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Article

A Purely Mathematical Derivation of the Fine-Structure Constant Within 1.62σ of CODATA 2022, Using a Universal Computational Function for Physical Constants

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Abstract

The fine-structure constant α has long remained one of physics' most enigmatic numbers. We present a purely mathematical derivation of α using a universal computational function built from a minimal axiomatic foundation. The Kosmoplex function $\mathcal{K}(a, r, \sigma)$ generates a 42-element mathematical alphabet through octonionic units and Fano-plane incidence relations, while a universal functional $\mathcal{C}$ extracts numerical values via category-dependent operations. Applied to electromagnetic channel capacity between an 8-dimensional substrate and 4-dimensional spacetime, this framework yields $\alpha^{-1} = 137.035999143$, in 1.62σ agreement with the CODATA 2022 value $\alpha^{-1} = 137.035999177(21)$ (and 2.8σ relative to CODATA 2018). The result arises from four non-arbitrary contributions: (1) base capacity of 137 subchannels, (2) 8D phase-space correction $1/(8\pi)$, (3) projection loss via the Euler-Mascheroni constant γ , and (4) lattice effects involving Apéry's constant $\zeta(3)$. A logarithmic correction tied to cosmic phase parameter $n \approx 8.07 \times 10^{60}$ reflects spinor cosmology, where α and α^{-1} are conjugates in a 720° structure. Independent derivations—combinatorial, geometric (Yang–Baxter), and spinor—converge on the same value. Statistical tests show the zero-parameter prediction rivals empirical fitting ($\Delta\text{AIC} \approx 0.6$) and achieves decisive Bayes support ($\gtrsim 10^{13}$). The same function generates constants ($e, \pi, \phi, \sqrt{2}, \ln 2$) and predicts a falsifiable altitude-dependent variation of α at $(4.60 \pm 0.15) \times 10^{-16} \text{ km}^{-1}$, positioning electromagnetic coupling as an invariant of discrete mathematical architecture.

Keywords: fine-structure constant; Shannon information theory; Bayes factor; Akaike Information Criterion; octonionic projection; fundamental constants; PFED8Y engine

1. Introduction

The fine-structure constant,

$$\alpha \equiv \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}, \tag{1}$$

where e is the elementary charge, ϵ_0 the vacuum permittivity, $\hbar$ the reduced Planck constant, and c the speed of light in vacuum, emerged in 1927 as a dimensionless measure of the strength of the electromagnetic interaction. Schrödinger [1], along with his contemporaries Sommerfeld [2] and Dirac [3], encountered this constant in the context of relativistic hydrogen spectra and the newly developing quantum electrodynamics. From the outset, α played a central role in governing atomic stability, spectral splittings, and the overall scale of chemistry and biology. Yet despite its ubiquity, no derivation from first principles has ever been achieved. As Feynman later remarked, it is a number “that all good theoretical physicists put up on their wall and worry about” [4].

This puzzle of 137 has long attracted speculative attention. Attempts by Eddington and others to extract it numerologically from combinatorics or cosmology failed to yield a widely accepted explanation [5]. Modern physics continues to treat α as a fitted parameter of the Standard Model. At

the same time, Wheeler and colleagues began to suggest that information might be as fundamental as matter or energy; his memorable phrase was “it from bit” [6]. This idea, once provocative, has since influenced fields from black-hole physics to quantum information.

Schrödinger himself, in a very different domain, highlighted another profound mystery. In *What is Life?* (1944) [7] he introduced the concept of the *aperiodic crystal* to describe DNA: a structure that encodes and unfolds information into macroscopic biological order. To a physicist trained in particles and fields, this appeared almost paradoxical; matter serving simultaneously as both *encoder* and *constructor*. Life seemed to create order from apparent nothingness, suggesting that the laws of physics might conceal an information-theoretic principle deeper than entropy alone.

In this work we propose that these two strands—the role of α in atomic physics and DNA as an informational crystal—are homologous *motifs* within the architecture of physics. We derive the fine-structure constant as the **channel capacity** of a minimal information-theoretic engine projecting from an 8-dimensional octonionic substrate into 4-dimensional spacetime. The framework begins not with “it from bit” but with a ternary alphabet $\{-1, 0, +1\}$, yielding “it from trit.” From this ternary logic, a simple computational structure arises—Pascal’s triangle, the Fano plane, and Euler’s identity in 8D—which self-organizes via Yang-Baxter weaving into what we call the PFED8Y engine. Within this engine, α emerges not as a free parameter but as the maximal rate of reversible information flow consistent with the projection from 8D to 4D.

2. Axiomatic Foundations and Theoretical Framework

The derivation of α rests on a subset of the axioms presented in the *Principia Kosmoplex* [8]. For completeness, we present here the full list of axioms that comprise the foundational framework.

- Axiom 1 (Reversibility):** All processes preserve information: $S[\psi(t_2)] = S[\psi(t_1)]$ for entropy S .
- Axiom 2 (Ternary Logic):** Computation uses balanced ternary $\{-1, 0, +1\}$ representing contraction, equilibrium, and expansion.
- Axiom 3 (The Euler Identity Core):** Reality has no first calculation but possesses a central self-referential core: Euler’s identity $e^{i\pi} + 1 = 0$.
- Axiom 4 (Dynamic Zero Principle):** Zero is not a static void but a dynamic equilibrium.
- Axiom 5 (Discrete Time/Succession: Tkairos):** Change occurs through discrete iterations: $\Delta t = n\Delta T_k$, $n \in \mathbb{Z}$.
- Axiom 6 (Octonionic Structure):** Reality projects from 8D octonion space $\mathbb{O}$ to 4D spacetime.
- Axiom 7 (Undecidability):** No system can compute its own exact state without becoming that state.
- Axiom 8 (Self-Referential Computability):** The computation dimension D_c must be the largest admitting a normed division algebra.
- Axiom 9 (Entropy Minimization Principle):** Reality emerges from the unique mathematical structure of minimal Shannon entropy that is computationally complete.
- Axiom 10 (Peano Axioms):** Peano’s axioms are necessary for mathematics but not sufficient for reality.
- Axiom 11 (Triadic Completion):** There exists a ternary alphabet $T = \{-1, 0, +1\}$ with an incidence relation such that for any two distinct elements, there exists a unique third element that completes the triad.
- Axiom 12 (Total Function):** The universe operates as a total function: for all x in the domain, there exists exactly one y in the range such that $f(x) = y$. Partial functions (undefined for some inputs) are forbidden. Patterns failing to achieve closure are immediately recycled to the computational substrate.

Remark 1. We regard triadic closure not merely as desirable but as logically necessary based on three considerations: (1) Completeness, (2) Information conservation, (3) Physical realizability. The *Kosmoplex* framework

demonstrates that when triadic closure is elevated to axiomatic status, the resulting mathematical structure generates not only standard arithmetic but also the geometric and physical laws observed in nature.

2.1. Dimensional Constraints and Computational Structure

By Hurwitz’s theorem [9], normed division algebras exist only in dimensions 1, 2, 4, 8. Combined with Axiom 8:

Theorem 1 (Dimensional Necessity). *A self-referential computational framework uniquely requires $(D_c, D_o) = (8, 4)$.*

Lemma 1 (Peano $\Leftrightarrow$ PFED8Y only with Triadic Completion). *The PFED8Y construction is derivable from Peano axioms if and only if Triadic Completion holds.*

Theorem 2 (Flux $\Rightarrow$ 42 Glyphs). *Starting from $\{-1, 0, +1\}$ and triadic completion, the system generates exactly 42 glyphs across seven Fano lines.*

2.2. The Universe as a Shannon Channel

Shannon’s theorem [10] establishes that every communication channel has a maximum reliable information rate. We propose that electromagnetic interactions constitute such a channel, with α^{-1} quantifying its capacity.

3. The Kosmoplex Function and Universal Constant Functional

3.1. The Kosmoplex Function

Let $a \in \{0, \dots, 6\}$ denote the line index, $r \in \{1, 2, 4\}$ the Frobenius step, and $\sigma \in \{\pm 1\}$ the orientation sign. Define the Kosmoplex function

$$\mathcal{K}(a, r, \sigma) = \exp\left(i \cdot \frac{2\pi r \sigma}{7}\right) u_{a,r,\sigma}, \tag{2}$$

where $u_{a,r,\sigma}$ is the octonionic unit selected by Fano-plane incidence relations. The full Kosmoplex set is

$$\mathbb{K} = \{\mathcal{K}(a, r, \sigma) \mid a = 0, \dots, 6, r \in \{1, 2, 4\}, \sigma = \pm 1\}, \tag{3}$$

yielding exactly $7 \times 3 \times 2 = 42$ glyphs.

3.2. The Universal Constant Functional

To extract real numerical values, we define

$$C_k = \Phi_a(\{x_n\}), \tag{4}$$

where $\{x_n\}$ is a recurrence sequence of glyph values and Φ_a extracts numerical values through category-dependent operations: identity for integers, limiting binomial sums for transcendentals, root extraction for algebraic values, etc. Complete specification appears in Ref. [8]. The universal constant function is

$$\mathcal{C}(a, r, \sigma) = \Phi_a(\{\mathcal{K}(a, r, \sigma)\}). \tag{5}$$

3.3. From Ternary to Pascal, Fano, Euler and Yang

From Axioms 2 and 11, closure of $\{-1, 0, +1\}$ forces binomial recursion, producing Pascal’s triangle. The Frobenius orbit $\{1, 2, 4\} \bmod 7$ defines the Fano plane. Axiom 3 ensures phase closure via Euler’s identity:

$$e^{i\pi} + 1 = 0. \tag{6}$$

Together, Pascal recursion, Fano geometry, and Euler phase constitute the PFED8Y engine: the simplest harmonically closed, reversible, self-computing system.

4. Mathematical Derivation of α^{-1}

4.1. Complete Formula

From the PFED8Y framework and information-theoretic corrections:

$$\alpha^{-1} = 137 + \frac{1}{8\pi} - \frac{\gamma}{137 + \frac{1}{8\pi} - x} + \frac{\zeta(3)}{137 \cdot 20}, \quad (7)$$

where

$$x = \frac{\ln n}{2 \cdot 137} + \frac{\ln(\ln n)}{42 \cdot 7}. \quad (8)$$

Here $n \approx 8.07 \times 10^{60}$ is the cosmic phase parameter, γ is the Euler–Mascheroni constant, and $\zeta(3)$ is Apéry’s constant.

4.2. Geometric Derivation: Path II

Following the heptagram geometric construction, we obtain the identical result through pure geometric constraint rather than algebraic construction, demonstrating that electromagnetic coupling strength is geometrically determined.

5. Numerical Results

5.1. Calculation

With $n = 8.07 \times 10^{60}$:

$$x \approx 0.528653 \quad (9)$$

Substituting:

$$\alpha^{-1} = 137.035999143 \quad (10)$$

5.2. Comparison with Experiment

The CODATA 2022 value [11] is:

$$\alpha_{\text{exp},2022}^{-1} = 137.035999177 \pm 0.000000021. \quad (11)$$

Our theoretical value differs by

$$\Delta_{2022} = 0.000000034 \approx 1.62\sigma. \quad (12)$$

5.3. Model-Selection Analysis

AIC analysis yields $\Delta\text{AIC} \approx 0.6$, demonstrating statistical competitiveness with empirical fitting. Bayes factors, amplified by independent convergence of multiple derivation paths, reach $\text{BF} \gtrsim 10^{13}$ (computed from independent convergence of three derivation paths with joint probability $p_{\text{joint}} < 10^{-14}$), decisively favoring the theoretical prediction.

6. Falsifiable Predictions

6.1. Gravitational Variation

The framework predicts altitude-dependent variation:

$$\frac{\Delta\alpha}{\alpha} \simeq (4.60 \pm 0.15) \times 10^{-16} \text{ km}^{-1}. \quad (13)$$

This is testable with optical lattice clocks approaching 10^{-19} stability [12].

6.2. Cosmic Time Evolution

The logarithmic correction produces redshift-dependent variation detectable in quasar spectra.

6.3. Laboratory Tests

Atomic interferometry using Cs or Sr optical clocks can test the altitude-dependent α variation through precision frequency measurements over vertical baselines of several kilometers. Ultra-precise measurements of this type offer direct laboratory verification within the coming decade [13].

7. Discussion and Conclusion

If experimentally confirmed, this derivation would have significant implications for the foundations of physics. It suggests that fundamental constants do not arise as arbitrary free parameters but instead emerge as invariants dictated by information-theoretic and geometric constraints.

The familiar notion of the Big Bang could be reframed as an ongoing projection through a cosmic event horizon. The requirement of 8D to 4D projection reinforces the mathematical necessity of $(3 + 1)$ -dimensional spacetime.

Our derivation required no fitted parameters, matched CODATA within 1.62σ , and made falsifiable predictions. This interpretation aligns with the trajectory of information-based physics that began with Shannon and Wheeler.

In this spirit, we suggest that α may be understood not as an arbitrary constant but as the invariant bandwidth of reality itself—a principle connecting Schrödinger's two puzzles: the spectral lines of atoms and the informational crystal of life.

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