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Communication

The Phase Velocity and Doppler Effect Associated with Moving Electromagnetic Sources and Moving Observers

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Abstract

This article provides a general expression for the phase velocity and the Doppler shift of the electromagnetic fields radiated from a uniformly moving Hertzian dipole measured by a uniformly moving observer. The results show that, when measured exactly in the direction pointing to the birthplace of the electromagnetic wave, the phase velocity of the wave is always equal to the light speed in the vacuum. The formula for the Doppler effect is of the same form of the Newtonian type classical formula, which implies that it might be not proper to consider the classical formula for the Doppler shift as the low-speed approximation of the conventional relativistic formula.

Keywords: Doppler shift; moving object; electromagnetic fields; Lorentz transformations

1. Introduction

We have derived in [1] the electromagnetic fields of a uniformly moving Hertzian dipole by making use of a simple model shown in Figure 1. The dipole consists of two anti-phase harmonic charges $q_{\pm} = \pm \rho_0 \cos(\omega_0 t_1)$ with small spacing l . According to the current continuity law, there is a very short current filament between the two charges, the density of which is $\mathbf{J}_{dip}(\mathbf{r}_1, t_1) = -\omega_0 \rho_0 l \sin(\omega_0 t_1) \hat{\mathbf{p}}$, where ω_0 is the oscillating angular frequency of the charges. The electric dipole moment is expressed by $\mathbf{p} = \rho_0 l \cos(\omega_0 t_1) \hat{\mathbf{p}}$ [2], in which the polarization unit vector $\hat{\mathbf{p}}$ points from q_- to q_+ .

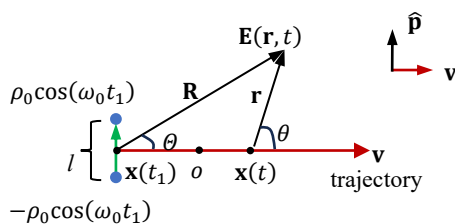


Figure 1. Trajectory of a moving Hertzian dipole for $\mathbf{v} \cdot \hat{\mathbf{p}} = 0$.

In the vacuum, when the dipole moves uniformly with velocity $\mathbf{v}$ in the direction perpendicular to $\hat{\mathbf{p}}$, the radiation fields can be derived utilizing the Lienard-Wiechert potentials [3–5]. The far fields at the position $\mathbf{r}$ and time instant t are expressed by [1]:

$$\mathbf{E}_{dip}^{far}(\mathbf{r}, t) \approx \frac{E_0 k_0^2}{R(1-\hat{\mathbf{n}} \cdot \boldsymbol{\beta})^3} [-\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \hat{\mathbf{p}}) - \hat{\mathbf{n}} \times (\hat{\mathbf{p}} \times \boldsymbol{\beta})] \cos(\omega_0 t_1) \quad (1)$$

$$\mathbf{B}_{dip}^{far}(\mathbf{r}, t) \approx \frac{1}{c_0} \hat{\mathbf{n}} \times \mathbf{E}_{dip}^{far}(\mathbf{r}, t) \quad (2)$$

The electromagnetic fields are generated by the source at the position $\mathbf{r}_1 = \mathbf{x}(t_1)$ and the time instant t_1 . In the expressions, $\mathbf{R} = \mathbf{r} - \mathbf{x}(t_1)$ is the radius vector from the source position to the observation position, $R = |\mathbf{R}|$, $\hat{\mathbf{n}} = \mathbf{R}/R$. The normalized velocity is denoted by $\boldsymbol{\beta} = \mathbf{v}/c_0 = \beta \hat{\mathbf{v}} = v/c_0 \hat{\mathbf{v}}$, where c_0 is the light velocity in the vacuum. The retarded time t_1 can be expressed in terms of the present time t and the distance R . They satisfy the following governing equation,

$$c_0(t - t_1) = R = |\mathbf{r} - \mathbf{x}(t_1)| \quad (3)$$

The amplitude $E_0 = \rho_0 l / (4\pi\epsilon_0)$ in (1) is a source-related constant. ϵ_0 and k_0 are respectively the permittivity and wavenumber in the vacuum.

As depicted in Figure 1, we have used θ to denote the observation angle with respect to the position $\mathbf{x}(t_1)$ where the fields are generated, namely, the birthplace of the fields. We have used θ for the observation angle with respect to the position $\mathbf{x}(t)$ where the source is currently located.

For motionless observers, the electromagnetic pulse generated at $(\mathbf{r}_1, t_1)$ will propagate away from its birthplace with c_0 in all directions. When the source moves along its trajectory, it will radiate continuously, and the electromagnetic pulses generated at each time instant will be superimposed to form a continuous electromagnetic wave in the vacuum. The phase velocity of the wave may change with respect to the reference point or the observation direction. However, the Doppler shift measured at a specified observation space-time point $(\mathbf{r}, t)$ is independent of the reference point and the observation direction.

In this paper, we consider a more general situation that the source moves uniformly with velocity $\mathbf{v}$, and in the meantime, the observer moves uniformly with velocity $\mathbf{v}_o$. We focus on discussing the phase velocity c and the Doppler shift of the fields far away from the birthplace $\mathbf{x}(t_1)$ of the source, that is, we choose the birthplace of the fields as the reference point.

2. The General Expressions for the Phase Velocity and Doppler Effect

Assume that the Hertzian dipole is located at $\mathbf{r}_1 = \mathbf{x}(t_1)$. It uniformly moves with velocity of $\mathbf{v} = \dot{\mathbf{x}}(t_1)$. We are to evaluate the phase velocity c and the angular frequency ω at the observation point $\mathbf{r}$ far away from the origin. The observer uniformly moves with velocity $\mathbf{v}_o = \dot{\mathbf{r}}(t)$. The far electric field of the Hertzian dipole is expressed with (1). Obviously, the propagation property of the wave is mainly described by the term $\cos(\omega_0 t_1)$. Denote the phase function of the wave as

$$\Phi(\mathbf{r}, t) = \omega_0 t_1 \quad (4)$$

For the fields measured at $(\mathbf{r}, t)$, the time t_1 is a function of $(\mathbf{r}, t)$ and satisfies the governing equation (3). Since we take the birthplace of the field as the reference position, that is, we take it as the origin of the coordinate system, the far field is approximately a spherical wave, so the phase function can be expressed by

$$\Phi(\mathbf{r}, t) = \omega_0 t_1 \cong \omega t - \mathbf{k} \cdot \mathbf{r} \quad (5)$$

The angular frequency ω measured at $(\mathbf{r}, t)$ can then be evaluated from the phase function with

$$\omega = \frac{\partial}{\partial t} \Phi(\mathbf{r}, t) = \omega_0 \frac{\partial t_1}{\partial t} \quad (6)$$

Generally, the phase velocity is defined as the propagation velocity of the equi-phase surface in its normal direction, which can be derived by making use of the gradient of the phase function. From (5) we obtain that

$$\mathbf{k} = -\nabla \Phi(\mathbf{r}, t) = -\omega_0 \nabla t_1 \quad (7)$$

For a moving source and observer, we can derive with the same techniques discussed in [1] that

$$\frac{\partial t_1}{\partial t} = \frac{1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta}_o}{1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta}} \quad (8)$$

$$\nabla t_1 = -\frac{1}{c_0} \frac{\hat{\mathbf{n}}(1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta}_o)}{1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta}} \quad (9)$$

where $\boldsymbol{\beta}_o = \mathbf{v}_o / c_0$. Substituting them into (6), we can obtain the formula for the Doppler effect

$$\omega = \frac{1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta}_o}{1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta}} \omega_0 \quad (10)$$

and from (7) we get the wave vector

$$\mathbf{k} = -\omega_0 \nabla t_1 = \frac{\omega_0}{c_0} \frac{1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta}_o}{1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta}} \hat{\mathbf{n}} = \frac{\omega}{c_0} \hat{\mathbf{n}} \quad (11)$$

Obviously, the normal direction of the equi-phase surface is $\hat{\mathbf{n}}$. Equation (11) clearly demonstrates that the phase velocity in the direction of $\hat{\mathbf{n}}$ is always c_0 and is independent of the moving velocity of the source and the observer.

3. About the Phase Velocity and the Pulse Propagation Velocity

In the derivation, we have taken the birthplace of the field $\mathbf{x}(t_1)$ as the origin. In practical measurements, we have to adjust the measurement system to ensure that we can point correctly to the birthplace of the fields that we are measuring. After each adjustment, we may manage to finish our measurement within a short time period $[0, t_1]$ during which the moving source can be considered to be very close to the origin. For example, when we measure the light from remote stars, we have to adjust the direction of the telescopes to ensure that it correctly points to the star during the measurement. The electromagnetic fields we are measuring can be approximately treated as a spherical wave pulse that propagates from the origin with phase velocity c_0 . We can estimate the time that the pulse reaches the observation point with $t \approx R/c_0$. Assume that when $t = 0$, the star is at the origin, both the observer Jia and Sang are at $\mathbf{r}_0$. Sang keeps staying at $\mathbf{r}_0$, while Jia moves uniformly with velocity $\mathbf{v}_o$. As shown in Figure 2. The electromagnetic pulse will take time $t_{sang} = r_0/c_0$ to reach Sang, and the phase velocity measured by Sang is c_0 . The pulse will take time $t_{jia} \approx |1 + \boldsymbol{\beta}_o \cdot \hat{\mathbf{r}}_0| r_0/c_0$ to reach Jia, and the phase velocity measured by Jia is still c_0 .

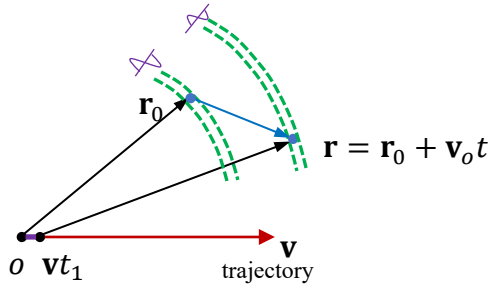


Figure 2. The electromagnetic pulse generated by the dipole in the period $[0, t_1]$.

We have emphasized in [1] that c_0 represents the propagation velocity of the electromagnetic pulse with respect to its birthplace. For observers who stay stationary relative to the birthplace, the propagation velocity of the electromagnetic pulse measured by them is definitely c_0 . Contrarily, for observers who are not motionless with respect to the birthplace of the pulse, the relative propagation velocity of the pulse may be not equal to c_0 . However, the phase velocity is always c_0 if we can manage to measure the star light in the direction exactly pointing to its birthplace.

Consider the problem in the laboratory frame where a star is assumed to stay motionless with respect to the earth. The light pulse from that star generated at $t_1 = 0$ travels directly to the earth with speed c_0 . The observer Sang measures the star light at her room on the earth while Jia does her measurement on a plane flying in the direction to the star with velocity $\mathbf{v}_o = -v_o \hat{\mathbf{r}}_0$, as shown in Figure 3. The time for the star light to reach Sang's detector is simply $t_{sang} = r_0/c_0$, while the time for the pulse to reach Jia's detector is $t_{jia} = r_0/(c_0 + v_o) < t_{sang}$. When measured by Jia, the relative speed of the light pulse with respect to her is $(c_0 + v_o)$. Obviously, it is larger than the light speed in the vacuum. Although not compatible with the theory of special relativity (TSR), this result is the natural outcome by solving the Maxwell's equations in the laboratory frame. Note that the phase velocity measured by the observers are both c_0 .

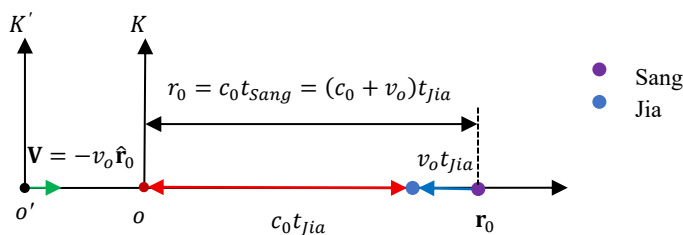


Figure 3. The relative motion of electromagnetic pulse and the observers.

If we try to explain it with TSR, we may need to transform the space-time events in the laboratory frame K to the inertial frame K' in which the observer Jia is at rest, as shown in Figure 3, where K' moves away from K with velocity $\mathbf{V} = -v_o \hat{\mathbf{r}}_0$. We choose the three events at the laboratory frame to

illustrate the explanation in TSR. The first event: the pulse is generated at $\mathbf{r}_1 = 0$ and $t_1 = 0$. The second event: the pulse reaches Jia at $\mathbf{r}_{jia} = \mathbf{r}_0 - v_o t_{jia} \hat{\mathbf{r}}_0$ and $t_{jia} = r_0/(c_0 + v_o)$. The third event: the pulse reaches Sang at $\mathbf{r}_0$ and $t_{sang} = r_0/c_0$. The three events can be respectively transformed to K' with the Lorentz transformations [6,7]

$$\begin{cases} \mathbf{r}' = \mathbf{r}_\perp + \gamma(\mathbf{r}_\parallel - \mathbf{V}t) \\ t' = \gamma\left(t - \frac{\mathbf{V} \cdot \mathbf{r}}{c_0^2}\right) \end{cases}, \quad (12)$$

where $\gamma = 1/\sqrt{1 - v_o^2/c_0^2}$ is the Lorentz factor. We can check that in the rest frame K' the pulse is generated at $\mathbf{r}'_1 = 0$ and $t'_1 = 0$. It reaches Jia at $\mathbf{r}'_{jia} = \gamma\mathbf{r}_0$ and $t'_{jia} = \gamma r_0/c_0$. It reaches Sang at $\mathbf{r}'_{sang} = \gamma(1 + v_o/c_0)\mathbf{r}_0$ and $t'_{sang} = \gamma(1 + v_o/c_0)r_0/c_0$. We can check that, when explained with TSR, the pulse travels with the propagation velocity c_0 to Sang and Jia even if Jia moves uniformly away from Sang. The phase velocity still remains to be c_0 . Note that we have used the standard Lorentz transformations for space-time and velocity in the derivation [8–11].

4. About the Formulae for Doppler Effect

Doppler effect in electromagnetic waves has been widely applied in radar systems for detecting flying objects or meteorological phenomena [12–15]. It is also a very powerful tool for astronomers to investigate the universe by measuring the Doppler shift in the stars [16–18].

However, when an object moves very fast, the relativistic effect has to be taken into account in the Doppler shift according to TSR. Conventionally, making use of the hypothesis of phase invariant of plane waves [19,20] among inertial frames, the relativistic formula for the Doppler shift is derived to be

$$\omega = \frac{1}{\gamma(1 - \hat{\mathbf{n}} \cdot \boldsymbol{\beta})} \omega_0 \quad (13)$$

Obviously, it is not same as the formula (10). The main differences between formula (10) and formula (13) are:

- (1) The Doppler shift due to the motion of the observer is included in (10) but not included in (13);
- (2) Even for motionless observer, the two formulae still differ by a Lorentz factor γ .
- (3) When the source and observer both move in directions perpendicular to the connecting line between them, namely, $\boldsymbol{\beta} \cdot \hat{\mathbf{r}}_0 = 0$ and $\boldsymbol{\beta}_o \cdot \hat{\mathbf{r}}_0 = 0$, we can obtain from (10) that $\omega = \omega_0$. However, the conventional relativistic formula (13) clearly shows that there is still a Doppler shift in this case due to the effect of the Lorentz factor. This is the transverse Doppler shift that is considered as a pure relativistic effect.

5. Conclusions

The electromagnetics of moving objects is still of great significance to revisit [1,10]. In particular, we have analyzed the electromagnetic properties of the fields of a moving source with the classical electromagnetic theory. We have two main findings through the analysis:

- (1) The electromagnetic pulse propagates away from its birthplace with velocity c_0 in all directions. The phase velocity of the electromagnetic wave is always c_0 when measured in the direction pointing exactly to the birthplace of the wave. The two velocities are independent of the observers, whether they are motionless or they are moving. However, the relative velocity between the electromagnetic pulse and the observer is not constant: the velocity of the electromagnetic pulse and the moving velocity of the observer should be added like vectors.
- (2) The derived expression for the Doppler shift is similar to the classical Newtonian type formula instead of the relativistic one. The motion of the observer also has significant impact on the Doppler shift.

We would like to remind that the Maxwell's equations can either be solved by staying in the laboratory frame and making use of the Lienard-Wiechert potentials, or be solved with the frame-hopping technique based on Lorentz transformations. The numerical examples provided in [21, 22]

have demonstrated that the two methods are equivalent. Consequently, equation (10) is valid for arbitrarily moving sources as their far fields share the same form of phase term as (5).

Since (10) is strictly derived based on the Maxwell's theory, we argue that its inconsistency with the conventional relativistic formula is just one of the many inconsistencies between the Maxwell's theory and the TSR.

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Conflicts of Interest: The authors declare no conflicts of interest.

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