Article

Not peer-reviewed version

Emergent Dynamics at Cosmological Scales

Yuanxin Li *

Posted Date: 9 June 2025

doi: 10.20944/preprints202504.2469.v6

Keywords: emergent dynamics; Kuramoto model; geometric flow equation

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 June 2025 doi:10.20944/preprints202504.2469.v6

Article

Emergent Dynamics at Cosmological Scales

Yuanxin Li

Independent Researcher; yuanxinli31@gmail.com

Abstract: In this paper, I investigated the emergent dynamics in the Newtonian limit by modeling a toy model of the universe with N masses coupled by gravity incorporating the cosmological constant. The dynamics are given by the equation of the Kuramoto-like form. This many-body system is drastically simpler by using a mean field approach. The equilibrium solution of the equation is consistent with the usual Milgrom's empirical law. It turns out that the state of the universe at large scales enters local dynamics in local small systems, such as galaxies. However, in the context of general relativity, gravity is described within a geometric framework. Thus, I constructed a geometric dynamical equation for the gravitational system, which describes a geometric flow.

Keywords: emergent dynamics; Kuramoto model; geometric flow equation

---

1. Introduction

The universe in its vast complexity harbors profound mysteries that challenge our understanding of fundamental physics. Observations of the flat rotation curves of galaxies reveal a striking discrepancy that galaxies rotate faster than can be accounted for by the visible matter alone, suggesting the presence of an invisible mass, known as dark matter. It is also hypothesized to explain a range of gravitational anomalies observed in gravitational lensing and the cosmic microwave background (CMB) [1]. Despite its success in accounting for these phenomena, the dark matter paradigm faces significant challenges. The lack of direct detection raises questions about its fundamental validity. The most widely accepted Cold Dark Matter (CDM) model struggles to account for observational discrepancies at smaller scales (for a review see Ref. [2]), such as the "missing satellites problem," where the predicted number of dwarf galaxies around massive galaxies exceeds observations [3–5], and the "cusp-core problem," where simulated dark matter halos exhibit central density cusps inconsistent with the flatter density profiles observed in some galaxies [6]. In addition, recent observational results including rapid galaxy growth [7] and flat velocity curves extending beyond the expected virial radii of dark matter halos [8] also suggest inconsistencies with the CDM model. These issues imply that the CDM framework may need refinement or that our understanding of gravitational dynamics requires revision.

An alternative theory is the Modified Newtonian Dynamics (MOND) proposed by Milgrom as a modification to Newtonian gravity at extremely low accelerations [9–11]. Although MOND struggles to account for phenomena at cosmological scales, the simplicity of MOND at smaller scales make it a valuable theoretical tool. Unlike the dark matter hypothesis, the MOND paradigm stipulates that the observed gravitational effects arise not from unseen mass but from a deviation in the law of gravity when accelerations fall below a critical threshold $a_0 \approx 1.2 \times 10^{-10}\mathrm{m/s}^2$. Milgrom's empirical law can be written as $\mu(a/a_0)a = a_N$, where a_N is the Newtonian acceleration produced by the visible matter, a is the true gravitational acceleration and the interpolating function $\mu(x)$ satisfies $\mu(x) \approx 1$ when $x \gg 1$, and $\mu(x) \approx x$ when $x \ll 1$. In the deep-MOND regime $(a \ll a_0)$, MOND modifies the gravitational force law to scale inversely with distance rather than the square of the distance, effectively reproducing the flat rotation curves of galaxies and Tully-Fisher relation without invoking dark matter. MOND can be interpreted as a modification of gravity or inertia. In the modified gravity interpretation, Newton's law of gravity should be modified at low accelerations, leading to a stronger gravitational effect than predicted by the usual inverse-square law. This approach has been formalized in theories such as the Tensor-Vector-Scalar (TeVeS) gravity [12] and bimetric theories [13,14], which attempts to provide a

 doi:10.20944/preprints202504.2469.v6

relativistic extension of MOND [2]. Alternatively, the modified inertia interpretation suggests that the response of a body to a given force depends on the acceleration regime. However, the origin of inertia remains an open question in modern physics. Some theories propose that inertia arises from interactions with the vacuum [15]. Alternative perspectives, like Mach's principle, suggest that inertia is a relational property, emerging from an object's interaction with the global distribution of matter in the universe.

A very noteworthy coincidence of MOND is that the value of $a_0 \approx 1.2 \times 10^{-10}\mathrm{m/s^2}$ determined from galaxy dynamics is of the order of some acceleration constants of cosmological significance. It is of the same order as H_0 (H_0 is today's Hubble constant) and $(\Lambda/3)^{1/2}$ (Λ is the cosmological constant). This mysterious "cosmic coincidence" raises questions about its fundamental significance and potential cosmic connections. The contents of the universe are typically modeled as a perfect fluid to derive the Friedmann equations. However, it effectively describes the overall state of the universe but may overlook potential emergent phenomena. In this paper, I derived a modified acceleration formula by modeling a toy universe with N masses coupled by gravity incorporating the cosmological constant. Studying this many-body system within the framework of general relativity would be a challenging task. Thus, I first investigate this model in the Newtonian limit. Upon using a mean field theory approach, it is possible to study a system with a large or infinite number of degrees of freedom. I define a mean field to represent the average effect of all the masses. The results suggest that the "cosmic coincidence" may be interpreted as "synchronization" at cosmological scales similar to the Kuramoto model [16,17]. The Kuramoto model is a mathematical framework that describes synchronization in systems of coupled oscillators. It predicts a phase transition where oscillators spontaneously synchronize, transitioning from disorder to a collective rhythm when the coupling strength exceeds a critical value. The difference is that the degree of freedom in this model is acceleration rather than the phase of the oscillator. However, on cosmological scales, the typical gravitational force that decays with distance is insufficient to induce collective emergence. But in a dark energy-dominated universe, the fact that the interaction strength increasing with distance inevitably leads to the state of the universe at large scales entering local dynamics in small systems when the cosmological constant is taken into account. On the other hand, the Hamiltonian H of a system can typically be divided into a free part and an interaction part, i.e., $H = H_0 + H_I$. When the coupling strength between the system's components is very strong, the free part of the Hamiltonian can be neglected, and the system's dynamics are entirely governed by the interaction part, leading to distinct behavior in the local system.

In the Kuramoto model, strong coupling strength between oscillators can lead to synchronization. One might expect that systems with strong gravitational coupling could also exhibit collective emergent phenomena. However, in the context of general relativity, gravity is described within a geometric framework. I will generalize the Kuramoto-like dynamical equations to the framework of general relativity based on results obtained in the Newtonian limit. This equation takes the form of a geometric flow. Common geometric flows include Ricci flow [18], mean curvature flow [19], and Yamabe flow [20]. The renowned Ricci flow, used to prove the Poincaré conjecture, has been applied in physics research [21–24]. The Ricci flow is defined by the partial differential equation $\partial g/\partial t = -2\mathrm{Ric}(g)$, where $\mathrm{Ric}(g)$ is the Ricci curvature tensor. It describes the evolution of a Riemannian metric g on a manifold over time t in a complicated way. Intuitively, the Ricci flow can be understood as a heat equation for geometry. The flow tends to make the manifold's geometry more uniform, contracting regions of positive curvature and expanding those of negative curvature.

This paper is organized as follows. Sec.2 is dedicated to the emergent modified Newtonian dynamics. In Sec.3, I investigate the dynamics equation of geometry. Finally, in Sec.4 I summarize the main results obtained. For convenience, I use natural units with $c = \hbar = k = 1$.

2. Emergent Dynamics at Cosmological Scales

Let us consider a toy model of the universe consisting of N masses coupled by gravity including the cosmological constant Λ. In the limit of small velocity and in the weak field approximation,

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 June 2025 doi:10.20944/preprints202504.2469.v6

Einstein's field equations should reproduce Newtonian gravity. In the Newtonian limit, Einstein's field equations with the cosmological constant reduce to a modified Poisson equation:

$$\nabla^2 \phi = 4\pi G\rho - \Lambda, \tag{1}$$

where ϕ is the gravitational potential, G is Newton's gravitational constant and ρ is the mass density. The equivalent force due to the cosmological constant is directly proportional to the distance. At cosmological scales, the Newtonian limit breaks down, and the effect of Λ is described by general relativity and the Friedmann equations. But this does not alter the final result because the repulsive effect still increases with distance, and the synchronization phenomenon of interest occurs as long as the coupling strength exceeds a critical value.

Similar to the Kuramoto model [16,17], we can sum over the acceleration vector $\boldsymbol{a}$ of all masses to obtain an average acceleration vector as the order parameter to describe the overall behavior of the universe. But assuming the universe is isotropic, it is more appropriate to describe the dynamics of the system using a scalar related to acceleration, such as the magnitude or the square of the acceleration. Furthermore, the cosmological constant is typically described as a homogeneous dark energy perfect fluid. However, here only the matter content is modeled. To describe a mass embedded in an expanding universe, we need to define an effective acceleration T that combines features of the mass and the expanding cosmological background. One might recall that 4-dimensional de Sitter spacetime can be embedded into a 5-dimensional flat Minkowski spacetime to generate a geometrical symmetry group $SO(4,1)$. Therefore, the acceleration of a constant accelerated observer in this flat spacetime would be $T = \sqrt{\boldsymbol{a}^2 + H_0^2}$ [25]. The fact that the couple strength increasing with distance inevitably leads to the "synchronization", namely that cosmic expansion is isotropic and homogeneous on large scales, which is consistent with observational data. In fact, we do not need to assume the cosmological principle to derive the Friedmann–Robertson–Walker (FRW) metric, and the homogeneity and isotropy of the universe are induced by the effects of Λ. The overall behavior of the universe encoded in Λ and H_0 enters local dynamics in small systems. The dynamics of the i-th mass are described by the equation of the following Kuramoto-like form:

$$\dot{T}_i = |\dot{\boldsymbol{a}}_N| + \frac{\varepsilon}{N}\sum_{j=1}^{N} \Lambda r_{ij} C_{ij}(T_j - T_i), \tag{2}$$

where the dot denotes the derivative with respect to time t, $T_i = \sqrt{a_i^2 + H_0^2}$ is the true effective acceleration of the i-th mass with $a_i = |\boldsymbol{a}_i|$ being the magnitude of true acceleration, $\boldsymbol{a}_N$ is the Newtonian expression for the acceleration vector of the i-th mass, ε is the positive coupling strength, r_{ij} is the distance and $C_{ij}(T_j - T_i)$ is a general coupling function for the interaction between the i-th and j-th masses. Here, I choose the coupling function $C_{ij}(T_j - T_i) = T_j - T_i$ because it naturally captures the acceleration differences between coupled masses in a simple and physically meaningful way. The coupling term in this model works to reduce acceleration differences, leading to synchronized behavior. The usual Newtonian acceleration is modified due to cosmological effects. I have neglected the contribution of the conventional Newtonian gravitational force that decays with distance to the second term on the right-hand side (RHS) of Equation (2). In the limit $r_{ij} \to 0$, i.e., for a local small system, the Hubble constant and cosmological effects can be neglected, and we have $|\dot{\boldsymbol{T}}_i| = |\dot{\boldsymbol{a}}_i| = |\dot{\boldsymbol{a}}_N|$. Let us define a mean field that represents the average effect of all the contents of the universe. Clearly, the mean acceleration is H_0. Every accelerated object appears to be subjected to a force that drives its effective acceleration toward H_0. The dynamics equation is expressed in terms of the magnitude of acceleration, this criterion allows us to determine the direction of acceleration. Upon using a mean field approach, all of the originally coupled differential equations become coupled only to the mean field quantity H_0. Let us consider periodic motion, such as uniform circular motion in a plane. We

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 9 June 2025 doi:10.20944/preprints202504.2469.v6

have $|\dot{\boldsymbol{a}}_N| = \omega|\boldsymbol{a}_N| = \omega a_N$ when the acceleration is expressed in complex exponential form. The simplest possible form of Equation (2) is

$$\dot{T}_i = \omega a_N + \eta(H_0 - T_i), \tag{3}$$

where η is the effective coupling strength. Assume that the Newtonian acceleration and the acceleration difference contribute equally to the variation in effective acceleration, we have $\omega = \eta$. Thus Equation (3) can be written in terms of the acceleration as

$$\frac{a\dot{a}}{\sqrt{a^2 + H_0^2}} = \omega a_N + \omega\left(H_0 - \sqrt{a^2 + H_0^2}\right), \tag{4}$$

where the subscript i has been omitted. The fixed point given by $\dot{a} = 0$ represents stable solutions of Equation (4) and one arrives at

$$a_N = \sqrt{a^2 + H_0^2} - H_0, \tag{5}$$

which leads to the Milgrom's empirical law $\mu(a/a_0)a = a_N$ with $a_0 = 2H_0$. It was interestingly noted that the local dynamics of small systems depend on the state of the universe at large scales and the Hubble constant varies during the evolution of the universe. Under the quasi-static approximation, the effective acceleration is $T_i \approx \sqrt{a^2 + H(t)^2}$. Therefore, we replace H_0 with the varying Hubble parameter $H(t)$ and Equation (5) becomes

$$\frac{H\dot{H}}{\sqrt{a^2 + H^2}} = \omega a_N + \omega\left(H - \sqrt{a^2 + H^2}\right). \tag{6}$$

One immediately obtains

$$a = \frac{a_N}{2} + \frac{1}{2}\sqrt{a_N^2 - \frac{4c^2 H\dot{H}}{\omega}}, \qquad \text{for } a \gg 2cH, \tag{7}$$

$$a = \sqrt{2cHa_N - \frac{2c^2 H\dot{H}}{\omega}}, \qquad \text{for } a \ll 2cH, \tag{8}$$

with

$$\omega = \frac{V_N}{R} = \sqrt{\frac{GM}{R^3}}, \tag{9}$$

where V_N is the Newtonian expression for the orbital velocity of a test particle in circular motion around a point mass M, R is the orbital radius and the speed of light c is restored in the final form. Note that the cosmological constant Λ is very small, so the acceleration enhancement due to $\dot{H} < 0$ is negligible at small scales (e.g., laboratory or planetary) and only significant at cosmological scales (e.g., galaxy clusters or larger). Thus, the emergent MOND does not violate the experimentally verified Newtonian mechanics and equivalence principle at small scales. In galaxy clusters, the usual MOND ($\sqrt{a_0 a_N}$) cannot fully account for the large dynamical mass inferred from velocity dispersions or X-ray gas temperatures because the acceleration regime in clusters is often above a_0, where MOND's corrections approach Newtonian behavior. But this model suggests that the $\dot{H}$ term can still provide the gravitational enhancement when the acceleration exceeds a_0. In addition, it is expected that the emergent MOND has a different impact on the evolution of the universe during the period of decelerated expansion compared to the present universe.

We have considered the simplest possible case. However, for complex local gravitational environments and more general cases in the real universe, the mean field approximation does not hold, and we should employ N-body simulations on Equation (2). Furthermore, the parameters used to describe the overall behavior of the universe may not be unique. In addition to $H(t)$, we may introduce further parameters, and thus the dynamics are governed by a set of Kuramoto-like equations. The key point is that the current state described by certain parameters at cosmic scales influences the behavior of local systems.

3. The Geometric Flow Equation

In the context of general relativity, gravity is described within a geometric framework. It is worth noting that Equation (3) may be generalized to the framework of general relativity, where the system is characterized by the curvature rather than acceleration. It is a geometric version of the dynamics describing the collective "synchronization" phenomena. I provide an intuitive and heuristic extension of the emergent MOND.

As in bimetric theory, let us introduce two auxiliary metrics $\hat{g}_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ in addition to the Riemannian metric tensor $g_{\mu\nu}$. The metric $\hat{g}_{\mu\nu}$ is associated with the true gravitational potential, $g_{\mu\nu}$ is associated with the local gravitational potential of the universe's contents, and $\bar{g}_{\mu\nu}$ related to the average gravitational potential represents the overall state of the universe and is exactly the FRW metric. Their corresponding connections are $\hat{\Gamma}^{\alpha}_{\beta\gamma}$, $\Gamma^{\alpha}_{\beta\gamma}$ and $\bar{\Gamma}^{\alpha}_{\beta\gamma}$, respectively. We can now construct nontrivial tensors from the differences of these connections. Let us define two tensors $K^{\alpha}_{\beta\gamma}$ and $C^{\alpha}_{\beta\gamma}$ as

$$K^{\alpha}_{\beta\gamma} = \hat{\Gamma}^{\alpha}_{\beta\gamma} - \Gamma^{\alpha}_{\beta\gamma} \tag{10}$$

and

$$C^{\alpha}_{\beta\gamma} = \bar{\Gamma}^{\alpha}_{\beta\gamma} - \hat{\Gamma}^{\alpha}_{\beta\gamma}. \tag{11}$$

We assume that the mean field approximation remains valid. Since connections act like gravitational accelerations, from Equations (2) and (3) the dynamic equation may be written in this form:

$$\frac{\partial K^{\alpha}_{\beta\gamma}}{\partial t} = \eta C^{\alpha}_{\beta\gamma}. \tag{12}$$

The geometric flow tends to smooth out the inhomogeneity of the metric distribution. In the Newtonian limit, Equation (12) can reproduce Equation (3). The connection $\bar{\Gamma}^{\alpha}_{\beta\gamma}$ can be computed from the FRW metric. Given the local gravitational field $g_{\mu\nu}$ and calculating the corresponding connection $\Gamma^{\alpha}_{\beta\gamma}$, we can solve Equation (12) to obtain the true metric field $\hat{g}_{\mu\nu}$. Note that the FRW metric $\bar{g}_{\mu\nu}$ and the local metric field $g_{\mu\nu}$ do not use identical coordinates. Their physical contexts and coordinate choices differ significantly. To describe a local gravitational field embedded in an expanding cosmological background, a composite metric combining the local geometry with the cosmological background is typically used. The McVittie metric is a common choice for embedding a Schwarzschild metric in an expanding universe [26]. However, strictly separating the local gravitational field and cosmological background contributions in the same coordinate system to calculate the connection difference is challenging due to the non-linear coupling of these effects.

4. Conclusion

In this paper, I investigate the emergent MOND of a toy model of the universe. The system is drastically simpler by using a mean field theory approach. The emergent MOND may still be insufficient to explain all phenomena on large scales. But it remains a valuable theoretical tool, prompting ongoing research into its foundations. Furthermore, I derive the geometric dynamical equation in a non-rigorous manner. The equation might be regarded as a possible relativistic extension of MOND. It is worth noting that the effective acceleration as a degree of freedom is exactly the Unruh temperature of the radiation seen by an accelerated observer in de Sitter spacetime, differing only by a factor of 2π [27]. It may suggest a connection between gravity and thermodynamics. In some studies, gravity is regarded as an entropic force [28–31]. In my personal view, this only means that gravity can be described in another framework at the macroscopic level, whereas at microscopic scales gravity may still be quantum. Furthermore, emergent phenomena at the cluster and cosmological scales arise from the gravitational interactions of small-scale cosmic constituents. For black holes, emergent phenomena are merely a consequence of excessively strong gravitational interactions. Therefore, it is gravity that gives rise to emergent phenomena on larger scales, rather than gravity originating from emergence.

References

1. Gianfranco Bertone and Dan Hooper. History of dark matter. *Reviews of Modern Physics*, 90(4):045002, 2018.
2. Benoit Famaey and Stacy S McGaugh. Modified newtonian dynamics (mond): observational phenomenology and relativistic extensions. *Living reviews in relativity*, 15:1–159, 2012.
3. Guinevere Kauffmann, Simon DM White, and Bruno Guiderdoni. The formation and evolution of galaxies within merging dark matter haloes. *Monthly Notices of the Royal Astronomical Society*, 264(1):201–218, 1993.
4. Anatoly Klypin, Andrey V Kravtsov, Octavio Valenzuela, and Francisco Prada. Where are the missing galactic satellites? *The Astrophysical Journal*, 522(1):82, 1999.
5. Ben Moore, Sebastiano Ghigna, Fabio Governato, George Lake, Thomas Quinn, Joachim Stadel, and Paolo Tozzi. Dark matter substructure within galactic halos. *The Astrophysical Journal*, 524(1):L19, 1999.
6. David J Sand, Tommaso Treu, and Richard S Ellis. The dark matter density profile of the lensing cluster ms 2137–23: a test of the cold dark matter paradigm. *The Astrophysical Journal*, 574(2):L129, 2002.
7. Stacy S McGaugh, James M Schombert, Federico Lelli, and Jay Franck. Accelerated structure formation: The early emergence of massive galaxies and clusters of galaxies. *The Astrophysical Journal*, 976(1):13, 2024.
8. Tobias Mistele, Stacy McGaugh, Federico Lelli, James Schombert, and Pengfei Li. Indefinitely flat circular velocities and the baryonic tully–fisher relation from weak lensing. *The Astrophysical Journal Letters*, 969(1):L3, 2024.
9. Mordehai Milgrom. Mond—theoretical aspects. *New Astronomy Reviews*, 46(12):741–753, 2002.
10. Mordehai Milgrom. Mond–a pedagogical review. *arXiv preprint astro-ph/0112069*, 2001.
11. Mordehai Milgrom. Md or dm? modified dynamics at low accelerations vs dark matter. *arXiv preprint arXiv:1101.5122*, 2011.
12. Jacob D Bekenstein. Relativistic gravitation theory for the modified newtonian dynamics paradigm. *Physical Review D—Particles, Fields, Gravitation, and Cosmology*, 70(8):083509, 2004.
13. Christopher J Isham and D Storey. Exact spherically symmetric classical solutions for the f- g theory of gravity. *Physical Review D*, 18(4):1047, 1978.
14. Mordehai Milgrom. Bimetric mond gravity. *Physical Review D—Particles, Fields, Gravitation, and Cosmology*, 80(12):123536, 2009.
15. Mordehai Milgrom. The modified dynamics as a vacuum effect. *Physics Letters A*, 253(5-6):273–279, 1999.
16. ME Morrison. A quantum kuramoto model. *Bachelor thesis, University of Otago, Dunedin, New Zealand*, 2013.
17. Juan A Acebrón, Luis L Bonilla, Conrad J Pérez Vicente, Félix Ritort, and Renato Spigler. The kuramoto model: A simple paradigm for synchronization phenomena. *Reviews of modern physics*, 77(1):137–185, 2005.
18. Bennett Chow and Dan Knopf. The ricci flow: an introduction. *Mathematical surveys and monographs*, 110, 2011.
19. Tobias Colding, William Minicozzi, Erik Pedersen, et al. Mean curvature flow. *Bulletin of the American Mathematical Society*, 52(2):297–333, 2015.
20. Ezequiel Barbosa and Ernani Ribeiro Jr. On conformal solutions of the yamabe flow. *Archiv der Mathematik*, 101(1):79–89, 2013.
21. Ioannis Bakas. Geometric flows and (some of) their physical applications. *arXiv preprint hep-th/0511057*, 2005.
22. E Woolgar. Some applications of ricci flow in physics. *Canadian Journal of Physics*, 86(4):645–651, 2008.
23. Matthew Headrick and Toby Wiseman. Ricci flow and black holes. *Classical and Quantum Gravity*, 23(23):6683, 2006.
24. Alexander Frenkel, Petr Horava, and Stephen Randall. Topological quantum gravity of the ricci flow. *arXiv preprint arXiv:2010.15369*, 2020.
25. Stanley Deser and Orit Levin. Accelerated detectors and temperature in (anti-) de sitter spaces. *Classical and Quantum Gravity*, 14(9):L163, 1997.
26. George C McVittie. The mass-particle in an expanding universe. *Monthly Notices of the Royal Astronomical Society, Vol. 93, p. 325-339*, 93:325–339, 1933.
27. Gary W Gibbons and Stephen W Hawking. Cosmological event horizons, thermodynamics, and particle creation. *Physical Review D*, 15(10):2738, 1977.
28. Erik Verlinde. On the origin of gravity and the laws of newton. *Journal of High Energy Physics*, 2011(4):1–27, 2011.

doi:10.20944/preprints202504.2469.v6

29. Chiu Man Ho, Djordje Minic, and Y Jack Ng. Cold dark matter with mond scaling. *Physics Letters B*, 693(5):567–570, 2010.
30. FR Klinkhamer and M Kopp. Entropic gravity, minimum temperature, and modified newtonian dynamics. *Modern Physics Letters A*, 26(37):2783–2791, 2011.
31. Chiu Man Ho, Djordje Minic, and Y Jack Ng. Quantum gravity and dark matter. *International Journal of Modern Physics D*, 20(14):2887–2893, 2011.