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[Viktor Gerasimenko](#) *

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Article

Kyiv Origins of Modern Mathematical Physics

Viktor Gerasimenko

Institute of Mathematics of the NAS of Ukraine, 3, Tereshchenkivs'ka Str., Kyiv, 01601, Ukraine; gerasym@imath.kiev.ua

Abstract

The article examines the origins of modern mathematical physics in Kyiv and provides an overview of M. Bogolyubov's manuscripts produced at the Institute of Mathematics.

Keywords: mathematical physics; scientific school of D. Grave; institute of mathematics in Kyiv; manuscripts by M. Bogolyubov

MSC: 01A60; 01A70; 01A74; 01A90

1. Introduction

This article presents new insights into the initial step in the development of modern mathematical physics in Kyiv, which have been hushed up or largely overlooked in the history of the Ukrainian Academy of Sciences. In fact, the scientific breakthrough discussed here occurred in the mid-20th century, when fundamental science could not develop in the imperial provinces and was under the "vigilant" control of the Academy of Sciences' party leadership. Unfortunately, these "traditions" remain tenacious even today, and as a result, science in Ukraine continues to develop in defiance of, rather than in accordance with, public priorities.

Below, we examine some issues related to the organization of the Institute of Mathematics in Kyiv [1], the first mathematical scientific school of Professor D.O. Grave in Ukraine [2,3], and the influence on their development of the Academy's permanent structural transformations aimed at party control over the scientists. In this context, we will discuss the origins of modern mathematical physics in Kyiv and provide a review of the manuscripts created by Professor M.M. Bogolyubov [4] at the Institute of Mathematics in this area.

2. Mathematical Institute of the Ukrainian Academy of Sciences and the Scientific School of D.O. Grave

The Mathematical Institute within the Ukrainian Academy of Sciences was officially founded during the following events. In February 1920, the General Meeting of the Ukrainian Academy of Sciences decided to elect Professor D.O. Grave (1863-1939) as a member of the Academy (the first mathematician member) at the next General Meeting on March 8, 1920. Professor D.O. Grave was unanimously chosen for the Department of Pure Mathematics with the responsibility of organizing the Mathematical Institute of the Ukrainian Academy of Sciences as quickly as possible, which "was founded at the same time and without delay" [1].

The social processes of that period were deeply intertwined with the long-standing history of Ukrainian culture. It is worth noting that during this time, scientists were finally permitted to defend their dissertations in the Ukrainian language. On March 23, 1918, the Central Council confirmed the status of the Ukrainian language as the language of official business. The alphabet developed by Metropolitan Illarion (Ivan) Ohienko was taken as the basis. This happened 42 years after the Ems decree banning the Ukrainian language. A little more than a year earlier, on March 12, 1917, the Central Council decided that the language of the first Ukrainian parliament's public address should be Ukrainian.

The historical atmosphere in which modern mathematical physics arose at the Institute of Mathematics is truthfully reflected in the work [5] of legendary Academician I.M. Dziuba. "... The early 1930s brought new challenges to the Academy. The atmosphere within it was oppressive due to political repression in the country, the deliberate "exacerbation of class struggle," and the consequences of the famine. The function of "irreconcilable struggle against the hostile elements of the All-Ukrainian Academy of Sciences (VUAS)" was performed by the VUAS communist faction, which transformed itself into a kind of official overseer of the activities of the entire Academy and each of its individual members under the pretext of "a comprehensive, specific, in-depth study of the scientific output of VUAS employees, aimed at assisting the overwhelming majority of scientists in methodological retraining and identifying clearly hostile ones" (Resolution of the VUAS communist faction on the political situation of the Academy, May 20, 1932)."

The turbulent period of the first three decades [6] and the early stages of the Institute of Mathematics' formation are covered in the essay [1].

The history of the Institute of Mathematics is closely intertwined with its founder, Academician D.O. Grave. It is difficult to separate the legacy of Grave from that of the mathematical institution within the Academy of Sciences. His remarkable contributions to mathematics, mechanics, and mathematical physics are recognized worldwide. Academician Grave played a pivotal role in the establishment of the Ukrainian Academy of Sciences and founded numerous scientific and educational institutions. These include the Institute of Mathematics, as well as various scientific societies and departments of mathematics and mechanics in Kyiv [3].

A significant role in the formation of a large mathematical school around Professor D.O. Grave was played by the scientific seminar he initiated in 1908, in which not only scientists but also students participated, starting from the first years of study. Grave's scientific seminar was devoted to the current problems of pure and applied mathematics and made significant progress during the difficult, turbulent years of the early twentieth century. Thus, in the 1920s, the seminar primarily focused on a variety of topics, including classical hydrodynamics, the mechanical theory of ether, Maxwell's theory of electricity, Hertz's mechanics, Einstein's theory, quantum mechanics, actuarial mathematics, and the mechanics of molecular structures. To grasp the extent of Graves' scientific interests, one needs only examine the list of books and textbooks he authored [3]. For a more in-depth analysis of Professor D. Grave's scientific creativity and his influences on the development of mathematics, we refer to the articles [2,3].

In the following, we will explore the scientific contribution of M.M. Bogolyubov to the development of mathematical physics in the 20th century, one of many renowned Ukrainian scientists whose research interests originated in the scientific school of Professor D.O. Grave.

Note that the Institute of Mathematics officially admitted M.M. Bogolyubov (1909-1992) as a senior researcher on March 1, 1945. On April 1, he assumed the head of the Department of Asymptotic Methods and Statistical Mechanics. The dismissal letter, dated November 1, 1956, was signed by his students, namely Yu.O. Mitropolsky, who was serving as the deputy acting director of the institute, and O.S. Parasyuk, the acting director of the Institute of Mathematics at that time. It is important to emphasize that until recently, M. Bogolyubov's contributions to the Institute of Mathematics were not acknowledged. His involvement was officially recognized during the celebration of his 100th anniversary in 2009 [4].

However, M.M. Bogolyubov's fate was intertwined with the Institute long before. He was a student of Professor D.O. Grave. From the age of thirteen, M.M. Bogolyubov became a participant in the well-known mathematical seminar of D.O. Grave. Under its influence, he is forming as a scientist in the field of mathematical and theoretical physics. Later in June 1925, at the request of D.O. Grave, the Small Presidium of Ukrgolovnauka decided to consider M.M. Bogolyubov as a graduate student of the research department of mathematics in Kyiv. In 1928, he defended his dissertation for the title of scientific worker on the topic "On the development of new methods of variational numerical analysis". The draft resolution stated that "the postgraduate student's work and its defense are brilliant, and

the postgraduate student fully meets the requirements for the department's research staff. Therefore, the department asks Ukrnauka to approve him as a research staff member of the department with an appropriate salary." Thus, Mykola Bogolyubov became the youngest research staff member of the Department of Mathematical Physics, which at that time was a structural unit of the VUAS under the leadership of Academician M.M. Krylov. Later, on April 6, 1930, after a report at the general meeting of the Physics and Mathematics Department of the All-Ukrainian Academy of Sciences (VUAS), at the proposal of Academicians D.O. Grave and M.M. Krylov, Mykola Bogolyubov was awarded the degree of Doctor of thematics without defending a dissertation.

We briefly remind you of the contribution to mathematics made by M.M. Bogolyubov during his tenure at the Department of Mathematical Physics of the Academy of Sciences in Kyiv before the beginning of World War II [4,7]. It is crucial to understand that the motivations and circumstances behind the publication of scholarly works related to the Soviet Empire period vary among researchers on historical topics. This variation likely stems from the fact that archival documents from that time have either "disappeared" or been "edited." Furthermore, many scholars from that era were not only "erased" from history, but their published works have also been destroyed in libraries [6]. This was a common practice at the time, including at the Kyiv Institute of Mathematics.

The first significant results in mathematical physics in Ukraine were obtained by M.M. Bogolyubov and M.M. Krylov and published in the late 1920s and early 1930s. Their work laid the foundations of nonlinear mechanics, which systematized mathematical methods for studying oscillatory processes using nonlinear differential equations with a small parameter. M.M. Bogolyubov proved the existence of an invariant measure in dynamical systems, which was crucial for the development of the general theory. Bogolyubov's most important contribution, the so-called averaging method, was made in the late 1940s. Later, together with Yu.O. Mitropolsky, asymptotic methods with applications to problems of modern physics and technology, which were included in their well-known monograph on asymptotic methods in the theory of nonlinear oscillations, published in 1961.

Thus, participation in the scientific seminar of Professor D.O. Grave shapes the outlook of M.M. Bogolyubov as a natural philosopher and mathematical physicist. This historical precedent convincingly illustrates the importance of the scientific school in shaping a scientist's development.

3. Manuscripts of M.M. Bogolyubov at the Institute of Mathematics

While working at the Institute of Mathematics in Kyiv, M.M. Bogolyubov authored many prominent manuscripts: [8–17]. We will briefly comment on some of these papers within the context of previously unknown facts about their writing during this period.

The 1945 report by the Institute of Mathematics, titled "Problems of Dynamical Theory in Statistical Physics", served as the manuscript [8] for the now-renowned monograph "Problems of Dynamical Theory in Statistical Physics," which was published in 1946. The results presented in this monograph marked a new age in the development of statistical physics.

In this manuscript, the relationship of the evolution equations that describe many-particle systems was established. The origins of this problem can be traced back to H. Poincaré [18] in 1905 and D. Hilbert [19] in 1912, when it was framed in the context of deriving hydrodynamic equations from the Boltzmann kinetic equation, and later to M.M. Bogolyubov in 1945 in deriving kinetic equations from the fundamental evolution equations governing many-particle systems.

The origins of the concept of a hierarchy of evolution equations for describing the state of many-particle systems date back to the works of J. Yvon in 1935 and M.M. Bogolyubov in 1945. Within this approach, the evolution of the state is described in a manner equivalent to the Liouville equation for the probability distribution function. In our time, this hierarchy of evolution equations is commonly referred to as the BBGKY hierarchy (Bogolyubov–Born–Green–Kirkwood–Yvon), owing that it was most consistently formulated in the work of M.M. Bogolyubov and independently by M. Born, H.S. Green, and J.G. Kirkwood in 1946. The advantages of this equivalent approach consist of the opportunity of rigorously describing the evolution of infinitely many particles whose collective behavior

exhibits statistical (thermodynamic) features, namely, the existence of an equilibrium state in such a system, as well as the kinetic or hydrodynamic behavior in corresponding scaling approximations. The BBGKY hierarchy describes both the non-equilibrium and equilibrium states. Non-equilibrium states are described by the solution of the initial value problem for this hierarchy, and, correspondingly, equilibrium states are solutions of the steady BBGKY hierarchy [20].

In his renowned monograph "Problems of Dynamic Theory in Statistical Physics," which, as we emphasize again, is a manuscript of a 1945 work report at the Kyiv Institute of Mathematics, M.M. Bogolyubov formulated a consistent approach to deriving kinetic equations from the dynamics of many-particle systems. Using perturbation theory methods, he developed for the first time an approach to constructing a generalization of the Boltzmann equation, now known as the Bogolyubov kinetic equation [10], and justified other kinetic equations. In particular, this result clarified the mechanism of irreversibility of the evolution of many-particle systems whose dynamics are described by time-reversible fundamental evolution equations.

The results regarding classical many-particle systems were extended to quantum systems [13,14]. As noted today, the rigorous justification of the kinetic Boltzmann equation remains an evergreen challenge. The current state and prospects of this area in mathematical physics are discussed in the review [21].

In the pioneering work [13] the superfluidity phenomenon was first described employing the quantum kinetic equation. In 1947, Bogolyubov's article was published in the Proceedings of the Institute of Mathematics. It has been translated from Ukrainian into several languages and is now the most cited work in condensed matter quantum theory:

◇ Bogolyubov, M. M.: On the theory of superfluidity. Proc. Inst. Math., 1947, **9**, 89–103. (in Ukrainian) (accepted 24.09.1946).

◇ Bogolyubov, N. N.: On the theory of superfluidity. Izv. AN SSSR, Phys. series, 1947, **11**(1), 77–90.

◇ Bogolyubov, N. N.: On the theory of superfluidity. J. Phys., 1947, **11**(1), 23–32.

◇ Bogoljubow, N. N.: Zur Theorie der Superflüssigkeit. Sowjetwissenschaft, 1948, **1**, 162–176.

M.M. Bogolyubov successfully constructed a consistent microscopic theory of superconductivity, demonstrating that superconductivity can be understood as the superfluidity of an electron gas [22]. The experimental discovery of the Bose condensate in rarefied gases (the Nobel Prize in Physics in 2001), fully confirmed Bogolyubov's theory from 1946. This validation also relates to the 2004 Nobel Prize in Physics, awarded for advancements in superconductivity and superfluidity.

In his work, M.M. Bogolyubov introduced a new mathematical technique, now known as the Bogolyubov canonical transformation. In 1957, he applied this transformation to advance the theory of superconductivity further, creating a consistent microscopic theory of superconductivity independently of J. Bardeen, L. Cooper, and J.R. Schrieffer. The subsequent advancement of the idea of superconductivity as the superfluidity of Fermi-particle systems in 1958 led M.M. Bogolyubov to discover the phenomenon of superfluidity in nuclear matter, which is now a crucial aspect of our understanding of atomic nuclei.

In 1948 in the Proceedings of the Institute of Mathematics [15], M.M. Bogolyubov published an article on deriving hydrodynamic equations from the BBGKY hierarchy. We notice that the rigorous derivation of hydrodynamic equations from the dynamics of many particles is still an open mathematical problem.

Let us also remember the textbook [17] published by M.M. Bogolyubov in 1949, titled "Lectures on Quantum Statistics. Issues of Statistical Mechanics of Quantum Systems." This book was based on lectures he delivered to students at Kyiv University and was originally published in Ukrainian. The textbook has become a classic and is considered decades ahead of its time.

To appreciate the amazing breadth of Bogolyubov's scientific interests during the Kyiv period of his work (March 1, 1945 – November 1, 1956), it is enough to look at the list of his work reports at the Institute of Mathematics [4].

The report for 1945 on the topic: "Problems of dynamical theory in statistical physics" (on 126 sheets!); for 1946 on the topics: "Methods of nonlinear mechanics in statistical physics", & "On positive completely continuous operators"; for 1947 on the topic: "The theory of the polar model of metal"; for 1948 on the topic: "On the elimination of self-energy divergence in non-relativistic field theory"; for 1949 on the topic: "The structure of distribution functions in quantum hydrodynamics" (never published!); for 1950 on the topic: "The theory of strong coupling of a particle with a quantum field".

The ideas generated by Bogolyubov at the Institute of Mathematics have laid the groundwork for the theory of kinetic equations and have become the cradle of modern statistical mechanics, as recognized today [23].

Considering the possibilities for sharing scientific information and its peculiarities under the totalitarian regime at the time, the uniqueness of creating these works becomes clear.

4. Synopsis of the Evolution of Bogolyubov Views on the Theory of Elementary Particles

In the early 1950s, the second quantization methods developed by M. M. Bogolyubov for describing quantum statistical systems drew his attention to the issues in quantum field theory. Below, we provide a brief overview of Bogolyubov's contributions to quantum field theory, which continue the ideas outlined in his previous works [4].

* The R -operation (Bogolyubov–Parasyuk theorem [24]) in the early 1950s. The theory of the R -operation is the basis of modern theories of renormalization in quantum field theory.

As is known, the development of quantum field theory began with quantum electrodynamics, the basic equations of which were formulated in the late 1920s. M.M. Bogolyubov was the first to note that in quantum field theory, certain objects arise that require the use of distribution (generalized) functions for an adequate description. This observation clarified the mathematical significance of the procedure for subtracting infinities. The discovery of the general form of the renormalization procedure and its justification played a crucial role in the further development of quantum field theory. As a result, this enabled us to prove the renormalizability of the unified theory of electroweak interactions, as well as supersymmetric theories.

* In 1961, an article by M.M. Bogolyubov was published, which introduced the fundamental concept of quasi-averages and, in essence, built a new theory of phase transitions.

The application of these ideas to elementary particle physics is known as spontaneous symmetry breaking.

* Axiomatic field theory (method of dispersion relations for scattering amplitude).

* The period 1964-1966 includes the works of M.M. Bogolyubov on the theory of symmetry and quark models of elementary particles.

Of great importance for the further development of the theory of elementary particles was the new quantum number for quarks proposed by him and his students (A.N. Tavkhelidze and B.V. Struminsky, Ukrainian Journal of Physics – golden pages of theoretical physics of Ukraine to the 90th anniversary of the Academy [24]), which is now known as color. It is interesting to note the symbolic connection between the colors of quarks: red, green, and blue, and the colors of the 12 volumes of Bogolyubov's works, published to commemorate his centenary in the series "Classics of Science." The first four volumes, which focus on mathematics, are red; the next four, dedicated to statistical mechanics, are green; and the final four, covering quantum field theory, are blue.

5. Discussion and Conclusion

One of the most productive periods in M.M. Bogolyubov's work took place in Kyiv, the Eternal City, especially with the Institute of Mathematics, where he was employed from 1945 to 1956. His fundamental research launched new directions in both theoretical and mathematical physics; notably,

he authored classic works on modern statistical physics. Later, he established the Institute of Theoretical Physics in Kyiv, serving as its first director from 1966 to 1973. This institute is now named after him. For more details on the phenomenon of Professor M.M. Bogolyubov's scientific creativity and its impact on the development of modern mathematical physics, refer to the book [4].

M.M. Bogolyubov's scientific work revealed the unity of the mathematical structure framework of theories from various branches of physics. He significantly encouraged the mutual exchange and influence of mathematical methods and physical concepts that emerged in different areas of natural science throughout the 20th century. M.M. Bogolyubov's student, Academician V.S. Vladimirov, noted [4]: "The organic fusion of mathematics and physics in the work of M.M. Bogolyubov enabled him to make a decisive contribution to the development of theoretical physics and actually lay the foundations of modern mathematical physics, which continues the traditions of Hilbert, Poincaré, Einstein, Dirac".

On the initiative of M.M. Bogolyubov, international conferences on the mathematical problems of quantum field theory and quantum statistics have been held regularly since 1972. These later evolved into congresses on mathematical physics – global gatherings of leading scientists in this field. In his program speech at the 1981 international conference on quantum field theory problems, M.M. Bogolyubov described the progress of mathematical and theoretical physics as follows: "Before our eyes, in recent years, a completely new branch of science has been formed, which is most appropriately called modern mathematical physics." Since 1972, the International Congress of Mathematical Physics has been represented by the logo $M \cup P$.

In 1966, M.M. Bogolyubov was awarded the most prestigious prize in the field of mathematical physics, the Danny Heinemann Prize, "For many outstanding achievements in the extension of the methods of modern mathematics to fundamental problems of physics and, in particular, for the first rigorous proof of dispersion relations for the indirect scattering of elementary particles." In his welcoming speech on this occasion, Professor R. Jost said: "You made an unforgettable impression on me. Most theorists at that time treated mathematics with disdain, and logical deduction was "trampled underfoot". Only the romantic influence of genius could have had any significance. And then you appeared, a man who knows both mathematics and physics, ready to take on complex problems that require their logical combination. It seems to me that this reflects the national character of your great people ..."

In honor of M.M. Bogolyubov's scientific feat, a coin of the Ukrainian national currency was minted to commemorate the 100th anniversary of his birthday, depicting the hierarchy of evolution equations he first formulated at the Institute of Mathematics. In our opinion, he who does not know his past is not worthy of the future; he who does not know about the glory of his ancestors is not worthy of respect. The National Academy of Sciences of Ukraine established the M.M. Bogolyubov Prize to recognize outstanding scientists for their achievements in mathematical physics.

Finally, it would not be an exaggeration to say that M.M. Bogolyubov continued his legacy in the works of his students and numerous followers, including those working at the Institute of Mathematics and the Bogolyubov Institute for Theoretical Physics of the National Academy of Sciences of Ukraine.

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References

1. Gerasimenko, V. I., Drozd, Yu. A., Maksymenko, S. I.: 100 years of the Mathematical Institute of the Ukrainian Academy of Sciences. In: Modern Problems of Mathematics and its Applications. I. Kyiv: IM, 2020, pp. 1–9.
2. Gerasimenko, V. I., Maksymenko, S. I., Timokha, O. M.: The founder of the first major mathematical school in Ukraine (to the 160th anniversary of Academician D.O. Grave). Visnyk NAS of Ukraine, 2023, No.10, pp. 78–86. doi:10.15407/visn2023.10.078
3. Gerasimenko, V. I., Maksymenko, S. I.: CLX years since the birthday of Academician D.O. Grave. In: Modern Problems of Mathematics and its Applications. III. Kyiv: IM, 2023, pp. 685–697. doi:10.3842/trim.v20n1.525

4. Gerasimenko, V. I., Zagorodny, A. G.: Genius of modern theoretical and mathematical physics. In: Creator of Theoretical and Mathematical Physics: to the 110th Anniversary of Academician M.M. Bogolyubov. Kyiv: PH "Akadempriodyka", pp. 13–30, 2019. doi:10.15407/akadempriodyka.387.532
5. Dziuba, I. M.: Natsionalna akademiia nauk Ukrainy. Kyiv: The NASU Institute of Encyclopedic Research, 2001, **1**, <https://esu.com.ua/article-43408>
6. Kratko, M. I.: Calvary of Academician Kravchuk. Lutsk: VIPDE, 2011.
7. Oliveira, A. R. E.: History of Krylov–Bogoliubov–Mitropolsky methods of nonlinear oscillations. *Advances in Historical Studies*, 2017, **6** (1), 40–55. doi:10.4236/ahs.2017.61003
8. Bogolyubov, M. M.: Problems of Dynamic Theory in Statistical Physics. M.: Gostekhizdat, 1946. (*IM report 1945, 7.02.1046*).
9. Bogolyubov, M. M.: On some Statistical Methods in Mathematical Physics. Kyiv: AN USSR, 1945.
10. Bogolyubov, M. M.: Kinetic equations. *JETP*, 1946, **10** (3), 265–274. (*submitted 07.1945*).
11. Bogolyubov, M. M.: Expansions by powers of a small parameter in the theory of statistical equilibrium. *JETP*, 1946, **16** (8), 681–690.
12. Bogolyubov, M. M.: The method of functional derivatives in statistical mechanics. *Proc. Inst. Math.*, 1947, **8**, 177–189. (in Ukrainian) (*submitted 25.12.1944*).
13. Bogolyubov, M. M.: On the theory of superfluidity. *Proc. Inst. Math.*, 1947, **9**, 89–103. (in Ukrainian) (*submitted 24.09.1946*).
14. Bogolyubov, M. M., Gurov, K. P.: Kinetic equations in quantum mechanics. *JETP*, 1947, **17** (7), 614–628. (*submitted 26.10.1947*).
15. Bogolyubov, M. M.: Hydrodynamics equations in statistical mechanics. *Proc. Inst. Math.*, 1948, **10**, 41–59. (in Ukrainian) (*submitted 25.03.1947*).
16. Bogolyubov, M. M., Khatset, B. I.: On some mathematical questions of the theory of statistical equilibrium. *Reports of Acad. Sci. USSR*, 1949, **66** (3), 321–324.
17. Bogolyubov, M. M.: Lectures on Quantum Statistics. Problems of Statistical Mechanics of Quantum Systems. Kyiv: Rad. Shkola, 1949. (in Ukrainian) (*Preface from 30.12.1948*).
18. Poincaré, H.: The principles of mathematical physics. *The Monist*, 1905, **15** (1), 1–24. doi:10.5840/monist190515137
19. Hilbert, D.: Begründung der kinetischen gastheorie. *Math. Ann.*, 1912, **72**, 562–577. doi:10.1007/BF01456676
20. Cercignani, C., Gerasimenko, V., Petrina, D.: Many-Particle Dynamics and Kinetic Equations. The Netherlands: Springer, 2012 (2nd ed) doi:10.1007/978-94-011-5558-8
21. Gerasimenko, V. I., Gapyak I. V.: Advances in theory of evolution equations of many colliding particles. *Proc. Inst. Math. NASU*, 2023, **20** (1), 729–804. doi:10.3842/trim.v20n1.528
22. Lieb, E. H., Solovej, J. P., Seiringer, R., Yngvason, J.: The Mathematics of the Bose Gas and its Condensation. Basel: Birkhäuser, 2005. doi:10.1007/b137508
23. Ernst, M. H.: Bogoliubov Choh Uhlenbeck theory: cradle of modern theory. In: *Progress in Statistical Physics*, W. Sung (Ed.), World Scientific, 1998.
24. Golden contribution of Ukrainian physics to modern science. *Ukr. J. Phys.*, 2008, **53**. Special Issue.

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