Communication

Not peer-reviewed version

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Posted Date: 23 March 2026

doi: 10.20944/preprints202603.1718.v1

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doi:10.20944/preprints202603.1718.v1

Article

Halo-like Scaling from Rotational Stresses in Relativistic Matter

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Abstract

It is shown that halo-like scaling can arise from the rotational sector of the matter energy-momentum tensor without modifying the Einstein equations. The mixed stress-energy components associated with vorticity generate a conserved Killing current describing angular-momentum transport in stationary axisymmetric systems. The corresponding stream potential admits a multipole expansion whose dominant odd mode determines the radial angular-momentum flux and its asymptotic amplitude is directly related to the angular-momentum current $\dot{J}_\infty = -4\pi R_\infty$. If this transport channel remains finite at large radii, the resulting flux scales as $F_r \propto r^{-2}$, producing an effective density profile $\rho_{eff} \propto r^{-2}$ and approximately flat rotation curves in the weak-field regime. Observational scaling relations constrain the asymptotic transport amplitude and are consistent with an interpretation in which the baryonic Tully-Fisher relation reflects large-scale angular-momentum transport generated by the baryonic disk. In this picture, MOND-like phenomenology can arise as an asymptotic consequence of rotational stresses in relativistic matter, without invoking modified gravity or additional dark matter components.

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1. Introduction

Galaxy rotation curves exhibit a well-known asymptotic behaviour: the circular velocity tends to approach a constant value at large radii, and the asymptotic velocity obeys the baryonic Tully-Fisher relation [1,2]

$$v_f^4 \propto G M_b a_0, \tag{1}$$

where M_b is the baryonic mass. These observations are typically interpreted either in terms of dark matter halos or in terms of modified low-acceleration dynamics [3,4]. An alternative possibility is that the observed phenomenology emerges from additional dynamical structure within the matter energy-momentum tensor itself, without introducing new particle species or modifying the Einstein field equations. Related approaches have explored the role of non-trivial energy-momentum exchange and effective dark sector behaviour within the stress-energy tensor itself [5]. In contrast to such scenarios, the mechanism proposed here attributes the effect specifically to the rotational sector of the matter energy-momentum tensor and the associated angular-momentum transport.

In this Letter it is pointed out that such phenomenology may arise naturally from the rotational sector of the Alena Tensor framework [6]. Although derived within the Alena Tensor framework, the discussed mechanism relies only on the existence of non-vanishing mixed stress-energy components $T^r{}_\phi$ associated with angular-momentum transport thus similar structures are expected in any relativistic description of rotating matter with anisotropic stresses. The mechanism discussed here does not modify the Einstein field equations, but exploits the rotational sector of the matter energy-momentum tensor already present in this framework. The key observation is that rotational stresses contribute to

doi:10.20944/preprints202603.1718.v1

the effective gravitating energy density of matter through the vorticity invariant. In the weak-field regime this contribution produces a halo-like density profile that leads to flat rotation curves and MOND-like scaling [7], similar in spirit to mechanisms where halo phenomenology emerges from additional stress-energy components rather than new particle species [8].

2. Rotational Stresses and Asymptotic Halo

The Alena Tensor framework can be viewed as an effective relativistic fluid description with intrinsic coupling between shear and vorticity, leading to additional anisotropic stress components. Recent results obtained within this framework e.g. show improved agreement with rotation curve data compared to single-parameter MOND [6]. In this construction the effective matter energy-momentum tensor derived from the Lagrangian density introduced in [6] can be written schematically as

$$T_{\mu\nu} = \rho U_\mu U_\nu - \Xi_{\mu\nu}, \quad \nabla^\mu T_{\mu\nu} = 0, \tag{2}$$

where $\Xi_{\mu\nu}$ encodes energy flux and rotational stresses associated with the vorticity of the flow. In this approach the system has a built-in anisotropic stress, but its source is not viscosity, but the coupling between shear and vorticity (between flow deformation and local spin angular momentum). The gravitational field in this framework satisfies the standard Einstein equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$, while the new effect arises from the rotational sector of the matter stress-energy tensor.

The explicit form of the effective stress-energy tensor obtained in the Alena Tensor framework can be written as

$$T_{\mu\nu} = \frac{\varepsilon}{c^2} U_\mu U_\nu + \frac{1}{c^2}(U_\mu q_\nu + U_\nu q_\mu) - E_{\text{rot}}\,\Delta_{\mu\nu} - \tau_{\mu\nu}, \tag{3}$$

where U^μ is the four-velocity of the flow, q^μ denotes the energy flux, $\tau_{\mu\nu}$ is the anisotropic stress tensor, $\Delta_{\mu\nu} = g_{\mu\nu} - U_\mu U_\nu / c^2$ is the spatial projector orthogonal to the flow and the rotational energy density is proportional to the vorticity invariant $E_{\text{rot}} \propto \omega_{\mu\nu}\omega^{\mu\nu}$, with $\omega_{\mu\nu}$ being the projected vorticity tensor.

The "halo" was already interpreted in [6] not as additional mass, but as a result of angular momentum transport via the energy flux q^μ and anisotropic stresses $\tau^\mu{}_\nu$, which generate non-zero fluxes $T^r{}_\phi$ and/or $T^\theta{}_\phi$.

For stationary axisymmetric systems the spacetime admits the rotational Killing vector $\xi^\mu_{(\phi)} = \partial_\phi$. The corresponding Killing current is

$$J^\mu_{(L)} = T^\mu{}_\phi. \tag{4}$$

Because $\nabla_\mu T^{\mu\nu} = 0$, the Killing current is automatically conserved

$$\nabla_\mu J^\mu_{(L)} = 0. \tag{5}$$

In rotating astrophysical systems angular momentum transport is known to arise from correlated stresses and torques in the disk, suggesting that a quasi-stationary transport channel may naturally develop on galactic timescales. In axisymmetric coordinates this conservation law reduces to

$$\partial_r(\sqrt{-g}F_r) + \partial_\theta(\sqrt{-g}F_\theta) = 0, \tag{6}$$

where the flux components are

$$F_r = \frac{q^r U_\phi}{c^2} - \tau^r{}_\phi, \qquad F_\theta = \frac{q^\theta U_\phi}{c^2} - \tau^\theta{}_\phi. \tag{7}$$

This equation has the structure of a two-dimensional continuity equation and therefore admits a stream potential $\Psi(r,\theta)$ defined by

$$\sqrt{-g}F_r = \partial_\theta \Psi, \qquad \sqrt{-g}F_\theta = -\partial_r \Psi. \tag{8}$$

The angular structure of Ψ can be expanded in Legendre modes

$$\Psi(r,\theta) = \sum_\ell R_\ell(r) P_\ell(\cos\theta). \tag{9}$$

On the galactic plane ($\theta = \pi/2$) only odd multipoles contribute to the radial flux because $P'_{2m}(0) = 0$. The lowest non-vanishing contribution therefore corresponds to $\ell = 1$

$$\Psi(r,\theta) \approx R_1(r)\cos\theta. \tag{10}$$

By analogy with standard multipole expansions in long-range fields, higher-order modes are expected to decay faster with radius [9,10], so that the asymptotic behaviour is dominated by the lowest non-vanishing odd multipole $\ell = 1$. In the weak-field regime the metric determinant behaves asymptotically as $\sqrt{-g} \sim r^2$, which implies

$$F_r \sim \frac{R_1(r)}{r^2}. \tag{11}$$

If $R_1(r)$ approaches a constant value at large radii, the resulting flux scales as r^{-2}. Such a scaling corresponds to an effective gravitating density profile $\rho_{\rm eff} \propto r^{-2}$, which is well known to generate approximately flat rotation curves in the weak-field Newtonian limit.

3. MOND-like Scaling, RAR and BTFR

The observational scaling relations place additional constraints on the allowed class of solutions. Galaxy rotation curves satisfy the radial acceleration relation (RAR) [11], which implies that the halo contribution must become negligible in the high-acceleration inner region while dominating at large radii. This requires that the radial function satisfies

$$R_1(r) = o(r^2) \quad (r \to 0), \tag{12}$$

so that the rotational sector does not affect the central baryon-dominated region.

A useful physical interpretation of the coefficient $R_1(r)$ follows from the conserved Killing current associated with rotational symmetry. The total radial flux of angular momentum through a sphere of radius r is given by

$$\dot{J}(r) = \int J^r_{(L)}\, dS = 2\pi \int_0^\pi d\theta\, \sqrt{-g}\, F_r. \tag{13}$$

Using the stream-potential representation

$$\sqrt{-g}F_r = \partial_\theta \Psi, \tag{14}$$

one obtains

$$\dot{J}(r) = 2\pi \int_0^\pi d\theta\, \partial_\theta \Psi = 2\pi[\Psi(r,\pi) - \Psi(r,0)]. \tag{15}$$

For the dominant odd multipole

$$\Psi(r,\theta) \approx R_1(r)\cos\theta, \tag{16}$$

this yields

$$\dot{J}(r) = -4\pi R_1(r). \tag{17}$$

 doi:10.20944/preprints202603.1718.v1

The coefficient $R_1(r)$ therefore has the direct physical meaning of the radial angular-momentum current carried by the rotational sector of the stress-energy tensor. In particular, the asymptotic amplitude is related to the asymptotic transport rate

$$R_\infty = -\frac{\dot{J}_\infty}{4\pi}. \tag{18}$$

The above assumption should be viewed not as a special choice but as the asymptotic fixed point of sustained angular-momentum transport. In a quasi-stationary regime, conservation of the Killing current implies that the total angular-momentum flux through spherical shells becomes approximately radius-independent at sufficiently large distances, unless balanced by external sinks or sources. Such quasi-stationary transport regimes, characterized by long-range angular-momentum redistribution and approximately constant fluxes, are well known in rotating astrophysical systems [12–15].

In the underlying Alena Tensor framework, the radial mode $R_1(r)$ is not a fundamental degree of freedom but the leading odd multipole of the stream potential associated with the conserved angular-momentum Killing current. Its dynamics is thus induced by the rotational sector of the stress-energy tensor, through the energy flux q^μ and anisotropic stress $\tau_{\mu\nu}$, which are generated by the vorticity, shear, and acceleration of the flow. In a fully dynamical treatment, $R_1(r)$ would therefore be determined by the coupled evolution of these quantities together with the spacetime geometry. In the present Letter, the full transport problem is not addressed explicitly. Instead, the focus is placed on the asymptotic regime, in which the radial angular-momentum current approaches, as expected from the conservation argument above, a finite value at large radii. This allows one to isolate the generic consequences of rotational stresses for the large-scale gravitational field.

At large radii the asymptotic behaviour $R_1(r) \to R_\infty$ leads to

$$F_r \sim \frac{R_\infty}{r^2}. \tag{19}$$

Such a scaling corresponds to an effective gravitating density profile $\rho_{\text{eff}} \propto r^{-2}$. In the weak-field limit the gravitational acceleration is determined by the enclosed effective mass through

$$g(r) = \frac{GM_{\text{eff}}(r)}{r^2}. \tag{20}$$

Importantly, this contribution is not merely kinematic: it enters the gravitational field equations through the stress-energy tensor, so that the effective density ρ_{eff} directly sources the metric in the weak-field limit. If $\rho_{\text{eff}} \propto r^{-2}$, the enclosed mass grows linearly with radius

$$M_{\text{eff}}(r) \propto r, \tag{21}$$

which leads to an asymptotic acceleration

$$g_h(r) \propto \frac{1}{r}. \tag{22}$$

The circular velocity then follows from the standard relation

$$v^2(r) = r\, g(r), \tag{23}$$

yielding an approximately constant asymptotic velocity

$$v(r) \to v_f = \text{const.} \tag{24}$$

Matching the asymptotic velocity with the baryonic Tully-Fisher relation suggests the scaling

$$v_f^4 \propto R_\infty, \tag{25}$$

doi:10.20944/preprints202603.1718.v1

Within this framework, this relation should be understood as a consistency condition on the asymptotic transport amplitude rather than a first-principles derivation. A simple order-of-magnitude estimate shows that the required transport amplitude is astrophysically reasonable. For a typical spiral galaxy with asymptotic circular speed $v_f \sim 2 \times 10^5\,\mathrm{m\,s^{-1}}$, one finds

$$v_f^4 \sim 1.6 \times 10^{21}\,\mathrm{m^4\,s^{-4}}. \tag{26}$$

In the present mechanism, (25) indicates that that the required transport amplitude corresponds to macroscopic galactic scales rather than microscopic processes. Equivalently, using (18), the corresponding asymptotic angular-momentum transport rate is also macroscopically large, as expected for a collective galactic-scale stress channel rather than a microscopic effect. Thus the required amplitude is not unnaturally small and lies in the range appropriate for rotating disk galaxies. This is consistent with the expectation that the amplitude of the dominant transport mode scales with the baryonic mass of the system

$$R_\infty \propto M_b. \tag{27}$$

A minimal radial profile satisfying both the central RAR constraint and the asymptotic scaling can therefore be written as

$$R_1(r) = R_\infty \frac{r^n}{r^n + r_c^n}, \qquad n > 2, \tag{28}$$

where r_c defines the transition scale between the baryon-dominated inner region and the rotationally supported halo regime.

4. Discussion

The analysis presented here suggests that halo-like phenomenology of galaxies may arise from the rotational sector of the matter energy-momentum tensor rather than from additional dark matter particles or modifications of gravity. In this picture the effective halo contribution originates from angular-momentum transport encoded in the mixed stress-energy components $T^r{}_\phi$ and $T^\theta{}_\phi$ and the resulting effective density distribution is generically anisotropic. In particular, the dominant transport mode is expected to produce flattened, disk-aligned halo structures rather than spherical halos. This contrasts with standard dark matter halos, which are typically modeled as approximately spherical, and with MOND-like theories, where the effective modification is isotropic. The framework therefore predicts inclination-dependent gravitational effects, including anisotropic lensing signatures, providing a potential observational test distinguishing this mechanism from both dark matter and modified gravity scenarios.

The conservation of the corresponding Killing current naturally introduces a stream potential describing the transport of angular momentum in the meridional plane. Its angular structure is governed by Legendre modes, with the lowest odd multipole $\ell = 1$ dominating the radial flux on the galactic plane. This leads to a natural r^{-2} scaling of the effective halo contribution in the weak-field regime.

The observational scaling relations further restrict the class of admissible solutions. The RAR requires that the rotational sector vanishes sufficiently rapidly toward the galactic centre, while the baryonic Tully-Fisher relation fixes the asymptotic amplitude of the dominant Legendre mode through the baryonic mass of the system. These constraints leave a narrow class of radial profiles characterized by a transition scale separating the baryonic and rotational regimes.

In this interpretation galactic halos correspond to macroscopic transport modes of angular momentum generated by rotational stresses in relativistic matter. The observed halo phenomenology may therefore reflect large-scale collective behaviour encoded in the structure of the energy-momentum tensor itself, rather than introducing new matter components.

This interpretation is consistent with the broader astrophysical picture in which correlated stresses and torques transport angular momentum outwards in rotating systems [15,16]. The efficiency of the

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transport channel could plausibly arise from correlated gravitational torques and vortical stresses in the disk-halo system, analogous to angular momentum transport mechanisms discussed in galactic disk dynamics. In the present framework, the same transport channel contributes to the gravitational source term, so the halo can be interpreted as a gravitating collective rotational mode of matter.

Interestingly, the present framework also suggests a possible dynamical route toward the baryonic Tully-Fisher relation. As shown, the asymptotic amplitude R_∞ is directly related to the radial angular-momentum current carried by the rotational sector. The asymptotic scaling derived in this Letter therefore implies $v_f^4 \propto R_\infty \propto \dot{J}_\infty$. If the outward transport of angular momentum operates on a characteristic timescale comparable to the orbital timescale of the baryonic disk $\tau_J \sim t_{\rm orb} \sim R_d/v_f$ (local orbital timescale) [17], the asymptotic transport rate scales as $\dot{J}_\infty \sim J_b/\tau_J$, where J_b is the total angular momentum of the baryonic component.

Using a simple scaling estimate $J_b \sim M_b R_d v_f$ for rotationally supported disks then leads to $\dot{J}_\infty \sim M_b v_f^2$. Since in the present mechanism the halo amplitude is controlled by the asymptotic transport coefficient R_∞, this scaling naturally approaches the baryonic Tully-Fisher relation. In this interpretation the observed BTFR reflects the efficiency of large-scale angular-momentum redistribution in the baryonic disk rather than a fundamental modification of gravity. Such a picture could also help explain deviations from the ideal BTFR in terms of variations in disk size, spin parameter, or transport efficiency. Deviations from the BTFR should e.g. correlate with variations in angular-momentum transport efficiency, providing a potential observational test of the mechanism. A quantitative derivation of this scaling requires a full dynamical model of angular-momentum transport and is left for future work.

The existence of a finite asymptotic transport amplitude should be viewed not as a fine-tuned assumption but as the generic outcome of sustained angular-momentum redistribution in rotating systems. A detailed quantitative analysis of the transport-driven scaling considered here, however, lies beyond the scope of the present Letter, and the full dynamical treatment of the transport mechanism and its coupling to the galactic disk structure will be presented elsewhere. The present analysis is also limited to the weak-field regime. A full relativistic treatment of the coupled metric-matter system, as well as a first-principles derivation of the transport amplitude R_∞ remains thus an open problem.

Summarizing, all above considerations suggest that rotational stresses in relativistic matter may play a previously overlooked role in the gravitational dynamics of rotating astrophysical systems which is consistent with recent developments in relativistic vortical hydrodynamics and galactic angular-momentum transport [18,19].

5. Statements

All data that support the findings of this study are included within the article (and any supplementary files).

During the preparation of this work the author did not use generative AI or AI-assisted technologies, except for continuous learning.

Author did not receive support from any organization for the submitted work.

Author have no relevant financial or non-financial interests to disclose

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