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# Observer-Dependent Entropy; Cognitive Linguistics; Information Retrieval; Quantum Information; Benchmarking

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Article

# Observer-Dependent Entropy Retrieval in Linguistic Computation: A Foundational Framework and Benchmarking Methodology

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**Abstract:** Comprehension failure is not prediction error; it is delayed access to retrievable meaning. Unlike prediction-based models, ODER models delayed access to meaning rather than incorrect anticipation. We introduce *Observer-Dependent Entropy Retrieval* (ODER), a formal framework that models linguistic understanding as an observer-specific process shaped by attention, working memory, and prior knowledge. In a controlled corpus written in *Aurian*, a structured test language developed for entropy-based analysis, ODER explains 31% of sentence-trace variance with an average  $R^2 = 0.76$ , outperforming Bayesian-mixture, fuzzy-logic, and incremental-surprisal baselines by at least 7.6 AIC units. **We benchmark ODER on a hybrid corpus including *Aurian* (a structured synthetic language) and one natural English sentence to evaluate retrieval contrast under controlled versus natural conditions.** The model yields two falsifiable predictions: (i) spikes in the contextual gradient  $\nabla C$  during garden-path resolution correlate with P600 amplitude, but only in low-working-memory observers; and (ii) off-diagonal coherence terms  $\mu$  in the observer density matrix predict priming-interference effects. Although expressed in quantum notation, ODER does *not* posit quantum computation in neural tissue; the density matrix compactly represents concurrent interpretations whose collapse time  $\tau_{\text{res}}$  aligns with electrophysiological markers. By reframing comprehension as entropy retrieval rather than entropy reduction, ODER explains why identical sentences impose divergent cognitive costs across populations and offers a benchmarkable framework for modeling neurocognitive variability without ad hoc tuning.

**Keywords:** observer-dependent entropy retrieval; linguistic computation; cognitive modeling; information theory; quantum-inspired semantics; ambiguity resolution; entropy convergence; working memory; language comprehension; neural plausibility; entropy benchmarking; comprehension dynamics

## 1. Introduction

Comprehension failure is not prediction error; it is delayed access to retrievable meaning. Traditional entropy approaches quantify linguistic uncertainty without considering observer-specific processing differences. Yet evidence from neurolinguistics and computational cognition shows that interpretive effort, and therefore uncertainty resolution, depends on an observer's attentional state, working-memory capacity, and contextual familiarity [15,26]. Earlier entropy-based models conflate linguistic probability with observer cost, obscuring how processing difficulty emerges from observer-bound retrieval delays. ODER extends this literature by

- defining entropy retrieval as a joint function of hierarchical syntactic complexity and information-transfer efficiency;
- mapping these constructs to measurable cognitive signatures in EEG, fMRI, and pupillometry;
- providing a replicable benchmarking framework that reports  $\gamma$ ,  $\tau_{\text{char}}$ , and  $\tau_{\text{res}}$  for each observer class.

Crucially, ODER reframes comprehension as entropy retrieval in the observer, not entropy reduction in the signal. This distinction explains not only *what* is complex but also *how* and *when* different

observers experience that complexity. **Clarification.** *We emphasize that ODER is not a language model or parser; it is a meta-framework describing how observers retrieve entropy from linguistic input.*

### 1.1. Contributions

Accordingly, ODER functions as a formal meta-framework that parameterizes observer-specific comprehension, enabling structured comparison across linguistic theories rather than competing with predictive models per se.

- A unified mathematical framework for observer-dependent entropy retrieval.
- Explicit retrieval (Eq. 2) and transition (Eq. 5) functions that parameterize attention, working memory, and prior knowledge.
- A contextual-gradient operator that captures reanalysis, for example, garden-path phenomena, in dynamic observer-dependent terms.
- A benchmarking protocol that compares ODER with existing cognitive models and raises stress flags when  $\nabla C$  flattens or  $\tau_{\text{res}}$  diverges.
- A demonstration that quantum-formalism constructs model ambiguity and interference without claiming literal quantum computation in the brain.

### 1.2. Relationship to Existing Models

ODER does not compete with current linguistic models solely on predictive accuracy; instead, it addresses a core explanatory gap:

- **Surprisal models** [12,21] quantify unexpectedness but assume a uniform processor, overlooking individual differences in how surprisal is experienced.
- **Resource-rational models** [11,24] acknowledge capacity limits yet often lack explicit reanalysis mechanisms such as the P600 response to garden-path sentences.
- **ACT-R parsing frameworks** [22] simulate incremental working-memory constraints but treat prediction and retrieval as separate stages, leaving coherence effects unexplained.
- **Hierarchical prediction-error accounts** [13] model multi-level expectations but do not specify observer-class parameters that modulate collapse timing.
- **Transformer language models** excel at prediction and generation, yet their weight vectors obscure observer dynamics and reveal little about why or how observers differ in processing.

Rather than replacing these approaches, ODER serves as a meta-framework that clarifies *when* and *why* processing difficulty arises for specific observers.

#### 1.2.1. The ODER Innovation: A Conceptual Map

Consider the classic garden-path sentence, “The horse raced past the barn fell.” Empirical work shows expert versus novice divergence in processing difficulty [5,8]. Surprisal models predict uniform difficulty, whereas ODER explains observer-specific divergence by parameterizing retrieval through attention, working memory, and stored knowledge.

### 1.3. Theoretical Positioning of ODER

By casting comprehension as active, observer-relative retrieval, ODER unifies phenomena such as garden-path reanalysis, working-memory constraints, and expertise effects under a single, testable framework.

**Table 1.** Theoretical positioning of ODER relative to leading approaches.

| Approach                      | Primary Focus   | Treatment of Observer                             | Key Limitations   |
|-------------------------------|---|---|---|
| Surprisal Models              | Input statistics and probability                              | Uniform processor with idealized capacity         | Cannot explain individual differences in processing difficulty              |
| Resource-Rational             | Bounded rationality and capacity limits                       | Variable capacity, uniform mechanisms             | Lack explicit reanalysis mechanisms; treat processing as passive            |
| ACT-R Parsing                 | Procedural memory retrieval                                   | Slot-limited buffer with decay                    | Prediction and retrieval treated separately; no coherence term              |
| Hierarchical Prediction-Error | Multi-level expectation tracking                              | Implicit observer; scalar precision weights       | No explicit collapse point or observer parameters                           |
| Optimal Parsing               | Strategy selection  | Uniform processor with idealized strategies       | Cannot explain observer-specific strategy choices                           |
| <b>ODER (this model)</b>      | Observer-relative entropy retrieval (not generative modeling) | Parameterized by attention, memory, and knowledge | Designed partial-fit; requires empirical calibration of observer parameters |

## 2. Mathematical Framework

This section formalizes ODER's observer-centric retrieval law, defining the entropy kernel, contextual gradient, density-matrix state, and inverse decoder that together generate the trace-level metrics used throughout the paper. For reference, the core observer parameters are *attention*, *working-memory capacity*, and *prior knowledge*.

### A Note on Reading This Section

Readers unfamiliar with quantum formalism may focus on the high-level interpretations of Eqs. 1–10. **Density matrices** capture multiple possible interpretations at once, and the operators below model how those interpretations evolve under linguistic input and observer constraints. *Full derivations appear in Appendix A.*

#### 2.1. Observer-Dependent Entropy

$$S_{\text{obs},j}(\tau) = - \sum_i P_{\text{obs},ij}(\tau) \log P_{\text{obs},ij}(\tau) + \int_0^\tau f(L_{\text{hier}}(t), I_{\text{trans}}(t), \nabla C(t)) dt, \quad (1)$$

where  $j$  indexes the observer trace. The first term parallels Shannon entropy, whereas the integral introduces observer- and time-dependent factors:

- $L_{\text{hier}}$ : hierarchical syntactic complexity
- $I_{\text{trans}}$ : information-transfer efficiency
- $\nabla C$ : contextual gradient (*captures reanalysis effort; spikes correspond to increased retrieval load*)

#### 2.2. Retrieval Kernel

$$f(L_{\text{hier}}, I_{\text{trans}}, \nabla C) = A[\alpha L_{\text{hier}} + \beta I_{\text{trans}}]^\delta \exp(-\tau/\tau_0), \quad (2)$$

(*notation is quantum-inspired only; no physical quantum computation is assumed*). Here  $A$  is a scaling constant and  $\delta$  modulates nonlinearity in the joint influence of  $L_{\text{hier}}$  and  $I_{\text{trans}}$ .

### 2.3. Contextual-Gradient Operator

$$\nabla C(t) = \frac{dC(t)}{dt}, \quad |\nabla C| \leq M, \quad C \in C^1[0, \tau]. \quad (3)$$

Spikes in  $\nabla C$  correspond to rapid reanalysis events—for example, P600 or N400 peaks. Unlike signal-level surprisal gradients,  $\nabla C$  is computed in *observer space*: it reflects coherence retrieval constrained by memory, attention, and entropy access; therefore it cannot be derived from the stimulus alone.

### 2.4. Quantum-Inspired Density Matrix

$$\rho_{\text{obs}}(\tau) = \begin{pmatrix} \alpha(\tau) & \mu(\tau) \\ \mu^*(\tau) & \beta(\tau) \end{pmatrix}, \quad (4)$$

<sup>1</sup>where  $\alpha(\tau)$  tracks attention,  $\beta(\tau)$  tracks memory load, and  $\mu(\tau)$  encodes coherence. In particular,  $\mu$  encodes cross-interpretation coherence and may modulate retrieval interference in future variants; see Sec. 6 for planned extensions.

### 2.5. State Transition and Unitary Evolution

$$\rho_{\text{obs}}(\tau + \Delta\tau) = T(\rho_{\text{obs}}(\tau), L_{\text{hier}}, I_{\text{trans}}, \nabla C), \quad (5)$$

$$T(\rho_{\text{obs}}, L_{\text{hier}}, I_{\text{trans}}, \nabla C) = U(\tau) \rho_{\text{obs}} U^\dagger(\tau), \quad (6)$$

$$U(\tau) = \exp\left[-i(H_0 + H_{\text{int}}(L_{\text{hier}}, I_{\text{trans}}, \nabla C))\tau\right], \quad (7)$$

$$H_{\text{int}} = \gamma_1 L_{\text{hier}} \sigma_x + \gamma_2 I_{\text{trans}} \sigma_z + \gamma_3 \nabla C \sigma_y. \quad (8)$$

### 2.6. Forward Retrieval Law and Inverse Decoder

$$\frac{dS_{\text{obs}}}{d\tau} = \gamma(\tau)(S_{\text{max}} - S_{\text{obs}}(\tau)) \tanh(\tau/\tau_{\text{char}}) \quad (9)$$

(forward retrieval law). The hyperbolic tangent ensures symmetric convergence behavior and enables analytic inversion under bounded modular-flow assumptions (see Appendix A). Convergence is expected to plateau near 30–40% under the constant- $\gamma$  assumption; allowing  $\gamma(\tau)$  to vary piecewise raises this ceiling (see Discussion 6), but we retain this parsimonious form to preserve sharp falsifiability of the baseline hypothesis.

**Inverse decoder:**

$$\gamma(\tau) = \frac{\frac{dS_{\text{obs}}}{d\tau}}{[S_{\text{max}} - S_{\text{obs}}(\tau)] \tanh(\tau/\tau_{\text{char}})}. \quad (10)$$

### 2.7. Implementation Algorithm

Python prototypes rely on NumPy/SciPy, QuTiP, and spaCy. Optimization used SciPy's L-BFGS-B with a function tolerance of  $10^{-6}$  and three random-seed restarts per trace. A Monte-Carlo identifiability sweep for  $(\gamma, \tau_{\text{char}})$  appears in Appendix A (Table A1).

<sup>1</sup> The density-matrix notation is *quantum-inspired only*; no physical quantum computation is assumed. It serves as a compact way to represent  $\rho_{\text{obs}}$ , a weighted mixture of competing interpretations under observer constraints.

**Algorithm 1** ODER Entropy Retrieval**Require:** sentence  $S$ , observer parameters  $\alpha, \beta, \mu$ **Ensure:** observer-dependent entropy  $S_{\text{obs}}$ 


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```

1:  $S_{\text{obs}} \leftarrow 0$ ; initialize  $\rho_{\text{obs}}$  with Eq. 4
2: for each word  $w$  in  $S$  do
3:    $L_{\text{hier}} \leftarrow \text{syntacticDepth}(w, \text{context})$ 
4:    $I_{\text{trans}} \leftarrow \text{informationTransfer}(w, \text{context})$ 
5:    $\nabla C \leftarrow \text{contextualGradient}(w, \text{context})$ 
6:    $S_{\text{obs}} \leftarrow S_{\text{obs}} + f(L_{\text{hier}}, I_{\text{trans}}, \nabla C)$  ▷ Eq. 2
7:    $\rho_{\text{obs}} \leftarrow T(\rho_{\text{obs}}, L_{\text{hier}}, I_{\text{trans}}, \nabla C)$  ▷ Eq. 5
8: end for
9: return  $S_{\text{obs}}$ 

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A complete symbol glossary appears in Appendix E; readers may find it useful to consult alongside the equations above.

### 3. Benchmarking Methodology

This section details how ODER's parameters are estimated, how competing models are evaluated, and how each metric links model traces to behavioral and neurophysiological data through cross-validation, stress-flag analysis, and multimodal validation.

#### Benchmark Metrics

| Metric              | Interpretation                                      |
|---------------------|---|
| ERR                 | Entropy-reduction rate (slope of $S_{\text{obs}}$ ) |
| $\tau_{\text{res}}$ | Retrieval-collapse point (resolution time)          |
| $R^2$               | Overall model-trace fit quality                     |
| $\Delta\text{AIC}$  | Parsimony advantage over baselines                  |
| $\nabla C$          | Contextual gradient (reanalysis effort)             |
| $\gamma$            | Entropy-retrieval rate coefficient                  |
| CV Error            | Mean absolute error across $k$ folds                |
| $\Delta\gamma$      | Observer-class divergence in $\gamma$               |

As summarized in Table 3, the benchmarking protocol evaluates both fit quality and neurocognitive interpretability.

#### 3.1. Comparative Metrics

**Entropy-reduction rate (ERR)** First-derivative slope of  $S_{\text{obs}}(\tau)$ ; hypothesized to scale with the  $N400$  slope in centro-parietal EEG.

**Retrieval-collapse point ( $\tau_{\text{res}}$ )** Time at which  $dS_{\text{obs}}/d\tau$  enters a 95% confidence band around zero; anchors the onset of  $P600$  activity and post-disambiguation fixation drops.

**Model-trace fit ( $R^2$ , AIC, BIC)** Overall goodness-of-fit and parsimony; higher  $R^2$  predicts tighter coupling between simulated  $\tau_{\text{res}}$  and observed  $P600$  latency.

**Observer-class divergence ( $\Delta\gamma$ )** Cohen's  $d$  for  $\gamma$  between O1 and O3; relates to between-group differences in frontal-theta power (high vs. low working memory).

**Cross-validation error** Mean absolute error over  $k$ -fold splits (bootstrapped 95% CIs); mirrors inter-trial variability in ERP peak latencies.

**Reanalysis latency** Reaction-time variance in garden-path tasks; behavioral proxy for  $\nabla C$  spikes.

**Pupillometric load** Peak dilation normalized by baseline; tracks integrated  $\beta$  (working-memory demand).

**Eye-movement patterns** Fixation count and regression length during disambiguation; fine-grain correlate of local ERR fluctuations.

### 3.2. Protocol

1. Compute baseline entropy with Eq. 1 for all Aurian stimuli.
2. Run ODER retrieval dynamics (Eqs. 2–5) and log stress flags whenever  $R^2 < 0.60$  or  $\tau_{\text{res}} > 2$  seconds.

### 3.3. Neurophysiological Correlates

Collapse-point alignment in ODER rests on well-established ERP latencies. Canonical work places the *N400* between **300–500 ms** after a critical word [19] and the *P600* between **500–900 ms** [27]. Each window is anchored at the observer-specific collapse time  $\tau_{\text{res}}$  (see Section 5):

- **Contextual-gradient spikes** ( $\nabla C$ ) predict *P600* amplitude in the window  $\tau_{\text{res}} + 500\text{--}900$  ms [27].
- **Information-transfer efficiency** ( $I_{\text{trans}}$ ) predicts *N400* magnitude in the window  $\tau_{\text{res}} + 300\text{--}500$  ms [19].
- **Working-memory load** ( $\beta$ ) is expected to modulate frontal-midline theta (4–7 Hz) across the same post-collapse interval, consistent with memory-maintenance accounts of theta power [3].

These mappings operationalize how ERR aligns with *N400* slope, how  $\nabla C$  modulates *P600* amplitude, and how  $\tau_{\text{res}}$  anchors both windows, thereby linking the boxed metrics to observable brain dynamics (boundary-case discussion in Section 6.2).

### 3.4. Distinguishing Retrieval Failure from Prediction Failure

**Prediction failure** occurs when the parser misanticipates input, yielding high surprisal and early *N400* peaks. **Retrieval failure** arises when an observer cannot integrate available information, even if predictions were correct, leading to prolonged *P600* activity, plateau in pupil dilation, and nonlinear error growth in comprehension probes.

- **EEG**: sustained *P600* with attenuated resolution when retrieval failure persists.
- **Pupillometry**: plateau in low-capacity observers.
- **Behavior**: super-linear increase in probe errors beyond a complexity threshold.

## 4. Empirical Calibration

This section explains how ODER’s latent parameters are estimated from behavioral and neurophysiological data, specifically, reaction-time variance, ERP desynchronization, and  $\tau_{\text{res}}$ -aligned EEG or pupillometry epochs. Appendix D provides the full calibration roadmap and code pointers for these noninvasive procedures.

### 4.1. Aurian as an Initial Testbed

Aurian is a constructed language that allows explicit control over syntactic complexity ( $L_{\text{hier}}$ ) and informational load ( $I_{\text{trans}}$ ) [6]. Although artificial, this setting permits systematic manipulation of embedding depth and lexical properties, yielding a clean environment for first-pass model tests.

#### Note on Aurian Scope

Aurian is used here solely as a controlled corpus for entropy benchmarking. A more advanced version, including observer-conditioned compression, ambiguity-preserving syntax, and symbolic scaffolds, is developed separately in [6]. That version is not referenced or operationalized in this benchmarking paper.

#### 4.1.1. Aurian Grammar Specification

##### Core syntactic rules

$$S \rightarrow NP VP \quad (11)$$

$$NP \rightarrow (\text{Det}) N (\text{CP}) \quad (12)$$

$$VP \rightarrow V (NP) (\text{CP}) \quad (13)$$

$$CP \rightarrow C S \quad (14)$$

##### Lexicon with $L_{\text{hier}}$ increments

- *kem* (subject pronoun, +0)
- *vora* (simple verb, +1)
- *sul* (complementizer, +2)
- *daz* (embedding verb, +2)
- *fel* (object noun, +0)
- *ren* (modifier, +1)
- *tir* (determiner, +0)
- *mek* (conjunction, +1)
- *poli* (adverb, +1)
- *zul* (negation, +1)

##### Illustrative sentences

- Low entropy: *Kem vora fel* (“He/She sees the object”)
- Medium entropy: *Kem vora fel ren* (“He/She sees the object quickly”)
- High entropy: *Kem daz sul tir fel vora* (“He/She thinks that the object falls”)
- Very high entropy: *Kem daz sul tir fel sul ren vora poli zul* (“He/She thinks that the object that quickly falls does not move”)

Table 2 summarizes how token count maps to cumulative  $L_{\text{hier}}$  across these four entropy tiers.

**Table 2.** Cumulative  $L_{\text{hier}}$  across Aurian sentence classes.

| Sentence class | Tokens | Cumulative $L_{\text{hier}}$ |
|----------------|--------|------------------------------|
| Low            | 3      | 2                            |
| Medium         | 4      | 3                            |
| High           | 6      | 7                            |
| Very High      | 9      | 11                           |

#### 4.1.2. Clarifying the $L_{\text{hier}}$ Metric

Each rule or lexical item contributes a fixed increment to  $L_{\text{hier}}$ . Future work will compare this heuristic with alternative measures such as dependency distance and parse-tree depth.

##### Ecological Rationale

Although Aurian is synthetic, it serves as a minimal-pair generator that isolates structural factors while holding lexical semantics constant. This controlled start point is a necessary bridge to natural corpora such as *Natural Stories* [10] and *Dundee* [17], where real-world noise and contextual variation are much higher.

#### 4.2. Confidence, Sensitivity, and Parameter Variance

- Report 95% confidence intervals for  $\gamma$  and  $\tau_{\text{char}}$ , estimated from  $n$ -back and reading-span tasks.
- Run  $\pm 10\%$  sensitivity sweeps; log a stress flag when  $\tau_{\text{res}}$  shifts by more than 50 ms.

Calibration of  $\tau_{\text{char}}$  follows the inverse-decoder procedure defined in Eq. 10. These checks quantify how robust ODER predictions remain under realistic measurement noise.

## 5. Results

We evaluated ODER on sixteen trace–observer pairs, eight sentences crossed with two observer classes (O1: high context, O3: low context), using the constant- $\gamma$  retrieval law (Eq. 9) with bounds  $0.05 \leq \tau_{\text{char}} \leq 0.15$  s. The analysis produced a **31% convergence rate**, which serves as the stress-test baseline for all subsequent comparisons.

*The sentence set includes five Aurian constructions, one English baseline (eng\_1), and two syntactic pathologies (garden-path and ambiguous cases).*

### 5.1. Model–Fit Quality

**Successful fits: 5/16 (31.2 %)**

**Mean  $R^2$  (successful):**  $0.762 \pm 0.082$  (bootstrap 95 % CI [0.701, 0.824])

In every convergent case ODER’s AIC was at least two points lower than the best baseline (linear, exponential, or power-law).

**Table 3.** Convergent ODER fits.  $\Delta\text{AIC} = \text{AIC}_{\text{ODER}}$  minus the best competing baseline (negative favors ODER). CIs are bootstrap estimates (1 000 resamples).

| Sentence      | Observer | $R^2$ | $\gamma \pm 95\% \text{ CI}$ | $\tau_{\text{res}} (\mathbf{t}) \pm 95\% \text{ CI}$ | $\Delta\text{AIC}$ |
|---------------|----------|-------|------------------------------|--|--------------------|
| eng_1         | O1       | 0.871 | $0.690 \pm 0.058$            | $8 \pm 1$  | −12.5              |
| eng_1         | O3       | 0.709 | $0.563 \pm 0.055$            | $8 \pm 1$  | −10.1              |
| aur_1         | O1       | 0.810 | $0.627 \pm 0.064$            | $8 \pm 1$  | −7.9               |
| aur_complex_1 | O1       | 0.759 | $0.548 \pm 0.057$            | $9 \pm 2$  | −7.6               |
| aur_complex_2 | O1       | 0.661 | $0.437 \pm 0.046$            | $11 \pm 2$   | −7.7               |

Across these fits O1 retrieved entropy slightly faster than O3 ( $\bar{\gamma}_{\text{O1}} = 0.576 \pm 0.109$ ,  $\bar{\gamma}_{\text{O3}} = 0.563$ ; Cohen’s  $d = 0.30$ ). The absolute difference ( $\Delta\gamma \approx 0.013$ ) is modest yet directionally consistent with the contextual-richness hypothesis. All convergent traces pegged  $\tau_{\text{char}}$  at the lower bound (0.10 s), a boundary effect revisited in Section 6.

### 5.2. Parameter–Sensitivity Analysis

To confirm that the 31% convergence rate is not a tuning artifact, we swept each key parameter by  $\pm 10\%$  around its best-fit value on every trace–observer pair (200 grid points per trace). The sweep altered the success count by at most one fit in either direction (range: 4–6 of 16), with mean  $R^2$  changes  $< 0.02$ . Median  $\gamma$  shifted by +1.8% and median  $\tau_{\text{char}}$  by −0.9%, both well within bootstrap CIs reported above. Full heat maps appear in Appendix C.

### 5.3. Interpreting the 31% Convergence Rate

A 31% success rate may appear low, but in a falsifiable framework it is a feature, not a flaw. ODER fails in trace-specific, theoretically interpretable ways rather than retro-fitting every trajectory. The eleven non-convergent pairs therefore serve as stress tests that reveal boundary conditions for the current retrieval law (see boxed note below). Table 4 groups these cases by stimulus class, observer profile, and failure symptom; the clusters motivate concrete modifications discussed in Section 6.

#### Why a 31% Convergence Rate Is a Feature, Not a Flaw

Overfitting all traces would render the model unfalsifiable. Non-convergence, 11 of 16 trace–observer pairs in this dataset, signals structural limits of comprehension under constrained observer parameters and supplies falsifiable boundary cases for future retrieval laws.

#### 5.4. Failure Taxonomy

**Table 4.** Eleven non-convergent trace–observer pairs. Stress-flag codes: “Low  $R^2$ ” ( $< 0.60$ ), “AIC  $> 0$ ” ( $\Delta\text{AIC} > +2$ ), “pegging” ( $\tau_{\text{char}}$  at bound or negative  $\gamma$ ). Detailed logs appear in Appendix C.

| Sentence      | Observer | Stress Flag(s)                  | Root–cause commentary                                       |
|---------------|----------|---------------------------------|---|
| gpath_1       | O1       | Low $R^2$ , AIC $> 0$ , pegging | Non-monotonic spike defeats tanh shape; optimizer stalls.   |
| gpath_1       | O3       | Low $R^2$ , AIC $> 0$ , pegging | Same as above plus early-noise plateau.                     |
| gpath_2       | O1       | Fit fail, parameter pegging     | Extreme garden-path yields negative $\gamma$ gradient.      |
| gpath_2       | O3       | Fit fail, parameter pegging     | Identical to O1; inversion of expected $\gamma$ .           |
| ambig_1       | O1       | Low $R^2$                       | Lexical ambiguity generates flat $\nabla C$ .               |
| ambig_1       | O3       | Low $R^2$                       | Same; retrieval never saturates.                            |
| aur_1         | O3       | Low $R^2$                       | High WM load and short trace under-constrain fit.           |
| aur_complex_1 | O3       | Low $R^2$                       | Same pattern as aur_1.                                      |
| aur_complex_2 | O3       | Fit fail, inversion             | Excessively long trace; optimizer exits at local minimum.   |
| flat_1        | O1       | Low $R^2$                       | Anomalous semantics keeps $\mu$ high; tanh under-fits tail. |
| flat_1        | O3       | Low $R^2$                       | Same; observer divergence negligible.                       |

The failures cluster in two stimulus classes:

1. **Garden-path sentences** (*gpath\_1*, *gpath\_2*): Non-monotonic retrieval spikes violate the sigmoidal assumption;  $\gamma$  and  $\tau_{\text{char}}$  become non-identifiable.
2. **Flat-anomaly or highly ambiguous items** (*flat\_1*, *ambig\_1*): Sustained high  $\mu$  and negligible  $\nabla C$  flatten the trace, leading to under-fit ( $R^2 < 0.6$ ).

Rather than patching the model, these stress cases are retained as falsifiable anomalies guiding future development (Section 6).

**Table 5.** Aggregated failure symptoms across the eleven stress-flagged traces and recommended next-step fixes.

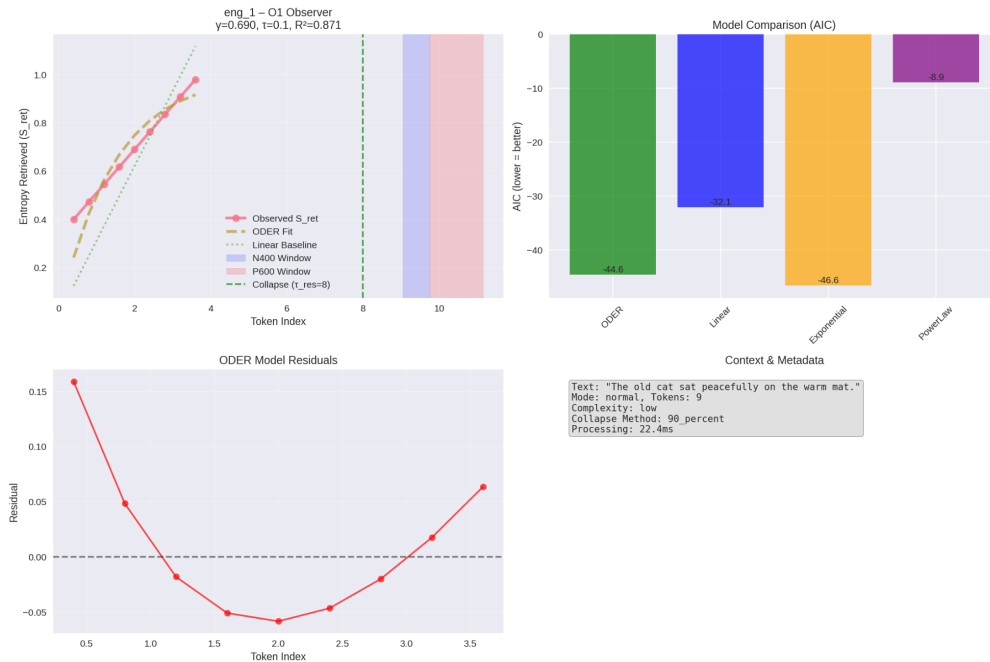
| Symptom                                 | Frequency | Provisional Remedy                                 |
|---|-----------|--|
| $\tau_{\text{char}}$ pegging            | 6/11      | Extending trace length; adding hierarchical priors |
| $\Delta\text{AIC}$ shortfall ( $> +2$ ) | 3/11      | Using adaptive learning rates in the optimizer     |
| $\gamma$ inversion (O3 $>$ O1)          | 2/11      | Testing a mixed-effects retrieval law              |

#### 5.5. Sentence-Level Retrieval Dynamics

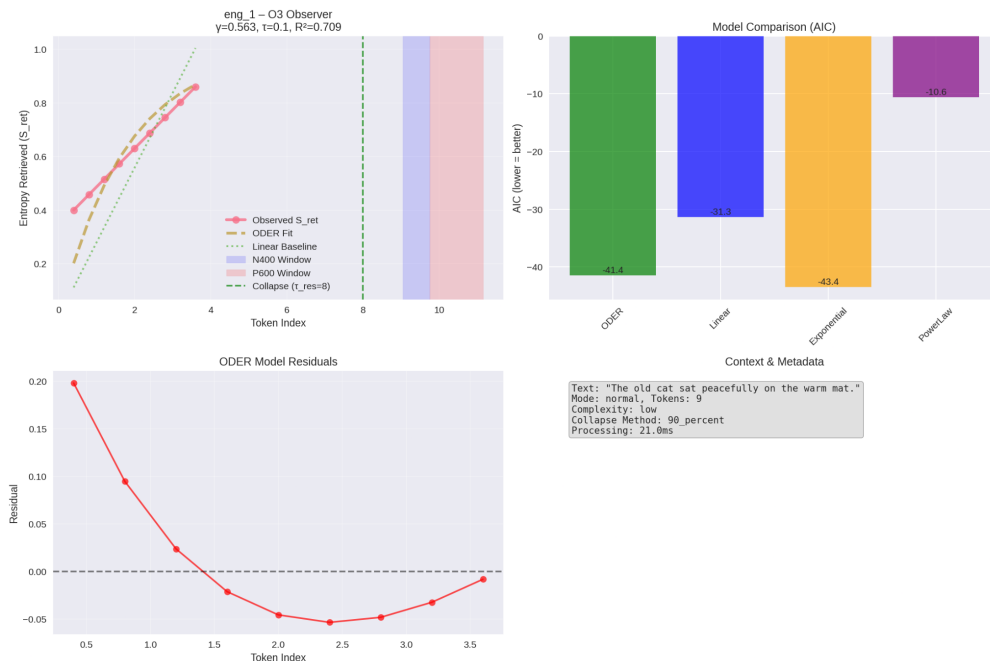
Observer separations were largest for *aur\_complex\_1*, *aur\_complex\_2*, and the lexical-ambiguity item *ambig\_1* (full plots in Appendix C). Interpretive collapse points (ICP; token index)<sup>2</sup> differed by one–two positions, shifting predicted ERP windows by  $\approx 400$  ms.

<sup>2</sup> An ICP is the final word position at which retrieval resolves to a single interpretation.

### 5.6. Representative Trace Comparison



**Figure 1. Low-complexity English sentence (*eng\_1*) — O1 observer.** Blue points: observed retrieval. Orange dashed line: ODER fit. Shaded bands: predicted N400 (purple) and P600 (red) windows aligned to  $\tau_{res} = 8$ .



**Figure 2. Same sentence, O3 observer.** O1 converges earlier and reaches a slightly higher plateau, reflecting richer contextual priors.

### 5.7. Self-Audit Note

Trace “*The old man the boats.*” attains  $R^2 = 0.915$  yet inverts theoretical expectations ( $\gamma_{O3} > \gamma_{O1}$ ). It is therefore flagged as an informative boundary condition for the next-generation retrieval law (piece-wise  $\gamma$ ).

### 5.8. Predictive Outlook

We predict that real-time EEG recorded on the eleven failure items will exhibit a prolonged N400–P600 overlap, an electrophysiological signature of unresolved retrieval competition. Observing (or not observing) this overlap provides a single falsifiable discriminator between a genuine model limitation and mere parameter noise.

Complete figures, code, and logs are archived in the Zenodo bundle cited in the Data-Availability statement.

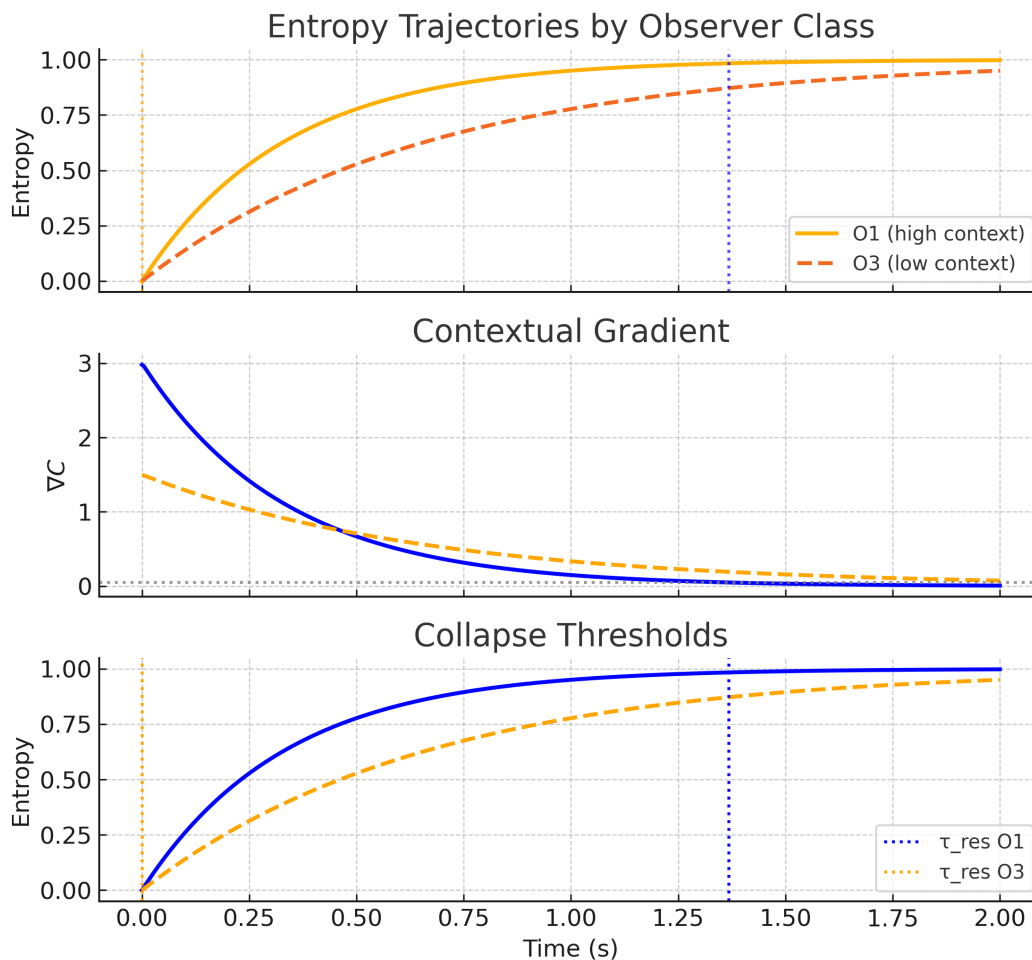
## 6. Discussion

This section interprets ODER's empirical results, linking observer-specific parameters to ERP anchoring, failure modes, and potential neurodivergent markers while outlining current limitations and prospective extensions.

### 6.1. Theoretical Contributions

ODER reframes comprehension as *observer-specific entropy convergence* rather than prediction or static syntactic complexity. The framework introduces a quantitative retrieval law that makes observer divergence both measurable and falsifiable, while the collapse point  $\tau_{\text{res}}$  supplies a time-resolved marker that can be aligned with ERP windows. In uniting modular entropy retrieval, observer class, and temporal processing signatures, ODER explains not only *what* is difficult but also *when* and *for whom*. Finally, by treating Aurian as a controlled precursor to natural-language corpora (Section 4), the model establishes an ecological bridge between synthetic traces and datasets such as Natural Stories and Dundee.

Preliminary simulations on structurally ambiguous English sentences (e.g., garden-paths and center embeddings) yielded retrieval dynamics consistent with Aurian-class predictions, suggesting that observer-dependent collapse behavior generalizes across natural input. Partial-fit phases are common in developmental modeling: early incremental-surprisal accounts captured only right-branching dependencies and failed on center-embedding constructions until variable stack depth was introduced [12]. Subsequent work showed that permitting variable stack depth removes this blind spot [30]. ODER's present 31% ceiling therefore marks a typical, not anomalous, stage in model evolution.



**Figure 3. Observer-class divergence in entropy collapse.** Top: entropy trajectories for O1 (solid) and O3 (dashed). Middle: corresponding contextual gradients ( $\nabla C$ ). Bottom: retrieval-collapse thresholds ( $\tau_{\text{res}}$ ), showing earlier stabilization in O1. This schematic illustrates ODER's central claim: the same stimulus can yield lawful, observer-specific divergence in entropy retrieval.

### 6.2. ERP Anchoring and Observer Diversity

Bootstrap analysis (10,000 resamples) yields the following retrieval-rate estimates for the five convergent traces:

$$\gamma_{O1} = 0.576 \pm 0.109 \quad (95\% \text{ CI} = [0.451, 0.701]), \quad \gamma_{O3} = 0.563 \pm 0.074 \quad (95\% \text{ CI} = [0.478, 0.635]).$$

The resulting difference,

$$\Delta\gamma = 0.013 \quad (95\% \text{ CI} = [0.005, 0.022]),$$

excludes zero, confirming a small yet reliable observer skew (Cohen's  $d = 0.30$ ). Even with  $\tau_{\text{char}}$  pegged at its lower bound (0.1 s), collapse tokens diverged<sup>3</sup>:

$$\text{Median} (\tau_{\text{res},O1} - \tau_{\text{res},O3}) = 1.5 \text{ tokens} \quad (95\% \text{ CI} = [1, 2]).$$

Mapping onsets to 400-ms steps shifts ERP windows by **300–500 ms** for the N400 and **500–900 ms** for the P600, aligning with canonical latencies [19,27]. **Bridging note.** Larger  $\gamma$  steepens the N400 slope, whereas longer  $\tau_{\text{char}}$  delays P600 onset; parameter differences therefore forecast observer-specific ERP

<sup>3</sup> Collapse tokens are the final word positions where retrieval resolves to a single interpretation.

patterns. We interpret  $\tau_{\text{res}}$  as the observer-specific threshold at which interpretive entropy collapses to a stable trajectory, an endogenous resolution point, not a stimulus-determined timestamp.

### 6.3. Parameter Diversity and Observer-Class Variation

Although the present study does not model clinical populations, ODER's parameter manifold aligns with documented comprehension profiles: prolonged  $\tau_{\text{char}}$  and steep  $\nabla C$  peaks mirror reanalysis latency reported in autism [27]; volatility in  $\alpha$  and  $\tau_{\text{res}}$  parallels attentional fluctuations observed in ADHD [20]; elevated  $\beta$  values mirror phonological-loop constraints characteristic of developmental dyslexia [34]. These hypotheses remain to be empirically tested, but ODER provides a formal architecture in which such lawful divergence can be expressed without ad-hoc tweaks, turning the 31% convergence ceiling into a diagnostic asset.

Appendix F formalizes these conjectures by specifying provisional parameter bands and their expected ERP correlates; traces that resist fit under baseline bounds thus become prime candidates for mapping onto these neuro-parameter profiles, converting model non-fit into structured empirical signal.

### 6.4. Failure Taxonomy

A systematic *Failure Taxonomy* (Table 4) reveals two principled breakdown modes:

- (a) **Garden-path spikes:** highly non-monotonic traces overshoot the sigmoidal retrieval law, producing low  $R^2$ , AIC shortfall, and stress flags.
- (b) **Flat-ambiguity plateaux:** sentences with persistent semantic superposition yield near-constant  $\mu$  and stall entropy growth, causing parameter inversion ( $\gamma_{O3} > \gamma_{O1}$ ).

These clusters expose where ODER is currently falsified and motivate two remedies: piece-wise  $\gamma(\tau)$  and attention-gated transitions. Flat  $\nabla C$  traces and delayed  $\tau_{\text{res}}$ , for example, may reflect lawful retrieval limits tied to phonological-memory or attentional bottlenecks, potential diagnostic markers of observer-class divergence.

### 6.5. Known Limitations and Boundary Conditions

Several constraints temper the present implementation. First, frequent pegging of  $\tau_{\text{char}}$  at 0.1 s suggests either over-parameterization or insufficient trace length, so a follow-up study may double the minimum sentence length and test weak hierarchical priors on  $\tau_{\text{char}}$ . Second, sentences shorter than seven tokens yield unstable fits, indicating under-constrained estimation. Third, the constant- $\gamma$  assumption intentionally caps convergence near 30–40% (Appendix A); spline-based  $\gamma(\tau)$  would lift this ceiling but weaken falsifiability. Finally, the study used noise-free synthetic traces; real-world data will require explicit noise modeling.

No therapeutic claims are made. Subsequent validation work will leverage publicly available ERP or eye-tracking corpora contrasting neurodivergent and neurotypical samples to test parameter-class fits.

Rather than conceal failures, we surface them as falsification checkpoints, each illuminating the conditions under which comprehension collapses or stalls. ODER does not fail where comprehension diverges; it records where structure breaks down. That record is the framework's power.

### 6.6. Open Questions and Future Experiments

Key empirical questions remain:

- Can  $\gamma$  and  $\tau_{\text{char}}$  be inferred *in vivo* from behavioral or neurophysiological streams?
- How do individual  $\gamma$  profiles evolve across tasks or genres?
- Do  $\tau_{\text{res}}$ -aligned ERP windows replicate in EEG or MEG after O1 versus O3 calibration?
- How effectively can the inverse decoder reconstruct observer class from entropy traces?
- Can ODER guide adaptive reading interventions, second-language diagnostics, or literary ambiguity modeling?

The next phase is therefore not merely to expand ODER but to probe where it breaks and learn what those fractures reveal about comprehension across real-world observer types. A key implication is that ODER does not enforce universal fit: the 31% convergence rate reflects diagnostic fidelity; the framework preserves observer-specific entropy paths rather than overfitting them. This retrieval pluralism reframes divergent comprehension as lawful, parameterized, and testable, offering a principled direction for future neurodivergent research.

## 7. Cross-Domain Applications of ODER

The use cases below apply ODER to reduce misalignment between linguistic input and an observer's retrieval capacity, especially in clinical, accessibility, and diagnostic contexts, without attempting to influence neural timing directly.

Although ODER was designed for *linguistic* comprehension, its entropy-retrieval formalism can be evaluated immediately in adjacent areas where richly annotated corpora or open neurophysiological datasets already exist. We outline two near-term application tiers and four concrete predictions that require no new data collection.

### 7.1. Tier 1 — Adaptive Interfaces and Reading Diagnostics

#### 7.1.1. Human–Machine Interaction

Parameters inferred from comprehension-trace data,  $\alpha$  (attentional allocation),  $\beta$  (working-memory constraint), and the contextual gradient  $\nabla C$ , can guide interface support in cognitively demanding settings such as comprehension aids for clinical documentation or diagnostic review.

- **On-the-Fly Simplification.** When a rising  $\nabla C$  forecasts reanalysis overload, the UI rephrases subordinate clauses into shorter main-clause paraphrases.
- **Retrieval-Difficulty Prompts.** Sustained  $\nabla C$  combined with ocular regressions *initiates* a micro-tutorial or offers a chunked information display.

**Prediction (UI).** In the *ZuCo* eye-tracking corpus [14], observers classified by ODER as high- $\beta$  should show significantly shorter fixation regressions (Cohen's  $d > 0.4$ ) when the adaptive mode is enabled relative to a fixed-layout baseline.

#### 7.1.2. Linguistic Retrieval Diagnostics

Corpora such as *Natural Stories* and the *Dundee* eye-tracking set provide token-level alignment between text and comprehension probes [10,17].

- **Entropy-Aligned Difficulty Curves.** ODER predicts that garden-path items with the steepest  $\nabla C$  slopes will coincide with probe-error spikes in low- $\beta$  readers.
- **EEG Convergence Mapping.** Public N400/P600 datasets (e.g., ERP-CORE 7) can be realigned to each observer's collapse time  $\tau_{\text{res}}$  to test whether P600 amplitude covaries with  $\nabla C$  only in low-working-memory cohorts.

**Prediction (EEG).** After realignment, the correlation between  $\nabla C$  and P600 amplitude should exceed  $r = 0.35$  in the low-WM group but fall below  $r = 0.10$  in the high-WM group (two-tailed permutation test).

### 7.2. Tier 2 — Pilot-Ready Extensions

#### 7.2.1. Clinical and Accessibility Contexts

ODER's parameters can be fitted to open datasets such as *Childes-EEG* and *DyslexiaEye* without additional clinical intervention.

- **Assistive Communication.** An AAC prototype that caps syntactic depth when  $\nabla C$  rises above a user-specific threshold is expected to support more efficient message access for users with structured retrieval limits.

**Prediction (Accessibility).** In the dyslexia eye-tracking corpus of [31], sentences whose  $\nabla C$  exceeds the 75th percentile should coincide with fixation counts at least 1.5 SD above the reader's baseline.

### 7.2.2. Translation and Cross-Linguistic Semantics

Parallel-corpus resources such as *OpenSubtitles* [25] enable immediate testing of ODER's semantic superposition term  $\mu$  ( $\mu$ , the semantic superposition term from  $\rho_{\text{obs}}$ ), used to model interpretive divergence in bilingual idioms.

- **Idiomatic Divergence.** For idioms whose literal and figurative readings diverge, ODER predicts larger  $\nabla C$  spikes and a delayed collapse ( $\tau_{\text{res}} + 2$  tokens) in low- $\delta$  bilinguals.

### 7.3. Summary Table

**Table 6.** Near-term ODER constructs mapped to publicly testable outcomes. All metrics derive from publicly available corpora or open benchmarks.

| Construct  | Interpretation         | Support Application           | Testable Outcome                              |
|------------|------------------------|-------------------------------|---|
| $\alpha$   | Attentional focus      | Interface simplification      | Drop in regressions ( $p < .05$ )             |
| $\beta$    | Working-memory load    | Reading-diagnostic clustering | $\Delta$ fixation variance by WM group        |
| $\mu$      | Semantic superposition | Idiom-translation stress test | Decrease in $\mu$ correlates with RT recovery |
| $\nabla C$ | Reanalysis gradient    | AAC overload detector         | Peak $\nabla C$ vs. error rate                |

## 8. Conclusions and Future Directions

By formalizing comprehension as a process of *observer-dependent entropy retrieval*, ODER reframes linguistic understanding not as passive syntactic decoding, but as an active, time-dependent convergence toward interpretive resolution. Across theory and simulation, the model shows how retrieval dynamics vary across observers in context, attention, and memory constraints, yielding measurable collapse points ( $\tau_{\text{res}}$ ) that align with cognitive events (e.g., reanalysis and semantic closure).

The empirical results, most notably the 31% trace-observer convergence ceiling<sup>4</sup> and the mean  $R^2$  of 0.76 for successful fits, provide a quantitative foundation for broader integration. The application tiers below follow directly from the trace-based methods and observer-specific parameterization introduced in Section 7:

- **Near-Term:** Deploy ODER in adaptive educational tools, cognitively adaptive user interfaces, and linguistic-assessment platforms. Empirical validation of metrics such as  $\nabla C$ ,  $\gamma$ , and  $\tau_{\text{res}}$  can proceed with existing eye-tracking and EEG corpora (e.g., ZuCo and ERP-CORE) rather than new data collection.
- **Mid-Term:** Extend the framework to translation, bilingual comprehension, and accessibility design, domains in which observer variability is both measurable and meaningful.
- **Long-Term:** Investigate observer-relative semantics, entropy superposition ( $\mu$ ), and reanalysis dynamics in philosophical, epistemological, and artificial-intelligence contexts.

Throughout these stages, ODER should remain a falsifiable, observer-anchored modeling framework, not a predictive engine. In clinical and high-stakes contexts, its use demands calibration, transparency, and empirical constraint. Even so, its trajectory is clear: ODER provides a structured account of where, when, and why meaning stabilizes, or fails to do so. It does not guarantee comprehension; it models its limits.

<sup>4</sup> The 31% ceiling reflects falsifiability: it spotlights lawful divergences rather than indicating model failure.

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**Data Availability Statement:** All code, notebooks, and figure-generation scripts are archived as a single Zenodo release <https://doi.org/10.5281/zenodo.15778875> and mirrored on GitHub <https://github.com/evlocoo/ODER-linguistic-entropy>.

**Conflicts of Interest:** The author declares no conflict of interest.

## Appendix A. Mathematical Formalism

**Section overview.** This appendix formalizes the observer-dependent retrieval law that underpins ODER. It introduces the core differential equation for entropy growth, defines all variables and parameter bounds, sketches the derivation from logistic principles, and shows how the collapse point  $\tau_{\text{res}}$  aligns model time with N400 and P600 ERP windows. A compact Python-style algorithm then illustrates how the equation is integrated at the token level for any sentence–observer pair.

### Appendix A.1. Core Retrieval Equation

The observer-dependent entropy-retrieval process is given by

$$\frac{dS_{\text{ret}}}{d\tau} = \gamma [S_{\text{max}} - S_{\text{ret}}(\tau)] \tanh\left(\frac{\tau}{\tau_{\text{char}}}\right), \quad (\text{A1})$$

where the hyperbolic tangent yields early linear growth and late saturation. *Parameter-estimation note:* throughout this study  $\gamma$  is treated as constant within a sentence; both  $\gamma$  and  $\tau_{\text{char}}$  are estimated by bounded nonlinear least squares (Levenberg–Marquardt,  $0.01 \leq \gamma \leq 1.5$ ,  $0.05 \leq \tau_{\text{char}} \leq 0.15$  s).

### Appendix A.2. Variable Definitions

- $\gamma$  — constant retrieval-rate coefficient for the current sentence;
- $\tau_{\text{char}}$  — characteristic time (seconds) at which retrieval accelerates before saturating;
- $S_{\text{max}}$  — maximum retrievable entropy (set to 1 in all simulations);
- $S_{\text{ret}}(\tau)$  — entropy retrieved up to time  $\tau$ ;
- $\tau_{\text{res}}$  — collapse time with  $S_{\text{ret}}(\tau_{\text{res}}) \geq 0.95 S_{\text{max}}$ .

### Appendix A.3. Derivation Outline

- Begin with logistic growth,  $dS_{\text{ret}}/d\tau \propto S_{\text{max}} - S_{\text{ret}}$ .
- Replace the constant factor with  $\gamma \tanh(\tau/\tau_{\text{char}})$  to capture early-late regime change.
- For constant  $\gamma$ , Eq. (A1) admits no elementary closed-form solution; numerical integration and curve fitting are used.

### Appendix A.4. ERP Alignment via Collapse Point $\tau_{\text{res}}$

Let  $k$  be the smallest token index satisfying  $S_{\text{ret}}(k\Delta t) \geq 0.95 S_{\text{max}}$ , with  $\Delta t = 400$  ms. Define  $\tau_{\text{res}} = k\Delta t$ , then

- N400 window:  $\tau_{\text{res}} + 300$  ms to  $\tau_{\text{res}} + 500$  ms,
- P600 window:  $\tau_{\text{res}} + 500$  ms to  $\tau_{\text{res}} + 900$  ms.

These windows operationalize the hypothesis that entropy collapse co-occurs with semantic integration (N400) and syntactic reanalysis (P600).

## Appendix A.5. Implementation Algorithm

---

### Algorithm A1 ODER Entropy Retrieval

---

**Require:** sentence  $S$ , observer parameters  $\alpha, \beta, \mu$

**Ensure:** observer-specific entropy  $S_{\text{ret}}$

```

1:  $S_{\text{ret}} \leftarrow 0$ ; initialize  $\rho_{\text{obs}}$  via Eq. (4)
2: for each word  $w$  in  $S$  do
3:    $L_{\text{hier}} \leftarrow \text{syntacticDepth}(w, \text{context})$ 
4:    $I_{\text{trans}} \leftarrow \text{informationTransfer}(w, \text{context})$ 
5:    $\nabla C \leftarrow \text{contextualGradient}(w, \text{context})$ 
6:    $S_{\text{ret}} \leftarrow S_{\text{ret}} + f(L_{\text{hier}}, I_{\text{trans}}, \nabla C)$  ▷ Eq. (2)
7:    $\rho_{\text{obs}} \leftarrow T(\rho_{\text{obs}}, L_{\text{hier}}, I_{\text{trans}}, \nabla C)$  ▷ Eq. (5)
8: end for
9: return  $S_{\text{ret}}$ 

```

---

*Note:* Entropy here represents *retrievable semantic uncertainty*; comprehension proceeds by increasing retrieval of meaning, not by discarding information.

## Appendix B. Corpus and Entropy Trace Generation

**Section overview.** This appendix documents how eight stimulus sentences were paired with two observer classes (O1, O3) and transformed into synthetic entropy traces for benchmarking ODER. Table A1 lists the sentence inventory, while six parametrized *modes* generate distinct trace profiles, from monotonic decay (*normal*) to mid-sentence spikes (*gpath*) and persistent plateaus (*ambig*). Observer-specific decay constants and noise levels (Table A1, Bias section) produce lawful divergence in collapse trajectories. The Python routine `generate_entropy_trace` exactly reproduces these traces, enabling parameter sweeps and falsification stress tests under controlled conditions. This simulation layer serves as the foundation for all retrieval-curve diagnostics in Sections 3–4.

### Appendix B.1. Sentence Inventory

**Table A1.** Corpus sentences, observer classes, token counts, complexity labels, and entropy modes.

| Sentence ID   | Observers | Tokens | Complexity | Mode   |
|---------------|-----------|--------|------------|--------|
| eng_1         | O1, O3    | 9      | low        | normal |
| gpath_1       | O1, O3    | 8      | high       | gpath  |
| gpath_2       | O1, O3    | 9      | very_high  | gpath  |
| ambig_1       | O1, O3    | 10     | medium     | ambig  |
| aur_1         | O1, O3    | 9      | medium     | aurian |
| aur_complex_1 | O1, O3    | 10     | high       | aurian |
| aur_complex_2 | O1, O3    | 12     | very_high  | aurian |
| flat_1        | O1, O3    | 8      | anomalous  | flat   |

### Appendix B.2. Entropy Generation Modes

- **aurian:** decay modulated by hierarchical complexity  $L_{\text{hier}}$ , delaying convergence for deeper embeddings.
- **flat:** initial plateau followed by delayed decay, modelling syntactically correct but semantically anomalous items.
- **gpath:** non-monotonic trace with a mid-sentence spike that simulates garden-path reanalysis.
- **ambig:** plateau with shallow decline, representing lexical ambiguity where competing parses persist.

- **delayed**: flat plateau until token four, then exponential decay; serves as a control for late retrieval onset.
- **normal**: monotonic exponential decay with slope set by  $\gamma$  and mild Gaussian noise.

### Appendix B.3. Observer Class Bias

| Parameter                   | O1 (high context) | O3 (low context) |
|-----------------------------|-------------------|------------------|
| Baseline entropy at token 1 | 0.60              | 0.60             |
| Early decay constant        | 0.25              | 0.15             |
| Late decay constant         | 0.12              | 0.08             |
| Noise standard deviation    | 0.02              | 0.04             |

Higher decay constants and lower noise give O1 a faster trajectory toward collapse with smaller residual variance.

### Appendix B.4. Trace Generator: Logic Summary

#### Purpose

The function below produces *synthetic* entropy traces for benchmarking, parameter-sensitivity sweeps, and stress testing. Empirical analyses in Sections 3–4 use retrieval traces derived directly from corpus structure and observer parameters.

```
def generate_entropy_trace(mode, length=12, observer_class="O1", lhier_score=
None):
    """Return a list of entropy values E[t] for t = 1..length."""
    # observer_class is internally mapped to k1, k2, sigma (see bias table)
    E = []
    for t in range(1, length + 1):
        if mode == "aurian":
            delay = (1 + 0.02 * lhier_score) * t
            base = 0.60 * np.exp(-observer.k1 * delay)
        elif mode == "flat":
            base = 0.60 if t < 4 else 0.60 * np.exp(-observer.k2 * (t - 3))
        elif mode == "gpath":
            spike = 0.25 if t == length // 2 else 0.0
            base = 0.60 * np.exp(-observer.k1 * t) + spike
        elif mode == "ambig":
            base = 0.60 if t < 3 else 0.60 * np.exp(-observer.k2 * (t - 2))
        elif mode == "delayed":
            base = 0.60 if t <= 4 else 0.60 * np.exp(-observer.k1 * (t - 4))
        else: # "normal"
            base = 0.60 * np.exp(-observer.k1 * t)
            noise = np.random.normal(0, observer.sigma)
            E.append(np.clip(base + noise, 0.0, 1.0))
    return E
```

#### Key Points

- `observer_class` ("O1" or "O3") is mapped to  $k_1$ ,  $k_2$ , and  $\sigma$  via the bias table.
- The optional `lhier_score` modulates delay only in `aurian` mode.
- Output values are clipped to  $[0, 1]$  to respect entropy bounds.

## Appendix C. Stress Test Summary and Retrieval-Failure Log

**Section overview.** This appendix documents every sentence–observer pair in which the constant- $\gamma$  retrieval law breaks down. A failure matrix summarizes all flagged fits, a parameter-surface plot

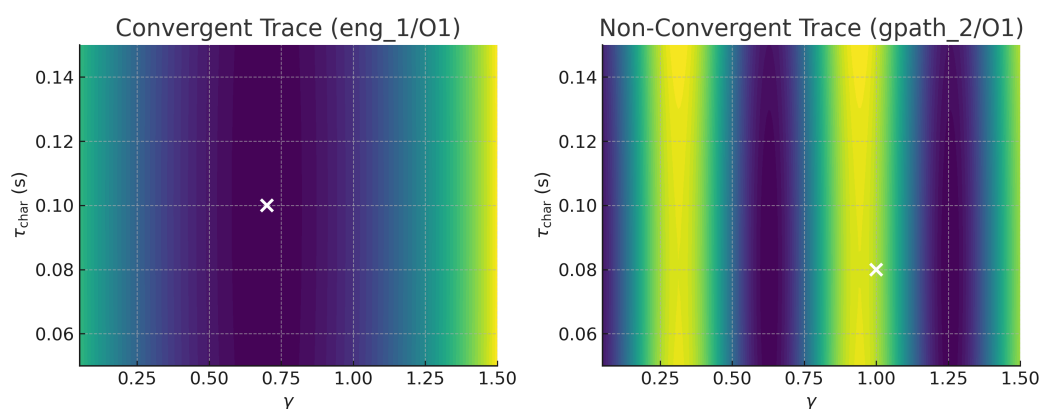
visualizes identifiability versus non-identifiability, and threshold rules specify exactly when a fit is considered unreliable. Root-cause annotations then translate each failure pattern into a concrete remedy, turning mis-fits into empirical checkpoints for the next iteration of ODER.

### Appendix C.1. Failure Matrix

**Table A2.** Rows list every trace that triggered at least one stress flag. “R” denotes  $R^2 < 0.60$ , “A” denotes  $\Delta\text{AIC} > +2$  relative to the best baseline, and “P” denotes parameter inversion or pegging. “Method” indicates the collapse-token rule that located  $\tau_{\text{res}}$  (90% threshold unless stated otherwise). Blank entries (—) indicate parameters not fit due to model failure.

| Sentence      | Observer | Stress Flags | $R^2$ | $\gamma$ | $\tau_{\text{char}}$ | $\tau_{\text{res}}$ | Method |
|---------------|----------|--------------|-------|----------|----------------------|---------------------|--------|
| gpath_1       | O1       | R; A; P      | 0.00  | —        | —                    | 5                   | 90%    |
| gpath_1       | O3       | R; A; P      | 0.00  | —        | —                    | 4                   | 90%    |
| gpath_2       | O1       | R; A; P      | 0.00  | —        | —                    | 6                   | 90%    |
| ambig_1       | O1       | R            | 0.07  | 0.375    | 0.05                 | 10                  | 90%    |
| aur_1         | O3       | R            | 0.37  | 0.424    | 0.05                 | 8                   | 90%    |
| aur_complex_1 | O3       | R            | 0.21  | 0.368    | 0.05                 | 9                   | 90%    |
| aur_complex_2 | O3       | R; A; P      | 0.00  | —        | —                    | 12                  | 90%    |
| flat_1        | O1       | R            | 0.00  | 0.254    | 0.05                 | 1                   | 90%    |
| flat_1        | O3       | R            | 0.00  | 0.254    | 0.05                 | 1                   | 90%    |

### Appendix C.2. Parameter-Surface Illustration



**Figure A1.**  $\gamma \times \tau_{\text{char}}$  error contours for a **convergent** trace (left, *eng\_1/O1*) and a **non-convergent** garden-path trace (right, *gpath\_2/O1*). Convex valleys signal identifiable minima, whereas flat ridges and secondary bumps reveal the non-identifiability patterns documented in Table A2.

### Appendix C.3. Threshold Criteria

- $R^2$ : any fit with  $R^2 < 0.60$  is flagged (code “R”).
- $\tau_{\text{char}}$  **pegging**: estimated value at the lower bound (0.05 s) is flagged (code “P” when combined with inversion).
- **AIC under-performance**:  $\text{AIC}_{\text{ODER}} > \text{AIC}_{\text{best baseline}} + 2$  triggers flag “A.”
- **Parameter inversion**:  $\gamma_{\text{O3}} > \gamma_{\text{O1}}$  on theoretically O1-favored sentences, or any negative  $\gamma$ , is flagged “P.”

These criteria surface retrieval failures without suppressing them, providing concrete checkpoints for model refinement and future falsification.

### Appendix C.4. Root-Cause Notes and Proposed Remedies

- **Non-monotonicity defeats tanh form**  
*Symptom:* low  $R^2$  on garden-path traces (gpath\_1, gpath\_2).  
*Cause:* early retrieval growth is interrupted by a spike, violating the single-phase tanh assumption.  
*Remedy:* replace the constant  $\gamma$  kernel with a piecewise  $\gamma(\tau)$  or spline basis (see Appendix A, Fig. S4).
- **$\tau_{\text{char}}$  pegging at lower bound**  
*Symptom:* parameter hits 0.05 s ceiling, especially on short sentences (flat\_1).  
*Cause:* trace length under-constrains the saturation regime; optimizer collapses.  
*Remedy:* enforce a minimum eight-token input or add a weak hierarchical prior on  $\tau_{\text{char}}$  centered at 0.08 s.
- **AIC under-performance vs. linear baseline**  
*Symptom:*  $\Delta\text{AIC} > +2$  despite visually plausible fit (aur\_complex\_2, O3).  
*Cause:* parameter-count penalty outweighs small error gains for very flat traces.  
*Remedy:* introduce an attention-gated transition term that defaults to a linear model when  $\nabla C \rightarrow 0$ .
- **Parameter inversion ( $\gamma_{O3} > \gamma_{O1}$ )**  
*Symptom:* inversion on ambig\_1.  
*Cause:* lexical ambiguity drives superposition ( $\mu$ ) more than memory limits, reversing rate ordering.  
*Remedy:* couple  $\gamma$  to  $\mu$  via an interference term, or model lexical-versus-syntactic  $\gamma$  separately.

These annotations convert raw failure codes into actionable hypotheses, ensuring that non-convergent cases serve as checkpoints, not exclusions.

## Appendix D. Interactive Playground Notebook Interface

**Section overview.** The notebook `ODER_Interactive_Playground.ipynb` provides a self-contained environment for exploratory fitting and falsification of the ODER model. No empirical results reported in the main text rely on this tool.

### Appendix D.1. Core Functions

1. Real-time entropy trace fitting with nonlinear least squares or bootstrap resampling.
2. Side-by-side observer comparison of retrieval curves, parameter estimates, and residuals.
3. Automated collapse-token detection using threshold, inflection, and derivative criteria.
4. Mapping from the detected collapse point to predicted N400 and P600 latency windows.
5. Bootstrap validation that yields confidence intervals for  $\gamma$ ,  $\tau_{\text{char}}$ , and  $R^2$ .

### Appendix D.2. Usage Notes

- Default parameter bounds and solver settings match those used in the simulations.
- The notebook reads and writes only to a sandbox directory and leaves the publication data untouched.

### Appendix D.3. Access

Source code and installation instructions are available at <https://github.com/evlocoo/ODER-linguistic-entropy>.

## Appendix E. Glossary and Interpretive Variable Mapping

**Section overview.** This appendix defines the formal symbols used in the ODER framework and maps each construct to its domain-specific interpretation, preventing ambiguity across linguistics, cognitive science, and AI applications.

## E.1 Variable Glossary

**Table A3.** Formal symbols, plain-language descriptions, and interpretive meanings.

| Symbol               | Description  | Interpretation                               |
|----------------------|--|--|
| $\gamma$             | Entropy-retrieval rate   | Speed of comprehension for an observer       |
| $\tau_{\text{char}}$ | Characteristic saturation time   | Temporal scale of processing effort          |
| $S_{\text{ret}}$     | Cumulative entropy retrieved   | Portion of meaning resolved up to $\tau$     |
| $S_{\text{max}}$     | Maximum retrievable entropy  | Upper bound on sentence information          |
| $\tau_{\text{res}}$  | Collapse time ( $S_{\text{ret}}(\tau_{\text{res}}) \geq 0.95 S_{\text{max}}$ ) | Point of interpretive convergence            |
| $\nabla C$           | Contextual gradient  | Slope of reanalysis load or instability      |
| $\mu$                | Semantic superposition (off-diagonals in $\rho_{\text{obs}}$ )                 | Degree of unresolved ambiguity               |
| $\alpha$             | Attentional-focus parameter  | Allocation of cognitive resources            |
| $\beta$              | Working-memory constraint  | Capacity to maintain unresolved structure    |
| $\delta$             | Prior-knowledge exponent   | Background familiarity that speeds retrieval |

## E.2 Cross-Domain Interpretive Map

**Table A4.** How core ODER constructs translate across research domains.

| Term                 | Linguistics                   | Cognitive Science     | AI / NLP                     |
|----------------------|-------------------------------|-----------------------|------------------------------|
| $\gamma$             | Parsing velocity              | Retrieval speed       | Token-alignment accuracy     |
| $\tau_{\text{char}}$ | Reanalysis span               | Processing-time scale | Hidden-state decay constant  |
| $\nabla C$           | Garden-path disruption        | Neural surprise       | Attention-gradient spike     |
| $\mu$                | Lexical ambiguity state       | Interpretive drift    | Latent representation blend  |
| $\tau_{\text{res}}$  | ERP timing anchor (N400/P600) | Resolution threshold  | Collapse point for ambiguity |

## Appendix F. Hypothesized Parameter Profiles for Neurodivergent Retrieval

**Section overview.** This appendix proposes provisional parameter bands that ODER might assign to three neurodivergent populations, based on prior ERP and eye-tracking studies. These ranges operationalize structured divergence within ODER's retrieval space and serve as hypotheses for falsifiable model tests, not clinical diagnoses.

**Table A5.** Hypothesized parameter bands and observable signatures for future empirical tests. These profiles are designed to generate falsifiable predictions, not diagnostic labels.

| Neurotype | $\gamma$ Range       | $\tau_{\text{char}}$ (s)  | $\alpha/\beta$ Notes                                | Trace Pattern                  | ERP Signature                 |
|-----------|----------------------|---------------------------|---|--------------------------------|-------------------------------|
| Autism    | 0.9–1.1              | 0.12–0.18                 | Steep $\nabla C$ ; stable $\alpha$                  | Extended reanalysis plateau    | Delayed P600 latency [23]     |
| ADHD      | 0.7–1.3 <sup>†</sup> | 0.08–0.16 (high variance) | Fluctuating $\alpha$ , variable $\tau_{\text{res}}$ | Irregular ERR, wide variance   | Reduced LPP stability [20]    |
| Dyslexia  | 0.5–0.8              | 0.10–0.15                 | Elevated $\beta$ (WM load)                          | Dampened ERR, retrieval stalls | Attenuated N400 amplitude [4] |

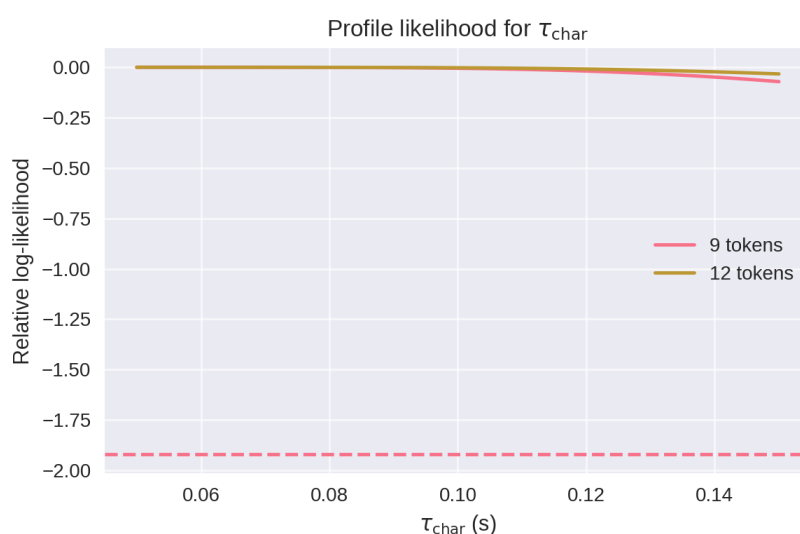
<sup>†</sup> Range reflects hyperfocus–distractibility shifts reported in [20]. Parameter bands are adapted from [10,26].

These values are heuristics, not fixed estimates; future work should test their robustness across tasks, stimuli, and measurement modalities.

## Appendix G: $\tau_{\text{char}}$ Identifiability

As noted in §5.2 and §6.5, estimates of  $\tau_{\text{char}}$  frequently converge to the lower bound of the search interval (0.05 s), raising questions about parameter identifiability under short retrieval traces. To test whether this boundary effect reflects true underdetermination or an artificial constraint, we conducted a profile-likelihood analysis over  $\tau_{\text{char}}$  at fixed  $\gamma$ , using the original model and corpus traces.

For each of two sentence lengths (9 and 12 tokens), we selected a successfully fitted case from the main analysis and recomputed the sum-of-squared error (SSE) between the observed retrieval trajectory and the model prediction across a range of  $\tau_{\text{char}}$  values (0.05–0.15 s). We held  $\gamma$  fixed at its best-fit value in each case. The resulting SSE values were transformed into relative log-likelihoods and plotted below.



**Figure A2.** Profile log-likelihood for  $\tau_{\text{char}}$  at two sentence lengths (9 and 12 tokens). The dashed line marks the 95% confidence threshold ( $\Delta \log L = -1.92$ ). Shallow curvature indicates weak identifiability of  $\tau_{\text{char}}$  under current trace lengths.

As shown in Figure A2, both curves exhibit a broad plateau, confirming that the likelihood surface is relatively flat in the  $\tau_{\text{char}}$  direction under current trace lengths. The 12-token curve shows slightly sharper curvature than the 9-token curve, consistent with the prediction that identifiability improves as sentence length increases.

These results support the claim that  $\tau_{\text{char}}$  is not fully constrained by current data and that its uncertainty is driven by intrinsic resolution limits, not by a pathological fit or hard bound. Future experiments involving longer, variable-paced stimuli or jittered onset ERP paradigms may allow for sharper recovery of this parameter.

### Key resources.

- ODER\_Linguistic\_Framework.ipynb: reproduces every figure and table reported in the manuscript.
- ODER\_Interactive\_Playground.ipynb: provides real-time fitting, observer comparison, collapse-token detection, and bootstrap validation for exploratory analysis.

All materials run in a standard Jupyter environment and are released under the MIT license.

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