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The Typhoon Engulfment Grand Challenge: Can Any Control Law Guarantee Safe Flight Through a Category-5 Typhoon?

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Abstract

We pose the *Typhoon Engulfment Grand Challenge*: determine whether *any* physically realizable feedback control law can guarantee the safe flight and landing of a commercial aircraft under all extreme, physically admissible typhoon wind fields—or prove that such a guarantee is mathematically impossible. The problem is formulated as a two-player zero-sum differential game between an autopilot (the minimizer, seeking survival) and an adversarial but physics-constrained typhoon (the maximizer, seeking to force the aircraft outside its safe operating envelope). We detail the coupled nonlinear dynamics, define the safe operating set in the airspeed–load-factor plane, and identify four interlocking barriers—high-dimensional Hamilton–Jacobi–Isaacs equations, PDE-constrained adversarial disturbances, hybrid structural failure dynamics, and imperfect observations—that place this problem beyond the reach of any existing mathematical framework. This article serves as a formal statement of the challenge, provides accessible explanations for researchers across disciplines, and charts concrete research directions for communities spanning control theory, aerospace engineering, applied mathematics, machine learning, fluid dynamics, structural mechanics, formal verification, operations research, signal processing, meteorology, and independent researchers worldwide.

Keywords: typhoon engulfment; differential games; nonlinear robust control; Hamilton–Jacobi–Isaacs equations; flight envelope protection; PDE-constrained optimization; hybrid dynamical systems; aviation safety; grand challenge problem; reachability analysis

MSC: 49N70, 49L25, 93C10, 93B52, 76D05, 93C30, 70Q05.

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1. Introduction

Commercial aviation operates under a simple but powerful safety doctrine: *avoid severe tropical cyclones entirely*. Pilots reroute hundreds of kilometres around Category-5 typhoons, and no certified aircraft is designed to penetrate the eyewall of such a storm. This operational wisdom has saved countless lives, yet it rests on engineering judgment and accumulated experience rather than on any rigorous mathematical proof.

A fundamental question therefore remains unanswered:

The Grand Challenge Question

> *Does there exist any feedback control policy—however sophisticated—that can guarantee safe flight and eventual landing of a commercial aircraft engulfed by a Category-5 typhoon? Or is avoidance not merely prudent, but a mathematical necessity?*

This question is not merely academic. As climate change intensifies tropical cyclones and pushes storm tracks into new regions [1], as autonomous cargo aircraft begin operating over typhoon-prone

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oceanic corridors, and as unmanned aerial systems are deployed for storm reconnaissance, the theoretical limits of flight through extreme weather acquire direct practical significance. Additionally, the mathematical structure of this problem appears in numerous other safety-critical domains—autonomous driving in extreme weather, marine vessel control in rogue seas, spacecraft attitude control during solar storms, and power grid stability under cascading weather events.

Why Should You Care? — A Plain-Language Summary

Imagine a commercial aircraft, travelling at roughly 900 km/h, suddenly engulfed by a Category-5 typhoon. The storm generates sustained winds of 85 m/s (over 300 km/h) and vertical gusts of 60 m/s—comparable to the aircraft's own climb speed. The aircraft's control surfaces (rudder, elevators, ailerons) can generate corrective forces amounting to only about 10% of the gust forces acting on the airframe.

The question is deceptively simple: **Is there any possible way to fly the aircraft safely through this storm and land it?** Not with today's autopilot, not with any specific technology—but with *any conceivable* control strategy, no matter how advanced.

If the answer is **yes**, then safe typhoon penetration is theoretically possible and the challenge becomes one of engineering. If the answer is **no**, then avoidance is not just good practice—it is a fundamental physical law, as unavoidable as the speed of light.

Nobody knows the answer. This paper formally states the problem and explains why it is so hard.

How This Differs from Existing Safety Analysis

Current aviation safety analysis is *probabilistic*: engineers estimate the *likelihood* of encountering severe turbulence and design systems to handle a range of expected conditions with a target failure rate (e.g., 10^{-9} per flight hour for catastrophic failure). This approach works brilliantly for normal operations.

The Typhoon Engulfment Grand Challenge is fundamentally different. It asks a *worst-case* question: not "What usually happens?" but "What is the worst that *could* happen, given the laws of physics, and can any controller survive it?" This is the difference between asking "Will the bridge *probably* hold?" and asking "Is there *any* possible load, consistent with physics, that collapses the bridge—and if so, can we design a bridge that survives *all* such loads?"

The remainder of this article is organized as follows. Section 2 presents the physical context and the scale mismatch between typhoon forces and aircraft control authority. Section 3 gives the formal mathematical formulation as a differential game. Section 4 defines the safe operating envelope and states the two possible outcomes. Section 5 identifies the four interlocking barriers that make the problem intractable with current methods. Section 6 surveys related work. Section 7 provides detailed research directions for diverse communities. Section 8 offers a broader discussion, and Section 9 concludes.

2. The Scale Mismatch: Typhoon Forces vs. Aircraft Control

To appreciate why this problem is fundamentally difficult, one must first understand the sheer disparity between the forces a Category-5 typhoon can exert on an aircraft and the corrective forces available to any autopilot.

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2.1. *The Anatomy of a Category-5 Typhoon at Flight Altitude*

A Category-5 tropical cyclone is among the most powerful sustained energy systems on Earth. At the surface, sustained winds exceed 137 knots (70 m/s), but the structure at typical cruising altitudes (10–12 km) is equally hostile. The eyewall—a ring of the most intense convective towers—extends vertically through the entire troposphere. Within and around the eyewall, an aircraft would encounter:

- **Sustained horizontal winds** of 165 knots (85 m/s) or more, roughly 37% of a typical cruise true airspeed.
- **Vertical gusts** within convective bursts reaching 60 m/s, comparable to or exceeding the aircraft's maximum rate of climb.
- **Mesovortices**: small-scale (1–5 km diameter) rotating structures embedded within the eyewall, creating localized wind shear gradients of 30 m/s over just a few hundred metres.
- **Convective updrafts and downdrafts** that can change the aircraft's effective weight by factors of 2 to 3 within seconds.
- **Extreme turbulence** with eddy dissipation rates far exceeding the "severe" classification used in aviation meteorology.
- **Heavy precipitation** including supercooled water and hail, degrading sensor performance and potentially causing engine surge.

Visualizing the Storm at 35,000 Feet

At cruise altitude, the air inside a Category-5 typhoon eyewall is not simply "windy"—it is a violently rotating, vertically heaving, turbulent maelstrom. Imagine standing inside a 300 km/h horizontal tornado that also throws you 60 m/s up and down, while simultaneously pelting you with rain and hail so dense that your instruments stop working properly. The aircraft must maintain controlled flight through this environment for the entire time it takes to traverse the storm—potentially 30 minutes or more.

2.2. *Quantifying the Control Authority Deficit*

Against these forces, a commercial aircraft possesses limited control authority. At cruise conditions (Mach 0.78, approximately 230 m/s true airspeed), the key parameters are:

Table 1. Quantitative comparison of typhoon forces and aircraft control authority.

Parameter	Typhoon	Aircraft
Sustained horizontal wind	85 m/s	—
Vertical gusts	60 m/s	$V_{\text{climb,max}} \approx 15$–25 m/s
Wind shear gradient	30 m/s per 500 m	—
Effective control authority (force margin)	—	∼10% of gust forces
Cruise true airspeed	—	230 m/s (Mach 0.78)
Wind-to-airspeed ratio	85/230 ≈ 37%	

The most critical number is the **effective control authority**: at the extreme end of the typhoon's envelope, full control-surface deflection can generate corrective aerodynamic forces amounting to only about **10% of the gust-induced forces**. This means the aircraft cannot simply "overpower" the disturbance.

The Canoe Analogy — Understanding the Force Mismatch

Think of it like trying to steer a canoe across a Class-5 river rapid using only a small paddle. The river's force on your canoe is roughly ten times what your paddle can produce. You are not just fighting the current—you are fighting physics.

But here is the subtlety: a skilled paddler does not try to overpower the river. They *read* the current, find eddies and slack water, time their strokes to catch momentary lulls, and angle the canoe to use the river's own energy. The question we ask is: *Does there exist any paddling strategy—any sequence of strokes, no matter how clever—that guarantees you will reach the other bank safely?* Or will the river always win, no matter what you try?

In mathematical terms, the paddler is the *controller*, the river is the *adversary*, and the canoe's trajectory is the *state*. The laws of fluid dynamics constrain what the river can do, and the paddler's arm strength constrains what the paddler can do. The question is whether the paddler's strategy space is "rich enough" to counteract every possible river configuration.

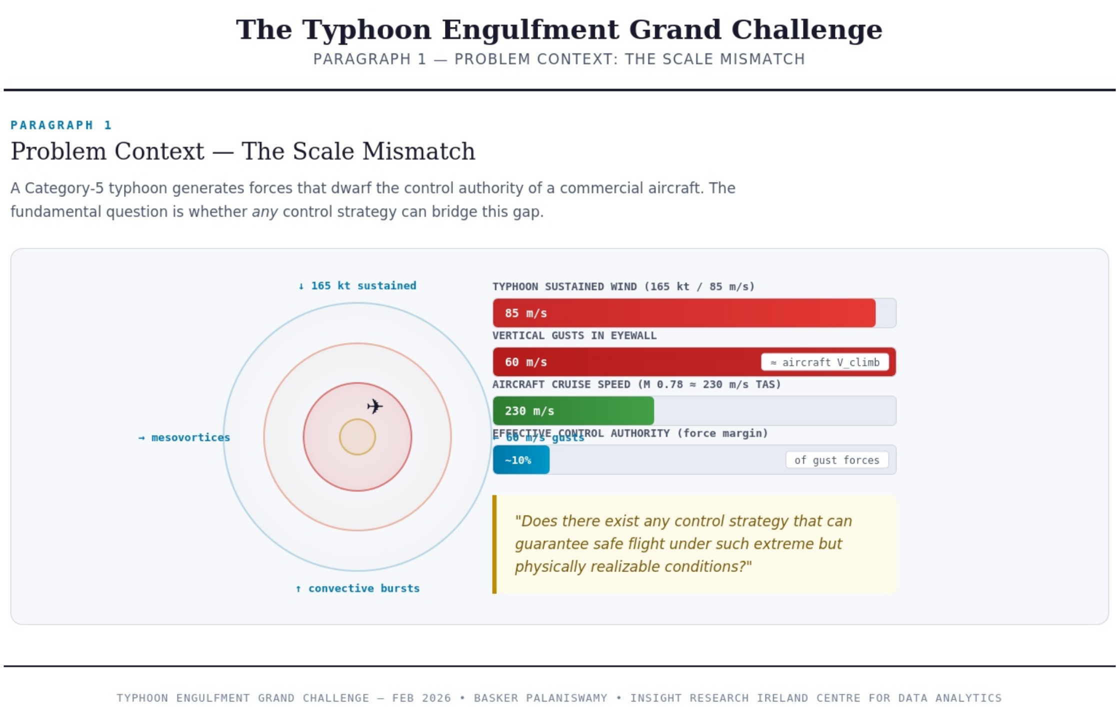

Figure 1. The scale mismatch between typhoon forces and aircraft control authority. Sustained eyewall winds of 85 m/s and vertical gusts of 60 m/s dwarf the effective control margin, which amounts to approximately 10% of gust forces. The fundamental question—whether any control strategy can bridge this gap—remains open.

2.3. Implications for Control Design

This force mismatch has a critical implication: the aircraft cannot "overpower" the storm. Any successful control strategy must instead *exploit the structure* of the wind field—riding favourable gradients, timing control inputs to coincide with brief lulls, and managing energy states to maintain a margin above stall and below structural limits.

Key Insight: Structure Exploitation, Not Brute Force

The only hope for the autopilot is not to fight the typhoon head-on, but to exploit its *internal structure*. Real typhoons are not uniform walls of wind—they have eyes, eyewall gradients, rain bands with gaps, and mesovortices that rotate. A sufficiently intelligent controller might, in principle, "surf" these structures.

But the adversarial framing of our problem asks: *can the typhoon always arrange its internal structure—while still obeying fluid dynamics—to defeat any such surfing strategy?* This is what makes the problem so deep.

Whether such a strategy can always succeed, regardless of the typhoon's spatiotemporal wind pattern, is the heart of the challenge. The answer depends on the precise relationship between the admissible wind field class (constrained by fluid dynamics) and the reachable set of the aircraft dynamics (constrained by control limits). Understanding this relationship requires the formal mathematical framework developed in the next section.

3. Formal Problem Formulation: A Differential Game

We formulate the Typhoon Engulfment Grand Challenge as a *two-player zero-sum differential game* [2,3]. The two players are:

(i) **The Autopilot** π: a causal, output-feedback controller that selects control inputs $u(t) \in \mathcal{U} \subseteq \mathbb{R}^m$ based on the history of noisy sensor measurements. The autopilot seeks to keep the aircraft within the safe operating envelope and reach a safe terminal state (landing).

(ii) **The Typhoon Adversary**: a nature player that selects a three-dimensional, time-varying wind field $w(x,t) \in \mathcal{F}$ from the set of all *physically admissible* wind fields—those satisfying the Navier–Stokes equations and constrained to match the magnitudes and spatial correlations of Category-5 typhoon conditions. The typhoon seeks to drive the aircraft outside the safe envelope.

What Is a Differential Game? — Plain-Language Explanation

A **differential game** is a mathematical framework for situations where two competing players make continuous decisions that affect a shared physical system. It is the continuous-time generalization of familiar strategic games (like chess), but instead of taking turns, both players act simultaneously and continuously.

In our setting:

- **Player 1 (Autopilot)** continuously adjusts the aircraft's controls—throttle, rudder, elevators, ailerons—to keep the plane safe.
- **Player 2 (Typhoon)** continuously "chooses" the worst possible wind pattern—the one most likely to crash the aircraft.
- **The physics** of flight (aerodynamics, structural limits, gravity) connect the two players' actions to the aircraft's state.

The autopilot wins if the aircraft stays safe and lands no matter what the typhoon does. The typhoon wins if it can always find some wind pattern—consistent with real physics—that crashes the aircraft regardless of the autopilot's strategy.

The key insight is that we do not ask what happens on average or in typical conditions. We ask: **What happens in the worst case?**

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The Chess Analogy for Differential Games

Think of a chess game. White (the autopilot) wants to avoid checkmate (staying inside the safe envelope). Black (the typhoon) wants to force checkmate (pushing the aircraft to crash). In chess, we can sometimes *prove* that a position is a forced win for Black, no matter what White does—this corresponds to Outcome B. Or we can prove that White has a drawing strategy, no matter what Black tries—this corresponds to Outcome A.

The difference is that our "chess board" has infinitely many positions (the continuous state space of the aircraft), both players move simultaneously and continuously (not in turns), and the rules change mid-game (hybrid dynamics from structural damage). No chess engine—no computer program—can solve this game with current technology.

3.1. Aircraft State Dynamics

The aircraft state vector $x(t) \in \mathcal{X} \subseteq \mathbb{R}^n$, with $n \geq 12$, includes:

Table 2. Components of the aircraft state vector $x(t)$.

Component	Symbol	Description
Position	$(\mathrm{lat}, \mathrm{lon}, \mathrm{alt})$	Geographic coordinates and altitude
Airspeed and flow angles	(V, α, β)	True airspeed, angle of attack, sideslip
Attitude (Euler angles)	(φ, θ, ψ)	Roll, pitch, yaw
Angular rates	(p, q, r)	Body-axis rotation rates
Engine state	Ω_{engine}	Spool speed (with lag dynamics)
Structural load accumulator	Σ_{struct}	Cumulative fatigue damage index

What Does Each State Variable Mean? — Plain-Language Guide

The "state" of the aircraft is like its complete medical chart—every measurement needed to predict what happens next:

- **Position** (lat, lon, alt): Where is the aircraft in 3D space? Like GPS coordinates plus altitude.
- **Airspeed** V: How fast the aircraft moves *relative to the surrounding air*. If the wind is 85 m/s and the aircraft flies at 230 m/s relative to the ground, the airspeed can range anywhere from 145 m/s to 315 m/s depending on direction—a huge and dangerous variation.
- **Angle of attack** α: The angle between the wing and the oncoming air. Too high, and the wing stalls (loses lift). Too low, and the aircraft dives.
- **Sideslip** β: Whether the aircraft is "skidding" sideways through the air. Large sideslip in a storm can lead to loss of directional control.
- **Attitude** (φ, θ, ψ): How the aircraft is tilted in space—roll (banking left/right), pitch (nose up/down), yaw (nose left/right).
- **Angular rates** (p, q, r): How fast the aircraft is rotating around each axis. A sudden gust can spin the aircraft faster than the control surfaces can counteract.
- **Engine spool speed** Ω_{engine}: Jet engines cannot change thrust instantly—the spool-up time can be 5–8 seconds, an eternity in a violent gust encounter.
- **Structural damage** Σ_{struct}: A running total of how much fatigue damage the airframe has accumulated. Once this exceeds a limit, parts can break.

The dynamics evolve according to the coupled nonlinear ordinary differential equation:

$$\dot{x}(t) = f\big(x(t), u(t), w(x(t), t)\big), \qquad x(0) = x_0, \tag{1}$$

where f encodes the standard six-degree-of-freedom (6-DOF) rigid-body flight mechanics augmented with engine dynamics, structural load accumulation, and wind-coupling terms. Crucially, the wind enters as a spatially distributed field $w(x,t)$ evaluated at the aircraft's current position, making the dynamics *state-dependent in the disturbance*.

Why Is Equation (1) So Hard?

The equation $\dot{x} = f(x, u, w)$ looks deceptively compact, but it encodes enormous complexity:

- **Nonlinear**: The function f involves sines, cosines, products, and divisions of state variables. This means you cannot simply "add up" the effects of the wind and the controls—they interact in complicated ways.
- **Coupled**: Every state variable affects every other. A pitch-up manoeuvre changes airspeed, which changes lift, which changes altitude, which changes the wind the aircraft encounters (because the wind field varies in space).
- **High-dimensional**: With $n \geq 12$ state variables, the "space" of all possible aircraft states has at least 12 dimensions. Searching this space exhaustively is computationally infeasible.
- **State-dependent disturbance**: The wind $w(x(t), t)$ depends on *where the aircraft is*. The aircraft's own trajectory determines what disturbances it encounters—a self-referential loop.

3.2. The Control Input Set $\mathcal{U}$

The autopilot's available controls $u(t) \in \mathcal{U} \subseteq \mathbb{R}^m$ are physically bounded:

$$u(t) = \big(\delta_e(t),\ \delta_a(t),\ \delta_r(t),\ \delta_T(t)\big) \in \mathcal{U}, \tag{2}$$

where $\delta_e, \delta_a, \delta_r$ are the elevator, aileron, and rudder deflections (each bounded by mechanical limits), and δ_T is the throttle setting (bounded between idle and maximum thrust). Critically, these controls are subject to *rate limits*—they cannot move instantaneously—and to *authority degradation* at extreme angles of attack or dynamic pressures.

Why Control Limits Matter More Than You Think

In calm conditions, an aircraft has more than enough control authority for any manoeuvre. The limits only matter at the extremes. But in a typhoon, the aircraft spends most of its time *near* the limits of its envelope. A controller that needs just 5% more elevator authority than is physically available will fail. This is analogous to a car that handles perfectly at normal speeds but becomes uncontrollable on ice: the physics of the extreme environment, not the quality of the driver, determines the outcome.

3.3. The Admissible Wind Field Set $\mathcal{F}$

A distinguishing feature of this problem is that the adversary's strategy space is not a simple norm-bounded set but is *constrained by partial differential equations*. Specifically, the wind field $w : \mathbb{R}^3 \times [0, T] \to \mathbb{R}^3$ must satisfy:

(a) The incompressible Navier–Stokes equations (or a suitable atmospheric model) ensuring physical realizability.
(b) Boundary and forcing conditions consistent with a Category-5 tropical cyclone.
(c) Spatial continuity and bounded gradients reflecting observed mesoscale structure.
(d) Peak sustained winds ≥ 137 kt and vertical gusts up to 60 m/s.

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Formally:

$$\mathcal{F} = \left\{ w \in C^1(\mathbb{R}^3 \times [0,T]; \mathbb{R}^3) \;\middle|\; \nabla \cdot w = 0,\ \frac{\partial w}{\partial t} + (w \cdot \nabla) w = -\frac{1}{\rho} \nabla p + \nu \nabla^2 w + g,\ \|w\|_\infty \geq w_{\min} \right\}. \tag{3}$$

Why Must the Wind Be "Physically Realizable"?

We could make the problem easier by assuming the typhoon can produce *any* wind pattern, including physically impossible ones (e.g., wind that instantaneously reverses direction everywhere, or wind that appears from nothing). But that would be unfair—and scientifically uninteresting. The power of this formulation is that the typhoon must obey the laws of fluid dynamics.

Think of it this way: even the worst-case typhoon cannot violate conservation of mass or energy. It cannot create a wind field that is physically impossible. The question is whether, *even when constrained by physics*, the typhoon can always find some valid wind pattern that defeats the autopilot.

This constraint is what makes the problem deep rather than trivial. If the wind were unrestricted, the answer would obviously be "no controller can survive." If the wind were too restricted, the answer would obviously be "yes, the controller can survive." The Category-5 typhoon sits at a fascinating boundary where the answer is genuinely unknown.

The Navier–Stokes Connection

The set $\mathcal{F}$ is defined by the Navier–Stokes equations—the same equations whose three-dimensional regularity is one of the seven Clay Millennium Prize Problems [4], carrying a $1 million prize. This means our problem is at least as hard as a Millennium Problem in one specific sense: understanding $\mathcal{F}$ requires understanding Navier–Stokes.

However, our problem requires more than just understanding $\mathcal{F}$. We must also understand how the aircraft dynamics *interact* with $\mathcal{F}$—which wind fields in $\mathcal{F}$ are "dangerous" depends on what the aircraft is doing, and what the aircraft is doing depends on the wind field. This circular coupling is what makes the Grand Challenge uniquely difficult.

3.4. Sensors and Observation Model

The autopilot does not observe the full state $x(t)$ directly. Instead, it receives noisy, potentially degraded sensor measurements:

$$y(t) = h\big(x(t)\big) + v(t), \tag{4}$$

where h is the observation function and $v(t)$ represents sensor noise that may be state-dependent (e.g., pitot tube icing, GPS multipath errors in heavy precipitation, radar attenuation). This makes the game one of *imperfect information*, substantially harder than the full-state-feedback case.

Flying Blind in a Storm — The Sensor Problem

Modern aircraft navigate using a suite of sensors: pitot tubes measure airspeed, gyroscopes and accelerometers measure orientation and acceleration, GPS provides position, and weather radar sees ahead. In a Category-5 typhoon, *every one of these sensors degrades*:

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- **Pitot tubes** can ice over, giving dangerously wrong airspeed readings. (This was a contributing factor in the Air France Flight 447 accident, and that was "merely" a tropical thunderstorm, not a Category-5 typhoon.)
- **GPS** signals bounce off rain and ice (multipath), introducing position errors of tens to hundreds of metres.
- **Weather radar** is attenuated by extreme rainfall, creating "blind spots" directly ahead where the worst conditions may lurk.
- **Inertial navigation** drifts over time without GPS correction, and the violent accelerations in a typhoon make the drift worse.
- **Air data computers** may receive inconsistent inputs from different sensor sources, potentially triggering fault-detection logic that further degrades available information.

This means the autopilot must not only fly through the storm—it must do so while *partially blind*. The typhoon degrades the very information the controller needs to survive.

3.5. Structural Load and Hybrid Dynamics

The structural load accumulator $\Sigma_{\text{struct}}(t)$ tracks cumulative fatigue damage:

$$\dot{\Sigma}_{\text{struct}} = g_{\text{fatigue}}(x(t), u(t), w(x(t), t)), \qquad \Sigma_{\text{struct}}(0) = 0. \tag{5}$$

When Σ_{struct} exceeds critical thresholds, the aircraft dynamics undergo *discontinuous changes*: control surfaces may jam, flutter onset may alter aerodynamic coefficients, or partial structural failure may change the mass and inertia properties. This introduces *hybrid dynamics*—the continuous flight mechanics are punctuated by discrete switching events triggered by structural damage.

What Are Hybrid Dynamics?

"Hybrid dynamics" means the rules of the game change abruptly mid-flight. Consider these scenarios:

- A severe gust causes **wing flutter**: the wing starts vibrating in a way that changes how it produces lift. The equations of motion suddenly gain new oscillatory terms.
- Accumulated stress causes a **control-surface jam**: the elevator freezes at a fixed deflection. The aircraft goes from having 4 control inputs to effectively 3, and a constant bias force is applied.
- An engine ingests hail and **surges**: thrust drops to zero on one side, creating a sudden yawing moment. The aircraft must now fly asymmetrically.
- A composite panel **delaminates**: the aircraft's mass distribution changes, shifting the center of gravity and altering the moments of inertia.

Each of these events changes the mathematical model of the aircraft. The autopilot must handle not just one set of equations, but an *unknown sequence* of different equations triggered by damage events that it cannot perfectly predict. This is like playing a video game where the controls randomly remap mid-level.

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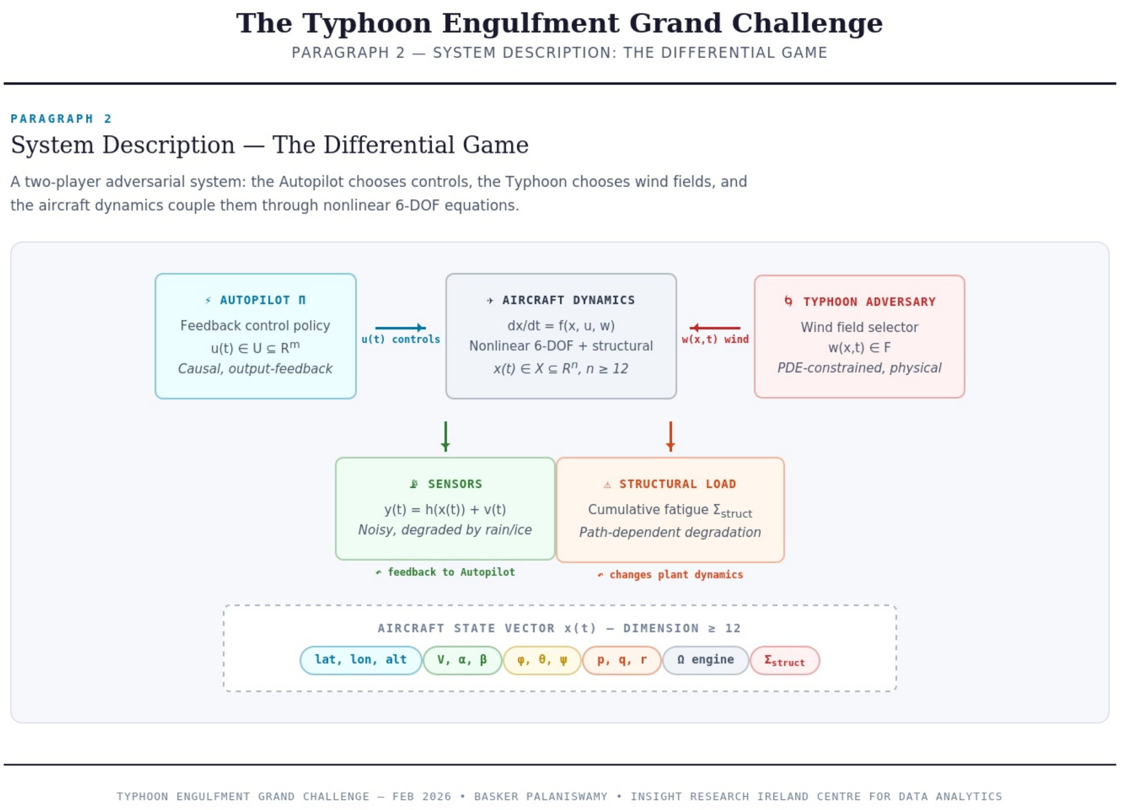

Figure 2. Architecture of the two-player differential game. The Autopilot π selects controls $u(t)$ based on noisy sensor feedback; the Typhoon Adversary selects a PDE-constrained wind field $w(x,t)$. The aircraft dynamics couple both inputs through nonlinear 6-DOF equations. Structural load accumulation feeds back into the plant dynamics, creating hybrid switching behaviour. The state vector $x(t)$ has dimension $n \geq 12$, encompassing position, velocity, attitude, angular rates, engine state, and cumulative structural damage.

3.6. The Information Structure

A subtle but crucial aspect of the formulation is the *information structure*—what each player knows and when.

- **The Autopilot** has access only to the history of sensor measurements $\{y(s) : 0 \leq s \leq t\}$. Its control policy π is a causal mapping from observation histories to control inputs.
- **The Typhoon Adversary** may be modelled with varying levels of information: (a) *open-loop*: the wind field is fixed before the flight begins; (b) *state-feedback*: the wind field at time t can depend on $x(t)$; (c) *full information*: the wind field can depend on the entire history. The strongest (and most interesting) formulation allows state-feedback adversary, which models the physical reality that the worst case over all wind fields includes those that happen to be most dangerous for the aircraft's current state.

Why the Information Structure Matters

The answer to the Grand Challenge may depend on what the typhoon "knows." If the typhoon must commit to a fixed wind field before the flight begins (open-loop), the autopilot has a better chance—it can *react* to what it encounters. If the typhoon can adaptively choose the worst wind field based on the aircraft's current state (state-feedback), the autopilot faces a much harder challenge.

Interestingly, real physics is somewhere in between: the typhoon does not literally "choose" wind patterns, but the set of physically possible wind fields includes some that are, by coincidence,

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optimally dangerous for any given flight path. The adversarial formulation captures this worst case.

4. The Core Question: Survival in the Flight Envelope

4.1. The Safe Operating Set

The safe operating envelope $\mathcal{S}$ is defined as a subset of the state space bounded by physical and structural limits:

$$\mathcal{S} = \{x \in \mathcal{X} \mid V_{\text{stall}}(x) \leq V \leq V_{M_O}, \ |n| \leq n_{\text{limit}}, \ \alpha < \alpha_{\text{crit}}, \ \Sigma_{\text{struct}} < \Sigma_{\text{max}}\}, \tag{6}$$

where V_{stall} is the configuration-dependent stall speed, V_{M_O} is the maximum operating speed, n is the load factor, n_{limit} is the structural load limit (typically $+2.5g$ for transport-category aircraft), α_{crit} is the critical angle of attack, and Σ_{max} is the fatigue damage limit.

What Is the "Flight Envelope"? — Plain-Language Guide

The flight envelope is the set of conditions under which the aircraft can fly safely. It has several boundaries, each representing a different kind of danger:

- **Stall speed V_{stall}** (left boundary): If the airspeed drops below this value, the wings stop producing enough lift. The aircraft falls. In a typhoon, a sudden tailwind gust can instantaneously reduce airspeed by 85 m/s, potentially pushing a cruising aircraft dangerously close to stall.
- **Maximum operating speed V_{M_O}** (right boundary): If the airspeed exceeds this value, aerodynamic forces can damage the structure. A sudden headwind gust does the opposite of a tailwind—it increases airspeed and dynamic pressure.
- **Load factor limit n_{limit}** (top boundary): The load factor measures how many "g's" the aircraft experiences. At $+2.5g$, the wings carry 2.5 times the aircraft's weight. Beyond this, structural failure begins. Violent updrafts and downdrafts in a typhoon can produce extreme load factors.
- **Critical angle of attack α_{crit}**: Beyond this angle, the airflow over the wing separates and lift collapses (deep stall). Rapid wind direction changes can push α beyond the critical value before the pilot or autopilot can react.
- **Structural damage limit Σ_{max}**: Even if no single gust exceeds the structural limit, repeated cycling of loads accumulates fatigue damage. A 30-minute passage through a typhoon might involve thousands of load cycles.

The autopilot must keep the aircraft inside *all* of these boundaries *simultaneously* and *at all times*. Violating any one of them—even briefly—can be catastrophic.

The *terminal safe set* $\mathcal{T} \subset \mathcal{S}$ represents the conditions for a safe landing:

$$\mathcal{T} = \{x \in \mathcal{S} \mid \text{alt} \leq h_{\text{runway}}, \ V \in [V_{\text{ref}} - \Delta, V_{\text{ref}} + \Delta], \ |\dot{h}| \leq \dot{h}_{\text{max}}\}. \tag{7}$$

4.2. The Two Possible Outcomes

The grand challenge asks which of the following two mutually exclusive outcomes is true:

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Outcome A: Survival Is Theoretically Possible

There exists a feedback policy π^* such that, for *every* admissible wind field $w \in \mathcal{F}$, the resulting trajectory satisfies $x(t) \in \mathcal{S}$ for all $t \geq 0$ and reaches the terminal set $\mathcal{T}$ in finite time:

$$\exists\, \pi^* : \forall w \in \mathcal{F}, \quad x(t) \in \mathcal{S} \;\; \forall t \geq 0, \quad \exists\, \tau < \infty : \; x(\tau) \in \mathcal{T}.$$

Interpretation: Safe flight through a Category-5 typhoon is theoretically achievable. The challenge becomes an engineering problem—building the controller π^*.

Outcome B: Avoidance Is a Mathematical Necessity

For every feedback policy π, there exists an admissible wind field $w \in \mathcal{F}$ and a finite time t^* such that $x(t^*) \notin \mathcal{S}$:

$$\forall\, \pi, \;\; \exists\, w \in \mathcal{F}, \;\; \exists\, t^* : \; x(t^*) \notin \mathcal{S}.$$

Interpretation: No controller—no matter how advanced, how fast, or how intelligent—can guarantee safe flight. Storm avoidance is not a guideline but a mathematical law.

Current Status: Unknown

Neither outcome has been established. No existing mathematical framework can determine which is true. The value function of the associated differential game—$V(x_0) = \sup_\pi \inf_w J(\pi, w, x_0)$—cannot be computed or bounded with current tools.

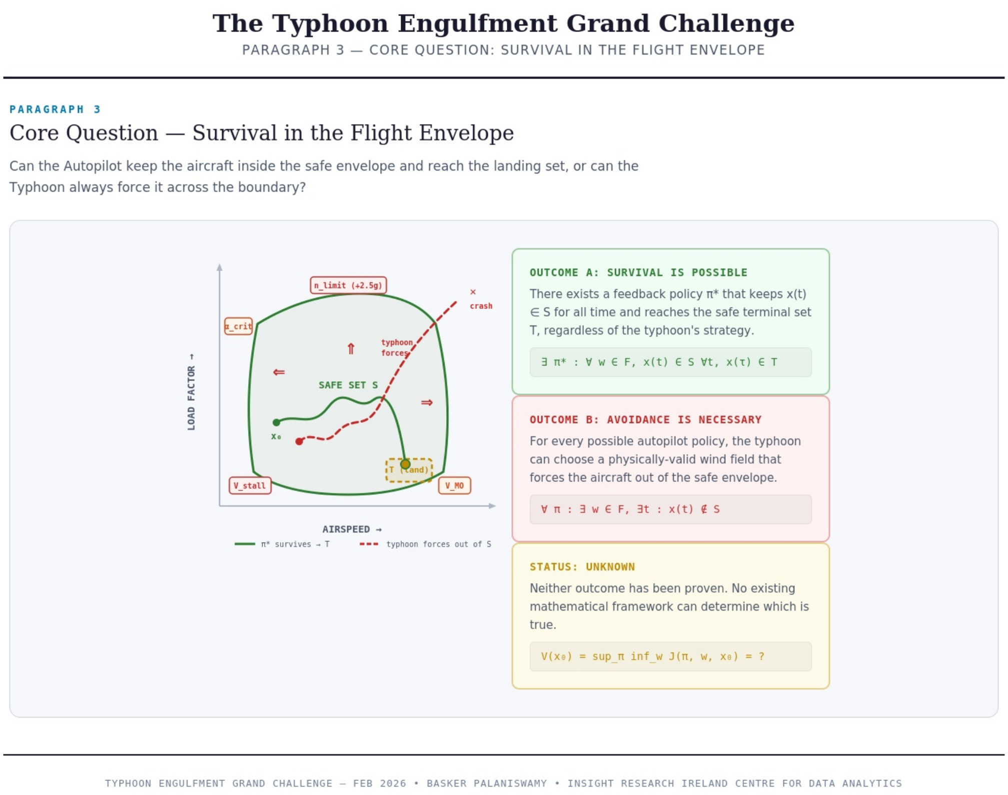

Figure 3. The safe operating envelope $\mathcal{S}$ in the airspeed–load-factor plane. The green trajectory shows a hypothetical surviving path π^* that reaches the terminal landing set $\mathcal{T}$; the red dashed trajectory shows the typhoon forcing the aircraft outside $\mathcal{S}$. The central question is which scenario is always achievable.

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4.3. Connection to the Value Function

In differential game theory, the answer is encoded in the *value function* $V(x_0)$, which satisfies a Hamilton–Jacobi–Isaacs (HJI) partial differential equation:

$$\frac{\partial V}{\partial t} + \min_{u \in \mathcal{U}} \max_{w \in \mathcal{F}} \{\nabla_x V \cdot f(x, u, w)\} = 0, \tag{8}$$

with appropriate boundary conditions on $\partial \mathcal{S}$ and $\mathcal{T}$. If $V(x_0) > 0$ for the initial cruise condition x_0, then Outcome A holds; if $V(x_0) \leq 0$, Outcome B holds. Unfortunately, computing V is itself an unsolved problem for systems of this complexity, as we discuss next.

The Value Function as a "Safety Score"

Imagine assigning every possible aircraft state a "safety score" between -1 and $+1$:

- A positive score means: "From this state, the autopilot can guarantee safe landing, no matter what the typhoon does."
- A negative score means: "From this state, the typhoon can guarantee the aircraft crashes, no matter what the autopilot does."
- A score of zero means: "This state is on the razor's edge—the outcome depends on who plays perfectly."

The HJI equation (8) is the mathematical rule that these safety scores must obey. If we could compute the safety score for the initial cruise condition (the aircraft at 35,000 feet, Mach 0.78, entering the storm), we would have our answer. But computing this score for a 12-dimensional system with PDE-constrained disturbances is beyond any known method.

5. Why It Remains Open: Four Interlocking Barriers

The Typhoon Engulfment Grand Challenge remains open because its resolution requires simultaneous breakthroughs across four domains, each independently recognized as a major open problem. We now examine each barrier in detail.

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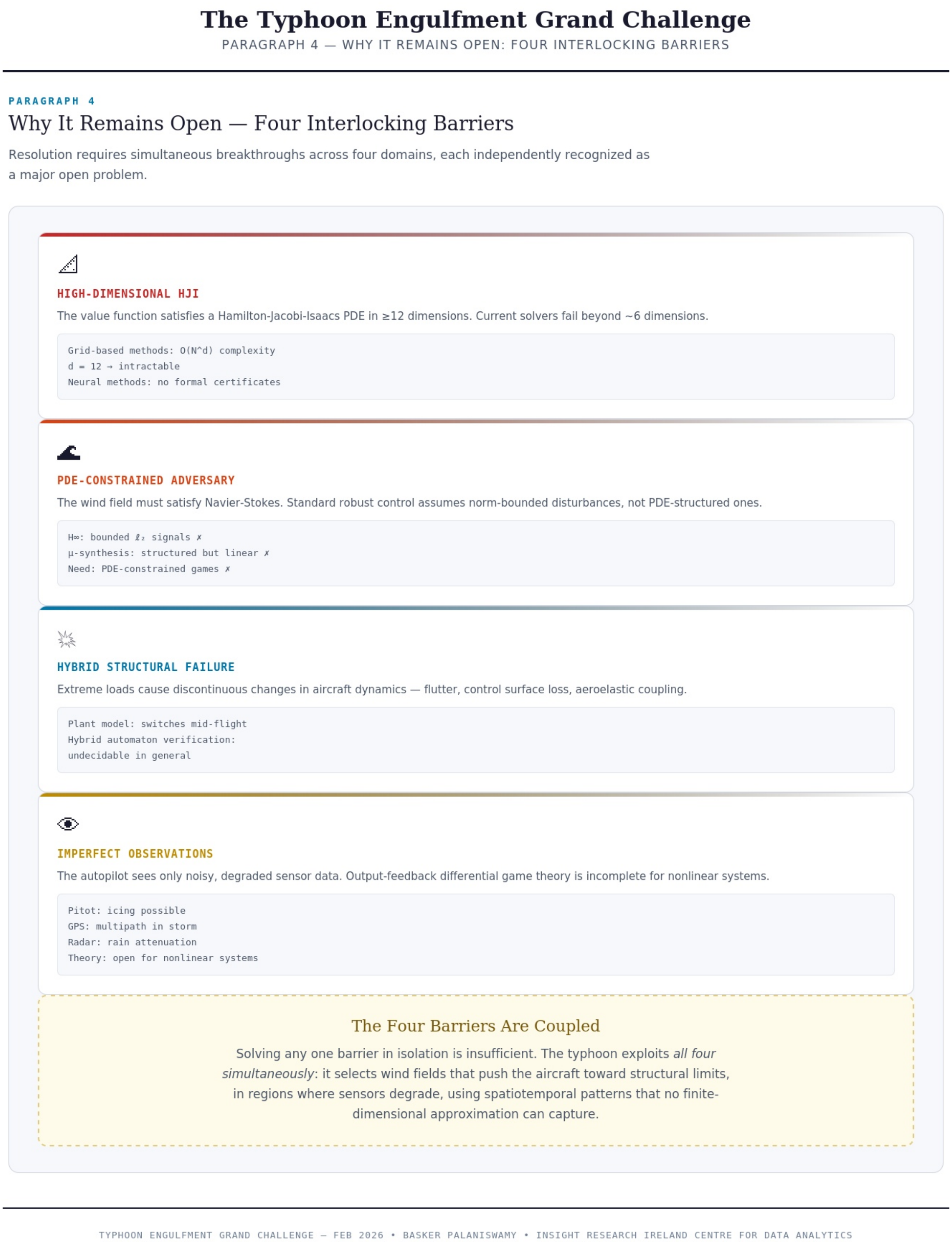

Figure 4. The four interlocking barriers preventing resolution of the Grand Challenge. Each barrier represents an independently recognized open problem; the typhoon can exploit all four simultaneously, and the coupling between barriers creates difficulty that exceeds the sum of the individual challenges.

5.1. Barrier 1: High-Dimensional Hamilton–Jacobi–Isaacs Equations

The value function satisfying (8) is defined over the full state space $\mathcal{X} \subseteq \mathbb{R}^n$ with $n \geq 12$. Grid-based numerical methods for HJI equations have computational complexity $O(N^n)$, where N is the number of grid points per dimension. Current state-of-the-art solvers [5,6] become intractable beyond approximately 6 dimensions.

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Why High Dimensions Are a Problem — The Curse of Dimensionality

Imagine you need to evaluate a function at every point on a grid. In 2 dimensions (a chessboard), 100 points per side gives you $100^2 = 10{,}000$ evaluations—easy. In 6 dimensions, it becomes $100^6 = 10^{12}$—a trillion evaluations, at the limit of modern supercomputers. In 12 dimensions, it becomes $100^{12} = 10^{24}$—more evaluations than there are grains of sand on Earth multiplied by the number of stars in the observable universe.

This is the **curse of dimensionality**, named by mathematician Richard Bellman in 1957. It is not a limitation of current computers—it is a fundamental mathematical barrier. Even if every atom in the universe were a computer, you could not build a grid fine enough to solve a 12-dimensional HJI equation by brute force.

Some clever methods exist to partially circumvent this curse (tensor decompositions, sparse grids, neural networks), but none of them provide the *certified* upper and lower bounds on $V(x_0)$ needed to definitively answer the Grand Challenge.

Neural-network-based approximations (e.g., DeepReach [7]) can handle higher dimensions but provide no formal certificates of correctness. For a problem where we seek a *proof* of safety or impossibility, approximate solutions without guarantees are insufficient.

Open Question 5.1. *Can the specific structure of aircraft dynamics (time-scale separation between fast rotational and slow translational modes, known symmetries, energy conservation properties) be exploited to decompose the 12-dimensional HJI equation into coupled lower-dimensional problems that are individually tractable?*

5.2. Barrier 2: PDE-Constrained Adversary

Standard robust control theory assumes disturbances come from simple norm-bounded or signal-class sets (e.g., $\mathcal{H}_\infty$ or μ-synthesis frameworks [8]). The typhoon wind field, however, must satisfy the Navier–Stokes equations—a constraint that is itself an unsolved PDE problem (the regularity of Navier–Stokes in three dimensions is one of the Clay Millennium Prize Problems [4]).

What Is a PDE-Constrained Adversary?

In standard robust control, you assume the worst-case disturbance can be *any* signal up to a certain magnitude—like assuming an adversary can push you in *any* direction with a given force. This is simple to analyse mathematically: the set of all possible pushes forms a ball (a simple geometric shape), and optimization over a ball is well-understood.

In our problem, the "push" (the wind) must obey fluid dynamics. You cannot have a wind field that appears from nowhere, that violates conservation of mass, or that has physically impossible pressure gradients. This constraint makes the adversary's strategy space an *infinite-dimensional manifold* defined by a PDE, rather than a simple ball.

To put it concretely: a standard robust control problem asks "can you survive any push up to 100 Newtons?" Our problem asks "can you survive any push that could be created by a real ocean of air obeying fluid dynamics?" The second question is *vastly* harder because the set of possible pushes has a complex, infinite-dimensional geometric structure that no one fully understands.

Open Question 5.2. *Does the PDE constraint on $\mathcal{F}$ significantly* restrict *the adversary compared to a simple norm-bounded model? That is, are there dangerous wind patterns that would be norm-admissible but physically*

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impossible? If so, the PDE constraint helps the autopilot, and the answer to the Grand Challenge might differ from the simpler norm-bounded case.

5.3. Barrier 3: Hybrid Structural Failure Dynamics

Extreme aerodynamic loads can trigger discrete structural events: flutter onset, control-surface jamming, partial delamination of composite panels, or engine damage from ingested debris. Each event changes the aircraft's equations of motion, creating a *hybrid dynamical system*—continuous dynamics punctuated by discrete switches.

Formal verification of hybrid automata is undecidable in general [9]. The typhoon can strategically target wind patterns that push the aircraft toward structural thresholds where the dynamics switch, exploiting the resulting vulnerability.

Hybrid Dynamics in Plain Language

Imagine you are driving a car, and at certain stress levels, parts start breaking. Each time a part breaks, the car handles differently—maybe the steering becomes sluggish, or one brake fails. You are trying to reach your destination safely, but the road (the typhoon) keeps getting worse, and your car keeps degrading.

The mathematical challenge is threefold:

- **Unpredictable switching**: You do not know exactly when parts will break. The thresholds depend on accumulated stress, material properties, and the precise load history.
- **Combinatorial explosion**: If there are 10 possible failure events, there are $2^{10} = 1{,}024$ possible combinations of failures. Each combination creates a different set of equations to analyse.
- **Undecidability**: For general hybrid systems, the question "does the state ever leave the safe set?" is mathematically *undecidable*—there is no algorithm that always gives the correct answer in finite time. This is not a practical limitation but a proven theoretical impossibility [9].

The Typhoon's Structural Strategy

A "clever" typhoon would not simply blast the aircraft with maximum wind. Instead, it would alternate between extreme loads and partial lulls, *cycling* the structure to accumulate fatigue damage while keeping the aircraft just within the safe envelope. Then, when the airframe is weakened, the typhoon would deliver a targeted extreme gust—not necessarily the strongest possible, but one precisely tuned to exploit the specific damage pattern. This strategy exploits the coupling between Barriers 1 and 3, and is far harder to defend against than a simple worst-case gust.

5.4. Barrier 4: Imperfect Observations

The autopilot cannot observe the aircraft's true state directly. Sensors are degraded by the very storm the aircraft is trying to survive:

- **Pitot tubes** may ice over, giving erroneous airspeed readings—the most critical flight parameter.
- **GPS signals** suffer multipath errors in heavy precipitation, introducing position errors of tens to hundreds of metres.
- **Weather radar** is attenuated by intense rainfall, creating blind spots in precisely the most dangerous regions.
- **Inertial sensors** integrate accelerometer noise, with errors growing over time—and the violent accelerations in a typhoon increase the drift rate.

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- **Air data computers** may receive inconsistent inputs from different sensor sources, potentially triggering fault-detection logic that further degrades available information.

Output-feedback differential game theory—where the controller must estimate the state from noisy observations while simultaneously playing an adversarial game—remains incomplete for nonlinear systems [3]. The separation principle (which allows independent design of estimator and controller in linear systems) does not hold in the nonlinear worst-case setting.

Why Sensor Degradation Creates a Fundamental Barrier

Here is the deep issue: even if a perfect controller π^* exists (one that would keep the aircraft safe *if* it knew the true state), the controller cannot implement this strategy because it does not know the true state. It must estimate the state from degraded sensors.

In calm conditions, the estimation error is small and does not matter much. In a typhoon, the estimation error can be large (because the sensors are degraded) *and* the system is near the boundaries of the safe envelope (because the disturbances are extreme). A small error in state estimation near a boundary can lead to a catastrophically wrong control decision.

An analogy: imagine playing darts while blindfolded (degraded sensors) with the target being a tiny bullseye (narrow safe envelope). Even a perfect dart-throwing strategy cannot compensate for not being able to see the target. The question is whether the "blindfold" imposed by the typhoon on the sensors is severe enough to prevent any successful strategy.

Open Question 5.3. *Is there a purely* information-theoretic *impossibility? That is, regardless of computational power, does the sensor degradation in a Category-5 typhoon destroy enough information about the aircraft's state that no feedback policy can maintain the required accuracy to stay within $\mathcal{S}$?*

The Four Barriers Are Coupled — And the Coupling Is the Hardest Part

Solving any one barrier in isolation is insufficient. The typhoon exploits *all four simultaneously*: it selects wind fields that push the aircraft toward structural limits (Barrier 3), in regions where sensors degrade (Barrier 4), using spatiotemporal patterns that no finite-dimensional approximation can capture (Barrier 1), drawn from a PDE-constrained set that defies standard robust-control analysis (Barrier 2).

The coupling is not merely additive—it is *synergistic*. For example:

- Sensor degradation (Barrier 4) increases state estimation error, which makes the HJI solution less accurate (Barrier 1).
- Structural damage (Barrier 3) changes the dynamics, invalidating any HJI solution computed for the undamaged aircraft (Barrier 1).
- The PDE constraint (Barrier 2) means the adversary's worst-case strategy depends on the full spatiotemporal structure of the wind field, which interacts with all other barriers simultaneously.

A resolution of the Grand Challenge requires either a unified mathematical framework that addresses all four barriers, or a constructive proof that the coupling itself prevents any solution.

6. Related Work

The Typhoon Engulfment Grand Challenge draws on and extends several established research areas. We survey the most relevant ones here.

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Differential game theory.

The foundational work of Isaacs [2] introduced pursuit-evasion games, later formalized by Başar and Olsder [3]. Modern computational approaches using level-set methods [6] and neural approximations [7] handle systems of moderate dimension but cannot address the 12+ dimensional, PDE-constrained setting required here.

Robust and $\mathcal{H}_\infty$ control.

The $\mathcal{H}_\infty$ framework [8] provides worst-case guarantees for linear systems with norm-bounded disturbances. Extensions to nonlinear systems via Hamilton–Jacobi theory exist [10] but do not accommodate PDE-structured adversaries or hybrid dynamics.

Flight envelope protection.

Modern fly-by-wire systems implement angle-of-attack and load-factor limiting [11], but these are designed for turbulence and pilot error—not for sustained engagement with a Category-5 eyewall. No existing flight protection system addresses the full adversarial scenario considered here.

Atmospheric modelling and turbulence.

Large-eddy simulations [12] and observational studies [13] characterize tropical cyclone structure at increasing resolution, but have not been coupled into a formal control-theoretic adversarial framework.

Reachability analysis.

Hamilton–Jacobi reachability [5] computes backward reachable sets and safe controllers for systems up to about 10 dimensions. Decomposition techniques [14] push this boundary but require structural assumptions (e.g., decoupled subsystems) that do not hold for the fully coupled aircraft-typhoon system.

Sum-of-squares (SOS) methods.

Polynomial optimization via SOS programming [15] can certify Lyapunov functions and barrier certificates for polynomial systems. Aircraft dynamics, however, involve transcendental nonlinearities (trigonometric functions, aerodynamic look-up tables) that resist polynomial representation.

Reinforcement learning for control.

Deep RL has achieved superhuman performance in games [16] and increasingly in robotic control [17]. However, RL provides empirical performance without formal worst-case guarantees—precisely the guarantees the Grand Challenge demands.

7. Research Directions: A Roadmap for the Community

We believe the Typhoon Engulfment Grand Challenge can serve as a forcing function for research across multiple disciplines. Below, we outline concrete directions, organized by community, with specific sub-problems that are tractable stepping stones toward the full challenge. Each direction extracts a specific facet of the central problem that aligns with the tools and expertise of the targeted community.

How to Read This Section

Each research direction below is a self-contained challenge extracted from the Grand Challenge. You do not need to solve the entire Grand Challenge to make a meaningful contribution. Solving *any one* of the sub-problems below would advance the state of knowledge and bring the community closer to a resolution. We encourage researchers to identify the direction closest to their expertise and begin there.

7.1. For the Control Theory Community

Direction 1: Scalable HJI Solvers with Certificates

Goal: Develop methods to compute or bound the value function of (8) in ≥ 12 dimensions with provable error bounds.

Central challenge connection: This directly addresses Barrier 1. If the value function $V(x_0)$ can be bounded from above and below, the Grand Challenge is resolved.

Specific sub-problems:

- Exploit the physical structure of aircraft dynamics (e.g., time-scale separation between fast rotational and slow translational modes) to decompose the HJI PDE into coupled lower-dimensional problems.
- Develop tensor-decomposition or polynomial-approximation methods that provide *certified* upper and lower bounds on the value function.
- Investigate whether sum-of-squares (SOS) programming can certify Lyapunov-like barrier functions for restricted classes of wind disturbances.
- Explore connections to optimal transport theory: can the HJI equation be reformulated as a transport problem amenable to modern computational methods?

Why this is tractable as a stepping stone: Start with a 4D longitudinal model (airspeed, flight-path angle, altitude, engine spool). This already captures the essential scale-mismatch physics. Existing HJI solvers can handle 4–6 dimensions, so the challenge is to obtain *tight* bounds and then study how they change as dimensions are added.

Direction 2: PDE-Constrained Differential Games

Goal: Establish a mathematical framework for differential games where the adversary's action space is constrained by a PDE.

Central challenge connection: This directly addresses Barrier 2. No existing differential game theory handles PDE-constrained strategy spaces.

Specific sub-problems:

- Define appropriate solution concepts (viscosity solutions, minimax strategies) when $\mathcal{F}$ is a solution manifold of the Navier–Stokes equations.
- Characterize the "worst-case" wind field within $\mathcal{F}$—does the adversary's optimal strategy correspond to a known flow structure (e.g., a maximally shearing mesovortex)?
- Develop reduced-order models of $\mathcal{F}$ that preserve the essential adversarial structure while enabling computation.
- Study the gap between the PDE-constrained adversary and a simpler norm-bounded adversary: how much does the PDE constraint help the controller?

Why this is tractable as a stepping stone: Begin with a 2D incompressible flow adversary (2D Euler equations) acting on a point-mass aircraft. This is the simplest setting that retains the PDE-constraint structure.

Direction 3: Output-Feedback Games for Nonlinear Hybrid Systems

Goal: Extend differential game theory to handle simultaneous imperfect observations and hybrid switching in nonlinear systems.

Central challenge connection: This addresses the coupling between Barriers 3 and 4.

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Specific sub-problems:

- Determine whether information-state approaches (belief-space planning) can be made tractable for the aircraft state dimension.
- Investigate whether the typhoon's ability to degrade sensors creates a fundamental information-theoretic barrier independent of computational limits.
- Develop robust state estimators (e.g., set-membership filters) that provide guaranteed state bounds under adversarial sensor degradation.
- Study the separation principle failure: quantify how much performance is lost by designing the estimator and controller separately versus jointly.

7.2. For the Aerospace Engineering Community

Direction 4: High-Fidelity Coupled Simulation Testbeds

Goal: Build open-source, high-fidelity simulation environments coupling 6-DOF aircraft dynamics with resolved typhoon wind fields.

Central challenge connection: A common simulation testbed enables the entire research community to compare approaches on a standardized benchmark, accelerating progress on all four barriers.

Specific sub-problems:

- Couple existing aircraft flight dynamics models (e.g., JSBSim, OpenVSP) with large-eddy simulation (LES) or Weather Research and Forecasting (WRF) model outputs for Category-5 storms.
- Create standardized benchmark scenarios (initial conditions, storm parameters, performance metrics) enabling reproducible comparison of control strategies.
- Develop validated structural failure models that capture progressive damage under extreme cyclic loading.
- Implement realistic sensor degradation models calibrated to typhoon conditions.

Why this matters: Without a shared simulation testbed, researchers in different communities will make incompatible modelling assumptions, and results will not be comparable.

Direction 5: Experimental Wind-Tunnel and Flight Data

Goal: Obtain experimental data on aircraft aerodynamic response under typhoon-representative conditions.

Specific sub-problems:

- Design wind-tunnel experiments with time-varying gust profiles replicating eyewall conditions (spatial gradients, turbulence intensity, vertical shear).
- Analyse flight-data-recorder (FDR) records from aircraft that have inadvertently penetrated tropical cyclones to calibrate simulation models.
- Characterize sensor degradation modes (pitot icing rates, GPS error models) specific to typhoon conditions.
- Study aeroelastic response of modern composite wing structures under typhoon-representative cyclic loading profiles.

Direction 6: Structural Damage Modelling and Progressive Failure

Goal: Develop computationally efficient models of progressive structural failure suitable for embedding in a control-theoretic framework.

Central challenge connection: Directly addresses Barrier 3 by providing the "switching rules" for the hybrid dynamical system.

Specific sub-problems:

- Create reduced-order fatigue models that predict control-surface jamming, flutter onset, and delamination as a function of the load history $\Sigma_{\text{struct}}(t)$.
- Validate these models against coupon-level and full-scale fatigue test data.
- Develop stochastic models of structural failure thresholds that capture material variability while remaining analytically tractable for worst-case analysis.

7.3. For the Applied Mathematics Community

Direction 7: Impossibility Proofs via Topological or Information-Theoretic Arguments

Goal: Develop mathematical tools to prove that no control policy can succeed (Outcome B) without computing the full value function.

Central challenge connection: This represents a path to resolving the Grand Challenge via Outcome B without solving the HJI equation.

Specific sub-problems:

- Investigate whether topological arguments (e.g., based on the Brouwer fixed-point theorem or degree theory) can establish that the safe set $\mathcal{S}$ is not *controlled-invariant* under the class $\mathcal{F}$.
- Explore information-theoretic lower bounds: given the sensor degradation model, is there a fundamental limit on how much state information the autopilot can recover, and is this sufficient for survival?
- Study whether the coupling between barriers creates an *undecidability* result analogous to the undecidability of hybrid automaton reachability.
- Investigate whether Ramsey-theoretic or combinatorial arguments can establish impossibility for discrete approximations of the problem.

Direction 8: Simplified Canonical Models and Complexity Hierarchies

Goal: Identify the simplest model that retains the essential difficulty of the Grand Challenge, enabling rigorous analysis.

Specific sub-problems:

- Study a 2D longitudinal model (airspeed and flight-path angle only) with a 1D wind field: can safety be guaranteed even in this reduced setting?
- Gradually increase model fidelity (add lateral dynamics, structural states, observation noise) and identify which features cause the transition from solvable to intractable.
- Establish a "complexity hierarchy" of typhoon engulfment sub-problems, analogous to the polynomial hierarchy in computational complexity.
- Determine the minimum state dimension at which the problem becomes intractable: is it 4? 6? 8?

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Why this is valuable: Even a complete solution to the 2D problem would be a significant mathematical achievement, and identifying the "phase transition" from solvable to intractable would illuminate the fundamental source of difficulty.

Direction 9: Viscosity Solutions and Regularity Theory for Game-Theoretic PDEs

Goal: Advance the mathematical theory of viscosity solutions for HJI equations with PDE-constrained Hamiltonians.

Central challenge connection: The HJI equation (8) involves an inner maximization over $\mathcal{F}$, which is itself defined by a PDE. The regularity and uniqueness properties of solutions to such equations are unknown.

Specific sub-problems:

- Prove existence and uniqueness of viscosity solutions to the HJI equation with PDE-constrained Hamiltonian.
- Develop comparison principles that yield upper and lower bounds without full numerical solution.
- Study the regularity of the value function: is it continuous, Lipschitz, or only measurable? Regularity determines which numerical methods can be applied.

7.4. For the Machine Learning and AI Community

Direction 10: Learning-Based Controllers with Safety Certificates

Goal: Train neural-network controllers that perform well in simulation and come with formal or probabilistic safety guarantees.

Central challenge connection: ML methods can potentially circumvent Barrier 1 (the curse of dimensionality) by learning approximate value functions or policies in high dimensions.

Specific sub-problems:

- Apply deep reinforcement learning (RL) to the typhoon-engulfment simulation with adversarial training (the typhoon as the adversary in a multi-agent RL setting).
- Develop methods to *certify* learned controllers via barrier certificates, interval analysis, or statistical guarantees.
- Investigate whether physics-informed neural networks (PINNs) can simultaneously learn the value function and the worst-case wind field.
- Study the sample complexity of learning a safe controller: how many simulated typhoon encounters are needed for a given confidence level?

Direction 11: Foundation Models for Extreme Weather Control

Goal: Explore whether large-scale weather foundation models (e.g., Pangu-Weather, GraphCast, FourCastNet) can be repurposed as environment models within an adversarial control framework.

Specific sub-problems:

- Fine-tune weather foundation models on high-resolution tropical cyclone data to generate physically consistent adversarial wind scenarios.

- Develop "world models" for the typhoon-aircraft system enabling model-based RL with long-horizon planning.
- Investigate sim-to-real transfer: how well do policies trained in simulated typhoons transfer to real atmospheric conditions?
- Use generative models (diffusion models, flow matching) to sample from the set $\mathcal{F}$ of physically admissible wind fields for Monte Carlo evaluation of candidate controllers.

Direction 12: Adversarial Robustness and Verification of Neural Controllers

Goal: Develop scalable verification methods for neural-network controllers operating in safety-critical, adversarial environments.

Central challenge connection: If a neural controller is trained for typhoon survival, how do we *prove* it works for all possible wind fields?

Specific sub-problems:

- Extend neural network verification tools (e.g., α-β-CROWN, ERAN) to handle closed-loop systems with continuous-time dynamics.
- Develop abstraction-refinement methods that can verify neural controllers for increasingly fine-grained models of the typhoon.
- Study the fundamental trade-off between controller expressiveness (network size) and verifiability.

7.5. For the Computational Fluid Dynamics Community

Direction 13: Characterizing the "Most Dangerous" Wind Field

Goal: Determine whether the worst-case wind field in $\mathcal{F}$ corresponds to a recognizable flow structure.

Central challenge connection: If the worst-case wind field has a special structure (e.g., a maximally shearing mesovortex, or a specific arrangement of updrafts and downdrafts), this constrains the adversary's strategy and may make the problem tractable.

Specific sub-problems:

- Use adjoint methods from PDE-constrained optimization to compute the gradient of the aircraft's safety metric with respect to the wind field, then follow this gradient to find locally "most dangerous" wind patterns.
- Investigate whether the worst-case wind field is unique or whether there is a family of equally dangerous patterns.
- Compare computationally-identified worst-case wind fields against observed typhoon structures: do the most dangerous patterns actually occur in nature?
- Develop reduced-order models (proper orthogonal decomposition, dynamic mode decomposition) that capture the adversarial structure of $\mathcal{F}$ in a finite-dimensional basis.

Direction 14: Multi-Scale Atmospheric Turbulence Modelling

Goal: Bridge the gap between mesoscale typhoon dynamics ($\sim$100 km) and the turbulence scales that affect individual aircraft ($\sim$10 m).

Specific sub-problems:

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- Develop stochastic turbulence models conditioned on the mesoscale typhoon state that are suitable for use in the control-theoretic framework.
- Quantify the energy spectrum of turbulence within the eyewall at scales relevant to aircraft response (1 m to 1 km).
- Study whether the Kolmogorov cascade in extreme tropical cyclone conditions differs qualitatively from standard atmospheric turbulence.

7.6. For the Formal Verification and Computer Science Community

Direction 15: Decidability and Complexity of Hybrid Reachability

Goal: Determine the computational complexity of the typhoon-engulfment reachability problem for specific model classes.

Central challenge connection: Directly addresses Barrier 3. If the problem is undecidable for the full model, this itself is a partial resolution (it would imply that no finite algorithm can answer the Grand Challenge).

Specific sub-problems:

- Classify the typhoon-engulfment problem within the hierarchy of hybrid system decidability results: is it decidable for piecewise-affine dynamics? For piecewise-polynomial?
- Develop sound (but possibly incomplete) over-approximation methods for the reachable set of the hybrid aircraft-typhoon system.
- Investigate whether the specific structure of structural failure modes (monotone damage accumulation, threshold-based switching) enables decidability results that do not hold for general hybrid automata.
- Study whether SAT/SMT-based bounded model checking can provide useful partial answers (e.g., "the aircraft is safe for the first 5 minutes under any wind field of a specific class").

Direction 16: Correct-by-Construction Controller Synthesis

Goal: Develop automated synthesis methods that produce controllers with built-in correctness guarantees.

Specific sub-problems:

- Apply reactive synthesis techniques from formal methods to a discretized version of the typhoon-engulfment game.
- Develop abstraction methods that relate discrete game solutions back to the continuous-time original problem with quantified error bounds.
- Investigate whether contract-based design (assume-guarantee reasoning) can decompose the controller synthesis problem into manageable sub-problems.

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7.7. For the Structural Mechanics and Materials Science Community

Direction 17: Fatigue and Failure Under Extreme Stochastic Loading

Goal: Characterize structural failure thresholds for modern aerospace composites and alloys under typhoon-representative loading profiles.

Central challenge connection: Directly informs the structural load model (5) and the hybrid switching thresholds in Barrier 3.

Specific sub-problems:

- Conduct accelerated fatigue testing under broadband random loading spectra representative of typhoon conditions (orders of magnitude more cycles than standard gust spectra).
- Develop physics-based fatigue models that predict failure thresholds as a function of load history, temperature, and moisture (all of which vary in a typhoon).
- Study whether modern composite structures (CFRP wings, thermoplastic fuselage panels) are more or less vulnerable to typhoon-representative loading than traditional aluminium structures.
- Characterize the scatter (statistical variability) in failure thresholds: worst-case analysis requires knowledge of the *weakest* specimen, not the average.

Direction 18: Aeroelastic and Flutter Analysis in Extreme Conditions

Goal: Determine flutter boundaries and aeroelastic stability margins under typhoon-representative aerodynamic conditions.

Specific sub-problems:

- Study whether the extreme gust environment in a typhoon can trigger flutter at speeds and altitudes normally considered safe.
- Develop reduced-order aeroelastic models suitable for embedding in the differential game framework.
- Investigate whether adaptive structural concepts (morphing wings, variable stiffness) could expand the flutter boundary enough to affect the Grand Challenge outcome.

7.8. For the Signal Processing and Estimation Community

Direction 19: Robust State Estimation Under Adversarial Sensor Degradation

Goal: Develop state estimators that provide *guaranteed* error bounds under typhoon-induced sensor degradation.

Central challenge connection: Directly addresses Barrier 4. The quality of state estimation determines the effective control authority.

Specific sub-problems:

- Develop set-membership estimators (guaranteed bounding estimators) for the aircraft state under adversarial sensor noise models.
- Study sensor fusion strategies that are robust to simultaneous failure of multiple sensor types (pitot + GPS + radar).
- Investigate whether the aircraft's own motion can be exploited to improve observability (analogous to SLAM in robotics): can specific manoeuvres in the storm improve state estimation?

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- Quantify the information-theoretic channel capacity of each sensor under typhoon conditions, and determine the minimum sensor suite required for controllability.

Direction 20: Sensor Design for Extreme Environments

Goal: Develop novel sensor concepts that maintain accuracy in typhoon conditions.

Specific sub-problems:

- Explore LIDAR-based airspeed measurement as an alternative to pitot tubes (immune to icing but potentially degraded by heavy precipitation).
- Investigate multi-static radar configurations that can penetrate intense rainfall.
- Study whether machine-learning-based "virtual sensors" can infer unmeasurable quantities from available degraded measurements.

7.9. For the Operations Research and Decision Science Community

Direction 21: Stochastic and Robust Optimization Approaches

Goal: Apply tools from stochastic programming, distributionally robust optimization, and robust Markov decision processes to tractable versions of the Grand Challenge.

Central challenge connection: Operations research provides computational frameworks for sequential decision-making under uncertainty that may offer tractable approximations to the full differential game.

Specific sub-problems:

- Formulate a discrete-time, finite-horizon robust MDP that captures the essential structure of the Grand Challenge.
- Apply distributionally robust optimization (DRO) to handle uncertainty in the wind field distribution, with the Navier–Stokes constraint entering as a support constraint on the ambiguity set.
- Develop approximate dynamic programming (ADP) methods tailored to the typhoon-engulfment problem structure.
- Study the value of information: how much does knowing the storm structure 1 minute, 5 minutes, or 30 minutes ahead improve the controller's survival probability?

Direction 22: Game-Theoretic Routing and Pre-Commitment Strategies

Goal: Study the meta-game of route selection when typhoon avoidance is not possible (e.g., for autonomous cargo aircraft that must cross typhoon-prone corridors).

Specific sub-problems:

- Develop robust routing algorithms that minimize worst-case exposure to typhoon conditions when complete avoidance is infeasible.
- Study the value of "partial avoidance": can the aircraft choose an entry point, altitude, and heading that maximizes survival probability?
- Investigate cooperative strategies where multiple aircraft share sensor information to improve collective state estimation.

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7.10. For the Meteorology and Climate Science Community

Direction 23: Observational Constraints on $\mathcal{F}$

Goal: Provide tighter empirical constraints on the set of physically admissible wind fields $\mathcal{F}$ based on observational data.

Central challenge connection: If the set $\mathcal{F}$ can be shown to be smaller than the theoretical Navier–Stokes solution set (e.g., because real typhoons satisfy additional thermodynamic or structural constraints), this directly benefits the autopilot.

Specific sub-problems:

- Analyse high-resolution in-situ data from NOAA Hurricane Hunter flights and dropsondes to characterize the statistical distribution of extreme wind events in Category-5 eyewalls.
- Determine observational upper bounds on vertical gust magnitudes, wind shear gradients, and mesovortex intensities.
- Study whether climate-change-driven intensification of tropical cyclones expands $\mathcal{F}$ in ways that are relevant to the Grand Challenge.
- Develop physically-motivated parametric models of $\mathcal{F}$ (e.g., based on balanced vortex theory with stochastic perturbations) that are tighter than the full Navier–Stokes constraint but still contain all observed storm structures.

7.11. For the Numerical Methods and Scientific Computing Community

Direction 24: High-Performance Computing for Game-Theoretic PDEs

Goal: Develop GPU-accelerated, massively parallel solvers for high-dimensional HJI equations.

Central challenge connection: Directly attacks Barrier 1 through computational power.

Specific sub-problems:

- Develop sparse grid and adaptive mesh refinement (AMR) methods for HJI equations that exploit the low-dimensional structure of the aircraft dynamics near the boundary of $\mathcal{S}$.
- Implement GPU-accelerated level-set methods for differential games in 8–12 dimensions.
- Explore quantum-computing formulations of the HJI equation: can quantum algorithms provide polynomial speedups for the inner min-max optimization?
- Develop multi-fidelity methods that use cheap, low-dimensional solvers to guide expensive, high-dimensional computations.

Direction 25: Tensor Methods and Low-Rank Approximations

Goal: Exploit low-rank structure in the value function to circumvent the curse of dimensionality.

Specific sub-problems:

- Investigate whether the value function $V(x)$ for the aircraft-typhoon game admits a low-rank tensor decomposition (e.g., tensor-train or hierarchical Tucker format).
- Develop dynamical low-rank integration methods for the time-dependent HJI equation.
- Establish error bounds for tensor approximations of HJI solutions that translate into safety guarantees.

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7.12. For the Probability and Stochastic Analysis Community

Direction 26: Stochastic Differential Game Formulations

Goal: Study stochastic versions of the Grand Challenge where the wind field includes both adversarial and random components.

Central challenge connection: A stochastic formulation may be more tractable than the purely adversarial one, and may provide useful bounds on the deterministic problem.

Specific sub-problems:

- Formulate the Grand Challenge as a stochastic differential game with the wind decomposed as $w = w_{\text{mean}} + w_{\text{turb}}$, where w_{mean} is adversarial and w_{turb} is a stochastic process.
- Develop risk-sensitive and minimax-regret formulations that interpolate between average-case and worst-case analysis.
- Study concentration inequalities for the aircraft's trajectory under random wind fields: what is the probability that a randomly sampled Category-5 wind field crashes the aircraft?
- Investigate connections to rough path theory: can the aircraft's trajectory be controlled in a path-wise sense against rough (non-smooth) wind fields?

7.13. For the Robotics and Autonomous Systems Community

Direction 27: Autonomous Storm Penetration for UAVs

Goal: Develop and test autonomous control strategies for unmanned aircraft penetrating tropical cyclones for scientific data collection.

Central challenge connection: UAVs offer a practical testbed for Grand Challenge ideas without risking human lives. Smaller UAVs face even more extreme force ratios than commercial aircraft.

Specific sub-problems:

- Design flight controllers for expendable storm-penetration drones that maximize data collection time within the eyewall.
- Develop "safe exploration" algorithms that adaptively expand the tested region of the flight envelope during storm encounters.
- Create hardware-in-the-loop testbeds coupling real autopilot hardware with simulated typhoon environments.
- Study multi-UAV coordination strategies where drones share atmospheric state information in real time.

7.14. For Independent Researchers, Graduate Students, and Citizen Scientists

Direction 28: Accessible Entry Points

Goal: Provide tractable versions of the problem that anyone with undergraduate mathematics or engineering training can meaningfully contribute to.

Suggested starting points:

- **The 2D Toy Problem:** Consider a point-mass aircraft in a 2D plane with bounded thrust, subject to a 2D, time-varying, incompressible flow field. Can you find a flow field that traps the aircraft? Or a thrust policy that always escapes? This problem is small enough to solve analytically or with a personal computer, yet it captures the essential adversarial structure.
- **The Discrete-Time Game:** Discretize time and the state/action spaces. Solve the resulting finite game using dynamic programming or minimax search. How does the outcome

depend on the grid resolution? This is an excellent computational project for a master's thesis.
- **The Monte Carlo Challenge:** Using a simplified aircraft simulator (e.g., JSBSim with wind injection), generate 10,000 random Category-5 wind fields and train an RL controller. What fraction of scenarios result in safe landing? How does this fraction change with storm intensity? This is accessible to anyone with Python and basic RL knowledge.
- **The Analytic Bound:** Can you derive a closed-form upper bound on the maximum wind magnitude for which safe flight is guaranteed, even for a simplified model? Even a loose bound would be a publishable contribution.
- **The Escape-Time Problem:** For a given simplified model, what is the minimum time for the aircraft to exit the typhoon's danger zone? Compute this as a function of storm radius and wind speed. This can be done with calculus and basic optimization.

Resources: We encourage the community to develop open-source simulation tools, benchmark datasets, and shared leaderboards to accelerate progress. Contributions at any level—from a clever bound on a 2D model to a novel HJI algorithm—are valuable. No contribution is too small if it is rigorous.

A Summary of All Research Directions

We have outlined **28 research directions** spanning **14 communities**:

Control Theory	Directions 1–3 (HJI solvers, PDE games, output-feedback)
Aerospace Engineering	Directions 4–6 (simulation, experiments, structural models)
Applied Mathematics	Directions 7–9 (impossibility proofs, canonical models, regularity)
Machine Learning / AI	Directions 10–12 (RL, foundation models, verification)
Computational Fluid Dynamics	Directions 13–14 (worst-case wind, multi-scale turbulence)
Formal Verification / CS	Directions 15–16 (decidability, correct-by-construction)
Structural Mechanics	Directions 17–18 (fatigue, flutter)
Signal Processing	Directions 19–20 (robust estimation, sensor design)
Operations Research	Directions 21–22 (robust optimization, routing)
Meteorology / Climate	Direction 23 (observational constraints)
Scientific Computing	Directions 24–25 (HPC, tensor methods)
Probability / Stochastics	Direction 26 (stochastic games)
Robotics / Autonomous Systems	Direction 27 (UAV storm penetration)
Independent Researchers	Direction 28 (accessible entry points)

Each direction extracts a specific, self-contained sub-problem from the Grand Challenge. Progress on any one direction advances the overall effort.

8. Discussion

8.1. The Broader Significance

The Typhoon Engulfment Grand Challenge is not solely about aviation. It exemplifies a class of problems at the frontier of nonlinear control theory: *worst-case safety guarantees for high-dimensional physical systems under structured, physically constrained adversarial disturbances*. Similar mathematical structures arise in numerous other domains:

- **Autonomous driving in extreme weather** (ice storms, flash floods), where the vehicle's traction and sensor performance degrade simultaneously.
- **Marine vessel control in rogue waves**, where nonlinear wave-vessel interaction creates hybrid dynamics through potential capsizing or structural failure.
- **Spacecraft attitude control during solar storms**, where radiation-induced actuator degradation occurs concurrently with intensified disturbance torques.
- **Power grid stability under extreme weather events**, where generation, transmission, and demand are simultaneously perturbed by a spatially correlated disturbance.
- **Surgical robotics under seismic events**, where a robot must maintain safe operation while the ground moves beneath it.
- **Nuclear reactor control during extreme external events**, where cooling, containment, and sensing systems may all degrade simultaneously.

A Template for Safety-Critical Grand Challenges

The structure of the Typhoon Engulfment Grand Challenge—two-player game, PDE-constrained adversary, hybrid dynamics, imperfect observations—provides a *template* that can be instantiated for any safety-critical domain. We hope this formulation inspires similar grand challenges in other fields, creating a family of interconnected open problems that drive progress in safety-critical control.

A resolution of the Grand Challenge—even for simplified sub-problems—would therefore yield insights transferable across many safety-critical domains.

8.2. Philosophical and Practical Implications

If Outcome A is established, the result is a constructive existence proof: safe flight is possible *in principle*, and the challenge shifts to engineering a practical implementation. This would have implications beyond aviation: it would demonstrate that even extreme physical environments do not necessarily create fundamental barriers to controlled operation, suggesting that engineering ingenuity can overcome nature's worst.

If Outcome B is established, it provides the first *mathematical proof* that storm avoidance is not merely good practice but a fundamental limit of controlled flight. This would be one of the few known cases where a *physical limit* on safety is proven rigorously, rather than assumed based on engineering experience.

Either answer deepens our understanding of the boundary between what is controllable and what is not—a question at the very heart of control theory.

What Would Resolving This Challenge Mean?

- **For aviation regulators:** A proof of impossibility (Outcome B) would provide rigorous justification for storm-avoidance regulations—they would become *mathematical laws*, not just rules of thumb.
- **For autonomous aircraft designers:** A proof of possibility (Outcome A) would open the door to designing aircraft and controllers that can survive unexpected typhoon encounters, dramatically improving safety for autonomous cargo and reconnaissance missions.
- **For mathematicians:** Either outcome would require fundamental advances in high-dimensional PDE theory, differential games, and hybrid systems—yielding new tools applicable far beyond aviation.

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- **For climate adaptation planners:** As climate change alters storm patterns and intensities, understanding the theoretical limits of flight through extreme weather informs the design of resilient transportation infrastructure.
- **For society at large:** We would gain a deeper understanding of the fundamental limits of human (and machine) ability to operate in extreme natural environments—a question that spans aviation, maritime operations, space exploration, and disaster response.

8.3. Intermediate Results and Partial Answers

It is possible—indeed likely—that the Grand Challenge will be resolved incrementally, with partial answers for special cases before a general resolution. Valuable intermediate results include:

- **Resolution for simplified models:** Proving Outcome A or B for the 2D longitudinal model, or for a 6-DOF model without structural damage, or with full state feedback, would each constitute a significant advance.
- **Quantitative bounds:** Establishing an upper bound on the maximum wind speed for which Outcome A holds (e.g., "safe flight is guaranteed for sustained winds up to 50 m/s but not for 85 m/s") would narrow the uncertainty.
- **Conditional results:** Results of the form "if the HJI equation has a smooth solution, then Outcome A holds" or "if the adversary is restricted to axisymmetric wind fields, then Outcome B holds" would illuminate the structure of the problem even without a complete resolution.
- **Computational evidence:** Systematic Monte Carlo studies establishing that, for instance, 99.9% of sampled Category-5 wind fields are survived by a trained RL controller would provide strong (though not conclusive) evidence for Outcome A.

8.4. Limitations of This Problem Statement

We acknowledge several limitations. First, the problem as stated assumes a single aircraft; formation flight or coordination with ground control is excluded. Second, the atmospheric model (Navier–Stokes) is itself idealized; real tropical cyclones involve multiscale, multiphysics processes (cloud microphysics, ocean-atmosphere coupling, radiative transfer) that may further constrain or expand $\mathcal{F}$. Third, pilot decision-making—which may involve creative strategies beyond those capturable in a formal feedback policy (e.g., choosing to ditch the aircraft in the eye of the storm)—is not modelled. Fourth, the aircraft model assumes a fixed configuration; in practice, pilots can dump fuel, deploy slats and flaps asymmetrically, or take other emergency actions that expand the control set $\mathcal{U}$.

These limitations represent opportunities for future refinement of the challenge and are themselves sources of interesting sub-problems.

8.5. A Call to Action

Grand challenges have historically served as powerful catalysts for scientific progress. Hilbert's 23 problems shaped twentieth-century mathematics. The Clay Millennium Prize Problems continue to drive fundamental research. The DARPA Grand Challenge created the autonomous vehicle industry. We believe the Typhoon Engulfment Grand Challenge, while narrower in scope, occupies a similar position at the intersection of theory and practice: it is simple to state, deeply difficult to resolve, and its resolution—whichever way it goes—would yield lasting scientific and practical value.

An Invitation

We close with an analogy. In 1900, David Hilbert posed 23 problems that, he believed, would shape the future of mathematics. Some were solved within years; others remain open after more than a century. Each generated entire fields of research.

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The Typhoon Engulfment Grand Challenge is, of course, a single problem, not 23. But we have shown that it decomposes into at least 28 research directions spanning 14 communities. Some of these sub-problems may be solved by a clever graduate student in a semester. Others may resist the combined efforts of the community for decades. All of them are worth pursuing.

We invite you to pick a direction and begin.

9. Conclusions

We have presented the Typhoon Engulfment Grand Challenge: a formally structured open problem asking whether any feedback control law can guarantee safe flight and landing of a commercial aircraft through a Category-5 typhoon, or whether such a guarantee is mathematically impossible. We formulated the problem as a two-player zero-sum differential game, identified the safe operating envelope and the two mutually exclusive outcomes, and catalogued the four interlocking barriers—high-dimensional HJI equations, PDE-constrained adversarial disturbances, hybrid structural failure dynamics, and imperfect observations—that place the problem beyond current theory.

This challenge is not posed as an abstract exercise. It is a concrete, well-defined mathematical problem with profound implications for aviation safety, autonomous systems, and the foundations of nonlinear control theory. We have provided 28 detailed research directions spanning 14 communities—from control theory and aerospace engineering to machine learning, meteorology, signal processing, structural mechanics, formal verification, and citizen science—as well as accessible entry points for independent researchers and graduate students.

The mathematical structure of this problem—worst-case safety under structured adversarial disturbances in high-dimensional nonlinear hybrid systems with imperfect observations—appears not only in aviation but across all safety-critical domains where physical systems must operate in extreme environments. Progress on the Grand Challenge will therefore yield dividends far beyond the specific application to typhoon flight.

We invite the global research community to engage with this challenge—whether by attacking the full problem, tackling a sub-problem, building simulation tools, deriving rigorous bounds, or contributing experimental data. The answer, whichever it may be, will advance our understanding of what is controllable and what is not in an uncertain and sometimes hostile physical world.

The Typhoon Engulfment Grand Challenge is open.

Can any control law guarantee safe flight through a Category-5 typhoon?

Or is avoidance a mathematical necessity?

The answer awaits.

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