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Communication

Generalized rest mass and Dirac's monopole in 5D theory and Cosmology

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Abstract: It is shown that the 5D geodetic equations and 5D Ricci identities give us a way to create a new viewpoint on some problems of Modern Physics, Astrophysics, and Cosmology. Specifically, the application of the 5D geodetic equations in (4+1) and (3+1+1) splintered forms obtained with the help of the monad and dyad methods made it possible to introduce a new, effective generalized concept of the rest mass of the elementary particle. The latter leads one to novel connections between the General Relativity and quantum field theories, as well as all of that, including the (4+1) splitting of the 5D Ricci identities, brings about a better understanding of the magnetic monopole problem and the vital difference in the origins of the Maxwell equations and gives rise to surprising connections between them. The obtained results also provide new insight into the mechanism of the 4D Universe's expansion and its following acceleration.

Keywords: monad and dyad methods; effective generalized rest mass concept; scalar gravitational field; 5D geodetic equations; cylindrical symmetry (cylindricity) conditions; 5D Ricci identities; Maxwell equations; magnetic monopole; topological second-order phase transition in cosmology.

1. Introduction

The article considers some old and new problems of modern physics, astrophysics, and cosmology in the framework of the 5D theory, using the well-developed and well-known for advanced researchers monad and dyad methods in General Relativity and 5D theory (See [1], pp. 184-207 and [2-4] for more details). Reviewing the papers of the different old and modern authors who have worked in the framework of the 5D theories [5-9], one can see that almost all of them, as it was rigorously proved by V.A. Fock [5] and Yu.B. Rumer [8], came to the 5D optics under the standard requirement that ratio e/m_0 has to be constant. In this article, we show that cancelation of this requirement leads to new, fundamental, and in some sense unexpected consequences, concerning the generalized effective rest mass concept $(m_0 \to \hat{m}_0)$, and to the new ratio requirement $(e/\hat{m}_0 \neq const)$. We reveal the physical grounds for the vital difference in the origins of the first and second pairs of the Maxwell equations and obtain unexpected functional connections between them. We also inspect the role of the 5D Ricci identities in the understanding of some of these problems. Moreover, we hope, that our findings can improve the present understanding of the magnetic monopole's problem, as well as the other important problems of modern physics, astrophysics, and cosmology, including the ones of dark matter and dark energy. The results obtained make it possible to single out two ages in the Universe's evolution, namely the magnetic monopoles' age and then the electrons' one.

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2. The Basic Elements of the Monad and Dyad Methods in 5D Theory

To begin with, I would like to tell that one of the first researchers who has developed and then successfully applied the full monad method together with the cylindrical symmetry (cylindricity) condition in the 5D theory was Prof. Peter Gabriel Bergmann [10], with whom – and it was a great Honor for me – I have had a long-lasting friendship and collaboration.

Here I want to briefly present the basic concepts of the monad method in 5D theory with the monad vector's chronometric gauging and the dyad method with the mixed gauging of the dyad vectors: first chronometric and then kinemetric. These methods allow us to employ the procedure of the V_5 reductions: $V_5 \mapsto V_4 \mapsto V_3$ more correctly or, in other words, the orthogonal $(4+1)$ splitting and then also the orthogonal $(3+1+1)$ splitting of the V_5 with the subsequent expression of the 5D theory in terms of the 4D and 3D physical-geometrical quantities. In the framework of the monad method with the chronometric gauging of the monad vectors we can represent the 5D metric G_{AB} as follows: $G_{AB} = g_{AB} - \lambda_A \cdot \lambda_B$. Here we have chosen the signature of the V_5 to be equal $(+----)$. Later we will be convinced, that this choice is a more preferable one. The indexes $A, B, C, ... = 0,1,2,3,5.$ The square of the 5D interval takes on the form $dI^2 = G_{AB} \cdot dx^A \cdot dx^B = ds^2 - d\lambda^2$. Here $ds^2 = g_{AB} \cdot dx^A \cdot dx^B$ and $d\lambda = \lambda_A \cdot dx^A$. The space-like monad vectors λ_A in chronometric gauging can be written as follows:

$$\lambda_A = \frac{G_{A5}}{\sqrt{-G_{55}}}, \lambda^A = \frac{G_5^A}{\sqrt{-G_{55}}}, \sqrt{-G_{55}} \equiv \varphi. \tag{1}$$

The λ_A is a tangential vector to the x^5 coordinate line and the orthogonal one to the space-time hypersurface V_4. Also g_{AB} is the metric tensor of V_4. Then, the orthonormality condition gives us: $\lambda^A \cdot \lambda_A = -1$; $\lambda^A \cdot g_{AB} = 0$; and, of course, we have the well-known relations: $G^{AB} \cdot G_{AB} = 5$; $G^{AB} \cdot G_{BC} = \delta_C^A$; $g^{AB} \cdot g_{AB} = 4$; $g^{AB} \cdot g_{BC} = \delta_C^A$. It should be also noted, that $g_{5A} \equiv 0$. Now we can construct the monad differentiation operators: $\lambda^A \cdot \partial_A \equiv \bar{\partial}^+_\Lambda$ and $g^A_\alpha \cdot \partial_A \equiv \bar{\partial}^+_\alpha$. Then, with help of the latter ones we can build the x^5 and V_4 projected directional normal and covariant derivatives: $\bar{\nabla}^+_\Lambda \equiv \bar{\partial}^+_\Lambda$ and $\bar{\nabla}^+_\alpha$, which have the standard form, being expressed through the operators $\bar{\partial}^+_\alpha$. In the case of the dyad method the 5D metric G_{AB} and the square of the 5D interval dI takes on the forms as it follows then here: $G_{AB} = \tau_A \cdot \tau_B - \lambda_A \cdot \lambda_B - h_{AB}$; $dI^2 = d\tau^2 - d\lambda^2 - dl^2$; $d\tau = \tau_A \cdot dx^A$; $dl^2 = h_{AB} \cdot dx^A \cdot dx^B$. Now we can also write down the orthonormal time-like vectors τ_A of the dyad in the kinemetric gauging as it follows below (the case of the space-like vectors λ_A of the dyad was commented above):

$$\tau^A = \frac{G^{0A}}{\sqrt{G^{00}}}, \tau_A = \frac{G_A^0}{\sqrt{G^{00}}}, \tau^A \cdot \tau_A = 1, \lambda^A \cdot \tau_A = 0, \tag{2}$$

and $\tau^A \cdot h_{AB} = \lambda^A \cdot h_{AB} = 0$. Note, that the 3D metric of the V_3 has the following well-known properties: $h^{AB} \cdot h_{AB} = 3, h^{AB} \cdot h_{BC} = -\delta_C^A$. It should be also noted, that $h^{0A} = h_{5A} \equiv 0$.′Vector τ^A is the tangential one to the x^0 coordinate line. We can also build the same type of differentiation dyad operators: $\bar{\partial}^+_\tau, \bar{\partial}^+_\Lambda$ are the τ^A and λ_A projected directional normal derivatives, and

$-h_i^A \cdot \partial_A \equiv \overline{\partial}_i^+ \to \overline{\nabla}_i^+$. Here $\overline{\nabla}_i^+$ stands for the V_3 projected covariant derivative of the standard above-mentioned form. The operator $\overline{\partial}_\tau^+$ is here defined as follows below:

$$\overline{\partial}_\tau^+ \overline{B}_{k\ldots}^{i\ldots} = \tau^A \cdot \partial_A \overline{B}_{k\ldots}^{i\ldots} + \overline{N}_k^l \cdot \overline{B}_{l\ldots}^{i\ldots} + \ldots - \overline{N}_l^i \cdot \overline{B}_{k\ldots}^{l\ldots} - \ldots \quad . \tag{3}$$

Here indexes $i, j, k, \ldots = 1,2,3,$ and τ^A connectivity (an analogy of the Christoffel symbols for the operator $\overline{\partial}_\tau^+$): $\overline{N}_i^k = \tau^A \cdot \overline{\partial}_i^+ h_A^k$. We will mark these projected quantities with a tilde over the operators. The $(4+1)$ splitting of the 5D relations one can produce then with the help of the monad method by making the inner product of the ones with the metric elements as λ_A, g_{AB}, and its combinations. The same $(3+1+1)$ splitting of the 5D relations one can produce with the help of the dyad method by making the inner product of the ones with the metric elements as τ_A, λ_A, h_{AB}, and its combinations. See [1-3], examples below, and also the Appendix for more details.

3. The New Rest Mass Concept and Some Cosmological Problems

Let us consider the geodetic equation in 5D theory, which has the following standard form: $G^A \equiv \dfrac{D^2 x^A}{dI^2} = 0$. Evidently, one can obtain it if one will use the 5D action $\Sigma = -m_0 \cdot c \cdot \int dI$ as it follows below (here m_0 is an initial rest mass of the 5D test particle and c is the light velocity) :

$$G^A \equiv \frac{d^2 x^A}{dI^2} + P_{BC}^A \cdot \frac{dx^B}{dI} \cdot \frac{dx^C}{dI} = 0 \quad . \tag{4}$$

Here the square of the 5D interval $dI^2 = G_{AB} \cdot dx^A \cdot dx^B$ and the indexes $A, B, C, \ldots = 0,1,2,3,5.$ G_{AB} is the 5D metric of the V_5 and P_{BC}^A stands for the 5D Christoffel symbols. Making here the $(4+1)$ splitting with the help of the monad method (see the examples below) and eliminating the 5th coordinate, then imposing the cylindrical symmetry (cylindricity) condition along the 5th coordinate (it means that we have the Killing equations for the 5D metric G_{AB} along the tangent vector to the 5th coordinate $\xi_A = \varphi \cdot \lambda_A$: the Lie derivatives $L_\xi G_{AB} = \xi_{A;B} + \xi_{B;A} = 0$) one can obtain from Eq. (4) the following system:

$$m_0 \cdot \lambda_A \cdot G^A = \frac{D^+}{ds}(\varphi \cdot \hat{p}) = 0 \Rightarrow \varphi \cdot \hat{p} = const. = \frac{n \cdot e}{2 \cdot \sqrt{k_0}} = n \cdot \hat{m}_{Pl}; \tag{5}$$

$$\hat{m}_0 \cdot \lambda_A \cdot G^A = 0 \Rightarrow \frac{D^+ \hat{\overline{p}}^\alpha}{ds} = \frac{2 \cdot \sqrt{k_0}}{c^2} \cdot \varphi \cdot \hat{p} \cdot \overline{u}^\beta \cdot F_{\beta \cdot}^\alpha + \overline{\partial}^{\alpha +} \hat{m}_0 . \tag{6}$$

Then, inserting the integral of Eq. (5) to Eq. (6) and choosing this integral to be corresponding to the Lorentz force one can easily obtain the 4D geodetic equation in 5D theory as it follows below (see Appendix for more details):

$$m_0 \cdot g_A^\alpha \cdot G^A = 0 \Rightarrow \frac{D^+ \hat{\overline{p}}^\alpha}{ds} = \frac{Q_0}{\hat{m}_0 \cdot c^2} \cdot \hat{\overline{p}}^\beta \cdot F_{\beta \cdot}^\alpha + \overline{\partial}^{\alpha +} \hat{m}_0 . \tag{7}$$

Here k_0 is the Newtonian gravitational constant, $Q_0 = n \cdot e$ (e is the electric charge of an electron), and also we have put here the hypothetical 5D "Plank mass" $\hat{m}_{Pl} = e/(2 \cdot \sqrt{k_0}) \approx 10^{-6} g$. It should be also added, that here we have: $\frac{D^+}{ds} = \bar{u}^\alpha \cdot \bar{\nabla}^+_\alpha - \hat{u} \cdot \bar{\partial}^+_\Lambda$, $F_{\alpha\beta} = \bar{\nabla}^+_\alpha A_\beta - \bar{\nabla}^+_\beta A_\alpha$ is a tensor of the electromagnetic field, and A_α is a 4D vector-potential of this field, $\hat{u} = d\lambda / ds$ is a "velocity" along the 5th coordinate, and the 4D velocity $\bar{u}^\alpha = d\bar{x}^\alpha / ds$. Here the indexes $\alpha, \beta, \gamma, ... = 0,1,2,3$. It should be noted, that by using here the monad method we have introduced by a natural way the effective generalized rest mass of the Lorentz type $\hat{m}_0$ in the following single-valued, unique form [2-4, 11]. We have again: $dI^2 = ds^2 - d\lambda^2; \hat{m}_0 = m_0 \cdot \hat{\beta}$; $\hat{p} = \hat{m}_0 \cdot \hat{u}$; $\hat{\bar{p}}^\alpha = \hat{m}_0 \cdot \bar{u}^\alpha$ and $\hat{\beta} = (1 - \hat{u}^2)^{-1/2}$ is a 5D Lorentz factor.

It should be also noted, that Eqs. (5) - (7) for the 5D test particle one can also obtain from the above-mentioned 5D action if vary it, due to the cylindricity condition, independently on x^α and x^5 [4]:

$$\Sigma = -m_0 \cdot c \cdot \int dI = -c \cdot \int \hat{m}_0 \cdot ds + \frac{2 \cdot \sqrt{k_0}}{c} \cdot \int \varphi \cdot \hat{p} \cdot A_\alpha \cdot d\bar{x}^\alpha - c \cdot \int \varphi \cdot \hat{p} \cdot dx^5. \tag{8}$$

Here one can easily obtain the standard action for the test charged particle up to the gage invariant term, inserting an integral of Eq. (5):

$$\Sigma = -c \cdot \int \hat{m}_0 \cdot ds + \frac{Q_0}{c} \cdot \int A_\alpha \cdot d\bar{x}^\alpha - n \cdot \hat{m}_{Pl} \cdot c \cdot x^5 = S_{\hat{m}_0} + S_e - n \cdot \hat{m}_{Pl} \cdot c \cdot x^5. \tag{9}$$

Now let us consider the 5D Hamilton-Jacoby equation for the charged test particle:

$$G^{AB} \cdot \partial_A \Sigma \cdot \partial_B \Sigma - m_0^2 \cdot c^2 = 0. \tag{10}$$

After (4+1) splitting this equation will take the following form:

$$g^{\alpha\beta} \cdot (\bar{\partial}^+_\alpha \hat{S} - \frac{Q_0}{c} \cdot A_\alpha) \cdot (\bar{\partial}^+_\beta \hat{S} - \frac{Q_0}{c} \cdot A_\beta) - \hat{m}_0^2 \cdot c^2 = 0, \tag{11}$$

where $\hat{S} \equiv S_{\hat{m}_0}$.

Here one can be convinced again, that in the 5D theory one can always obtain the effective generalized rest mass $\hat{m}_0$ if one makes (4+1) splitting. One more confirmation of this Transition Principle in the 5D theory one can obtain when one considers the transition to the quantum theory. In this case, as it is well-known, the 5D equation for the wave function Ψ has the following Klein-Gordon-Fock form:

$$G^{AB} \cdot \nabla_A \nabla_B \Psi + \frac{m_0^2 \cdot c^2}{\hbar^2} \cdot \Psi = 0. \tag{12}$$

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It should be noted here, that this equation should have the Hamilton-Jacoby Eqs. (10), (11) as the characteristic ones.

In the approximation of 5D optics one has the following relation for the wave function Ψ :

$$\Psi = C \cdot \psi \cdot \exp\left\{ \frac{i}{\hbar} \cdot \frac{Q_0 \cdot c}{2 \cdot \sqrt{k_0}} \cdot x^5 \right\}.$$

Here function ψ does not depend on the x^5 coordinate. Then, one can make the (4+1) splitting again and as a result, one can obtain the equations for the 5D or 4D wave functions Ψ or ψ as it follows below:

$$g^{\alpha\beta} \cdot \overline{\nabla}_\alpha^+ \overline{\nabla}_\beta^+ \Psi + \Phi^\alpha \cdot \overline{\nabla}_\alpha^+ \Psi + \frac{\hat{m}_0^2 \cdot c^2}{\hbar^2} \cdot \Psi = 0; \quad (13)$$

$$(\overline{\nabla}^{+\alpha} + \frac{i \cdot Q_0}{\hbar \cdot c} \cdot A^\alpha + \Phi^\alpha) \cdot (\overline{\nabla}_\alpha + \frac{i \cdot Q_0}{\hbar \cdot c} \cdot A_\alpha) \cdot \psi + \frac{\hat{m}_0^2 \cdot c^2}{\hbar^2} \cdot \psi = 0. \quad (14)$$

The Transition Principle for the effective generalized rest mass is valid also in this case: $V_5 \to V_4 \Rightarrow m_0 \to \hat{m}_0$. It should also be added, that namely, Louis de Broglie was the first one who has closely approached the understanding of this Transition Principle of Nature in the 5D theory (See [7] for more details). Unfortunately, he has chosen the wrong signature $(+---+)$ of the V_5 instead of the right one $(+----)$, as well as the restrictive requirement for the relation e/m_0 to be constant. Because of that he failed to establish the Transition Principle in its complete form and could not obtain the *exact* formula (18) for $\hat{m}_0$ (See below). Moreover, he has tried to develop the 5D theory together with this requirement beyond the 5D optics, though it was shown, as we have mentioned above, by V. A. Fock [5] and Yu.B. Rumer [8], that this requirement ($e/m_0 = const.$) leads one only to the 5D optics. Because of this, he has come to the contradiction and, as a result, he has obtained the incomplete formula for the effective generalized rest mass, that valid just in a certain special case solely. However, it is important to note that it was exactly L. de Broglie, the one who later, considering the 5D wave equation and its connection with the quantum theory, correctly understood that in the case of a charged test particle in 5D wave Eq. (12) a certain invariant I should be employed instead of the usual rest mass m_0. This invariant (Eq. (41) in Ref. [7]):

$$I = \sqrt{m_0^2 + \frac{e^2}{16 \cdot \pi \cdot k_0}}.$$

is just a special case of the effective generalized rest mass $\hat{m}_0$ introduced in the present paper, see Eq. (18) below. The latter is reduced to I at $n = 1$ and $\varphi = const = 2 \cdot \sqrt{\pi} : \hat{m}_0 = \omega(n;\varphi) \Rightarrow I = \omega(1;2 \cdot \sqrt{\pi})$.

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Thus, the effective generalized rest mass $\hat{m}_0$ could be considered as a generalization of L. de Broglie's hypothesis. It's very important, that in his article he has also made a very interesting and deep prophesy: «Si l'on parvient à interpréter la façon dont interviennent, dans l'équation de'onde a cinq dimensions, les constantes $e, m_0, c, \hbar$ et k_0, on sera bien près d'avoir compris quelques-uns des secrets les plus troublants de la Nature» (in French, see below an English translation)[2]. One can easily be convinced, that only the introduction of the effective generalized mass $\hat{m}_0$ leads one to the compact and complete form of the geodetic Eq. (7) and fulfills it with a large and important physical sense. When P.A.M. Dirac visited the Lomonosov Moscow State University (MSU), he took a piece of chalk, and wrote on the wall of the Chair of Theoretical Physics: "The physical law should have the mathematical beauty." It is easy to see, that the geodetic Eq. (7) has this mathematical beauty.

It is worth mentioning that the first time the author obtained Eqs. (5)-(7) and relation (18) in 1978 and then these results were reported at the seminar of my scientific adviser Prof. Yu.S. Vladimirov in the Lomonosov MSU in Moscow. Next, in 1979, they were reported and published in Russian in Minsk [11] and one year later, in 1980, in English in Germany (Jena, GDR) [2]. Unfortunately, the author has found an article [7] only a few years later, about 1982, and it's necessary to stress, that this great article of the outstanding physicist was not appreciated with dignity till now. It should be also noted here, that only many years later, in 2015, T.X. Zhang in his very interesting article [9] has found the correct expression for the so-called certain factor f, but unfortunately, he has not connected this factor with the particle's mass (this connection follows from the results of the present paper and reads: $\hat{m}_0 = m_0 \cdot f$). Therefore, the (4+1) splintered geodetic equations in his article [9] are very cumbersome and also contain several terms which do not have any clear physical meaning. Moreover, their form is also very far from the clear and compact form of Eq. (7) (See [9], Eqs. (77), (78) for comparison).

Probably, T.X. Zang was not familiar either with L.de Broglie's article [7] or with my articles [2,11,13], since in his paper [9] he claimed that he was the first who obtained the factor f in the 5D theory. This is not the case. The first, in some sense, was L. de Broglie. Then, these results were independently obtained in Refs. [2-4,11,13]. Thus, to the best of my knowledge, T.X. Zhang was only the third in this row. But in any case, we need to note, that it has occurred to be very important and nontrivial, that three independent researchers at the different times and using different ways came to very close results. It permits one to believe earnestly that this new concept of the rest mass $\hat{m}_0$ is highly likely correct and makes our scientific horizons wider.

Then, let us briefly demonstrate below, as an example, the $(4+1)$ splitting of the above-mentioned 5D

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[2] "If one can understand how the constants $e, m_0, c, \hbar$ and k_0 may be interpreted conspicuously in the 5D wave equation, then one will be able to discover the most mysterious secrets of Nature."

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Lie derivatives of the 5D metric G_{AB}. We will start with these Killing equations (See Appendix below for more details and also examples):

$$\lambda^A \cdot \lambda^B \cdot L_\xi G_{AB} = \lambda^A \cdot \lambda^B \cdot (\xi_{A;B} + \xi_{B;A}) = -2 \cdot \overline{\partial}_\Lambda^+ \varphi = 0 \Rightarrow \overline{\partial}_\Lambda^+ \varphi = 0; \tag{15}$$

$$\lambda^A \cdot g_\alpha^B \cdot (\xi_{A;B} + \xi_{B;A}) = \varphi^2 \cdot \overline{\partial}_\Lambda^+ (\lambda_\alpha / \varphi) = \frac{2}{c^2}\sqrt{k_0} \cdot \varphi^2 \cdot \overline{\partial}_\Lambda^+ A_\alpha = 0 \Rightarrow \overline{\partial}_\Lambda^+ A_\alpha = 0; \tag{16}$$

$$g_\alpha^A \cdot g_\beta^B \cdot (\xi_{A;B} + \xi_{B;A}) = \varphi \cdot \overline{\partial}_\Lambda^+ g_{\alpha\beta} = 0 \Rightarrow \overline{\partial}_\Lambda^+ g_{\alpha\beta} = 0.$$

(17)

In the case of the 5D geodetic equations, the process of the $(4+1)$ splitting one can make the same way. Of course, when one is making this splitting of Eq. (4) one should take into account the content of Section 2 and the above-mentioned relations. Thus, to obtain Eqs. (5) and (6) one need to project firstly Eq. (4) with the help of the monad vector λ_A and then with the help of the metric tensor g_A^α.

The evaluation of the above-mentioned integral of Eq. (5) leads one directly to the following *exact* expression for the effective generalized rest mass:

$$\hat{m}_0 = \frac{m_0}{\sqrt{1-\hat{u}^2}} = \sqrt{m_0^2 + \frac{Q_0^2}{4 \cdot k_0 \cdot \varphi^2}} = \sqrt{m_0^2 + \frac{n^2 \cdot \hat{m}_{Pl}^2}{\varphi^2}}, \tag{18}$$

It should be also stressed, that Eq. (7) has the same form as the corresponding expression in the geodetic equations in a scalar-tensor theory [12], but up to the Lorentz force. Then, in this theory, it was just declared, following Mach's principle, that the rest mass must be a certain unknown function of the scalar field φ and it should interact with this scalar field through the hypothetical scalar charge Q_{Sc}. The explicit form of this function remains unknown till now and the scalar charge has not been detected yet and remains a hypothetical quantity. In contrast, Eq. (18) provides the definite, single-valued explicit expression for $\hat{m}_0$. Note, that Eq. (18) also agrees with the Economy Principle of Nature: $Q_0 \equiv Q_{Sc}$. Thus, one can believe, that rather there is not any scalar charge Q_{Sc} in Nature. It means that an electric charge is simultaneously the scalar one. Thus, one can conclude, that the neutral particles in the scalar-tensor theory interact neither with a scalar field nor with an electromagnetic one. In an opposite case, one needs to generalize the well-known Equivalence Principle in General Relativity, spreading it also on the scalar field and being under the hypothesis, that almost all particles, excepting photon and neutrino, are the composite ones. Here we also should repeat, that the relation (18) generalizes the above-mentioned expression for invariant I obtained by L. de Broglie [7].

Expression (18) may be rewritten in a more compact and convenient form by introducing the mass angle:

$$\chi_n = Arsh\frac{n \cdot \hat{m}_{Pl}}{m_0 \cdot \varphi}: \quad \hat{m}_0 = m_0 \cdot ch\chi_n. \tag{19}$$

Besides, one may represent this expression (18) as a modulus of a complex quantity:

$$\hat{m}_0 = \sqrt{\hat{m}_{0z} \cdot \hat{\bar{m}}_{0z}} \tag{20}$$

where $\hat{m}_{0z} = m_0 + i \cdot n \cdot \hat{m}_{Pl} / \varphi$. It may be written also in the exponential form as $\hat{m}_{0z} = \hat{m}_0 \cdot e^{i \cdot \psi_n}$, where the phase $\psi_n = \arctan\{n \cdot \hat{m}_{Pl} / (m_0 \cdot \varphi)\}$. Here $\hat{\bar{m}}_{0z} = m_0 - i \cdot n \cdot \hat{m}_{Pl} / \varphi$ is a complex conjugate to the $\hat{m}_{0z}$ quantity. It motivates one to assume in it a possible connection with the quantum properties of the elementary particles. In these terms, one may consider, for example, a photon as a complex null-particle $(0;0)$, a neutrino as a real particle $(m_\nu;0)$, and a hypothetical tachyon as an imaginary particle $(0;m_\tau)$ [13]. Thus, one must stress again, that it seems to be natural to spread out the Equivalence Principle of the General Relativity for gravitational field additionally also on the scalar gravitational field and therefore, to put forward the hypothesis that almost every verily 5D particle must contain an electric charge, leastwise in a latent form. Basing on this idea one can also to suppose, that the neutral 5D particles must be the composite ones. In this sense, as we believe, there are rather only two verily 4D particles: the photon and the neutrino. Specifically, the existence of the hypothetical tachyon one can connect just with the 5th coordinate. An additional argument to introduce the effective generalized rest mass is also such a fact that only in this case it is possible to perform the further $(3+1+1)$ splitting of the 5D geodetic Eq. (7) with an additional elimination of the time coordinate, using the dyad method [3,4,11]. Only in this case, we can obtain the well-known system of the geodetic equations, which is very familiar to the same system, that we usually obtain in 4D theory, making the similar (3+1) splitting of the 4D geodetic equation with help of the 4D monad method with the elimination of the x^0 coordinate but up to the 4D scalar force. Thus, in the 5D case we can obtain the following system from Eq.(7), making the further (3+1+1) splitting of it [2-4, 11, 13]:

$$m_0 \cdot \tau_A \cdot G^A = 0 \Rightarrow \frac{D^+ \hat{m}}{d\tau} = \hat{\bar{p}}^i \cdot (\bar{F}_i - \bar{v}^k \cdot \bar{D}_{ik}) - \frac{Q_0}{c^2} \cdot \bar{v}^i \cdot \bar{E}_i + \frac{Q_0 \cdot \sqrt{1-\bar{v}^2}}{2 \cdot \sqrt{k_0} \cdot \varphi} \cdot \bar{\partial}_\tau^+ \hat{m}_0 \;; \tag{21}$$

$$-m_0 \cdot h_A^i \cdot G^A = 0 \Rightarrow$$

$$\Rightarrow \frac{D^+ \hat{\bar{p}}^i}{d\tau} = \frac{Q_0}{c^2} \cdot (\bar{E}^i - \bar{v}^k \cdot H_{.k}^i) - \hat{m} \cdot \bar{F}^i + 2 \cdot \hat{\bar{p}}^k \cdot \bar{D}_k^i + \frac{Q_0 \cdot \sqrt{1-\bar{v}^2}}{2 \cdot \sqrt{k_0} \cdot \varphi} \cdot \bar{\partial}^{+i} \hat{m}_0 . \tag{22}$$

Here $\dfrac{D^+}{d\tau} = \bar{\partial}_\tau^+ + \hat{v} \cdot \bar{\partial}_\Lambda^+ + \bar{v}^i \cdot \bar{\nabla}_i^+$ and the indexes $i, j, k, ... = 1,2,3$.

Specifically, it occurred that it is impossible in this case to perform further $(3+1+1)$ splitting of Eq. (7) without introducing an effective generalized rest mass $\hat{m}_0$ because in an opposite case Eqs. (21) and (22) are here coupled so strongly that there is not any possibility to disconnect them. The last terms in Eqs. (21) and (22) are the components of the scalar 4D force which are caused by the dependence of the effective generalized rest mass $\hat{m}_0$ on the scalar gravitational field. Here we have also put the well-known designations: $\hat{m} = \hat{m}_0 / \sqrt{1-\bar{v}^2}$, $\bar{v}^i = d\bar{x}^i / d\tau$, $\hat{v} = d\lambda / d\tau$. Then, $\hat{\bar{p}}^i = \hat{m} \cdot \bar{v}^i$, $\hat{p} = \hat{m} \cdot \hat{v} = \hat{m}_0 \cdot \hat{u}$, and $d\tau = c \cdot dt$. It should be also noted, that the scalar 4D force in Eqs. (21) and (22) vanishes for the zero rest mass particles, as it was predicted in all of the scalar-tensor theories, due to the Lorentz factor $\sqrt{1-\bar{v}^2}$ [12].

Note here, that the effective generalized rest mass $\hat{m}_0$ in the 5D theory appears just in the same way as the relativistic mass $m = m_0 / \sqrt{1-\bar{v}^2}$ in the 4D theory:

$$dl \to ds \Rightarrow ds^2 = d\tau^2 - dl^2; m = m_0 \cdot \beta; \bar{v} = dl / d\tau; d\tau = c \cdot dt,$$

where the usual 4D Lorentz factor $\beta = (1-\bar{v}^2)^{-1/2}$. Thus, one can believe the $\hat{m}_0$ to be, in some sense, the 5D "relativistic" effective generalized rest mass.

It is easy to prove that the same results one can obtain whenever we increase the space dimensions of the space-time: $V_n \mapsto V_{n+1}$. As a consequence, one can obtain here the following result: $\hat{m}_{0,n} \mapsto \hat{m}_{0,n+1}$. Thus, following the idea of P. Ehrenfest [15], one can be convinced here that the increase in the dimension adds some new features to the physical nature of the particles and to Nature itself.

The geodetic equation (7) we can easily bring to the following form:

$$\hat{m}_0 \cdot \frac{D^+ \bar{u}^\alpha}{ds} = f_L^\alpha + f_B^\alpha. \tag{23}$$

Here

$$f_L^\alpha = \frac{Q_0}{c^2} \cdot \bar{u}^\beta \cdot F_{\beta}^{\alpha} \tag{23a}$$

and

$$f_B^\alpha = -\frac{Q_0^2 \cdot P^{\alpha\beta} \cdot \Phi_\beta}{4 \cdot k_0 \cdot \hat{m}_0 \cdot \varphi^2} \tag{23b}$$

are the 4D Lorentz force and the 4D scalar one or, as we have proposed [13], the Brance (or maybe Brance-Dicke) force. We may leave the decision of this question on the world physical community. Here $P^{\alpha\beta} = g^{\alpha\beta} - \bar{u}^\alpha \cdot \bar{u}^\beta$ is a tensor of the orthogonal projection upon the direction of the 4D velocity $\bar{u}^\alpha$ and $\Phi_\beta = \bar{\nabla}_\beta^+ \ln \varphi$. It should be noted, that the 4D Lorentz force for big masses is equal to zero because it depends on the Q_0 linearly. But the 4D Brance (or Brance-Dicke) force depends on the Q_0^2 and, although it is very weak [12], it can be very significant, being accumulated for the big masses. One can call it the big masses effect. Besides, the 4D scalar force is rather also negative and so it may have the repelling property. Thus, it may be one of the reasons for the Universe expansion. I would like to note also again, that the expression (18) for the effective generalized rest mass permits us to think that an electric charge e superposes simultaneously, as it was mentioned above, the role of the scalar one, so it can explain to us why we could not find this scalar charge so far. Also, one can hope that an interplay between the scalar gravitational field and an electric charge can explain how the dark matter may contribute to the total mass of the 4D Universe, which is immersed, in some sense, into the scalar boson ocean and also, highly likely, to the dark energy of Universe. It should also be added, that the scalar gravitational field contributes to the braking (some authors prefer here the German term bremsstrachlung) radiation force [13] and permits one to generalize the concept of the last one: $g^{\alpha} = g^{\alpha}_{E} + g^{\alpha}_{Sc} + g^{\alpha}_{ESc}$. Here we should inform, that the total braking (bremsstrschlung) radiation force may be represented as a sum of the electromagnetic, scalar, and mixed parts, where the first one has the following well-known form:

$$g^{\alpha}_{E} = \frac{2 \cdot e^{3} \bar{u}^{\gamma}}{3 \cdot \hat{m}_{0} \cdot c^{3}} \cdot \left(\bar{u}^{\beta} \cdot \overline{\nabla}^{+}_{\gamma} F^{\alpha}_{\beta \cdot} + \frac{e \cdot P^{\alpha\delta}}{\hat{m}_{0} \cdot c^{2}} \cdot F_{\beta\delta} \cdot F^{\beta}_{\gamma \cdot} \right); \tag{24}$$

and the second one has the form as below:

$$g^{\alpha}_{Sc} = -\frac{e^{4} \cdot P^{\alpha\beta} \cdot \bar{u}^{\gamma}}{6 \cdot c \cdot k_{0} \cdot \varphi^{2} \cdot \hat{m}^{2}_{0}} \cdot \left(\overline{\nabla}^{+}_{\gamma} \Phi_{\beta} - 2 \Phi_{\beta} \cdot \Phi_{\gamma} + \frac{3 \cdot e^{2} \cdot \Phi_{\beta} \cdot \Phi_{\gamma}}{4 \cdot k_{0} \cdot \phi^{2} \cdot \hat{m}^{2}_{0}} \right). \tag{25}$$

At last, the third one has the next form:

$$g^{\alpha}_{ESc} = -\frac{e^{5}}{6 \cdot c^{3} \cdot k_{0} \cdot \varphi^{2} \cdot \hat{m}^{3}_{0}} \cdot \left(g^{\beta\delta} \cdot P^{\alpha\gamma} - 3 \cdot g^{\alpha\gamma} \cdot \bar{u}^{\beta} \cdot \bar{u}^{\delta} \right) \cdot F_{\beta\gamma} \cdot \Phi_{\delta}. \tag{26}$$

(See [13] for more details). Of course, the electromagnetic part is here the most significant.

4. The 5D Ricci Identities and Some Problems of the Physics, Astrophysics, and Cosmology

We should remember some interesting and perspicacious ideas of the scientific classics about the connections between World geometry and physical interactions. Following one of the greatest mathematicians of the XIXth century – W.K. Clifford [16] – let us consider the well-known in Riemannian geometry Ricci identities in the case of the 5D theory:

$$R^{A}_{(BCD)} \equiv R^{A}_{\cdot BCD} + R^{A}_{\cdot DBC} + R^{A}_{\cdot CDB} = 0. \tag{27}$$

The $(4+1)$ splitting of the V_5 (see above the analogous procedures in relations (5)-(7)) gives us the following relations between the 4D physical-geometrical quantities [13], starting with the 4D Ricci identities:

$$g^{\alpha}_{A} \cdot g^{B}_{\beta} \cdot g^{C}_{\gamma} \cdot g^{D}_{\delta} \cdot R^{A}_{(BCD)} \equiv R^{\alpha}_{(\beta\gamma\delta)} \equiv R^{\alpha}_{\cdot\beta\gamma\delta} + R^{\alpha}_{\cdot\delta\beta\gamma} + R^{\alpha}_{\cdot\gamma\delta\beta} = 0 \ . \tag{28}$$

It means that the 4D World also has the Riemannian geometry. Then, we can obtain the following very important connections between the electromagnetic tensor and the curls of the scalar gravitational field's gradients. It gives us a very specific "trigger" of the changing electromagnetic process:

$$\lambda_{A} \cdot g^{B}_{\alpha} \cdot g^{C}_{\beta} \cdot g^{D}_{\gamma} \cdot R^{A}_{(BCD)} = 0 \Rightarrow F_{(\alpha\beta;\gamma)} + 2 \cdot F_{(\alpha\beta} \cdot \Phi_{\gamma)} = 0; \tag{29}$$

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$$\lambda_A \cdot \lambda^D \cdot g_\alpha^B \cdot g_\beta^C \cdot R^A_{(BCD)} = 0 \Rightarrow \frac{2 \cdot \sqrt{k_0}}{c^2} \cdot F_{\alpha\beta} \cdot \overline{\partial}^+_\Lambda \varphi = \Phi_{\alpha;\beta} - \Phi_{\beta;\alpha} \equiv \mathrm{M}_{\alpha\beta}. \tag{30}$$

Here we have introduced the curls of the scalar gravitational field's gradients $\mathrm{M}_{\alpha\beta}$, which one may consider as the quasi-particles of the soliton-type or magnetic monopole-type. These quasi-particles, in some sense, are similar to solitons [17,18]. Importantly, that the imposition of the cylindricity condition gives rise to the vanishing of these scalar curls, since only in this case the necessary and sufficient condition for the scalar gravitational field to be laminar may be satisfied (it is, as well-known, the following condition: $\Phi_{\alpha;\beta} = \Phi_{\beta;\alpha} \Rightarrow \mathrm{M}_{\alpha\beta} \equiv 0$).

At last, making use of Eqs. (29) and (30), one can obtain the first pair of the Maxwell equations with a nonzero r.h.s. of the magnetic monopole-type, namely:

$$F_{(\alpha\beta;\gamma)} \equiv F_{\alpha\beta;\gamma} + F_{\gamma\alpha;\beta} + F_{\beta\gamma;\alpha} = \frac{2}{\sqrt{\kappa_0} \cdot \overline{\partial}^+_\Lambda \varphi} \cdot \begin{vmatrix} \Phi_\alpha & \overline{\nabla}^+_\alpha & \Phi_\alpha \\ \Phi_\beta & \overline{\nabla}^+_\beta & \Phi_\beta \\ \Phi_\gamma & \overline{\nabla}^+_\gamma & \Phi_\gamma \end{vmatrix}. \tag{31}$$

The imposition of the cylindricity condition here also gives rise to the vanishing of the r.h.s of Eq. (31), which brings this equation to the conventional form of the first pair of the Maxwell equations:

$$F_{(\alpha\beta;\gamma)} \equiv F_{\alpha\beta;\gamma} + F_{\gamma\alpha;\beta} + F_{\beta\gamma;\alpha} = 0. \tag{32}$$

Thus, one can conclude that the first pair of the Maxwell equations is obliged to the Riemannian structure of the V_5 and has another origin than the second one, which we have obtained from the 5D variation principle [4,19]. The r.h.s. in Eq. (31) one can rewrite employing the above-mentioned curls $\mathrm{M}_{\alpha\beta}$ in the following form:

$$r_{(\alpha\beta\gamma)} = \frac{2}{\sqrt{\kappa_0} \cdot \overline{\partial}^+_\Lambda \varphi} \cdot (\Phi_\alpha \cdot \mathrm{M}_{\beta\gamma} - \Phi_\beta \cdot \mathrm{M}_{\alpha\gamma} + \Phi_\gamma \cdot \mathrm{M}_{\alpha\beta}). \tag{33}$$

This gives us the possibility to represent the second pair of the Maxwell equations, which we have obtained, as we have mentioned above, from the 5D variation principle [4,19], with the same monopole-type r.h.s., if we believe that this r.h.s. is the linear combination of the magnetic monopole-type currents. With this purpose, we will write down the mixed, x^5 and V_4 projected, 5D field equations with the energy-momentum tensor of the 5D dust as a r.h.s. [4,19]:

$$\overline{\nabla}^+_\nu (\varphi^3 \cdot F^{\mu\nu}) = -\frac{8 \cdot \pi}{c^2} \cdot \sqrt{k_0} \cdot \varphi \cdot Q^\mu_5. \tag{34}$$

Here the energy-momentum tensor of the 5D dust has the following form [4,19]:

$$Q^{AB} = \mu_0 \cdot c \cdot \frac{dx^A}{dI} \cdot \frac{dx^B}{d\tau}, \tag{35}$$

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where μ_0 is a matter density. However, before the cylindricity condition to be imposed, one may easily bring Eq. (34) to the following form, casting temporarily aside the r.h.s. in Eq. (34) and using Eq. (30):

$$\overline{\nabla}^+_\nu F^{\mu\nu} = -\frac{3 \cdot c^2 \cdot \Phi_\nu \cdot \mathrm{M}^{\mu\nu}}{2 \cdot \sqrt{k_0} \cdot \overline{\partial}^+_\Lambda \varphi} = -\frac{4 \cdot \pi}{c} \cdot \bar{j}^\mu_m. \quad (36)$$

We can interpret here, that the r.h.s. in (36) as a magnetic monopole-type current density, where we can believe that curl $\mathrm{M}_{\mu\nu}$ corresponds, for example, to the «north»-particle of the magnetic monopole-type and $\mathrm{M}_{\nu\mu}$ corresponds to the «south» one, or vice versa:

$$j^\mu_m = \frac{3 \cdot c^3 \cdot \Phi_\nu \cdot \mathrm{M}^{\mu\nu}}{8 \cdot \pi \cdot \sqrt{k_0} \cdot \overline{\partial}^+_\Lambda \varphi}. \quad (37)$$

Let us call them «*n*-monopole» and «*s*-monopole». Thus, we believe that there exist two types of the magnetic monopoles «charges»: *n* and *s* ones.

Finally, we can establish that just after we have imposed the cylindricity condition, as a result of it, the curls of the scalar gravitational field disappear and then Eq. (5) gives us an integral of the movement along the 5th coordinate connected with an electric charge of the 5D test particle, so thus the energy-momentum tensor of the 5D dust (the r.h.s. in Eq. (34)) easily transforms in the second pair of the Maxwell equations and gives us the new r.h.s. of the conventional type as follows below [4,19]:

$$\nabla^+_\nu F^{\mu\nu} = -\frac{4 \cdot \pi}{c \cdot \varphi^2} \cdot j^\mu_e. \quad (38)$$

Here $j^\mu_e = \rho_0 \cdot \bar{v}^\mu$ is the 4D electric current (ρ_0 is an electric charge density). We have used here the relation (34) and then we have also used the evident relation: $\rho_0 = \mu_0 \cdot Q_0 / m_0$.

One can join Eq. (36) and (38) by using the Heviside's unit-jump-function $H(t)$ as it is defined below:

$$H(t) = 0 \text{ at } t < 0 \text{ and } H(t) = 1 \text{ at } t \geq 0. \quad (39)$$

Thus, one can write down now the second pair of the Maxwell equations (I have also proposed to call it the Maxwell-Dirac equations [13]) as follows below:

$$\overline{\nabla}^+_\nu F^{\mu\nu} = -\frac{4 \cdot \pi}{c} \cdot \left[H(t_0 - t) \cdot \bar{j}^\mu_m + H(t - t_0) \cdot \varphi^{-2} \cdot \bar{j}^\mu_e \right] \quad (40)$$

Here t_0 is the moment of the time, when the x^5 cylindricity condition was established. The first pair of the Maxwell-Dirac equations one can also write down employing the Heaviside function:

$$F_{(\alpha\beta;\gamma)} \equiv F_{\alpha\beta;\gamma} + F_{\gamma\alpha;\beta} + F_{\beta\gamma;\alpha} = H(t_0 - t) \cdot r_{(\alpha\beta\gamma)}. \quad (41)$$

The analysis of these results leads us to the fundamental idea about the evolution of the 4D Universe. The process of the transition to the cylindrical symmetry condition in the 5D Universe $(V_5 \to V_4)$ seems to be, in some sense, very familiar with the second-order phase transition in helium. As well-known, the cooling of the helium leads to the second-order phase transition [20] of the last and this process attends with the disappearance of the collective curl excitations (rotons) and, as a result of the transition to the laminar state, with the appearance of the phonons. Then, we may hypothesize that here we have some kind of a second-order phase transition in cosmology. Let us call it the topological one [18]. It is quite possible that this phase transition is connected with the cooling of the expanding Universe and leads to the more stable state of the Universe's aggregation. Note also, that in work [21] was obtained the energetical spectrum of some kind of the scalar gravitational field that looks very similar to the energetical spectrum of helium. Thus, one can suppose that, as a result of this phase transition, we have obtained some kind of a superfluid state of the scalar matter and it may accelerate the expansion of the 4D Universe [18]. This acceleration, as well-known, was discovered more than twenty years ago with help of the observations after the type Ia supernovas' explosions. Also, we may add, that this transition and, then, further compactification of the 5th coordinate is possibly caused by the Casimir effect [21]. Besides, one can assume that in modern time we are not able to find these magnetic monopoles in the 4D Universe, maybe only a few of the relict ones [13,18]. Also, one can hypothesize, basing on the expression (18), that the effective rest mass of the particles, being dependent on the scalar $\varphi = \eta(t)$ which in turn is connected with the 5D metric G_{55} (See above Section 2, formula (1)), possibly slightly (as an adiabatic invariant) varies with the cosmological time t. In work [23] the authors have considered a 5D metric of the Kasner type, which depends on the cosmological time. Then, maybe one can estimate the age of the 4D Universe, comparing the values of the elementary particles' masses at different moments of the cosmological time t.

Thus, hopefully, someday we will be able, among many other questions, to answer the old question: "Where has the magnetic monopole gone to?"[24].

5. Discussion

This article was written because for a long time the author has accumulated a lot of results in the 5D theory of the Kaluza type, which has been almost forgotten during the last fifty years. It has occurred that if one goes beyond the 5D optics, certain new and very non-trivial properties of the matter and 4D Universe may come into being. Following this approach, the author has succeeded to generalize the rest mass concept following the ideas of L. de Broglie and has understood more deeply the quantum nature of the matter. The implementation of the monad method to the (4+1) splitting of the 5D Ricci identities makes it possible to understand how the Riemannian structure of the World affects its physical properties. It permits one to approach closer to the understanding of the magnetic monopole's problem and the difference in the origins of the first and the second pairs of the Maxwell equations, and at the same time the surprising connections between them. The obtained results also provide new insight into the mechanism of the expansion of the 4D Universe and its acceleration. The author believes that the application of the developed approach extends far beyond the specific problems discussed in the present paper. Its generalization beyond the 5D case may be also worthwhile and fruitful.

6. Conclusions

The author would like to express the hope that these results may open a new age in the investigations of the 5D theory, which was appreciated by one of the greatest physicists L. de Broglie as a preferable one, and will attract more attention of the world's physical community. It should be also noted, that the possible discovering of the 5th force with help of the circular accelerator in Batavia may inspire the new generations of researchers and stimulate their interest to investigate deeper the 5D theory.

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Appendix

Here we consider in detail several examples of the obtaining of the above-mentioned relations. Let us start with the Killing equations (7)-(9):

$$\lambda^A \cdot \lambda^B \cdot L_\xi G_{AB} = \lambda^A \cdot \lambda^B \cdot (\xi_{A;B} + \xi_{B;A}) = -2 \cdot \bar{\partial}^+_\Lambda \varphi \Rightarrow \bar{\partial}^+_\Lambda \varphi = 0; \tag{7}$$

The detailed calculations give us the same result as in relation (7):

$$2 \cdot \lambda^A \cdot \lambda^B \cdot (\varphi \cdot \lambda_A)_{;B} = 2 \cdot \varphi^{-2} \cdot (-\partial_5 \varphi^2 - P_{55,5} \cdot \varphi^{-1} \cdot \varphi) =$$

$$-4 \cdot \varphi^{-1} \cdot \varphi_{,5} - \varphi^{-2} \cdot (2 \cdot G_{55,5} - G_{55,5}) = -4 \cdot \bar{\partial}^+_\Lambda \varphi + 2 \cdot \bar{\partial}^+_\Lambda \varphi = -2 \cdot \bar{\partial}^+_\Lambda \varphi = 0;$$

$$\lambda^A \cdot g^B_\alpha \cdot (\xi_{A;B} + \xi_{B;A}) = \varphi^2 \cdot \bar{\partial}^+_\Lambda (\lambda_\alpha / \varphi) = \frac{2}{c^2} \sqrt{k_0} \cdot \varphi^2 \cdot \bar{\partial}^+_\Lambda A_\alpha = 0 \Rightarrow \bar{\partial}^+_\Lambda A_\alpha = 0; \tag{8}$$

and also one can obtain an analogical result for relation (8) as follows below:

$$\lambda^A \cdot g^B_\alpha \cdot [\partial_A (\varphi \cdot \lambda_B) + \partial_B (\varphi \cdot \lambda_A) - 2 \cdot P_{AB,C} \cdot \varphi \cdot \lambda^C] = \varphi \cdot (g^A_\alpha \cdot \bar{\partial}^+_\Lambda \lambda_A - 2 \cdot \Phi_\alpha) -$$

$$-2 \cdot \varphi^{-1} \cdot g^A_\alpha \cdot P_{5A,5} = \varphi^2 \cdot \bar{\partial}^+_\Lambda (\lambda_\alpha / \varphi) - 2 \cdot \varphi \cdot \Phi_\alpha - \varphi^{-1} \cdot g^A_\alpha \cdot \partial_A G_{55} =$$

$$\varphi^2 \cdot \bar{\partial}^+_\Lambda (\lambda_\alpha / \varphi) - 2 \cdot \varphi \cdot \Phi_\alpha + 2 \cdot \varphi \cdot \Phi_\alpha = \frac{2 \cdot \sqrt{k_0}}{c^2} \cdot \bar{\partial}^+_\Lambda A_\alpha = 0 \Rightarrow \bar{\partial}^+_\Lambda A_\alpha = 0.$$

It should be reminded here, that $G_{55} = -\varphi^2$ and A_α is a 4D vector-potential of the electromagnetic field.

$$g^A_\alpha \cdot g^B_\beta \cdot (\xi_{A;B} + \xi_{B;A}) = \varphi \cdot \bar{\partial}^+_\Lambda g_{\alpha\beta} = 0 \Rightarrow \bar{\partial}^+_\Lambda g_{\alpha\beta} = 0. \tag{9}$$

At last, one can also obtain the above-mentioned relation (9) as follows below:

$$g^A_\alpha \cdot g^B_\beta \cdot [\partial_A (\varphi \cdot \lambda_B) + \partial_B (\varphi \cdot \lambda_A) - 2 \cdot P_{AB,C} \cdot \varphi \cdot \lambda^C] = g^A_\alpha \cdot \bar{\partial}^+_\beta (\varphi \cdot \lambda_A) + g^B_\beta \cdot \bar{\partial}^+_\alpha (\varphi \cdot \lambda_B) -$$

$$-g^B_\beta \bar{\partial}^+_\alpha (\varphi \cdot \lambda_B) - g^A_\alpha \cdot \bar{\partial}^+_\beta (\varphi \cdot \lambda_A) + \varphi \cdot g^A_\alpha \cdot g^B_\beta \cdot \bar{\partial}^+_\Lambda (g_{AB} - \lambda_A \cdot \lambda_B) = \varphi \cdot \bar{\partial}^+_\Lambda g_{\alpha\beta} = 0 \Rightarrow \bar{\partial}^+_\Lambda g_{\alpha\beta} = 0.$$

Now we will pay attention to the geodetic equations (5), (6), and (7). It's suitable to represent these calculations to be divided into two parts. For the first part of the relation (5) one can obtain as follows below:

$$m_0 \cdot \lambda_A \cdot \frac{d^2 x^A}{dI^2} = \hat{\beta} \cdot \lambda_A \cdot \frac{d}{ds}(m_0 \cdot \hat{\beta} \cdot \frac{dx^A}{ds}) = \hat{\beta} \cdot \lambda_A \cdot \frac{d}{ds}(\hat{m}_0 \cdot u^A) = \hat{\beta} \cdot \lambda_A \cdot \left(\hat{m}_0 \cdot \frac{du^A}{ds} + u^A \cdot \frac{d\hat{m}_0}{ds} \right) =$$

$$= \hat{\beta} \cdot \left(\hat{m}_0 \cdot \lambda_A \cdot \frac{du^A}{ds} + \lambda_A \cdot u^A \cdot \frac{d\hat{m}_0}{ds} \right) = \hat{\beta} \cdot \left[\hat{m}_0 \cdot \left(\frac{d\hat{u}}{ds} - u^A \cdot \frac{d\lambda_A}{ds} \right) + \hat{u} \cdot \frac{d\hat{m}_0}{ds} \right] =$$

$$= \hat{\beta} \cdot \left(\frac{d\hat{p}}{ds} - \hat{m}_0 \cdot u^A \cdot \frac{d\lambda_A}{ds} \right);$$

here $\lambda_A \cdot u^A = \hat{u}$; and $\hat{m}_0 \cdot \hat{u} = \hat{p}$; then for the second part of the relation (5), one can also obtain as it follows below:

$$m_0 \cdot \lambda_A \cdot P^A_{BC} \cdot \frac{dx^B}{dI} \cdot \frac{dx^C}{dI} = \hat{m}_0 \cdot \hat{\beta} \cdot \lambda^A \cdot P_{BC,A} \cdot u^B \cdot u^C = \hat{\beta} \cdot \hat{m}_0 \cdot \frac{P_{BC,5}}{\varphi} \cdot u^B \cdot u^C =$$

$$= \frac{\hat{\beta} \cdot \hat{m}_0}{2 \cdot \varphi} \cdot (2 \cdot \partial_B G_{C5} - \partial_5 G_{BC}) \cdot u^B \cdot u^C = \hat{\beta} \cdot \hat{m}_0 \cdot \left[\frac{\partial_B(\varphi \cdot \lambda_C)}{\varphi} - \frac{\bar{\partial}^+_\Lambda (g_{BC} - \lambda_B \cdot \lambda_C)}{2} \right] \cdot u^B \cdot u^C =$$

$$= \hat{\beta} \cdot \hat{m}_0 \cdot \left(u^A \cdot \frac{d\lambda_A}{ds} + \hat{u} \cdot \frac{d \ln \varphi}{ds} + \hat{u} \cdot u^A \cdot \bar{\partial}^+_\Lambda \lambda_A - \frac{1}{2} \cdot \bar{u}^\alpha \cdot \bar{u}^\beta \cdot \bar{\partial}^+_\Lambda g_{\alpha\beta} \right).$$

Adding two parts of the relation (5), one can easily obtain the following result:

$$m_0 \cdot \lambda_A \cdot G^A = \hat{\beta} \cdot \left(\frac{d\hat{p}}{ds} - \hat{m}_0 \cdot u^A \cdot \frac{d\lambda_A}{ds} + \hat{m}_0 \cdot u^A \cdot \frac{d\lambda_A}{ds} + \hat{p} \cdot \frac{d \ln \varphi}{ds} + \frac{0}{1} \right) = \frac{\hat{\beta}}{\varphi} \cdot \frac{d}{ds} (\varphi \cdot \hat{p}) = 0.$$

Here the last term within the brackets means, as a certain symbol, the sum of the next two quantities below, which are both equal to zero because of the cylindricity condition relative to the 5th coordinate:

$$\hat{p} \cdot u^A \cdot \bar{\partial}^+_\Lambda \lambda_A = 0; \frac{1}{2} \cdot \bar{p}^\alpha \cdot \bar{u}^\beta \cdot \bar{\partial}^+_\Lambda g_{\alpha\beta} = 0 \Rightarrow \frac{0}{1}$$

Finally, one can consider and obtain by the same way the relation (6) as follows:

$$m_0 \cdot g^\alpha_A \cdot \frac{d^2 x^A}{dI^2} = \hat{\beta} \cdot g^\alpha_A \cdot \frac{d}{ds} (m_0 \cdot \hat{\beta} \cdot u^A) = \hat{\beta} \cdot g^\alpha_A \cdot \frac{d}{ds} (\hat{m}_0 \cdot u^A) = \hat{\beta} \cdot \frac{d\bar{p}^\alpha}{ds};$$

$$m_0 \cdot g^\alpha_A \cdot P^A_{BC} \cdot \frac{dx^B}{dI} \cdot \frac{dx^C}{dI} = m_0 \cdot \hat{\beta}^2 \cdot g^{\alpha A} \cdot P_{BC,A} \cdot u^B \cdot u^C = \hat{m}_0 \cdot \hat{\beta} \cdot g^{\alpha A} \cdot (\partial_B G_{CA} - \partial_A G_{BC} / 2) \cdot u^B \cdot u^C =$$

$$= \hat{m}_0 \cdot \hat{\beta} \cdot g^{\alpha A} \cdot [\partial_B (g_{CA} - \lambda_C \cdot \lambda_A) - \partial_A (g_{BC} - \lambda_B \cdot \lambda_C) / 2] \cdot u^B \cdot u^C =$$

$$= \hat{m}_0 \cdot \hat{\beta} \cdot (g^{\alpha\beta} \cdot u^B \cdot \bar{u}^\gamma \cdot \partial_B g_{\beta\gamma} - \hat{u} \cdot u^B \cdot g^{\alpha A} \cdot \partial_B \lambda_A - \bar{u}^\beta \cdot \bar{u}^\gamma \cdot \bar{\partial}^{+\alpha} g_{\beta\gamma} / 2 + \hat{u} \cdot u^A \cdot \bar{\partial}^{+\alpha} \lambda_A) =$$

$$= \hat{m}_0 \cdot \hat{\beta} \cdot \left[g^{\alpha\beta} \cdot \bar{u}^\gamma \cdot \bar{u}^\delta \cdot \left(\bar{\partial}^+_\gamma g_{\beta\delta} + \bar{\partial}^+_\delta g_{\beta\gamma} - \bar{\partial}^+_\beta g_{\gamma\delta} \right) / 2 - \hat{u} \cdot \bar{u}^\beta \cdot g^{\alpha A} \cdot \bar{\partial}^+_\beta \lambda_A + \hat{u} \cdot g^{\alpha\beta} \cdot u^A \cdot \bar{\partial}^+_\beta \lambda_A \right] =$$

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$$= \hat{m}_0 \cdot \hat{\beta} \cdot \left(\hat{\Gamma}^{\alpha}_{\beta\gamma} \cdot \bar{u}^{\beta} \cdot \bar{u}^{\gamma} - \frac{2 \cdot \sqrt{k_0}}{c^2} \cdot \varphi \cdot \hat{u} \cdot \bar{u}^{\beta} \cdot F^{\alpha}_{\beta \cdot} + \hat{u}^2 \cdot \Phi^{\alpha} \right);$$

here we have used the following evident relations:

$$g^{\alpha A} \cdot \bar{\partial}^{+}_{\beta} \lambda_A = \varphi \cdot \bar{\partial}^{+}_{\beta} \left(\frac{\lambda_{\alpha}}{\varphi} \right) = \varphi \cdot \frac{2 \cdot \sqrt{k_0}}{c^2} \cdot \bar{\partial}^{+}_{\beta} A_{\alpha};$$

$$u^A \cdot \bar{\partial}^{+\alpha} \lambda_A = \varphi \cdot \bar{u}^{\beta} \cdot \bar{\partial}^{+\alpha} \left(\frac{\lambda_{\beta}}{\varphi} \right) + \hat{u} \cdot \Phi^{\alpha} = \varphi \cdot \frac{2 \cdot \sqrt{k_0}}{c^2} \cdot \bar{\partial}^{+\alpha} A_{\beta} + \hat{u} \cdot \Phi^{\alpha}.$$

Then one can add these two parts and obtain the relation (6) as it follows below:

$$m_0 \cdot g^{\alpha}_{A} \cdot G^A = \hat{\beta} \cdot \left(\frac{D^{+} \bar{p}^{\alpha}}{ds} - \frac{2 \cdot \sqrt{k_0}}{c^2} \cdot \varphi \cdot \hat{p} \cdot \bar{u}^{\beta} \cdot F^{\alpha}_{\beta \cdot} - \bar{\nabla}^{+\alpha} \hat{m}_0 \right) = 0.$$

It should be also noted, that here we have used the relations (5) and (18). Then, it helped us to obtain one more evident and very important relations as it follows below:

$$\hat{m}_0 \cdot \hat{u}^2 \cdot \Phi^{\alpha} = \left(\frac{n \cdot \hat{m}_{Pl}}{\varphi} \right)^2 \cdot \frac{\Phi^{\alpha}}{\hat{m}_0} = -\bar{\nabla}^{+\alpha} \hat{m}_0.$$

More information and more samples of the calculations in 5D theory, and more references about it see also in [1] and [4]. The author would like to hope that these samples of the calculations with help of the monad method in 5D theory will lighten the readers an understanding of the obtained results.

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