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Not peer-reviewed version

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Posted Date: 17 December 2025

doi: 10.20944/preprints202512.1545.v1

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Article

Beyond a Naive Absolute Infinite

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Abstract

This paper proposes an idealized, philosophical axiomatization of the absolute infinite in a meta-formal class theory, called MK^{meta} , that can be back-translated to the formal Morse–Kelley with the axiom of global choice (GC). First, class ordinals and class cardinals are introduced, which avoid the Burali-Forti paradox. Second, GC is assumed to make class cardinals well-orderable. Third, the Hamkinsian multiverse M_h is defined as the meta-formal collection of all the models v of any relatively consistent, formal theory. Fourth, a meta-formal theory is rigorously defined by ranging over all the sets $x \in v \in M_h$. Fifth, V^{meta} is the unique model of any meta-formal theory. At last, the absolute infinite Ω^{meta}_{card} is the proper class cardinality of V^{meta} . Moreover, truth relativism can be countered in a GC-consistent branch of M_h , by accepting the axioms that maximize V^{meta} . Consequently, the definition of M_h can be used as a rebuttal of both height and width potentialism, when combined with the argument that only the meta-formal level can capture the entire mathematical reality.

Keywords: Cantor’s absolute infinite; set-theoretic multiverse; foundations of mathematics; axiomatic truth; Morse–Kelley set theory; potentialism

1. Introduction

In a letter to Dedekind, Cantor called his absolute infinite Ω^1 an inconsistent, absolutely infinite multiplicity and he associated it with God (Cantor 1962, Thomas-Bolduc 2016). The idea of an absolute infinite can be used for various other purposes than theology: in metaphysics and modal realism to describe the size of the plenitude (Blondé 2024) or Lewis’ (1986) logical space, in computer science to have the ultimate oracle (Burgin 2017) or ordinal² machine (Koepeke and Seyfferth 2009), in epistemology to formulate omniscience (Heylen 2020), and in the philosophy of mathematics to count the number of abstract entities that have a formal definition in the rich mathematical landscape (Sutto 2024).

Cantor’s idea continues to provoke controversy. Welch and Horsten (2016) review Cantor’s conception of the set-theoretic universe as a completed infinity and prefer it above Zermelo’s (1908) conception, because Cantor’s universe includes the modern large cardinals via reflection principles. Livadas (2020) also discusses Cantor’s absolute infinite in light of the modern large cardinals and argues that it is proof-theoretically unattainable. Gutschmidt and Carl (2024) maintain that the foundational problem with Cantor’s absolute infinite calls for humility, rather than negative theology.

Set-theoretic potentialism, which has garnered significant attention in the last decade, is the view that the process of set formation is incompletable or inexhaustible and that the classical universe of sets V and the absolute infinite Ω cannot be fully captured or defined as actual, completed totalities (Zermelo 1930, Putnam 1967, Parsons 1983, Hamkins 2012, Linnebo 2013, Linnebo and Shapiro 2019, Brauer et al. 2022, Sutto 2024). Height potentialism can be distinguished from width potentialism. Height potentialism is the claim that certain large collections, such as the collection of all natural

¹ Not to be confused with Woodin’s (2011) Ω nor any other non-maximally large infinite numbers.
² While ordinals are inherently ordered, cardinals focus on the notion of ‘how many’ without regard to order. Assuming the axiom of choice, every set x can be well-ordered. In this case, every cardinal $|x|$ can be identified with the first ordinal that has size $|x|$.

numbers or the collection of all sets, are not actual, but only potential collections. Width potentialism, on the other hand, asserts that there are no privileged axiomatic truths and that there is, therefore, no privileged universe of sets. Instead, there is a set-theoretic multiverse of potential multiverse-universes that are equally legitimate (Hamkins 2012, Scambler 2020, Meadows 2021, Gorbow 2022). For example, a multiverse-universe in which the Continuum Hypothesis (CH) is true is not better or worse than one in which CH is false.

In spite of this, the aim of this paper is to show that an absolute infinite Ω^{meta} , that can be proven nor defined in any formal theory, can be proven in an idealized meta-formal second-order theory.³ The definitions of Ω^{meta} and V^{meta} can be obtained via a definition of the set-theoretic multiverse and counter both height and width potentialism.

In the next section, class ordinals and class cardinals are formalized in Morse–Kelley set theory with the axiom of Global Choice (GC), henceforth referred to with the acronym MK (Wang 1949). In Section 3, meta-formal concepts are introduced as an idealization of formal concepts. Section 4 contains the definitions of MK^{meta} , V^{meta} , and the absolute infinite Ω^{meta} . After that, in Section 5, a range of objections to these definitions is rebutted. In Section 6, it is shown how the definition of a set-theoretic multiverse establishes a theoretical principle to determine the truth of an axiom. At last, the conclusions follow in Section 7.

2. Class Ordinals and Class Cardinals

Class ordinals and *class cardinals* will first be defined in MK as an extension of the more widely studied notion of ordinals and cardinals, which will be called *set ordinals* and *set cardinals* for clarity. By adding the axiom of global choice (GC) to Morse–Kelley set theory (hence creating MK), class cardinals become well-orderable (Jech 2006).

2.1. Class Ordinals

A set α is a set ordinal if and only if (henceforth iff):

1. α is a transitive set.
2. $(\alpha, \in)$ is a well-ordering.

The Burali-Forti paradox shows that the collection of all set ordinals Ord cannot be a set. If such a collection would be a set, it would exceed itself in size. However, a collection that is too large to be a set, such as Ord , can still be a class (von Neumann 1928). Every set is a class, namely a class that is an element of another class, but not every class is a set. In particular, proper classes, which are classes that are not an element of any other class, are not sets. In this paper, MK will be used when a formal second-order theory about sets and classes is needed. With this notion of classes, class ordinals can be defined as an extension of set ordinals. A class C is a class ordinal iff:

1. C is a transitive class.
2. $(C, \in)$ is a well-ordering.

A class C is transitive iff whenever

$$x \in C \wedge y \in x \rightarrow y \in C \quad (1)$$

$(C, \in)$, with C some class, is a well-ordering iff for every non-empty subset of C , there is a least element according to the relation $\in$

$$\forall X_i \subseteq C, X_i \neq \emptyset : \exists x_i \in X_i, \forall y \in X_i, x_i \neq y : x_i \in y \quad (2)$$

This means that the relation $\in$ can be used as an ordering relation $<$. The successor $\alpha + 1$ of a set ordinal α is defined as $\alpha \cup \{\alpha\}$. For the proper class ordinal Ord , however, the successor $\text{Ord} \cup \{\text{Ord}\}$

³ The requirement of a meta-formal theory makes Ω^{meta} philosophical in nature.

does not exist, because $\{\text{Ord}\}$ does not exist. Indeed, the proper class Ord is not an element of any class. Therefore, Ord does not have a successor. The following three theorems declare that a class C_p is a proper class ordinal 1) iff C_p is unique in being a proper class ordinal, 2) iff C_p is the least upper bound of all set ordinals, and 3) iff C_p is a maximal class ordinal: no class ordinal exceeds it:

Theorem 1. $\text{ProperClassOrdinal}(C_p) \leftrightarrow \forall C(\text{ProperClassOrdinal}(C) \leftrightarrow C = C_p)$

Proof. Left to right: Since all class ordinals are well-ordered, for any two class ordinals α and β , we have *comparability*: exactly one of $\alpha \in \beta$, $\alpha = \beta$, $\beta \in \alpha$, holds. If we assume that both C and C_p are proper class ordinals, then both $C \in C_p$ and $C_p \in C$ are impossible, because a proper class cannot be a member of anything. Therefore, $\text{ProperClassOrdinal}(C) \rightarrow C = C_p$. Conversely, we find that if $C = C_p$, that C must be a proper class ordinal.

Right to left: Instantiating C at $C = C_p$ forces C_p to be a proper class ordinal. $\square$

Theorem 2. $\text{ProperClassOrdinal}(C_p) \leftrightarrow \forall x(\text{SetOrdinal}(x) \leftrightarrow x \in C_p)$

Proof. Left to right: Suppose C_p is a proper class ordinal and suppose x is a set ordinal. Then x must be $\in$ -less than any proper class ordinal, in particular, $\in$ -less than C_p – so $x \in C_p$. Conversely, if $x \in C_p$, then x is a class ordinal $\in$ -less than C_p . But since C_p is unique in being a proper class ordinal according to Theorem 1, every x must be a set ordinal.

Right to left: Suppose $\forall x(\text{SetOrdinal}(x) \leftrightarrow x \in C_p)$. Then we must show that C_p is a class ordinal, hence that it is transitive and well-ordered by $\in$. Transitive: Let $y \in x \in C_p$. Because x is a set ordinal, it is transitive, such that $y \subseteq x$. Every element of y is therefore an element of the set ordinal x , and therefore itself a set ordinal. Thus $y \subseteq C_p$. Because y was an arbitrary element of an arbitrary $x \in C_p$, C_p is transitive.

Well-ordered by $\in$: Let $X \subseteq C_p$ be any nonempty subclass and pick some $\alpha \in X$. This α is then a set ordinal. Now we can construct the set $S = \{\beta \in X : \beta \in \alpha\} \cup \{\alpha\}$, which consists of all the elements in X that are $\in$ -equal-or-less than α . Because α is well-ordered by $\in$, S has an $\in$ -least element γ that is also the $\in$ -least element of X . Since X was an arbitrary nonempty subclass of C_p , this shows that C_p is well-ordered by $\in$. Together with the transitivity, this shows that C_p is a class ordinal. Seen that C_p contains every set ordinal, it is a proper class ordinal. $\square$

Theorem 3. $\text{ProperClassOrdinal}(C_p) \leftrightarrow \text{ClassOrdinal}(C_p) \wedge \forall C(\text{ClassOrdinal}(C) \rightarrow C_p \notin C)$

Proof. Left to right: Because a proper class ordinal C_p is not an element of any class, it is not an element of any class ordinal C .

Right to left: By comparability of ordinals, either $C \in C_p$, $C = C_p$, or $C_p \in C$. The last option is ruled out by the hypothesis on the right. By Theorem 1, the middle option is ruled out for set ordinals. Therefore, $\forall x(\text{SetOrdinal}(x) \leftrightarrow x \in C_p)$. Using Theorem 2, C_p must be a proper class ordinal. $\square$

Because of these properties of proper class ordinals, if Ω can be defined as a class ordinal, it must be equal to the proper class ordinal Ord .

2.2. Class Cardinals and Global Choice

In order to prove similar results for cardinals, GC is needed, because it warrants well-orderability of all classes. Without well-orderability, comparability among all classes, including proper classes, is ill-defined (Halbeisen and Shelah 2001). Cardinality comparison being lost, the idea of a maximally large collection with class cardinality Ω -as-cardinal (henceforth Ω_{card}) collapses. For example, frameworks like those supporting Reinhardt cardinals explicitly reject GC, leaving cardinal comparability – and

thus the very notion of ‘largest size’ – underdetermined (Reinhardt 1974). This is the reason not to consider extensions of ZF (Zermelo–Fraenkel set theory) without the axiom of choice (AC) in this paper, but extensions of the choice-consistent ZFC (ZF + AC) (Zermelo 1908). Morse–Kelley (without GC) and MK (with GC) are such extensions.

In the presence of GC, every class cardinal is identified with its initial class ordinal, such that class cardinals inherit the well-ordering – and comparability – from class ordinals. More precisely, under GC every set cardinal is identified by its initial set ordinal and the unique proper class cardinal by the unique proper class ordinal. The proofs of the theorems about class ordinals can then be reused for similar theorems that assert that proper class cardinals are also unique, the least upper bound of all set cardinals, and maximal. Consequently, if Ω_{card} can be defined, it must be a proper class cardinal.

3. Meta-Formal Concepts

An axiomatic theory is *formal* iff all its axioms can be recursively enumerated by a Turing machine. A Turing machine is a computer (or an ordinal machine) that can only make a countable number of computational steps. Consequently, a formal theory cannot have more than a countable number of axioms. Alternative expressions are that a formal theory is algorithmically enumerable or effectively axiomatizable. The most common examples of formal theories about sets are ZF and ZFC, and about classes are NBG (von Neumann–Bernays–Gödel; von Neumann 1928) and Morse–Kelley.⁴

Many technical consequences follow from the study of set-theoretic infinities via formal theories (Jech 2006, Kunen 2011): Gödel’s (1931) incompleteness theorems, Tarski’s (1936) undefinability of arithmetical truth, the Löwenheim–Skolem theorem about the cardinality of infinite models, the difference between proof-theoretic and model-theoretic definability, the great number of actually investigated theories, infinitely many models of a single theory, formal theories reasoning about other formal theories, an incomplete large cardinal hierarchy, relative consistency, independence, forcing, and many more. Even though formal theories have enabled set theorists to prove an abundance of mathematical theorems (not just meta-theoretical theorems), they have one major drawback: there is no ultimate formal theory. For every formal theory, a stronger formal theory can be built. This is to say that no formal theory can capture the entire mathematical reality.

The inability of formal theories to capture the entire mathematical reality is a serious drawback for their utility in certain domains of philosophy, such as metaphysics. Moreover, formal theories reuse the same axioms and their negations, making them mutually inconsistent. This problem – consistently defining the collection of all mathematical or set-theoretic objects in a single theory – can only be overcome by leaving the requirement of formality for an axiomatic theory aside.

For this reason, an idealized non-formal but rigorous theory is proposed that is more theoretical and abstract, although less technical, as compared to formal theories: its number of syntax rules, symbols, formulas, axioms, and theorems can be unrestricted and can thus be as large as the number of absolutely all abstract entities. This means that every set x can be proven to exist by its own explicit axiom or theorem that states: “ x exists.” Let us call such an idealized theory a *meta-formal* theory. The forementioned technical consequences of formal theories, from Gödel’s incompleteness theorems to forcing, are not readily applicable to a meta-formal theory. These technicalities typically arise from the limitation on the number of axioms and theorems in formal theories or from taking the perspective of one formal theory on another formal theory.

Because a meta-formal theory lies outside the scope of Gödel’s incompleteness theorems, it can be both consistent and complete (Franzén 2005). A meta-formal theory can achieve completeness by including all true statements about a domain, such as arithmetic, as axioms. It will also remain consistent as long as no contradictory statements are included. Gödel’s diagonal argument for constructing self-referential statements depends on recursive enumerability, which is unavailable in a meta-formal theory. Even though it is clear that a meta-formal theory is even more theoretical than a formal theory

⁴ Both NBG and Morse–Kelley extend ZFC by introducing classes alongside sets, but they differ in strength: NBG is a conservative extension of ZFC (i.e., it proves no new theorems about sets), while Morse–Kelley is strictly stronger.

with infinite models, it does precisely what formal theories cannot do, namely capturing the entire set-theoretic reality in a single theory.

Three levels of genericness can be distinguished with respect to formality. For the universe of all sets V , we have:

- V : The universe of sets that is a model of some formal or meta-formal theory $\mathcal{T}$.

$$(\text{Formal}(\mathcal{T}) \vee \text{MetaFormal}(\mathcal{T})) \wedge V \models \mathcal{T} \quad (3)$$

- V^{meta} : The classical universe of absolutely all sets that is a model of a meta-formal theory $\mathcal{T}$.

$$\text{MetaFormal}(\mathcal{T}) \wedge V^{\text{meta}} \models \mathcal{T} \quad (4)$$

- V^{form} (also referred to as v): A multiverse-universe, that is a model of some formal theory T .

$$\text{Formal}(T) \wedge V^{\text{form}} \models T \quad (5)$$

The same distinctions can be made between Ω , Ω^{meta} , and Ω^{form} (and hence between Ord , Ord^{meta} , and Ord^{form}). For theories, we can distinguish a meta-formal theory $\mathcal{T}^{\text{meta}}$ from formal theories T^{form} . Let us now define a Hamkinsian multiverse as follows:

$$M_h = \{v \mid T_1 \vdash v \models T_2 \wedge \text{Formal}(T_1) \wedge \text{Formal}(T_2)\} \quad (6)$$

By ranging over all the sets $x \in v \in M_h$ via a meta-formal universal quantifier $\forall^{\text{meta}}$, we can create a rigorous meta-mathematical definition of what it means to be a meta-formal theory in the context of this paper:

$$\begin{aligned} \text{MetaFormal}(\mathcal{T}) &\leftrightarrow \text{MK} \subseteq \mathcal{T} \wedge \\ \forall^{\text{meta}} x (\exists S [\text{Formal}(S) \wedge \text{Con}(S, \mathcal{T}) \wedge \text{MK} \subseteq S \wedge S \vdash \exists x] &\leftrightarrow \mathcal{T} \vdash \exists x) \end{aligned} \quad (7)$$

Because $\forall^{\text{meta}} x (\mathcal{T} \vdash \exists x)$ ranges over a number of sets that is not recursively enumerable, $\mathcal{T}$ cannot itself be formal. $\text{Con}(S, \mathcal{T}) \leftrightarrow \mathcal{T} \vdash 'S \text{ is consistent}'$. Using $\mathcal{T}$ as meta-theory yields $\text{Con}(S) \leftrightarrow \text{Con}(S, \mathcal{T})$ and $(v \models S) \leftrightarrow (\mathcal{T} \vdash v \models S)$. At last, *meta-consistency* is defined as being consistent with a meta-formal theory. It will be shown to be a synonym of consistency in the next section.

4. Defining the Absolute Infinite

Let us introduce the absolute/meta concepts in the previous section more rigorously. There are different paths toward a definition of the absolute infinite, although all make use of the key idea to leave formality behind and to choose for the meta-formal level. Three paths will be taken to illustrate this. The first path starts with a definition of a unique maximally consistently extended MK^{meta} as a meta-formal theory that has V^{meta} as its unique model. The second path begins with a definition of a meta-consistent multiverse M_m (having only models of meta-consistent theories) that is in a bijection with V_m^{meta} . The third path departs from the Hamkinsian multiverse M_h , in a bijection with V_h^{meta} . It is then shown that $V^{\text{meta}} = V_m^{\text{meta}} = V_h^{\text{meta}}$, with a proper class cardinality equal to $\Omega_{\text{card}}^{\text{meta}}$.

4.1. Via Maximally Consistent Extendedness

We can define MK^{meta} as the maximally consistent extension of MK (Enderton 2001), with no restrictions on the number or enumerability of axioms,⁵ such that:

1. $\text{MK} \subseteq \text{MK}^{\text{meta}}$
2. MK^{meta} is complete: $\forall^{\text{meta}} \phi (\text{MK}^{\text{meta}} \vdash \phi \vee \text{MK}^{\text{meta}} \vdash \neg \phi)$

⁵ Because of this non-formality, the properties of MK^{meta} do not violate Gödel's incompleteness theorems.

3. MK^{meta} proves its own consistency: $MK^{meta} \vdash \neg MK^{meta} \vdash \perp$

It can then be shown that MK^{meta} is meta-formal:

Theorem 4. $MetaFormal(MK^{meta})$

Proof. $MK \subseteq MK^{meta}$ is given by definition. Because of the maximal extendedness, we have that MK^{meta} proves the existence of a set x if it can be proven to exist in a meta-consistent, formal extension S of MK . This proves left-to-right in the $\forall^{meta} x$ part of Equation (7). As a result of MK^{meta} 's consistency, it does so *only* if this is the case. Otherwise, we can construct a formal but meta-inconsistent definition of the smallest set ordinal that cannot be proven to exist in any meta-consistent, formal theory, for instance, via a fixpoint construction using the Diagonal Lemma (Kunen 2011). This proves right-to-left. Therefore, $MetaFormal(MK^{meta})$. $\square$

That MK^{meta} can be both consistent and complete follows from the next theorem:

Theorem 5. $MK^{meta} \vdash$
 $(\forall^{meta} \phi (MK^{meta} \vdash \phi \vee MK^{meta} \vdash \neg \phi) \wedge \neg MK^{meta} \vdash \perp)$

Proof. Because there is no restriction on the number and the enumerability of axioms in a meta-formal theory, it can be maximally extended. This means that it can include every theorem ϕ or its negation as an axiom, leaving no theorem undecided. This proves the completeness of MK^{meta} . Moreover, MK is not provably inconsistent. Given that a maximally consistent extension does not add any inconsistent axioms, also MK^{meta} is not provably inconsistent. Because of the completeness, and because MK^{meta} contains arithmetic and can thus express its own consistency, the independence of its consistency is not an option: it has to explicitly assert either its own consistency or its own inconsistency via a dedicated sentence. Since a not provably inconsistent theory can never assert its own inconsistency, it follows that MK^{meta} asserts and internally proves it is consistent via this dedicated sentence. $\square$

The following theorem shows that MK^{meta} has a model:

Theorem 6. $\exists \mathcal{M} (\mathcal{M} \models MK^{meta})$

Proof. By construction, MK^{meta} is meta-consistent. Therefore, it is satisfiable in some universes of the Hamkinsian multiverse M_h . Hence there exists a model $\mathcal{M}$ such that $\mathcal{M} \models MK^{meta}$. $\square$

Let us say that a theory $\mathcal{T} \supseteq MK$ is an MK-theory. The following theorem asserts that any (Hamkinsian or meta-level) universe satisfying a meta-formal theory is the same universe:

Theorem 7. $MetaFormal(\mathcal{T}_1) \wedge MetaFormal(\mathcal{T}_2) \wedge \mathcal{M}_1 \models \mathcal{T}_1 \wedge \mathcal{M}_2 \models \mathcal{T}_2 \rightarrow \mathcal{M}_1 = \mathcal{M}_2$

Proof. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be meta-formal theories and suppose $\mathcal{M}_1 \models \mathcal{T}_1$ and $\mathcal{M}_2 \models \mathcal{T}_2$. Assume toward a contradiction that $\mathcal{M}_1 \neq \mathcal{M}_2$. Then there exists a set $x \in v \in M_h$ such that $x \in \mathcal{M}_1$ but $x \notin \mathcal{M}_2$. Since x exists in a model of a meta-formal theory, there exists a formal, meta-consistent MK-theory S such that $S \vdash \exists x$. By Definition 7, meta-formality of $\mathcal{T}_2$ implies $\mathcal{T}_2 \vdash \exists x$. Hence every model of $\mathcal{T}_2$ must satisfy $\exists x$. But $\mathcal{M}_2 \not\models \exists x$, contradicting $\mathcal{M}_2 \models \mathcal{T}_2$. Therefore $\mathcal{M}_1 = \mathcal{M}_2$. $\square$

By Theorem 6, MK^{meta} has a model. Hence every meta-formal theory has as unique model the same unique model that MK^{meta} has. Now, we can define V^{meta} as the unique model of MK^{meta} :

$$\exists! V^{meta} (V^{meta} \models MK^{meta}) \quad (8)$$

4.2. Via a Meta-Consistent Multiverse

In this section, a model V_m^{meta} and a theory MK_m^{meta} are defined starting from a definition of a meta-consistent multiverse M_m (Hamkins 2012). However, this M_m differs from Hamkins' multiverse M_h , as it makes a distinction between three categories of formal theories and their models: 1) meta-consistent, 2) relatively consistent, and 3) internally inconsistent theories.⁶ While M_m contains only models of category 1), M_h contains those of 1) and 2). Still using a meta-formal theory as background theory, M_m is defined as follows:

Definition: The meta-consistent multiverse M_m is the proper class of all the models of any meta-consistent, formal⁷ extension of MK. Any of these models is a multiverse-universe v :

$$M_m = \{v \mid v \models T \wedge \text{Con}(T) \wedge \text{Formal}(T) \wedge T \supseteq \text{MK}\} \quad (9)$$

Via M_m , we can define V_m^{meta} as having a set for each multiverse-universe in M_m , such that there is a bijection between M_m and V_m^{meta} :

$$\exists V_m^{meta} \left(\exists f \subseteq V_m^{meta} \times M_m \left[\begin{array}{l} \forall^{meta} x \in V_m^{meta}, \exists! y, ((x, y) \in f) \wedge \\ \forall^{meta} y \in M_m, \exists! x, ((x, y) \in f) \wedge \\ \forall^{meta} x \in V_m^{meta}, \text{set}(x) \end{array} \right] \right) \quad (10)$$

Now it can be shown that V_m^{meta} is not a model of any meta-consistent, formal MK-theory T , thereby motivating the need to give it a meta-formal definition:

Theorem 8. $\forall^{meta} T (\text{Con}(T) \wedge \text{Formal}(T) \wedge T \supseteq \text{MK} \rightarrow V_m^{meta} \not\models T)$

Proof. Assume, for contradiction, that $V_m^{meta} \models T$, with T a meta-consistent, formal extension of MK. Since T is formal, Gödel's incompleteness theorems apply. In particular, we can construct a meta-consistent, formal MK-theory T_2 that reasons about the models of T , by representing them as syntactic objects in the language of T_2 . Given GC in the MK-theories T and T_2 , this T_2 can be designed to describe a collection of models of T whose cardinality strictly exceeds $\|V_m^{meta}\|$. However, because T is meta-consistent, all such models must be elements of M_m , and $\|M_m\| = \|V_m^{meta}\|$ by the assumed bijection. This yields the contradiction that $\|M_m\|$ and $\|V_m^{meta}\|$ exceed themselves. Therefore, our assumption must be false, and $V_m^{meta} \not\models T$. $\square$

Let us then consider the following theory:

$$\text{MK}_m^{meta} := \text{MK} \cup \{\phi_v = \exists x_v \mid v \in M_m\} \quad (11)$$

where x_v is a set for each multiverse-universe v in M_m . These sets x_v can all be integrated in a definable way in $V^{MK} \models \text{MK}$, such that $V_m^{meta} \models \text{MK}_m^{meta}$. Because MK_m^{meta} has a model, it is meta-consistent. It will now be shown that MK_m^{meta} is meta-formal:

Theorem 9. $\text{MetaFormal}(\text{MK}_m^{meta})$

Proof. $\text{MK} \subseteq \text{MK}_m^{meta}$ is true by definition. In the left-to-right direction of the last part of Equation (7) (the definition of MetaFormal), $\text{Formal}(S)$, $\text{Con}(S, \text{MK}_m^{meta})$, and $S \vdash \exists x$ imply that S has a model $v \in M_m$ for which $v \models \exists x$. This shows that the universal quantification $\forall^{meta} x$ in Equation (7) is not more than the number of v 's, such that there is an $\exists x$ proven by MK_m^{meta} for each $\exists x$ proven by a formal, MK_m^{meta} -consistent MK-theory S .

⁶ These could be translated to 1) (temporally successively) actual worlds, 2) possible worlds, and 3) impossible worlds in modal logic.

⁷ Excluding models of non-formal theories avoids self-containment paradoxes akin to the Russell (1908) paradox.

For the right-to-left direction, we need to show that the number of v 's is not more than the number of x 's in the $\forall^{meta}x$ in Equation (7). If the number of v 's were more, then there exists a v that is not the model of any MK_m^{meta} -consistent (and hence meta-consistent), formal MK-theory S according to Equation (7). However, according to Equation (9) (the definition of M_m), every v is a model of such theory. Therefore, for each $\exists x_v$ proven by MK_m^{meta} , there exists a formal theory $S = MK \cup \{\exists x_v\}$, such that $\text{Formal}(S)$, $\text{Con}(S, MK_m^{meta})$, $MK \subseteq S$, and $S \vdash \exists x_v$. That proves right-to-left. $\square$

4.3. Via a Hamkinsian Multiverse

Let us consider the following definition of a theory that is derived from the Hamkinsian multiverse M_h (see Equation (6)):

$$MK_h^{meta} := MK \cup \{\phi_v = \exists x_v \mid v \in M_h\} \quad (12)$$

where x_v is a set for each multiverse-universe v in M_h . The model V_h^{meta} can then be constructed as being in a bijection with M_h . An analogue of Theorem 8 cannot be constructed for V_h^{meta} , because there exist meta-inconsistent theories in which, for example, $\|V_h^{meta}\| = \aleph_0 < \|M_h\|$. In order to show that MK_h^{meta} is meta-formal, it suffices to show that the meta-formal $MK_m^{meta} = MK_h^{meta}$. This in turn follows from this theorem:

Theorem 10. $\|M_m\| = \|M_h\|$

Proof. Direction $\leq$: M_m admits any $v_m \models T$, where T is a meta-consistent, formal extension of MK. Given that every model v_m of such an extension also occurs as a Hamkinsian v_h , we have $M_m \subseteq M_h$ and $\|M_m\| \leq \|M_h\|$.

Direction $\geq$: Let $v_h \in M_h$. Then we construct the theory $S_m = MK \cup \{\phi_{v_h}\}$, where ϕ_{v_h} is a single sentence asserting "there exists a set x_{v_h} coding the structure v_h ." ϕ_{v_h} is non-contradictory, given that v_h has a model in some relatively consistent, formal theory. This S_m extends MK and is formal and meta-consistent, given that it adds a single, non-contradictory sentence to MK. Moreover, $S_m \vdash \exists x_{v_h}$ by definition of ϕ_{v_h} . Therefore, because MK_m^{meta} is meta-formal (see Theorem 9), we can use Equation (7) to show that $MK_m^{meta} \vdash \exists x_{v_h}$, so $x_{v_h} \in V_m^{meta}$. Finally, since V_m^{meta} is in bijection with M_m , each x_{v_h} , and thus each v_h , picks out a unique $v_m \in M_m$. Therefore, $\|M_m\| \geq \|M_h\|$. Both directions then prove $\|M_m\| = \|M_h\|$. $\square$

Together, Equation (11), Equation (12), and Theorem 10 prove that $MK_m^{meta} = MK_h^{meta}$, such that also MK_h^{meta} is meta-formal.

4.4. The Axiom of Absolute Infinity

It was shown that MK^{meta} , MK_m^{meta} , and MK_h^{meta} are all meta-formal theories. Then we can use Theorem 7 and Equation (8) to show that they have the same unique model $V^{meta} = V_m^{meta} = V_h^{meta}$. This enables us to axiomatize a unique absolute infinite Ω_{card}^{meta} as the proper class cardinality of V^{meta} :

$$\text{Axiom of absolute infinity (AI)} : \exists \Omega_{card}^{meta} (\Omega_{card}^{meta} = \|V^{meta}\|) \quad (13)$$

4.5. Theorems

In this section, we continue to use a meta-formal background theory. Let us define $\text{Cofom}(T)$ as shorthand notation:

$$\text{Cofom}(T) := \text{Con}(T, MK^{meta}) \wedge \text{Formal}(T) \wedge T \supseteq MK \quad (14)$$

The following theorem shows that in MK^{meta} , Ω_{card}^{meta} exceeds a class cardinal κ iff some meta-consistent, formal MK-theory T proves κ exists:

Theorem 11. $MK^{meta} \vdash \forall^{meta} \kappa ([\exists T, X(\text{Cofom}(T) \wedge T \vdash \exists X \wedge \kappa = \|X\|)] \leftrightarrow \kappa < \Omega_{card}^{meta})$

Proof. Using GC in the MK-theory T , every class can be well-ordered. For every class cardinal κ that some meta-consistent, formal MK-theory can prove to exist, MK^{meta} proves the existence of a stronger meta-consistent, formal MK-theory in which κ is used to proof-theoretically define a cardinal that is larger in this well-ordering. Using the cardinality version of Theorem 3, a class cardinal is a proper class cardinal iff it is not exceeded by any class cardinal in this well-ordering. This means that κ is a set cardinal in MK^{meta} iff κ can be formally proven, and, because every class $X \subseteq V^{meta}$ in MK^{meta} , that $||V^{meta}|| = \Omega_{card}^{meta}$ is a proper class cardinal in MK^{meta} . Using the cardinality version of Theorem 2, it follows that $\kappa < \Omega_{card}^{meta}$ iff there is a meta-consistent, formal MK-theory T that can prove κ exists. $\square$

Because of the corresponding well-ordering between class cardinals and class ordinals, as warranted by GC, Theorem 11 results in the proper class ordinal Ω^{meta} that exceeds a class ordinal α iff there exists a meta-consistent, formal MK-theory T in which α can be proven to exist.⁸ This counters height potentialism. The following theorem expresses that the axiom of absolute infinity is independent of every meta-consistent, formal MK-theory T :

Theorem 12. $MK^{meta} \vdash \forall^{meta} T (Cofom(T) \rightarrow (T \not\vdash AI \wedge T \not\vdash \neg AI))$

Proof. For proving this meta-independence, we have to show both that:

1. A meta-consistent, formal MK-theory T cannot prove AI.
2. A meta-consistent, formal MK-theory T cannot disprove AI.

Step 1: According to Theorem 11 (left to right), MK^{meta} proves that all the class cardinals that any meta-consistent, formal MK-class theory T can prove to exist, are smaller than Ω_{card}^{meta} in MK^{meta} . Therefore, MK^{meta} proves that AI can never be proven within T .

Step 2: A meta-consistent, formal MK-theory T only contains axioms that are about the existence of class cardinals that can be formally defined, such that, according to Theorem 11 (again left to right), MK^{meta} proves that these axioms are about the existence of class cardinals that are smaller than Ω_{card}^{meta} . Therefore, MK^{meta} proves that the theory T cannot disprove AI.

Step 1 and step 2 together show that MK^{meta} proves that AI is independent of every meta-consistent, formal MK-theory T . $\square$

Because the axiom of absolute infinity is independent of every meta-consistent, formal MK-theory T , it can be added to any such theory T without introducing internal inconsistencies within T . However, the resulting $\Omega_{card}^{meta}(v)$'s are equal to the $||v||$'s. From the perspective of MK^{meta} , they do not deserve to be called $\Omega_{card}^{meta}(v)$'s, because they are MK^{meta} set cardinalities that are all smaller than the MK^{meta} proper class cardinality Ω_{card}^{meta} . Here we see that strict, formality-level-independent distinctions between Ω^{meta} , Ω , and Ω^{form} run into problems. The distinctions can be made in MK^{meta} , but not in any meta-consistent, formal MK-theory. Nevertheless, given the undeniable value of formal theories and their perspective in mathematics and philosophy, this calls for the following generalization of AI (GAI):⁹

$$\exists \Omega_{card} (\Omega_{card} = ||V||) \quad (15)$$

GAI is a level-independent expression of AI that can be used in both formal and meta-formal theories $\mathcal{T}$, with V being the universe of all sets as defined by $\mathcal{T}$.

5. Objections

In this section, it is argued that the provided definition of Ω is robust by answering a range of objections to it. Any competing definition has to deal with most of these objections.

⁸ The ordinal Ω is technically equal to Ord . While Ω is used as an ordinal number here, Ord is still used in this paper as the class of all set ordinals.

⁹ Various other symbols are used for this cardinal: $|V|$, $||\text{Ord}||$, $|\text{Ord}|$, κ_{On} , and $\beth_{\text{Ord}}$.

5.1. Ω Succumbs to the Burali-Forti Paradox

The Burali-Forti paradox demonstrates that naively constructing the set of all set ordinals leads to a contradiction, namely that the constructed set is both an element of itself and not an element of itself (Burali-Forti 1897). By introducing class ordinals in Section 2.1, Ω can be constructed as the proper class of all set ordinals Ord , rather than the set of all set ordinals (Jech 2006). This avoids the contradiction.

5.2. Ω Is Smaller Than $\Omega + 1$

An ordinal $\Omega + 1$ is not well defined in any of the formal or meta-formal axiomatic theories that are meta-consistent. (Without this requirement, it becomes possible to redefine Ω as, for example, ω .) Since any successor construction requires $\{\Omega\}$, which is not defined, $\Omega + 1$ cannot be formed. If both a theory-specific Ω^{form} and $\Omega^{\text{form}} + 1$ are well defined in the same Cofom-theory, then they are both MK^{meta} sets and therefore smaller than Ω^{meta} .

5.3. Ω 's Definition Is Inconsistent

By defining a class ordinal Ω^{meta} that is not an MK^{meta} set, there is no obvious self-reference in AI. Moreover, it follows directly from Theorem 12, about the independence of AI, that this axiom does not create inconsistency when added to any meta-consistent, formal axiomatic theory about classes.

5.4. Ω Is Indefinable and Inexhaustible

Gödel has described the universe of sets V^{meta} as structurally undefinable (Wang 2016, p. 280). A related objection is that V^{meta} is inexhaustible (Maddy 1988, pp. 501-3). Such remarks can also be made about Ω^{meta} . However, what Gödel means is that Ω^{meta} and V^{meta} cannot be *formally* defined. The definitions in this paper are meta-formal.

5.5. Non-Formal Theories Should Be Avoided

MK^{meta} is a non-formal axiomatic theory that is rigorously and consistently defined as meta-formal. This theory is needed to define and prove the existence of absolutely every class in a single theory and to acquire several desirable meta-theoretical properties, such as completeness, provable consistency, uniqueness, and maximality. Set theorists use formal theories all the time, as they make proofs achievable by humans. However, being manageable or directly constructible by humans cannot be invoked as a principle in defining abstract entities (Quine 1948, Putnam 1971, Feferman 1991). A meta-formal theory like MK^{meta} is indispensable, because the set-theoretic reality is too large to be captured by a single formal theory. Therefore, theoretical definitions must be accepted for abstract entities, regardless of how theoretical they are. After all, any rigorous definition of a set-theoretic multiverse is not formal either, even though it is equally indispensable in the philosophy of mathematics. This paper only completes the meta-formal level with entities like MK^{meta} , V^{meta} , Ω^{meta} , $\Omega_{\text{card}}^{\text{meta}}$, and M_m .

6. Absolute Truths in Hamkins' Multiverse

In this section, a further argument is made against width potentialism and relativism of truth by asserting that an MK-consistent Hamkinsian multiverse M_h^{mk} enables a theoretical criterion that determines which axioms are true and which are false.

6.1. The Principle of Maximizing Truth

Let us define M_h^{mk} as follows:

$$M_h^{\text{mk}} = \{v \mid T_1 \vdash v \models T_2 \wedge \text{Formal}(T_1) \wedge \text{Formal}(T_2) \wedge T_1, T_2 \supseteq \text{MK}\} \quad (16)$$

We can define maximal ordinal height (MOH) as follows:

Definition: The *maximal ordinal height* of an axiom ϕ is the least upper bound (or supremum) of the class of all set ordinals α in Ord^{meta} for which there exists a multiverse-universe $v \in M_h^{mk}$ in which ϕ holds and $\alpha \in \text{Ord}^v$, where Ord^v denotes the class of set ordinals of v :

$$\text{MOH}(\phi) := \sup\{\alpha \in \text{Ord}^{meta} \mid \exists v \in M_h^{mk} (v \models \phi \wedge \alpha \in \text{Ord}^v)\} \quad (17)$$

This supremum is always taken in MK^{meta} , in which the supremum of any class of set ordinals is a class ordinal. GC warrants that the class of all these Ω_{card}^{meta} -many set ordinals is well-orderable. The following theorem expresses that for any axiom ϕ , the MOH of either ϕ , or, non-exclusively, $\neg\phi$, is equal to Ω^{meta} :

Theorem 13. $\forall^{meta} \phi (\text{MOH}(\phi) = \Omega^{meta} \vee \text{MOH}(\neg\phi) = \Omega^{meta})$

Proof. Hamkins' multiverse M_h^{mk} is partitioned into three collections based on an axiom ϕ :

- C_T : multiverse-universes where ϕ holds,
- C_F : multiverse-universes where $\neg\phi$ holds, and
- C_I : multiverse-universes where ϕ is independent.

Because adding an independent axiom ϕ to a base theory preserves the set ordinals that were already definable in the base theory, C_I does not contain any set ordinals that are not also in C_T and C_F . That is what it means for both ϕ and $\neg\phi$ to be relatively consistent extensions of the base theory. Assume now for contradiction that the MOHs of both ϕ and $\neg\phi$ are less high than Ω^{meta} . This means that Ord^{meta} contains set ordinals that are in none of the three collections: neither in C_T nor C_F , which are less high, nor in C_I , which is even less high. This is a contradiction, because all the set ordinals in Ord^{meta} can be found in some v in M_h^{mk} . Therefore, the MOHs of ϕ and $\neg\phi$ cannot both be less high than Ω^{meta} . Given that every set ordinal is in Ord^{meta} , they can also not be higher than Ω^{meta} , such that the MOH of either ϕ , or, non-exclusively, $\neg\phi$, is equal to Ω^{meta} . $\square$

The following definition states that an axiom ϕ is MOH-decidable iff it can be decided in MK^{meta} that its MOH is either higher or less high than the MOH of $\neg\phi$:

$$\begin{aligned} \text{Definition: } \text{MOH-Decidable}(\phi) &:= \\ \text{MK}^{meta} \vdash (\text{MOH}(\phi) > \text{MOH}(\neg\phi)) \vee (\text{MOH}(\phi) < \text{MOH}(\neg\phi)) \end{aligned} \quad (18)$$

This brings us to the principle of maximizing truth (MT): A MOH-decidable axiom ϕ is true in MK^{meta} iff its MOH exceeds that of $\neg\phi$. It provides the theoretical criterion that determines whether the axiom ϕ is true or not:

$$\text{MK}^{meta} \vdash \text{MOH-Decidable}(\phi) \rightarrow (\text{MOH}(\phi) > \text{MOH}(\neg\phi) \leftrightarrow \text{True}(\phi)) \quad (19)$$

6.2. Arguments for the Principle of Maximizing Truth

Three arguments are provided for MT in this section: a classical V^{meta} argument, the agreement with MK, and a maximal definability argument. Let us assume that the MOH of ϕ exceeds that of $\neg\phi$ and argue that ϕ is true and $\neg\phi$ is false:

6.2.1. A Classical V^{meta} Argument

Because of Theorem 13, Ω^{meta} , and therefore V^{meta} , are maximized for either ϕ or $\neg\phi$. Given that the MOH of ϕ exceeds that of $\neg\phi$, Ω^{meta} must coincide with the MOH of ϕ . This means that ϕ is true in V^{meta} . Therefore, if we make the classical assumption that V^{meta} is defined by all the true axioms, we have that ϕ is true (Shoenfield 1977).

6.2.2. The Agreement with MK

Apart from axioms like extensionality, regularity, and global choice, which do not seem to be MOH-decidable, there are six axioms and one axiom schema in MK that appear to follow from MT: the

axiom of empty set, the axiom of pairing, the axiom of union, the axiom of power set, the axiom of infinity, the axiom of separation, and the axiom schema of replacement. For most of these axioms, an argument can be made that their MOHs exceed that of their negation. For example, the MOH of the axiom of infinity is Ω^{meta} , while the MOH of its negation is ω , the smallest set ordinal that is infinite. Another example is that the MOH of the axiom of empty set is Ω^{meta} , while that of its negation is the height of the empty set itself. Therefore, according to MK, the axioms with the highest MOHs are the true axioms.¹⁰

6.2.3. A Maximal Definability Argument

If the MOH of ϕ is strictly higher than that of $\neg\phi$, then the universe where ϕ holds enables the definitions of more ordinals than the universe in which $\neg\phi$ holds. Consequently, no combination of axioms together with $\neg\phi$ can ever provide an equally complete description of V^{meta} , and therefore of the entire mathematical reality, as compared to ϕ . As a result, ϕ is preferred over $\neg\phi$.

7. Conclusions

This paper defines several meta-formal concepts: 1) M_h , the Hamkinsian multiverse, 2) MK^{meta} , a meta-formal theory, 3) V^{meta} , the unique universe model of MK^{meta} , 4) $\Omega_{card}^{meta} = \kappa_{On}^{meta}$, the proper class cardinality of V^{meta} , and 5) $\Omega^{meta} = Ord^{meta}$, the collection of all the set ordinals in V^{meta} . The maximality and the uniqueness of V^{meta} counter height and width potentialism, respectively, when combined with the claim that the meta-formal level is a superior level because it can capture the entire set-theoretic reality. The proposed principle of maximizing truth, which can be resolved on the meta-formal level, further counters width potentialism and relativism about set-theoretic truth. This principle prescribes that an axiom is true if MK^{meta} can prove that its maximal ordinal height exceeds that of its negation. Since these rebuttals can start from definitions of set-theoretic multiverses, we can conclude that multiversism is no safe refuge for potentialists.

One of the reasons why the set-theoretic multiverse and width potentialism appear viable is that relative consistency is sometimes abbreviated as consistency. Only meta-consistency can be considered a synonym of consistency, given that only meta-formal theories can provide an appropriate proof of consistency.

While Gödel’s incompleteness theorems are important and sound results, their applicability is restricted to a notion of formality that requires recursive enumeration on Turing machines that can perform only countably many computational steps. Formal theories are inevitable tools for mathematicians, but they cannot provide a complete description of the whole mathematical reality. Completeness, consistency, a unique model, and maximality can be obtained for MK^{meta} , which is not formal in this restricted sense. Even though a meta-formal theory can be perceived as an extreme ideal, it is philosophically relevant.

Funding: This research received no external funding.

Data Availability Statement: No data was created for this research.

Acknowledgments: I wish to thank Ludger Jansen and anonymous referees for their feedback on earlier versions of this paper. During the preparation of this manuscript, I used ChatGPT as teacher, intellectual sparring partner, proof checker, and Latex expert. I have reviewed and edited the output and take full responsibility for the content of this publication.

Conflicts of Interest: The author declares no conflicts of interest.

¹⁰ A precise analysis of all the foundational axioms of MK is beyond the scope of this paper.

Abbreviations

The following abbreviations are used in this manuscript:

AC	The Axiom of Choice
card	cardinal
CH	The Continuum Hypothesis
Cofom	Meta-consistent, formal MK-extension
Con	Consistent
GC	The axiom of Global Choice
MK	Morse–Kelley set theory with GC
MOH	Maximal Ordinal Height
MT	The principle of Maximizing Truth
Ord	The class of all set ordinals
ZF	Zermelo–Fraenkel set theory
ZFC	ZF with AC

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