

Review

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Review

Three Generations of SM Gauge Symmetry from Non-SUSY Compactification on $CP^2 \times S^2$

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Abstract: This paper investigates the dynamics of D-branes and tachyon condensation in non-supersymmetric string theory, focusing on how these processes realize Standard Model (SM) gauge symmetries. We discuss the distance-dependent potentials arising from D-brane separation and their implications for the Coleman-Weinberg potential. When D-branes coincide, an attractive force emerges due to massless states, reflecting symmetry-breaking phenomena. Tachyon condensation modifies the particle spectrum, providing masses to gauge bosons and fermions through interactions reminiscent of the Higgs mechanism. We show that the gauge symmetries are tied to the isometries of compactified spaces, with masses determined by the compactification scale. Our results indicate that non-supersymmetric string theory can yield a viable realization of the SM, illuminating the relationship between brane dynamics, tachyon behavior, and mass generation.

Keywords: Non-SUSY string; topology; D-brane; tachyon condensation; symmetry breaking

Introduction

In the Standard Model of particle physics, there are three generations of quarks (up/down, strange/charm, and top/bottom), along with three generations of leptons (electron, muon, and tau). All of these particles have been observed experimentally, and no additional generations have been detected.

The requirement for gauge anomaly cancellation in the Standard Model places constraints on the number of generations. Specifically, triangle anomalies from the quark and lepton sectors must cancel to ensure the theory's consistency. This anomaly cancellation works perfectly with three generations of fermions [1].

We consider the RNS model of string theory, setting aside the $U(1)$ gauge symmetry, which necessitates an additional dimension beyond ten for its realization. The most natural way to incorporate the remaining gauge symmetries $SU(3) \times SU(2)$ of the Standard Model into the theory is to compactify the six-dimensional internal space K on the Kähler manifold $CP^2 \times S^2$. Here, CP^2 is isomorphic to $SU(3)$, and S^2 corresponds to $SU(2)$. This manifold is not a Calabi-Yau manifold [2,3] and admits only a $spin^c$ [4] structure, corresponding to the three generations of fermions [5]. Since the compactified manifold is not Ricci-flat, we cannot expect $N = 1$ supersymmetry in this case. The corresponding closed strings can either be non-supersymmetric or of type IIA, which can be obtained by dimensional reduction of eleven-dimensional supergravity [6].

In our non-chiral model, similar to type IIA, the presence of D0-branes coupled to R-R 1-form potentials indicates the emergence of a $U(1)$ gauge symmetry, associated with an 11th dimension compactified on a circle.

The Chan-Paton factors of $SU(2)$ and $U(1)$ can be introduced through two coincident D-branes positioned away from orientifold fixed planes, ensuring the cancellation of the fixed plane charges after toroidal compactification of the coordinates tangent to M^4 only. The Chan-Paton factors for $SU(3)$ correspond to fully Neumann boundary conditions.

The fermions transform in the fundamental representation, while the bosons are in the adjoint representation of the gauge groups. The full open string spectrum, though not supersymmetric, contains equal numbers of bosonic and fermionic excitations at each level, provided that the GSO conditions [7] are satisfied. Consequently, the zero-point energy cancels between the bosons and fermions.

Fermions and $SU(2)$ gauge bosons can acquire masses in 4D through the condensation of tachyons attached to a stack of two unstable D-branes, transforming in the adjoint representation of $U(2) \cong SU(2) \times U(1)$. The $SU(2)$ component is removed via gauge fixing, breaking the $SU(2)$ gauge symmetry. The remaining $U(1)$ part undergoes tachyon condensation, imparting masses to the $SU(2)$ gauge bosons through its vacuum expectation value (VEV). The $SU(3)$ gauge bosons remain massless due to their fully Neumann Chan-Paton factors.

Main Text

It is a well known fact that the index of the Dirac operator on a six-dimensional compactified manifold is given by $\frac{\chi}{2}$, where χ is the Euler Characteristic of the manifold. Since the Euler Characteristic of a product manifold is the product of the Euler Characteristics of its components, the index of the Dirac operator on the product manifold $CP^2 \times S^2$ is

$$\nu_+ - \nu_- = \frac{\chi_{CP^2} \cdot \chi_{S^2}}{2} \quad (1)$$

Given that $\chi_{CP^2} = 3$ and $\chi_{S^2} = 2$:

$$\begin{aligned} \nu_+ - \nu_- &= \frac{3 \cdot 2}{2} \\ &= 3 \end{aligned} \quad (2)$$

$CP^2 \times S^2$ is a six-dimensional Kähler manifold possessing a $spin^c$ structure inherited from CP^2 . However, it is not a Calabi-Yau manifold. This is because it does not admit a Ricci-flat Kähler metric, and the canonical bundle is not trivial, as the first Chern class of CP^2 does not vanish.

To ensure that the gauge symmetries introduced in the 4D effective theory via D-branes are consistent with the isometries of the compactified manifold, we must perform a T-duality along the coordinates tangent to M^4 . T-duality does not apply directly to manifolds like $CP^2 \times S^2$ without global $U(1)$ isometries. Generalizations of T-duality might offer ways to explore dualities in spaces more complex than tori, but these are advanced topics with their own sets of challenges and requirements.

There are 2^{D-1-p} fixed planes with tensions $\tau'_p = \mp 2^{p-\frac{D}{2}} \tau_p$ [8], where the total fixed-plane source tension is $2\tau_p$, given that $D = 4$ in this context. This configuration involves two Dp-branes with tensions of τ_p , so that the fixed-planes charge cancels out the total charge for the branes.

When two coincident D-branes are placed away from the fixed plane, the gauge symmetry that arises is typically $U(2)$, which decomposes as a direct product $SU(2) \times U(1)$. The fixed $U(1)$ charge of unity aligns with the structure of the electroweak theory.

In contrast, the Chan-Paton factors for $SU(3)$ correspond to fully Neumann boundary conditions. This means that the gauge symmetry arises independently of the D-brane configuration and instead reflects the ordinary gauge symmetry associated with the free motion of open strings along the brane.

From the $SU(3)$ matrix, a factor $e^{\frac{2in\pi}{3}}$ can always be extracted and added to the $U(1)$ factor $e^{i\phi}$, where $n = 1, 2$, and 3 . As $\phi \rightarrow \phi + 2\pi$, a state acquires a phase $e^{\frac{2in\pi}{3}}$, which results in allowed $U(1)$ charges of $\frac{1}{3}$, $\frac{2}{3}$ and 1 .

Since the representations are complex and both particles and antiparticles appear in the spectrum, we compute the degrees of freedom (DOF) for right-handed iso-scalar fermions. Excluding the neutrino and up quark, right-handed fermions contribute $2(1+3)=8$ DOF, while the three colors of the right-handed up quarks add another $2 \cdot 3 = 6$ DOF. Left-handed fermions, being iso-doublets, contribute $2 \cdot 2(1+3) = 16$ DOF.¹

Therefore, the total number of fermionic DOF for left-handed and right-handed fermions is $16+8+6=30$.

For the bosonic sector, the total number of DOF for gluons, W bosons, Z bosons, and photons is $2 \cdot (8+3+1) = 24$. Additionally, the vectors on $CP^2 \times S^2$, contribute six components transforming as massless scalars under $SO(1,3)$, resulting in a total of 30 DOF for bosons.

In the case of unoriented strings, the ground state transforms in the adjoint representation of the gauge group, while the excited states transform either in the symmetric or the antisymmetric representations of the same group, depending on the action of the twist operator (Ω). The operator Ω imposes a projection that determines the symmetry properties of the states: ($\Omega = +1$) leads to states in the symmetric representation, while ($\Omega = -1$) corresponds to states in the antisymmetric representation.

For oriented strings, there is no twist operator, and therefore, such constraints do not apply. In a model like the Standard Model (SM), without supersymmetry, all states in the bosonic sector of the open string are assumed to transform in the adjoint representation of one of the gauge groups, corresponding to the gauge bosons that mediate the fundamental forces.

In the fermionic sector, both the ground state and the excited states belong to the fundamental representations of the gauge groups. This aligns with the structure of the SM, where matter fields such as quarks and leptons, which are fermions, transform in the bifundamental representation of the gauge groups $SU(3) \times SU(2) \times U(1)$, with each quark or lepton carrying charges under two or more of these groups simultaneously.

The degeneracies at level- n in the NS and R sectors are given by the coefficients of ω^n in the generating functions:

$$\begin{aligned} f_{NS}^{SM}(\omega) &= f_R^{SM}(\omega) \\ &= 30(1 + 16\omega + 144\omega^2 + \dots). \end{aligned} \quad (4)$$

Since generation number is not one of the quantum numbers used in the Pauli exclusion principle, the contribution to the zero-point energy from a particular set of quantum numbers of fermions comes from a single generation, as dictated by the exclusion principle. Thus, adding more families of fermions does not alter the partition function.

Consequently, the zero-point energy cancels between the bosons and fermions. This cancellation occurs not only for the ground states but also for the excited states, as evident from (4). Such comprehensive cancellation is crucial for maintaining the stability of the model, ensuring that the energy contributions from bosonic and fermionic excitations remain balanced. This balance is particularly important in non-supersymmetric models, as it mitigates potential instabilities that could arise from unbalanced quantum fluctuations.

¹ We can also construct Majorana neutrino from ν_L as

$$\nu_M = \begin{bmatrix} \nu_L \\ \nu_L^c \end{bmatrix}, \quad (3)$$

where ν_L^c is the right-handed, sterile neutrino.

In string theory, the separation of D-branes introduces a distance-dependent potential between them, which can manifest in non-trivial one-loop corrections captured by the Coleman-Weinberg potential [9–11]. The gauge symmetries $U(1)^2$ introduced by two separated D-branes are trivial, as reflected by the Chan-Paton factor $(\text{Tr } 1)^2 = 1$. In this scenario, the additional degeneracies of the open-string states contributing to the Coleman-Weinberg potential, carried by the open-string loop, are determined by the dimensions of the representations of the isometry groups of the internal space to which the states belong, rather than the non-Abelian gauge group $U(2)$, which would typically arise when the two branes coincide.

The open string states stretching between two coincident D-branes fill out the adjoint representation of $U(2)$, which corresponds to the direct sum of the adjoint of $SU(2)$ and an additional $U(1)$ factor. Fermions transform in the fundamental representation of the gauge group, so the primary focus is on the bosons in the adjoint representation when the branes coincide. This configuration leads to an attractive force, mainly due to the negative contribution of the massless states to the one-loop potential.

In the context of the Standard Model as the underlying symmetry, the accidental matching of degrees of freedom between bosons and fermions can result in the cancellation of the force between separated D-branes. However, this cancellation arises not from supersymmetry but from the specific structure of the SM and the isometries of the internal space [12].

When the D-branes coincide, the enhanced gauge symmetry typically induces an attractive force due to the contribution of the massless bosonic states. Since the underlying symmetry is based on the Standard Model rather than supersymmetry, a permanent force cancellation is not required. The abrupt change in the force as the branes coincide can be interpreted as a phase transition or a symmetry-breaking event, which is a familiar phenomenon in field theory.

Since the model is non-chiral, it supports only even-dimensional branes, including D0-branes. The Ramond-Ramond (R-R) charge of the D0-brane can be interpreted as a $U(1)$ charge, as it couples to the R-R 1-form potential. The string coupling constant is approximately $g \approx \tau_0^{-1}$, where τ_0 is the tension of the D0-brane, and the 4D Yang-Mills coupling constant is related by $g_{YM}^2 \approx g$.

In cubic string field theory (SFT), the tachyons associated with open strings attached to D-branes signify the instability of the D-branes [13–15], reflecting their potential for complete annihilation. In the process of tachyon condensation, the condition $M + V(T) = 0$ holds, where M is the total mass of the D-branes to which the tachyons are attached, and $V(T)$ represents the tachyon potential on the brane, with $V(T) = Mf(T)$, where $f(T)$ is given by the SFT expression:

$$f(T) = 2\pi \left(\frac{1}{2} \langle I \circ T(0) Q_B T(0) \rangle + \frac{1}{3} \langle h_1 \circ T(0) h_2 \circ T(0) h_3 \circ T(0) \rangle \right). \quad (5)$$

At level zero, the tachyon field is approximated by

$$|T\rangle = tc_1 |0\rangle, \quad (6)$$

where $c_1 |0\rangle$ is a level-zero state with ghost number one. Substituting this into (5) we get the zeroth approximation to the zero momentum tachyon potential

$$f^{(0)}(t) = 2\pi^2 \left(-\frac{1}{2} t^2 + \frac{1}{3} \frac{t^3}{r^3} \right), \quad (7)$$

where the mapping radius of the disc defining the three string vertex is given by

$$r = \frac{4}{3\sqrt{3}}. \quad (8)$$

The potential has a stationary point at $t = t_c = r^3 \approx \frac{1}{2}$. We shift the tachyon field around this minimum by substituting $t = t' + t_c$. As the coupling strength is absorbed into t , the field theoretic mass of t' is obtained from the coefficient of the term quadratic in t' in the expression:

$$-\frac{t^2}{2} + \frac{t^3}{3} = -\frac{1}{2}(t' + t_c)^2 + \frac{(t' + t_c)^3}{3} \quad (9)$$

Thus,

$$m_{t'}^2 \approx 0, \quad (10)$$

where $m_{t'}^2$ is the mass squared of the shifted field (a Higgs-like mode after condensation).

Though the above expression for $f(T)$ was derived from tachyons of the bosonic string theory, the states in the F_1 picture in the RNS formulation are constructed in the same way as those in the bosonic string theory. Specifically, we have $|\tilde{\phi}\rangle = \lim_{z \rightarrow 0} z^{-1} V(k, z) |0; 0\rangle$. Thus, for the tachyon: $|\tilde{t}\rangle = \lim_{z \rightarrow 0} z^{-1} \psi.k e^{ik.X}(z) |0; 0\rangle$. Since $k^2 = 1$ and $\psi.k = \psi^+ = \frac{\psi^3 + i\psi^0}{\sqrt{2}}$, we obtain $|\tilde{t}\rangle = e^{iy} e^{ik.x} |0; 0\rangle$, where Y is the bosonization of ψ^+ . By absorbing the unit momentum conjugate to y into k through local rotations of the momentum in the (x, y) plane to align it with the x directions, we obtain $|\tilde{t}\rangle = e^{ik.x} |0; 0\rangle = |t\rangle$, where $k^2 = 2$, consistent with the bosonic string theory.

Thus, it is apparent that if the tachyon in bosonic string theory acquires a VEV after condensation, the tachyon in the RNS model on unstable D-branes will also acquire the same VEV. The equivalence of the vertex operators under rotation and the shared CFT structure ensure that tachyon condensation behaves consistently in both cases.

The requirement for modular invariance prohibits tachyons and odd- G -parity states in closed loops. However, this restriction does not apply to tree amplitudes, where such states can couple to physical degrees of freedom, especially in the context of unstable systems like non-BPS D-branes.

Open strings carrying tachyons, which are stretched between a pair of unstable D-branes, transform in the adjoint representation of $U(2)$. The cubic interaction is constrained by gauge invariance, allowing contributions only from the $U(1)$ singlet component of the tachyon. Gauge fixing can be used to eliminate unphysical degrees of freedom associated with the $SU(2)$ symmetry, leaving the physical components intact.

CP^2 has the holonomy group $U(2) \cong SU(2) \times U(1)$. Vectors tangent to it, $A = A_m dz^m + A_{\bar{m}} d\bar{z}^{\bar{m}}$, transform in the representations $(2, 1)$ and $(\bar{2}, -1)$ of the holonomy group. After dimensional reduction, these components can be identified with the Higgs fields in the effective 4D theory [16]. Gauge fixing of the $SU(2)$ symmetry removes three unphysical real components from $(2 + \bar{2})$ of A_m , leaving the necessary physical DOF in the 4D Higgs fields.

We identify the remaining real component $H = c^m (A_m + A_{\bar{m}})$, where m is one of the two complex indices of A_m , as the Higgs boson associated with t' . The 4D vacuum expectation value (VEV) of t is determined by the 10D VEV $t_c^{10} = t_c^6 t_c^4$, where t_c^6 is set by the size of the compactified manifold and t_c^4 represents the 4D effective VEV. Consequently the 4D mass of H , which arises from the interaction term tH^2 , is ultimately controlled by the tachyon VEV and the compactification geometry. The tachyon can also impart mass to fermions through Yukawa-like couplings and to $SU(2)$ gauge bosons via four-point functions derived from three-point interactions in the low energy limit.²

The simultaneous removal of the $SU(2)$ -component of A_m and the corresponding tachyon field is crucial for fully breaking the symmetry via tachyon condensation. This requires the alignment of

² The three-point function involving the tachyon and the $U(2)$ gauge bosons includes a term like $f_{abc} t^a \partial_\mu t^b A^{\mu c}$. The antisymmetry of f_{abc} ensures that if a corresponds the $U(1)$ generator, then $f_{abc}=0$ unless b and c are indices of the $SU(2)$ subalgebra. Consequently, the tachyon couples exclusively to the $SU(2)$ gauge bosons.

the gauge field and the tachyon in the internal space, enabling both to be removed by the same gauge transformation. Failure to achieve this would result in incomplete symmetry breaking and possible residual instabilities, as the two fields are dynamically linked through their interactions.

In conventional spontaneous symmetry breaking (SSB) mechanisms, such as the Higgs mechanism, Goldstone bosons arise from broken symmetries and contribute longitudinal components to gauge bosons. In contrast, tachyons in this context do not function as physical degrees of freedom post-condensation. Their elimination does not necessitate a compensatory mechanism typical of Goldstone bosons, as the longitudinal modes of the massive W and Z bosons account for the lost degrees of freedom from A_m .

Open strings attached to unstable D-branes initially carry massless fermions and gauge bosons as part of their spectrum. However, as tachyon condensation occurs—indicated by the tachyon field acquiring a vacuum expectation value (VEV)—the instability of the D-branes leads to significant transformations in the system. This process results in the collapse of the open strings, causing the massless fermions and gauge bosons to potentially cease being physical degrees of freedom.

Conclusion

In summary, the vanishing of the zero-point energy in the RNS model, regardless of the number of fermionic families, reveals a profound structural stability in the vacuum of the theory. This result indicates the absence of spontaneous symmetry breaking under the current framework while also highlighting the potential for tachyonic instabilities that could lead to SSB in alternative scenarios.

This finding opens up several intriguing possibilities for further investigation. While the current analysis shows robustness against family multiplicity, it prompts a re-examination of how tachyons might interact within non-supersymmetric models to induce symmetry breaking. The universality of the vanishing effective action suggests a fundamental constraint that may still be circumvented by the presence of tachyonic fields, leading to new vacuum configurations and physical implications.

Furthermore, the non-perturbative nature of tachyon condensation poses significant challenges, particularly regarding the complexity of the infinite number of interacting fields in cubic SFT. While gauge fixing issues have been resolved, understanding the new stable vacuum formed after tachyon condensation—especially in relation to unstable D-branes—remains an important task.

Ultimately, this work emphasizes a unique feature of the RNS model that could have broader consequences for our understanding of vacuum stability in quantum field theories, particularly in string theory frameworks. Exploring whether the cancellation persists under different compactifications or in the presence of additional interactions—and how tachyons might facilitate SSB—remains an open and pressing question. Addressing these inquiries could provide crucial insights into the construction of realistic, non-supersymmetric models in high-energy physics.

Data Availability Statement: Data available on request.

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