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Not peer-reviewed version

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Posted Date: 9 May 2024

doi: 10.20944/preprints202405.0607.v1

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Article

The Simplification of Gauge Conditions in General Relativity Theory

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Abstract: We proposed an equation of motion as a constraint condition on the gauge degree of freedom in general relativity theory based on a co-moving framework. This constraint equation transformed the gauge degree freedom of general relativity theory into an natural equation of motion, and provide an explicit theoretical understanding on the Mach's principle and rotating bucket experiment. The galaxy rotation curves is derived directly from general relativity theory under this constraints equation. This work may inspires different understanding on dark matter, dark energy, and the quantization of gravity.

Keywords: general relativity; gauge freedom; Mach's principle; galaxy rotation curve

1. Introduction

The choice of a gauge is believed to be arbitrary [1,2] in general relativity (GR), due to the confusing conceptions between gauge transformation and the transformation of coordinates. But there are also different perspectives. Eddington clarified the difference between the transformation of gauge with the transformation of coordinates[3], and believed that gauges cannot be arbitrarily chosen. His solution is to adopt "The natural gauge": $\lambda g_{\mu\nu} = R_{\mu\nu}$. Fock[4], Zhou , and others argued that the harmonic gauge is the only physically valid gauge. Lo realized the problem of gauge freedom and proposed using experimental judgment[5], but failed to provide a theory to eliminate gauge freedom.

Mach's principle[6] has not been contained in GR. Theorists have made various efforts to achieve Mach's principle[7–13]. The best solution is to achieve Mach's principle while fully preserving GR, which means selecting solutions of field equations that comply with the Mach's principle[7,8]. The origin why Mach's principle was not included in GR is the existence of gauge freedom, which makes geometry becoming unconstrained by matter.

In addition, the motion of galaxies and clusters were not well explained[14–17]. Most introduce dark matter for this[17,18], while a few seek for modified gravity[19–22]. But no one wants to give up on the achievements of GR. If these problem can be solved within the theory of GR[23], it will be a great success for GR.

Section 2 elaborates on the problem of gauge freedom of GR. Section 3 provides a solution: adding a constraint to eliminate the gauge freedom of GR. As inferences, Section 4 5 discusses the Mach's principle and the problem of galaxy rotation curves.

2. The Redundant Gauge Degree of Freedom in General Relativity Theory

In differential geometry, we distinguish the abstract index[2] (such as g_{ab}) and specific index (such as $g_{\mu\nu}$). Spacetime is a set of events. We believe that spacetime is a manifold. On this manifold, a coordinate system can be artificially established by giving each event a coordinate (a string of real numbers). There is a metric field g_{ab} and other tensor fields on this manifold. After fixing the coordinate system $\{x^\mu\}$, a tensor (such as g_{ab}) can be represented using coordinate components (such as $g_{\mu\nu}$).

As is well known, due to the Bianchi identity, there are only 6 independent equations in Einstein field equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$, making it impossible to determine the 10 components $g_{\mu\nu}$ of the metric. Theorists generally believe that this ensures the principle of general covariance, while also leading to the gauge freedom of GR. We emphasize that principle of general covariance and gauge freedom are two different things. General covariance is: the coordinate components $g_{\mu\nu}$ change under a coordinate

transformation, while the metric field g_{ab} on the manifold remains. Gauge freedom is: the metric field g_{ab} changes under a diffeomorphism, while "geometry" and "physics" remain. The traditional solution is to write down a coordinate condition.

The understanding of this "gauge freedom" is not correct. This so-called "gauge freedom" leads to problems in our calculations. Let's take solar system as an example. Firstly, establish a coordinate system, which means giving each event a coordinate. We use the most common coordinate system, with the sun at the origin and the earth at 1 A.U.. The parameters in the field equation are the mass of the sun and its world line, which are known. The boundary conditions are spherical symmetry and tend to the Minkowski metric in the distance. The initial condition is static. We cannot determine $g_{\mu\nu}$ since there are infinite solutions, including Schwarzschild's solution,

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (1)$$

Fock's solution,

$$ds^2 = - \frac{r-M}{r+M} dt^2 + \frac{r+M}{r-M} dr^2 + \left(1 + \frac{M}{r}\right)^2 r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (2)$$

and so on. Don't think this is caused by general covariance, it is caused by gauge freedom. We set a coordinate system from the beginning, it is the metric field g_{ab} uncertain. This leads to uncertainty of geodesic. The physical processes related to geodesics, such as whether a bullet hit the target, are also uncertain. We have set the coordinate system, but we don't know the corresponding coordinate condition. If you recklessly believe that the coordinate system we set correspond to harmonic coordinate condition, then you will get the Fock's solution, not the Schwarzschild's solution you familiar with.

Due to the use of Newtonian approximation, we were not aware of the failure in solving the solar system. But I believe that similar errors also occur in solving galaxies, which directly leads to the problem of galaxy rotation curve and the introduction of dark matter. The origin of the problem is the gauge freedom of GR.

3. The Constraints Equation on the Metric of Spacetime

3.1. Constraints from the Equation of Motion

We suggest adding a constraint to eliminate the gauge freedom of GR. For general, fluids with gravitational and other interactions and pressure, **the constraint is the equation of motion of the fluid element:**

$$F^a = m U^b \nabla_b U^a \quad (3)$$

F^a is the 4-force acting on the element, originating from other interactions and pressures. U^a is the 4-velocity of the flow element. m is the mass of the element.

For fluids with only gravitational interaction and no pressure, this constraint degenerates into geodesic equation:

$$U^b \nabla_b U^a = 0 \quad (4)$$

Which means matter is not only the source of the gravitational field, but also moves in the gravitational field, and its motion must be consistent with the gravitational field it generates. We are not solving the gravitational field under the known motion of matter, but rather solving both the gravitational field and the motion of matter simultaneously, which requires simultaneous field equation and motion equation (4).

3.2. Revisit of the Gauge Degree of Freedom

Establishing a coordinate system is to give each event a coordinate, so that the coordinates of each world line of the flow element has been determined, and the 4-velocity U^μ and density ρ are known. ρ is observed by co-moving observers and is independent of the metric, known as proper energy density. We are different from traditional views. Traditionally, U^μ and ρ are unknown, as they didn't fix a coordinate system really. U^μ and ρ are known, but the motion of matter is still unknown because $g_{\mu\nu}$ is unknown and the distance between events is unknown. There are only 10 unknown functions $g_{\mu\nu}$ to be solved.

The $T_{\mu\nu} = \rho U_\mu U_\nu$ of dust is still unknown, as U_μ is related to $g_{\mu\nu}$. Conservation of energy-momentum

$$\nabla^\mu T_{\mu\nu} = 0 \quad (5)$$

is not an identity, unable to reduce the number of independent equations with Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ together. Field equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \quad (6)$$

are 10 independent algebraic equations that precisely determine 10 unknown functions $g_{\mu\nu}$. g_{ab} has no gauge freedom at all, once a coordinate system is fixed, $g_{\mu\nu}$ is unique.

In another way, (5) and the Bianchi identity form $\nabla^\mu (G_{\mu\nu} - 8\pi T_{\mu\nu}) = 0$, resulting in (6) having only 6 independent equations. But in this way, (5) becomes 4 independent constraints. There are still a total of 10 independent equations. You can also equivalently say that the constraint we added is the conservation of energy-momentum.

The elimination of gauge freedom of g_{ab} does not affect the general covariance of GR. The reason is that the general covariance is not guaranteed by the Bianchi identity at all, but by the Einstein field equation $G_{ab} = 8\pi T_{ab}$ being a equation of tensor. Any equation of tensor is general covariant, including the constraint (3) we add.

We actually do not add any constraints for dust without other interactions. The origin is the constraints of the field equation. But considering other interactions, the equation of motion is not entirely derived from the field equation, so we still use (3) as an additional constraint.

3.3. Derivation of Coordinate Conditions

In section 2, we mentioned the problem of not being able to write down coordinate conditions after fixing a coordinate system, and now coordinate conditions are derived from motion equations. Taking co-moving coordinate system as an example. Establish a co-moving coordinate system, where the world line of the element is the t -coordinate line, and the 4-velocity of the element is the coordinate basis vector: $(\partial/\partial t)^a = U^a = (1, 0, 0, 0)$. (3) is simplified as:

$$F^\mu = m\Gamma^\mu_{00}. \quad (7)$$

Eq. (4) simplified as:

$$0 = \Gamma^\mu_{00} \quad (8)$$

This is actually the coordinate condition corresponding to co-moving coordinate system.

For dust with only gravitational interaction. The co-moving coordinate system can be upgraded to a Gaussian normal coordinate system. This requires adjusting t to the proper time of the element, requiring normalization $g_{\mu\nu}U^\mu U^\nu = -1$, which leads to $g_{00} = -1$. And perform a clock synchronization,

which requires existing an equal t surface Σ orthogonal to the t -coordinate line. For the coordinate basis vector $(\partial/\partial x^i)^a$, there is $(\partial/\partial x^i)^i = 1$, $(\partial/\partial x^i)^j = (\partial/\partial x^i)^0 = 0$. Orthogonality means:

$$\begin{aligned} 0 &= g_{\mu\nu} U^\mu \left(\partial/\partial x^i \right)^\nu \Big|_\Sigma \\ &= g_{0i} \Big|_\Sigma U^0 \left(\partial/\partial x^i \right)^i \\ &= g_{0i} \Big|_\Sigma \end{aligned} \quad (9)$$

Next, we need to prove that there is also $g_{0i} = 0$ outside of Σ , just prove $\partial g_{0i}/\partial t = 0$.

$$\begin{aligned} \frac{\partial}{\partial t} g_{0i} &= (\partial/\partial t)^a \nabla_a g_{0i} \\ &= U^a \nabla_a \left[g_{bc} U^b \left(\partial/\partial x^i \right)^c \right] \\ &= U^a g_{bc} \left(\nabla_a U^b \right) \left(\partial/\partial x^i \right)^c \\ &\quad + U^a g_{bc} U^b \nabla_a \left(\partial/\partial x^i \right)^c \end{aligned} \quad (10)$$

Note that the coordinate basis vectors are commutative:

$$\begin{aligned} 0 &= \left[\left(\partial/\partial x^i \right), U \right]^b \\ &= \left(\partial/\partial x^i \right)^a \nabla_a U^b - U^a \nabla_a \left(\partial/\partial x^i \right)^b \end{aligned} \quad (11)$$

Substitute (4) and (11) into (10):

$$\begin{aligned} \frac{\partial}{\partial t} g_{0i} &= g_{bc} U^b \left(\partial/\partial x^i \right)^a \nabla_a U^c \\ &= \left(\partial/\partial x^i \right)^a \nabla_a \left(g_{bc} U^b U^c \right) / 2 \\ &= \left(\partial/\partial x^i \right)^a \nabla_a g_{00} / 2 \\ &= 0 \end{aligned} \quad (12)$$

This is exactly the coordinate condition corresponding to Gaussian normal coordinate system:

$$g_{00} = -1, \quad g_{0i} = 0 \quad (13)$$

In coordinate system we select, our constraints (3) can derive the coordinate condition correctly. Once the coordinate system is selected, the coordinate condition should be derived. You cannot manually specify coordinate condition while fixing coordinate system.

In fact, if you have ever solved the Einstein's field equation (for a fluid with only gravitation and no pressure) in a co-moving coordinate system, then you have successfully eliminated the gauge freedom of GR and obtained the correct solution. The solution you obtain in other coordinate systems (or without selecting a coordinate system but fixing coordinate conditions) can be transformed into a co-moving coordinate system to check it.

4. Mach's Principle

There has been no unified expression for Mach's principle for a long time. We acknowledge Mach's own description[6] only, not the later inferences including the anisotropy of inertia, Brans-Dicke theory[10] and so on.

We suggest an expression of Mach's principle: **the metric g_{ab} at a point is jointly determined by all matter in the universe.**

The GR with no gauge freedom conforms to Mach's principle, as matter completely determines geometry. As early as 1975, researchers studied which background spacetimes comply with Mach's principle using approximations[8]. They still adopt the background spacetime and have not fully incorporated Mach's principle into GR. And we don't have the concept of background. We now demonstrate how all matter in the universe affect the water in a rotating bucket. We start at the cosmological principle. The universe is composed of homogeneously isotropic dust, and this boundary condition limits the form of metric. The metric is the well-known Robertson-Walker metric in the co-moving coordinate system $\{t, l, \theta, \varphi\}$:

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dl^2}{1 - kl^2} + l^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right] \quad (14)$$

The laboratory on Earth can be seen as a small neighborhood, so t expands at $t = t_0$ and l expands at $l = 0$. Make an approximation $1 - kl^2 \rightarrow 1$, $a^2(t) \rightarrow a^2(t_0)$. The metric approximates to:

$$ds^2 = -dt^2 + a^2(t_0) \left[dl^2 + l^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right] \quad (15)$$

The transformation from co-moving coordinate system to laboratory coordinate system $\{t, \eta, \theta, \varphi\}$ is: $\eta = a(t_0)l$. The metric in the laboratory coordinate system is:

$$ds^2 = -dt^2 + d\eta^2 + \eta^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (16)$$

We transform to a cylindrical coordinate system $\{t, r, \varphi, z\}$ for convenience. The transformation is $r = \eta \sin \theta$, $z = \eta \cos \theta$. Metric writes:

$$ds^2 = -dt^2 + dr^2 + r^2 d\varphi^2 + dz^2 \quad (17)$$

The transformation from laboratory coordinate system to bucket coordinate system $\{\tilde{t}, \tilde{r}, \tilde{\varphi}, \tilde{z}\}$ is: $\tilde{t} = t$, $\tilde{r} = r$, $\tilde{\varphi} = \varphi + \omega t$, $\tilde{z} = z$. The metric in the bucket coordinate system is:

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} -1 + \tilde{r}^2 \omega^2 & 0 & \tilde{r}^2 \omega & 0 \\ 0 & 1 & 0 & 0 \\ \tilde{r}^2 \omega & 0 & \tilde{r}^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (18)$$

Consider a drop of water stay still in the bucket coordinate system, with 4-velocity $\tilde{U}^\mu = (1, 0, 0, 0)$. The 4-force acting on it is:

$$\tilde{F}^\mu = m \tilde{U}^\nu \tilde{\nabla}_\nu \tilde{U}^\mu = (0, -m\tilde{r}\omega^2, 0, 0) \quad (19)$$

We will see a depression on the water surface to provide a force of $m\tilde{r}\omega^2$ pointing towards the origin as a result.

Distant stars affect the inertia by influencing the local metric g_{ab} to determine which reference frames are inertial, not by influencing the inertial mass. The implementation of Mach's principle does not break equivalence principle, that is, it does not distinguish inertial mass and gravitational mass. GR conforms to Mach's principle after eliminating gauge freedom.

Our answer to the origin of inertia: **Inertia originates from distant galaxies.**

Our concept of spacetime is same as Einstein's in his later years: **spacetime is a manifestation of the extensibility of matter**. It's not that matter exists in space. The "empty spacetime" is meaningless. The Minkowski spacetime in the laboratory does not appear as a "stage" before matter exists, but is determined by all matter in the universe, as derived above. The vacuum at infinity is not the boundary condition of the field equation, matter is.

5. The Galaxy Rotation Curves

It is incorrect to use Newtonian theory in calculating the galaxy. We translate those into the language of GR: the spacetime of the galaxy is quasi-Euclidean. In the coordinate system we commonly use in astronomy, the metric is:

$$ds^2 = (-1 - 2\phi)dt^2 + (1 - 2\phi)dr^2 + (1 - 2\phi)r^2d\varphi^2 + (1 - 2\phi)dz^2 \quad (20)$$

This means establishing a coordinate system on the scale of galaxy and assuming that it satisfies the harmonic coordinate condition. This conflicts with our ideas.

Below, we will show that this metric does not satisfy condition (13) in co-moving coordinate system. The gravitational potential $\phi(r, z)$ is known by observing stars. The angular velocity $\omega(r, z)$ is calculated from potential and is also known. The transformation from common coordinate system $\{t, r, \varphi, z\}$ to co-moving coordinate system $\{\tilde{t}, \tilde{r}, \tilde{\varphi}, \tilde{z}\}$:

$$\begin{cases} \tilde{t} = \frac{t}{\sqrt{1 + [1 - 2\phi(\tilde{r}, \tilde{z})]\tilde{r}^2\omega^2(\tilde{r}, \tilde{z})}} \\ r = \tilde{r} \\ \varphi = \tilde{\varphi} - \omega(\tilde{r}, \tilde{z})\tilde{t} \\ z = \tilde{z} \end{cases} \quad (21)$$

In the transformed metric:

$$\begin{cases} \tilde{g}_{00} = -1 \\ \tilde{g}_{01} = \frac{\tilde{t}[\phi_{,\tilde{t},\tilde{r}} + \tilde{r}\omega^2(-1 + 4\phi^2 + 2\tilde{r}\phi_{,\tilde{t},\tilde{r}})]}{1 + 2\phi} \\ \tilde{g}_{02} = (2\phi - 1)\tilde{r}^2\omega \\ \tilde{g}_{03} = \frac{\tilde{t}(1 + 2\tilde{r}^2\omega^2)\phi_{,\tilde{t},\tilde{r}}}{1 + 2\phi} \end{cases} \quad (22)$$

The requirement $\tilde{g}_{0i} = 0$ adds 3 more equations. They are 3 new constraints, not automatically established. Therefore, conflicts arise. Even if we further limit the coordinate patch within a small neighborhood of $\tilde{t} = 0$, we have $\tilde{g}_{01} \approx \tilde{g}_{03} \approx 0$, but still $\tilde{g}_{02} = (2\phi - 1)\tilde{r}^2\omega \neq 0$, which is still contradictory.

In fact, researchers used GR to calculate the galaxy rotation curve in co-moving coordinate system as early as 2005[23], and obtained good results, indicating that dark matter does not need to be introduced. They first write the metric in the common coordinate system $\{t, r, \varphi, z\}$:

$$ds^2 = -e^{\nu-\omega}(dz^2 + dr^2) - r^2e^{-\omega}d\varphi^2 + e^{\omega}(dt - Nd\varphi)^2 \quad (23)$$

Then establish a co-moving coordinate system, whose the coordinate patch is only a small neighborhood of $t = 0$. Their article emphasizes that it's "a purely local (r, z held fixed at each point when taking differentials) transformation, not a global one". In our language, we establish the co-moving coordinate system only within a small neighborhood of $t = 0$. Calculating in such a coordinate patch is convenient and sufficient. The transformation is:

$$\tilde{\varphi} = \varphi + \omega(r, z)\tilde{t} \quad (24)$$

They require the metric to be diagonal in this co-moving coordinate system, which corresponds exactly to our constraint above. They obtain the equation:

$$\omega = \frac{Ne^{\omega}}{r^2 e^{-\omega} - N^2 e^{\omega}} \quad (25)$$

The only correct metric to describe galaxies is obtained by solving this equation in conjunction with the field equation. However, they think the reason is the nonlinearity of the field equation, while we think the reason is the incorrect selection of coordinate condition.

6. Conclusion

We have added a constraint to GR: the equation of motion (3) (or conservation of energy-momentum). If a Gaussian normal coordinate system is chosen, this constraint can derive coordinate condition $g_{00} = -1$, $g_{0i} = 0$. This eliminates the gauge freedom of GR, making the solution g_{ab} of the Einstein field equation uniquely determined by boundary and initial conditions. This does not break general covariance, nor does it affect the freedom of choosing coordinate systems. We also provided an expression of Mach's principle. The GR without gauge freedom is in accordance with Mach's principle and well reflect Einstein's concept of spacetime in his later years. Our theory supports the calculation in [23] and suggests that there is no need to introduce dark matter.

Acknowledgments: The author thanks Professor Tieyan Si for helpful discussions and support.

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