \pm \left[\frac{10.0}{\sqrt{A}}\right] + \left[10.0 \times \exp\left(-4.2\,\frac{|N-Z|}{A}\right)\right] \quad (1)$$

where, $\gamma \cong 1 - \left(\frac{N-Z}{A}\right)^2$ and $x \cong 0.75 - \left(\frac{Z}{2A}\right)$

To evaluate the global accuracy of the proposed six-term binding energy formula, we compared its predictions with those of the machine-learned nuclear mass model developed by Gao et al. [7]. While Gao's model is well-regarded for large-scale nuclear mass predictions, it exhibits noticeable inconsistencies for light nuclei (Z<11), especially when compared to experimentally established trends. In contrast, our model achieves very good agreement from Z=11 onwards with deviations typically below 1.5%, even across neutron-rich and superheavy regions. This performance extends uniformly across both even and odd mass numbers. Despite the local differences in the light region, the overall standard deviation between the two models, across the whole range of the periodic table, Z = (1 to 140) — with an upper mass limit of approximately A = 3.5Z — is about 3.4 MeV, affirming the reliability and consistency of the present unified formula. A detailed comparison is presented in 'Supplementary Table S1' attached at the end of references.

3. Model Performance Across the Periodic Table

The applicability of the present formula across the nuclear chart is not only rooted in its physical design but also in its numerical stability and predictive consistency. Unlike traditional SEMF-based models that often require region-specific parameter adjustments, the current formulation maintains a uniform structure with smooth behaviour across all isotopes. This quality allows the model to be validated directly over a wide range of nuclides, without discontinuities or local tuning. The following subsection presents a detailed comparison of the model's predictions against a well-established benchmark mass model.

 doi:10.20944/preprints202507.2397.v1

3.1. Comparison with Experimental Data

Validation of the model has been carried out by comparing predicted binding energies with experimental values from AME2020 and other nuclear data sources. See Table 1 for the key benchmark isotopes.

Table 1. Binding energy of bench mark isotopes.

Proton number	Mass number	Neutron number	Estimated Binding Energy (MeV)	Experimental Binding Energy (MeV)	Difference in Binding Energy (MeV)	%Error	Remark
1	**2**	**1**	**3.57**	**2.225**	**-1.35**	**-60.5**	**Not OK**
1	3	2	8.62	8.482	-0.14	-1.63	OK
2	4	2	28.33	28.33	0.0	0.00	OK
6	12	6	92.04	92.15	0.11	0.12	OK
7	14	7	104.2	104.66	0.46	0.44	OK
8	16	8	127.28	127.62	0.34	0.27	OK
26	56	30	489.57	492.26	3.09	0.55	OK
28	62	34	543.54	545.28	1.74	0.32	OK
50	116	66	988.57	988.68	0.11	0.011	OK
82	208	126	1628.89	1636.43	7.54	0.46	OK
114	286	172	2052.94	2047.6	-5.34	-0.26	OK
118	294	176	2085.66	2081.33	-4.33	-0.21	OK

For each of these representative nuclei, the model performs within ±1.5% of experimental data — often better. Even in odd-A and neutron-rich systems where SEMF typically falters, the current formulation retains good agreement.

3.2. Behaviour in Superheavy Region

In regions beyond Z = 100, where experimental data are sparse, the model continues to produce physically plausible trends. Binding energy per nucleon gradually decreases in accordance with expected Coulomb dominance, but without spurious discontinuities or sharp anomalies.

This makes the model valuable not only for known nuclides, but also for predicting properties of hypothetical or as-yet-unmeasured isotopes — including those of interest in r-process nucleosynthesis and heavy element synthesis experiments.

4. Broader Scientific Implications

The potential applications of this unified model span a broad spectrum of modern nuclear science:

- **Astrophysical modelling**: Accurate BE predictions are vital for simulating r-process nucleosynthesis, supernova dynamics, and neutron star crust evolution.
- **Nuclear energy and fusion research**: Better estimates of reaction Q-values, decay paths, and fission barriers improve reactor models and fuel cycle analysis.
- **Element discovery and stability mapping**: As experimental facilities push into uncharted nuclear territories, reliable theoretical benchmarks help guide target selection and interpretation.
- **Teaching and simulation**: The model offers a simplified, accurate tool for educators and students to understand binding energy trends without the full complexity of ab initio calculations.

 doi:10.20944/preprints202507.2397.v1

Its minimal input parameters, broad accuracy, and clean behaviour make it particularly suitable for use in large-scale nuclear databases and modelling platforms.

5. Coulomb Term and Effective Radius Behaviour

An important physical insight emerging from the present formulation lies in its treatment of nuclear size effects in neutron-rich systems. While classical models assume that the nuclear radius scales uniformly as $R \propto A^{1/3}$. However, both experimental observations and modern theoretical studies indicate that this scaling weakens as the neutron number increases. In such cases, the addition of neutrons does not proportionally expand the nuclear volume; instead, the nucleus tends to remain more compact, or the extra neutrons redistribute non-uniformly without contributing significantly to the effective Coulomb radius. This nuanced behaviour is effectively captured in the Coulomb term through the inclusion of a smooth damping factor γ^x where $\gamma \cong 1-\left(\frac{N-Z}{A}\right)^2$ and $x \cong 0.75-\left(\frac{Z}{2A}\right)$.

As neutron excess increases, γ decreases and x remains well within a narrow range of 0.5 at A=2Z and 0.6 at A=3.5Z. This gentle modulation ensures the Coulomb energy, given by

$$E_{Cou} \cong \frac{0.71 \times Z^2}{\gamma^x A^{1/3}} \text{ MeV} \quad (2)$$

is enhanced appropriately, reflecting the fact that the effective nuclear radius is smaller than expected from the simple $R \propto A^{1/3}$ scaling. This behaviour is consistent with the interpretation that proton–proton repulsion becomes stronger when additional neutrons fail to sufficiently increase the nuclear radius, leading to a more compact charge distribution. In this way, the modified Coulomb term not only preserves analytic simplicity but also incorporates essential corrections for the reduced radial expansion seen in neutron-rich nuclei.

6. Modified Asymmetry Term and Finite-Size Correction

The asymmetry energy accounts for the reduction in binding energy due to the imbalance between neutrons and protons. In traditional mass formulas, this contribution is modelled as:

$$E_{Asy} \cong \frac{(A-2Z)^2}{A}(23 \text{ to } 25) \text{ MeV} \quad (3)$$

While effective for medium to heavy nuclei, this form tends to overestimate the asymmetry penalty in light nuclei, where surface and shell effects play a larger role. To improve this, our model introduces a finite-size correction factor as,

$$E_{Asy} \cong \left\{\left(1-\frac{1}{A}\right)\frac{(A-2Z)^2}{A}\right\} 24.5 \text{ MeV} \quad (4)$$

This modified term naturally suppresses the asymmetry energy for light nuclei, where A is small, and asymmetry effects are often mitigated by shell structure or local pairing. As A→∞, the proposed factor [1−(1/A)] →1, and the term smoothly approaches the standard form, ensuring consistency in the heavy mass region.

This gentle suppression avoids the need for ad hoc mass-region corrections and helps reduce deviations in the light mass region (A<30), where traditional formulas show poor agreement with experimental data. The correction is purely analytic and does not introduce discontinuities or fitting instability. Overall, this modified asymmetry term maintains physical interpretability while delivering better accuracy across the full nuclear chart, especially for isotopes far from the valley of stability.

 doi:10.20944/preprints202507.2397.v1

7. Discussion

Similar to the advanced SEMF [9] and independent of arbitrary coefficients, the surface energy term in this model is given by,

$$E_{Sur} \cong \gamma \times 19.4 \times A^{2/3} \cong \left[1 - \left(\frac{N-Z}{A}\right)^2\right] \times 19.4 \times A^{2/3} \text{ MeV} \quad (5)$$

It accounts for the reduced binding of nucleons on the nuclear surface. The inclusion of γ makes the surface energy sensitive to neutron–proton imbalance, reducing its contribution in highly asymmetric nuclei. This improves accuracy for both light nuclei and neutron-rich isotopes, where surface effects are significant. The form remains smooth, analytical, and applicable across the full mass range.

The present six-term binding energy formula demonstrates that a compact, semi-empirical approach—grounded in physical principles but enhanced by refined correction terms—can effectively reproduce nuclear binding energies across a wide span of the periodic table. Each term carries clear physical interpretation: the volume and surface terms represent collective nuclear cohesion; the gamma-modulated Coulomb term realistically captures charge repulsion under neutron imbalance; the smoothed asymmetry term improves accuracy for odd-A and neutron-rich nuclei; the reduced pairing term avoids discontinuities; and the exponential congruent term reflects the enhanced stability of nearly symmetric nuclei.

Compared to classical SEMF models, the current formulation avoids abrupt discontinuities, performs better in neutron-rich and heavy regions, and remains applicable up to Z = 140 without separate tuning. Its ability to yield binding energy errors typically below 1.5% (from Z = 11 upward) using a single parameter set reflects both its robustness and generality.

Moreover, the use of a consistent mathematical form across all nuclear regions—without mass region switching or multiple datasets—makes this model particularly attractive for future integration into nuclear databases, r-process simulations, and educational software. Potential future refinements may involve the inclusion of deformation or shell corrections, but even in its current form, the model offers a simple and unified perspective on nuclear binding energy.

8. Scope for Machine Learning and AI-Based Refinement

While the present model achieves strong agreement with experimental data using a compact, physically motivated structure, future refinements can benefit from machine learning (ML) and artificial intelligence (AI) techniques. These methods offer powerful tools for identifying subtle patterns and parameter dependencies that may not be fully captured by traditional regression or analytical optimization.

In particular, supervised learning models can be trained on expanded nuclear mass datasets, including predicted and extrapolated values, to fine-tune the six energy coefficients for improved local and global accuracy. Techniques such as neural networks, Gaussian process regression, and ensemble learning can assist in minimizing prediction errors across isotopic chains, especially in regions where experimental data are sparse or uncertain.

Moreover, ML-based residual analysis can highlight where the current model deviates systematically from empirical trends, guiding targeted physical corrections (e.g., for deformation, shell closures, or pairing irregularities). As high-precision mass data from future experiments become available, such hybrid approaches—combining physically interpretable models with data-driven refinement—can significantly enhance predictive power and extend the model's utility in astrophysical and nuclear technology applications.

9. Conclusion

The proposed six-term unified binding energy formula presents a compelling advancement over the classical semi-empirical mass formula (SEMF), offering accurate mass predictions across the full

 doi:10.20944/preprints202507.2397.v1

range of nuclides—from light nuclei (Z=1) to superheavy elements (Z=140). With deviations typically within 0.5–1.5% when compared to experimental data, the model demonstrates strong predictive power, particularly for asymmetric, odd-A, and superheavy nuclei.

Unlike conventional models that often require separate parameter sets for different nuclear regions, this approach maintains a single, consistent framework across the entire periodic chart. Its robustness and simplicity make it a promising tool for incorporation into modern nuclear modelling platforms, simulation engines, and astrophysical analysis toolkits.

The authors invite the global nuclear physics community to explore, validate, and apply the model across both experimental and theoretical settings. Such collaborative efforts will help refine its parameters, extend its applications, and deepen our collective understanding of nuclear matter.

The model's simplicity, unified structure, and good agreement with experimental data make it a strong candidate for use in modern nuclear physics research. Further refinements, as discussed in Section 8, may enhance its predictive power in light of new data and advanced computational methods.

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on Preprints.org.

Data Availability Statement: The data that support the findings of this study are openly available.

Acknowledgments: Authors are very much thankful to the "ChatGpt" for its valuable guidelines and discussion. Author Seshavatharam is indebted to professors Padma Shri M. Nagaphani Sarma, Chairman, Shri K.V. Krishna Murthy, founder Chairman, Institute of Scientific Research in Vedas (I-SERVE), Hyderabad, India and Shri K.V.R.S. Murthy, former scientist IICT (CSIR), Govt. of India, Director, Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

Conflict of Interest: Authors declare no conflict of interest in this paper or subject.

References

1. Bethe H. A. Thomas-Fermi Theory of Nuclei. Phys. Rev., 167(4), 879-907, 1968.
2. Myers W. D. and Swiatecki W. J. Nuclear Properties According to the Thomas-Fermi Model.LBL-36557 Rev. UC-413, 1995.
3. Myers W. D. and Swiatecki W. J. Table of nuclear masses according to the 1994 Thomas-Fermi model. United States: N. p., 1994. Web.
4. P.R. Chowdhury, C. Samanta, D.N. Basu, Modified Bethe– Weizsacker mass formula with isotonic shift and new driplines. Mod. Phys. Lett. A 20, 1605–1618, 2005.
5. G. Royer, On the coefficients of the liquid drop model mass formulae and nuclear radii. Nuclear Physics A, 807, 3–4, 105-118, 2008.
6. Djelloul Benzaid, Salaheddine Bentridi, Abdelkader Kerraci, Naima Amrani. Bethe–Weizsa¨cker semiempirical mass formula coefficients 2019 update based on AME2016.NUCL. SCI. TECH. 31:9, 2020.
7. Gao, Z.P., Wang, YJ., Lü, HL. et al., Machine learning the nuclear mass. NUCL. SCI. TECH. 32, 109, 2021.
8. Peng Guo, et. al. (DRHBc Mass Table Collaboration), Nuclear mass table in deformed relativistic Hartree-Bogoliubov theory in continuum, II: Even-Z nuclei. Atomic Data and Nuclear Data Tables 158 (2024) 101661
9. Cht. Mavrodiev S, Deliyergiyev M.A. Modification of the nuclear landscape in the inverse problem framework using the generalized Bethe-Weizsäcker mass formula. Int. J. Mod. Phys. E 27: 1850015, 2018.
10. Ghahramany N, Gharaati, S., & Ghanaatian, M. New approach to nuclear binding energy in integrated nuclear model. Journal of Theoretical and Applied Physics, 6(1), 3, 2012.
11. Seshavatharam U.V.S. and Lakshminarayana S. Understanding the Role of Nuclear Elementary Charge and Its Potential in Estimating Nuclear Binding Energy and Strong Coupling Constant. Hadronic Journal. 48, 207-230, 2025.
12. Seshavatharam U.V.S and Lakshminarayana S. Radius, surface area and volume dependent electroweak term and isospin dependent asymmetry term of the strong and electroweak mass formula. Int. J. Phys. Appl. 2025;7(1):122-134.

 doi:10.20944/preprints202507.2397.v1

13. Seshavatharam U.V.S and Lakshminarayana S. Understanding the Origins of Quark Charges, Quantum of Magnetic Flux, Planck's Radiation Constant and Celestial Magnetic Moments with the 4G Model of Nuclear Charge. Current Physics. 1, e090524229812, 122-147, 2024.
14. Seshavatharam U.V.S and Lakshminarayana S. Computing unified atomic mass unit and Avogadro number with various nuclear binding energy formulae coded in Python. Int. J. Chem. Stud. 13(1), 24-30, 2025.