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Dimensional Analysis in Finite Ring Cosmology

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Abstract

This paper explores the role of dimensional analysis as the fundamental grammar that decides which physical expressions can be meaningfully compared before any dynamics is established. We develop this grammar inside the Finite Ring Cosmology framework. In this setting, spatial and temporal dimensions arise as frame readings of a finite symmetry space of admissible reference frames, while conventional units such as metres and seconds enter as observer-assigned modular domains. The shell structure itself is invariant under changes of observer frame and unit convention, although its numerical values change with the chosen scale. The resulting unit-domain algebra reproduces the familiar rules of dimensional analysis: quantities can be added only within the same domain, products and ratios move between domains, and physical invariants are precisely the expressions with neutral total domain. The construction gives a finite-domain reading of the constants that connect mechanics, quantum phase, and gravitation. The speed of light appears as a finite, observer-invariant upper boundary relating spatial and temporal scale assignments: its numerical value changes with units, but the boundary does not. The Planck relation forces energy to share the temporal recurrence domain: h converts frequency measured relative to the chosen time scale into frequency measured relative to the complete phase cycle, not into a separate modular unit domain. The mass domain is derived from energy and speed, and Newton's constant is identified as the conversion domain for gravitational geometry.

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1. Introduction

Dimensional analysis is one of the oldest and most robust consistency principles in physics. Before a physical equation is tested, solved, or interpreted, it must satisfy dimensional homogeneity: only quantities of the same physical kind can be added, while products and ratios move between physical kinds according to fixed algebraic rules. This requirement is often treated as bookkeeping. In practice, it is closer to a grammar of physical law. It determines which expressions can carry physical meaning before any particular dynamics is specified.

The classical formulation of this grammar is the theory of dimensions and similarity. Rayleigh's discussion of similitude and Buckingham's formulation of the Pi theorem made explicit that physically meaningful laws are constrained before their numerical coefficients are known [1,2]. If a quantity has dimension $L^r T^s M^u \cdots$, then the exponents record how it transforms under changes of unit scale. Dimensionless monomials are the invariant combinations of this exponent map. Later treatments by Bridgman and Barenblatt developed the same idea into a systematic method for physical scaling, model comparison, and similarity analysis [3,4]. In this tradition, dimensional analysis is not merely notation. It is a structural test for admissible comparison.

This viewpoint belongs to the broader symmetry tradition in physics: physical content is often identified with what remains invariant under an admissible class of transformations. Noether's theorem gives the classical variational form of this idea, connecting continuous symmetries with

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conservation laws, while later accounts of symmetry emphasize its broader role as a principle for identifying meaningful physical structure [5–7].

The same issue appears in the status of physical constants. Planck's construction of natural units showed that c, G, $\hbar$, and k_B can be used to pass between conventional units and natural scale assignments [8]. Later discussions emphasized that the number and role of dimensionful constants depend on what is taken as primitive and what is treated as a conversion convention [9,10]. For the present paper, the important point is modest: the speed of light connects spatial and temporal measurement, Newton's constant connects mass-coupling to gravitational geometry, Planck's constant fixes the conventional energy scale of one phase cycle, and Boltzmann's constant connects thermal scale to energy scale. These constants do not merely insert numerical values into equations. They translate between domains of comparison.

The mathematical setting used below belongs to finite algebra rather than continuum analysis. Finite fields provide exact cyclic additive and multiplicative structures, with a well-developed theory of primitive elements, characters, and extensions [11]. Finite geometries and finite Fourier analysis show that finite algebraic systems can carry geometry and harmonic structure without first passing to an actual continuum [12–14]. Finite-dimensional quantum and phase-space constructions likewise show that finite fields can serve as coordinate systems for exact discrete phase labels [15–17].

In this paper, "Finite Ring Cosmology" denotes the physical interpretation of the Finite Ring Continuum (FRC) framework [18–20] as a finite algebraic symmetry space of admissible reference-frame positions, equipped with frame charts for units and physical comparison domains; no phenomenological cosmological model is proposed here. The FRC framework reconstructs arithmetic and geometry from framed finite fields rather than from an actual continuum. In the reading used here, shell residues are coordinate primitives. They acquire physical roles only when a frame assigns an origin, an additive unit, a phase generator, and scale readings. This is relational in the limited sense used here: physical roles are not assigned to isolated residues, but to residues placed within a frame of admissible comparisons [21,22]. We work with a framed symmetry-complete shell

$$\mathbb{F}_{\mathsf{p}}(\mathsf{t};0,1,e_{\mathsf{t}}), \qquad \mathsf{p}=4\mathsf{t}+1,$$

where 0 fixes the observer origin, 1 fixes the spatial unit scale of the additive meridian chart, and e_{t} fixes the temporal phase step of the multiplicative latitude chart. The additive and multiplicative frame actions are unit-free coordinate structures. A choice of meters, millimeters, seconds, or another conventional unit changes the observer's scale assignment; it does not change the cardinality of the shell.

The construction uses two physical-interface unit-domain assignments: $[L]$ for the space-like meridian chart, and $[T]$ for the time-like latitude or phase-step chart. Together they record how the same framed shell is read additively through 1 and exponentially through e_{t}. For a prime shell, finite modular unit domains are generated by expressions

$$[L]^r[T]^s, \qquad (r,s)\in\mathcal{D}_{\mathsf{p}},$$

where $\mathcal{D}_{\mathsf{p}}=\mathbb{Z}_{\mathsf{p}}\times\mathbb{Z}_{\mathsf{p}-1}$. Thus the spatial-domain exponent follows the order-p additive meridian period, while the temporal-domain exponent follows the order-$(\mathsf{p}-1)$ multiplicative phase period. The associated homogeneous quantity algebra is

$$\mathcal{A}_{\mathsf{p}}=\bigoplus_{(r,s)\in\mathcal{D}_{\mathsf{p}}}\mathbb{F}_{\mathsf{p}}[L]^r[T]^s.$$

Addition is fibrewise: a nonzero physical sum is homogeneous only when all nonzero summands lie in the same domain. Multiplication composes domains by adding exponents, and physical invariants are precisely neutral-domain monomials.

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The same framework gives a finite-domain reading of the constants that mediate between mechanics, quantum phase, and gravitation. In particular, the analysis below derives the native energy domain from the Planck relation $E = hf$, then derives the domains of action, mass, basic mechanical quantities, and gravitational coupling from the two generators $[L]$ and $[T]$. It also separates two roles that are often merged in continuum notation: conversion from conventional energy-action scales to phase count, and expression of that phase count in quarter-turn notation.

The objective of this paper is deliberately bounded. We construct the modular unit-domain algebra generated by $[L]$ and $[T]$, prove its homogeneity and invariant criteria, and then derive the basic mechanical and gravitational domains. The broader classification problem - whether all standard physical unit classes reduce to FRC shell domains once conventional metrological constants are expressed natively - is left for subsequent work.

It should be also explicitly stated that the proposed formalism is distinctively different and unrelated to the idea of "natural units", where some of the fundamental constants are set to 1 by fiat. The present construction does not set any constants to 1, but rather identifies the geometric interpretation of the modular unit domains that they connect. The numerical values of the constants change with the choice of conventional units, but the geometric constructs and modular domains they represent do not.

The paper is organized as follows. Section 2 recalls the framed FRC shell charts used in the construction. Section 3 separates residue phase labels from framed-complex labels and identifies the role of the finite quarter-turn. Section 4 defines the modular unit-domain group and homogeneous quantity algebra. Section 4.2 proves fibrewise homogeneity and neutral-domain invariance. Section 4.3 derives the energy and action domains and separates phase-count conversion from quarter-turn labeling. Section 4.4 records the speed domain and its packet normalization. Section 4.5 derives the mechanical and gravitational domains. Section 5 explains finite periodicity and gives examples. Section 6 summarizes the interpretation and scope.

2. FRC Shell Charts and Scale Readings

This section fixes the FRC frame-chart structure and scale readings needed for dimensional analysis. No global coordinate chart is assumed beyond the framed shell notation already used in the FRC programme.

For orientation, only four FRC facts are used here. A shell is a finite algebraic symmetry space of coordinate primitives, and a frame is the observer datum that assigns readable roles to them. In the symmetry-complete prime case $\mathsf{p} = 4\mathsf{t} + 1$, the additive meridian action has order p, while the multiplicative phase action on $\mathbb{F}_{\mathsf{p}}^{\times}$ has order $4\mathsf{t} = \mathsf{p} - 1$ and contains the internal quarter-turn subgroup $\{\pm 1, \pm i_{\mathsf{t}}\}$. The frame $(0, 1, e_{\mathsf{t}})$ fixes the origin, additive unit, and phase generator. Additive action gives meridian depth; multiplicative powers give the phase cycle $\mathbb{Z}_{\mathsf{p}-1}$. These two cyclic orders supply the two domain-label periods below. The Lorentzian quadratic-extension layer is background context only [18–20].

Definition 1 (Framed symmetry-complete shell)**.** *A framed symmetry-complete shell is a finite field with frame data*

$$\mathbb{F}_{\mathsf{p}}(\mathsf{t}; 0, 1, e_{\mathsf{t}}), \qquad \mathsf{p} = 4\mathsf{t} + 1,$$

where p is prime, $0 \in \mathbb{F}_{\mathsf{p}}$ is the chosen origin, $1 \in \mathbb{F}_{\mathsf{p}}$ is the chosen spatial scale anchor, and $e_{\mathsf{t}} \in \mathbb{F}_{\mathsf{p}}^{\times}$ is a primitive generator of the multiplicative group. The finite field $\mathbb{F}_{\mathsf{p}}$ is the coordinate shell, while $(0, 1, e_{\mathsf{t}})$ is the observer frame in which the additive and multiplicative charts are written.

The elements of $\mathbb{F}_{\mathsf{p}}$ are coordinate primitives. Physical roles are assigned by the frame and by the scale readings placed over that frame.

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The additive chart is the cyclic group

$$(\mathbb{F}_p, +),$$

which has cardinality p. Relative to the chosen origin 0, it supplies the meridian or space-like frame chart. If the observer assigns the scale label "meter" to the unit 1, then a coordinate $x \in \mathbb{F}_p$ receives an x-meter spatial reading in that frame. If the observer assigns the scale label "millimeter" instead, the same coordinate receives a millimeter-scale reading. The field and its cardinality remain fixed.

The multiplicative chart is the unit group

$$\mathbb{F}_p^\times,$$

which has order

$$p - 1 = 4t.$$

Relative to the chosen primitive generator e_t, it supplies the latitude or phase-action chart. The element e_t is the fine phase step. Its powers enumerate the nonzero phase cycle

$$1, e_t, e_t^2, \ldots, e_t^{p-2}.$$

A change of phase generator is written

$$e_t' = e_t^\epsilon,$$

where $\epsilon \in \mathbb{Z}_{p-1}^\times$ if e_t' is required to remain primitive.

Figure 1 (left) shows these data on the phase cycle for the finite ring $p = 13$, $t = 3$. The primitive frame value e_t, the induced half-period $\pi_t = 6$, and the induced quarter-turns $\pm i_t$ all appear directly on the same finite cycle. The right panel shows a concrete combinatorial sphere visualization in the case $p = 13$ [18]. The gray meridians and orange latitudes display the longitude-latitude cellulation induced by the orbit complex, while N marks the observer's origin, and the top orange latitude records the terminal shell boundary for which its image in the external spherical comparison is the antipode.

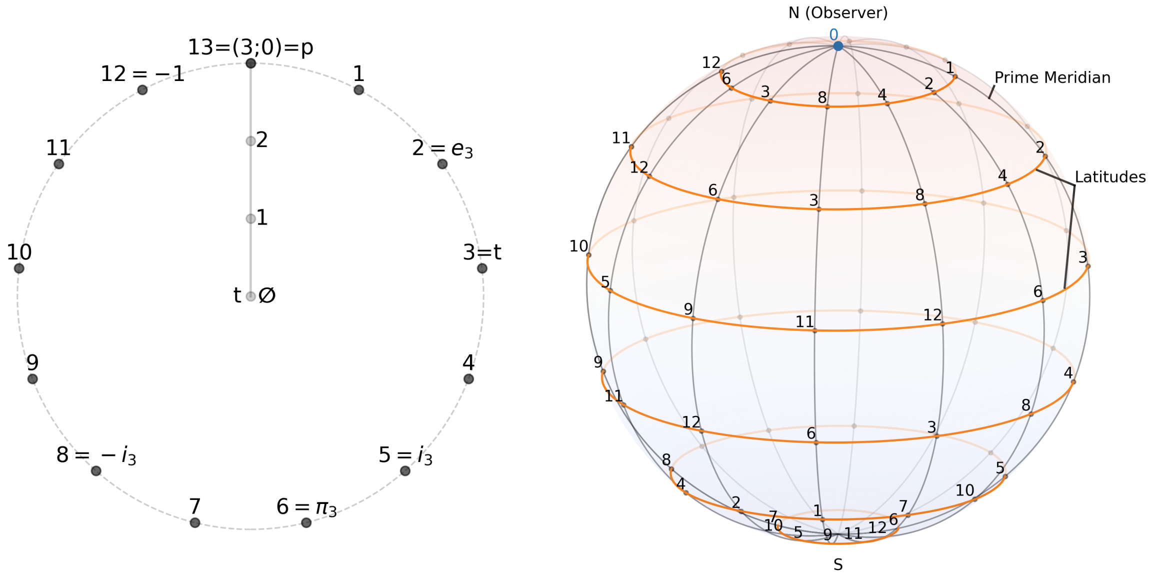

Figure 1. Illustration of the framed finite field Euclidean datum (left) and the corresponding combinatorial sphere (right) of the symmetry-complete shell $\mathbb{F}_p(3; 0, 1, e_t)$ with $p = 13$ [18]. The primitive generator value is $e_t = 2$, and the same cycle determines the induced values $\pi_t = 6$, as well as $\pm i_t$.

The paper uses the following convention.

Definition 2 (Unit-free coordinate actions)**.** *The additive meridian action* $(\mathbb{F}_p, +)$ *and multiplicative phase action* $\mathbb{F}_p^\times$ *are unit-free coordinate actions. A physical unit is a scale assignment to a frame chart of these actions. A measure is a coordinate reading relative to the assigned scale.*

Definition 3 (Typed modular reading)**.** *A physical-interface reading is not a bare residue. It is a coordinate or label together with the modular domain in which it is read. Additive meridian readings belong to the order-*p *domain, while phase-step readings belong to the order-*$p-1$ *domain. These domains have different periods, so their values cannot be added unless an explicit admissible conversion has first been specified.*

This distinction is essential. The shell supplies finite coordinate actions. Meters, seconds, joules, and other conventional units are scale readings attached to frame roles or to modular combinations of such assignments.

Proposition 1 (Scale assignment preserves shell cardinality)**.** *Changing the physical scale assigned to the additive meridian or multiplicative latitude chart does not change the finite shell. In particular,*

$$\operatorname{card}(\mathbb{F}_p) = p, \qquad \operatorname{card}(\mathbb{F}_p^\times) = p - 1$$

are invariant under such scale assignments.

Proof. A scale assignment labels the interpretation of coordinates already present in $\mathbb{F}_p$ or $\mathbb{F}_p^\times$. It does not alter the underlying finite set, its addition, its multiplication, or its group order. Hence the cardinalities remain p and $p-1$, respectively. □

The affine part of the frame is expressed by transformations

$$x' = mx + a, \qquad a \in \mathbb{F}_p, \quad m \in \mathbb{F}_p^\times.$$

Here a changes origin and m changes spatial scale. This is the same frame logic as the framed finite-field algebra of FRC. The phase part is expressed by reindexing

$$e_t' = e_t^{\epsilon}.$$

The dimensional analysis developed below uses these frame charts but does not require any additional coordinate ontology.

3. Residue Phase Labels and Framed-Complex Labels

Quantum notation uses an imaginary unit to express phase evolution. In FRC the relevant quarter-turn is already present as a finite-field residue [18]. The purpose of this section is to separate two label-level formalisms for the same shell phase geometry.

Since $p = 4t + 1$, the multiplicative group $\mathbb{F}_p^\times$ has order $4t$. Therefore, it has a unique subgroup of order four. In a chosen clockwise frame this subgroup is written as

$$\mathcal{Q}_p = \{1, -i_t, -1, i_t\},$$

where

$$i_t^2 = -1.$$

If e_t is the chosen primitive generator, then e_t^t is a generator of this order-four subgroup. Following the established clockwise orientation convention, one may write $e_t^t = -i_t$. The present paper uses only the fact that e_t^t is a cardinal quarter-turn and that its square is -1.

There are two convenient ways to label this structure.

Definition 4 (Residue phase label)**.** *The residue phase label writes a phase as a power*

$$e_{\mathsf{t}}^{k} \in \mathbb{F}_{\mathsf{p}}^{\times}.$$

In this notation the cardinal packet is

$$1, \quad e_{\mathsf{t}}^{\mathsf{t}}, \quad e_{\mathsf{t}}^{2\mathsf{t}}, \quad e_{\mathsf{t}}^{3\mathsf{t}}, \quad e_{\mathsf{t}}^{4\mathsf{t}} = 1.$$

Definition 5 (Framed-complex label)**.** *The framed-complex label writes the same quarter-turn structure using expressions of the form*

$$a + b i_{\mathsf{t}},$$

with $i_{\mathsf{t}}^2 = -1$. This is not an external imaginary extension at a symmetry-complete prime shell. It is a coordinate language for a finite-field residue already present in $\mathbb{F}_{\mathsf{p}}$.

Figure 2 shows this label-level presentation for the finite framed field $\mathbb{F}_{13}(3;0,1,2)$. The red labels give the clockwise framed-imaginary notation $z i_{\mathsf{t}}$, while the purple labels give the corresponding residues in $\mathbb{F}_{13}$. The vertical axis is therefore a framed presentation of shell residues within the same finite field.

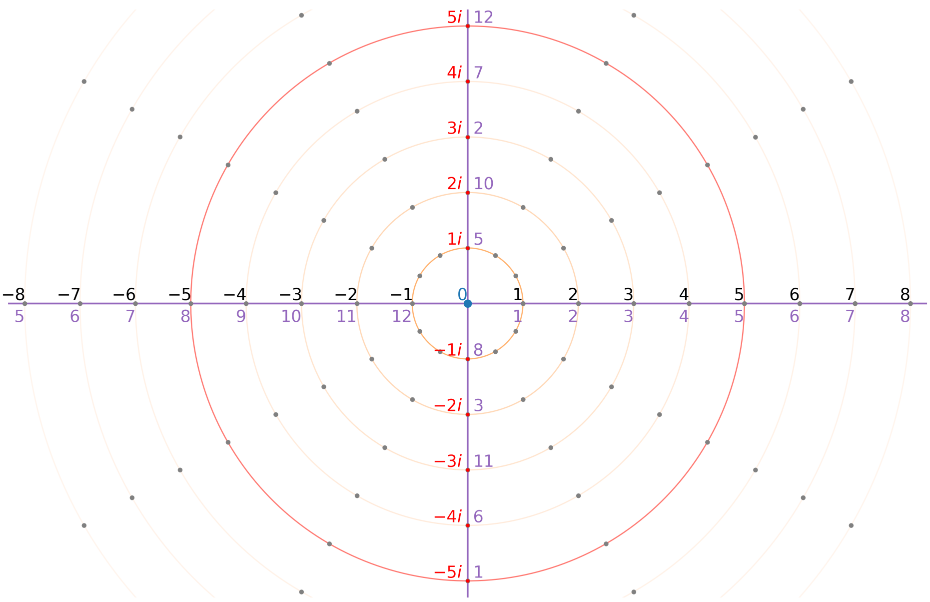

Figure 2. Framed complex number plane ${}^{\mathsf{f}}\mathbb{C}$ in the finite framed field $\mathbb{F}_{13}(3;0,1,2)$. The horizontal axis represents the f-reals ${}^{\mathsf{f}}\mathbb{R}$ on the prime meridian. The vertical axis represents the clockwise framed-imaginary labels $z i_{\mathsf{t}}$, shown in red, and the corresponding residues in $\mathbb{F}_{13}$, shown in purple.

Proposition 2 (Quarter-turn label)**.** *In a symmetry-complete framed shell $\mathbb{F}_{\mathsf{p}}(\mathsf{t};0,1,e_{\mathsf{t}})$, the element i_{t} is the finite shell quarter-turn label. Multiplication by i_{t} is the shell analogue of the quarter-turn represented by i in the continuum complex notation of quantum mechanics.*

Proof. The group $\mathbb{F}_{\mathsf{p}}^{\times}$ is cyclic of order $4\mathsf{t}$. Hence it contains a unique subgroup of order four, generated by $e_{\mathsf{t}}^{\mathsf{t}}$. Any generator i_{t} of that subgroup satisfies $i_{\mathsf{t}}^4 = 1$ and $i_{\mathsf{t}}^2 \neq 1$. Since the unique element of order two in $\mathbb{F}_{\mathsf{p}}^{\times}$ is -1, it follows that $i_{\mathsf{t}}^2 = -1$. Thus i_{t} is the quarter-turn element of the shell. □

This proposition clarifies the role of the imaginary unit in the quantum phase factor

$$\exp\left(-i\frac{E\tau}{\hbar}\right).$$

The expression contains two different conversions. The factor $\hbar$ converts the conventional energy-time product $E\tau$ into a phase count. The symbol i expresses that phase count in quarter-turn framed-complex notation. The phase count is a frame-readable quantity; i_{t} supplies the quarter-turn label in which that count is written. In FRC the second role is represented by the actual shell residue i_{t}. Thus $\hbar$ and i_{t} are not identified. The first is an action-to-phase-count conversion; the second is a quarter-turn phase label.

4. Modular Unit-Domain Algebra

We now define the finite unit-domain system generated by the two shell charts. The space-like generator is denoted $[L]$, and the time-like generator is denoted $[T]$. These are formal interface generators attached to the framed shell charts.

Definition 6 (Unit-domain assignments)**.** *The symbol $[L]$ denotes the domain generator attached to the additive meridian chart of 1. The symbol $[T]$ denotes the domain generator attached to the multiplicative latitude or phase-step chart of e_{t}. They record two presentations of one framed shell structure: additive presentation through 1, and exponential or phase presentation through e_{t}. They form the two-generator interface used for dimensional analysis and belong to the physical-interface language rather than to $\mathbb{F}_{\mathsf{p}}$ itself.*

4.1. Finite Scale-Character Exponents

The notation $\mathbb{Z}_{\mathsf{p}-1}$ denotes the cyclic exponent label system with $\mathsf{p}-1$ elements. In the FRC shell it records the finite scale-character exponent attached to the multiplicative latitude group $\mathbb{F}_{\mathsf{p}}^{\times}$. Once a primitive frame value $e_{\mathsf{t}} \in \mathbb{F}_{\mathsf{p}}^{\times}$ is chosen, the nonzero shell residues are labelled by powers of e_{t} modulo the phase period $\mathsf{p}-1$. Thus, the time-like domain exponent is a finite scale-character label. The space-like exponent is attached instead to the additive meridian period $\mathbb{Z}_{\mathsf{p}}$, giving the domain-label set $\mathcal{D}_{\mathsf{p}}$.

Definition 7 (Modular unit-domain labels)**.** *Let*

$$\mathcal{D}_{\mathsf{p}} := \mathbb{Z}_{\mathsf{p}} \times \mathbb{Z}_{\mathsf{p}-1}.$$

For $(r,s) \in \mathcal{D}_{\mathsf{p}}$, define the modular unit-domain generator

$$U_{r,s} := [L]^r [T]^s.$$

The neutral domain is

$$U_{0,0} = 1.$$

Theorem 1 (Modular unit-domain group)**.** *The set*

$$\{U_{r,s} : (r,s) \in \mathcal{D}_{\mathsf{p}}\}$$

forms a finite abelian group under the product

$$U_{r,s} U_{r',s'} = U_{r+r',s+s'}.$$

The inverse of $U_{r,s}$ is $U_{-r,-s}$, and the group is isomorphic to $\mathcal{D}_{\mathsf{p}}$.

Proof. Closure follows from componentwise cyclic addition in $\mathcal{D}_{\mathsf{p}}$: the first component is reduced modulo p, and the second component is reduced modulo $\mathsf{p}-1$. Associativity and commutativity follow from associativity and commutativity of addition in the two cyclic factors. The identity is $U_{0,0}$. The inverse of $U_{r,s}$ is $U_{-r,-s}$, since

$$U_{r,s} U_{-r,-s} = U_{0,0}.$$

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The map $(r,s) \mapsto U_{r,s}$ is a group isomorphism from the additive group $\mathcal{D}_\mathsf{p}$ to the multiplicative domain-label group. □

Definition 8 (Homogeneous quantity algebra)**.** *The finite homogeneous quantity algebra generated by the shell domains is*

$$\mathcal{A}_\mathsf{p} := \bigoplus_{(r,s)\in\mathcal{D}_\mathsf{p}} \mathbb{F}_\mathsf{p} U_{r,s}.$$

The homogeneous fibre of domain (r,s) is

$$\mathcal{A}_\mathsf{p}^{(r,s)} := \mathbb{F}_\mathsf{p} U_{r,s}.$$

An element of $\mathcal{A}_\mathsf{p}^{(r,s)}$ has the form

$$Q = qU_{r,s}, \qquad q \in \mathbb{F}_\mathsf{p}.$$

The domain label is part of the type of the quantity. Thus $qU_{r,s}$ is not merely the coefficient $q \in \mathbb{F}_\mathsf{p}$; it is the coefficient q read in the homogeneous fibre $\mathbb{F}_\mathsf{p} U_{r,s}$. Addition is defined as physical-interface arithmetic only inside one fixed fibre.

Theorem 2 (Graded quantity algebra)**.** *The algebra $\mathcal{A}_\mathsf{p}$ is graded by $\mathcal{D}_\mathsf{p}$. More precisely,*

$$\mathcal{A}_\mathsf{p}^{(r,s)} \mathcal{A}_\mathsf{p}^{(r',s')} \subseteq \mathcal{A}_\mathsf{p}^{(r+r',s+s')}.$$

Proof. Let $Q = qU_{r,s} \in \mathcal{A}_\mathsf{p}^{(r,s)}$ and $R = q'U_{r',s'} \in \mathcal{A}_\mathsf{p}^{(r',s')}$. Define multiplication by

$$(qU_{r,s})(q'U_{r',s'}) = qq'U_{r+r',s+s'}.$$

Since $qq' \in \mathbb{F}_\mathsf{p}$, the product lies in $\mathcal{A}_\mathsf{p}^{(r+r',s+s')}$. This proves the grading law. □

The interpretation is direct: addition is expected to be internal to a homogeneous fibre, while multiplication composes domains. The next section makes the additive statement exact.

4.2. Homogeneous Addition and Neutral-Domain Invariants

The algebra $\mathcal{A}_\mathsf{p}$ is a direct sum of homogeneous domain fibres. This direct-sum structure is the finite FRC version of dimensional homogeneity.

This typed structure prevents cross-domain addition. A value read in the additive order-p domain and a value read in the phase order-$\mathsf{p}-1$ domain are not summands of one physical-interface quantity. The same obstruction appears after multiplication. If $Q = qU_{r,s}$, then

$$Q^2 = q^2 U_{2r,2s}.$$

Thus Q and Q^2 generally lie in different homogeneous fibres. The formal expression $Q + Q^2$ is a direct-sum expression, not a homogeneous physical sum, unless $U_{r,s} = U_{2r,2s}$.

Definition 9 (Domain of a homogeneous quantity)**.** *A nonzero element $Q \in \mathcal{A}_\mathsf{p}$ is homogeneous of domain (r,s) if*

$$Q \in \mathbb{F}_\mathsf{p} U_{r,s} \quad \text{and} \quad Q \neq 0.$$

In that case we write

$$\mathrm{dom}_\mathsf{p}(Q) = (r,s).$$

The zero element is assigned a domain only by context, since it belongs to every homogeneous fibre.

Theorem 3 (Fibrewise addition)**.** *Let*

$$Q_j = q_j U_{r_j,s_j} \qquad (1 \le j \le N)$$

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be homogeneous elements of $\mathcal{A}_{\mathsf{p}}$. The sum

$$Q_1 + \cdots + Q_N$$

is either zero or homogeneous if all nonzero summands have the same domain. In general, the component of the sum in the fibre $\mathbb{F}_{\mathsf{p}} U_{a,b}$ is the sum of all summands whose domain is (a, b). Hence the sum is nonzero and homogeneous of domain (r, s) exactly when its (r, s)-component is nonzero and all other fibre components vanish. As a physical-interface addition rule, a sum is admissible without cross-domain cancellation only when all nonzero summands are supplied in the same domain.

Proof. If all nonzero summands lie in the same fibre $\mathbb{F}_{\mathsf{p}} U_{r,s}$, then their sum also lies in that fibre. If the sum is nonzero, it is a homogeneous element of domain (r, s).

For the general statement, group the summands by their domain labels. Since $\mathcal{A}_{\mathsf{p}}$ is a direct sum, every element has a unique decomposition into components in the fibres $\mathbb{F}_{\mathsf{p}} U_{a,b}$. The component in fibre (a, b) is precisely the sum of the terms $q_j U_{a,b}$ with $(r_j, s_j) = (a, b)$. Therefore the total sum is homogeneous of domain (r, s) exactly when the (r, s)-component is nonzero and every component in a different fibre is zero. The final admissibility statement is the corresponding physical-interface rule that addition is defined inside one homogeneous fibre. □

Thus, a formal expression such as

$$3[L]^2 + 5[L]^3$$

may be written in the direct sum, but it is not one homogeneous physical quantity. The usual rule that areas cannot be added to volumes is the fibrewise addition theorem in this setting.

Theorem 4 (Neutral-domain invariant criterion). *Let $Q_1, \ldots, Q_N$ be nonzero homogeneous quantities with*

$$\mathrm{dom}_{\mathsf{p}}(Q_j) = (r_j, s_j).$$

For exponent choices k_j, define the monomial

$$M = \prod_{j=1}^{N} Q_j^{k_j}.$$

Then

$$\mathrm{dom}_{\mathsf{p}}(M) = \sum_{j=1}^{N} k_j (r_j, s_j).$$

In particular, M is a neutral-domain invariant if and only if

$$\sum_{j=1}^{N} k_j (r_j, s_j) = (0, 0)$$

in $\mathcal{D}_{\mathsf{p}}$.

Proof. Write $Q_j = q_j U_{r_j, s_j}$. By repeated use of the multiplication rule,

$$Q_j^{k_j} = q_j^{k_j} U_{k_j r_j, k_j s_j}.$$

Multiplying these terms gives

$$M = \left(\prod_{j=1}^{N} q_j^{k_j} \right) U_{\sum_j k_j r_j, \sum_j k_j s_j}.$$

The domain formula follows. The neutral-domain condition is exactly the statement that the resulting domain label is $(0, 0)$. □

This theorem is the modular FRC counterpart of the usual dimensionless-invariant condition in classical dimensional analysis. The analogy with Noether's theorem is structural rather than variational: the present paper has no action principle, but it uses the same guiding logic that admissible transformations determine invariant physical content [5].

4.3. Energy, Action, and Phase Conversion

Energy is the first derived domain that differs substantially from the standard mechanical presentation. In the present shell-domain reading, energy is not first introduced as mass times velocity squared. It is forced into the temporal recurrence domain by the Planck relation.

Definition 10 (Temporal recurrence domain)**.** *A frequency f counts phase cycles per temporal frame step. Its domain is*

$$[f] = [T]^{-1}.$$

The physical bridge to energy is the Planck relation

$$E = hf.$$

This relation does not merely compare two measured quantities. It says that every energy value is proportional to a temporal recurrence rate by one universal constant h. In the modular unit-domain algebra, domains distinguish comparison fibres. A single universal conversion constant cannot define a distinctive fibre between E and f: any such separation would be carried entirely by h, but h is the same scale for every frame-local expression and fixes only the conventional energy assigned to one phase cycle. Hence, h is a metrological scale conversion, not an independent modular domain.

A complete phase turn can be counted in two equivalent ways. In cycle units it is one cycle; in angular units it is 2π. In a finite cyclic phase representation, the complete turn is not represented by a real continuum magnitude but by the order of the phase cycle. For the shell phase cycle that order is $\mathsf{p} - 1$, so

$$2\pi \quad \text{is represented by} \quad \mathsf{p} - 1.$$

Propagation over a finite cycle of order N is written

$$\exp\left(-i2\pi\frac{nk}{N}\right).$$

Setting $N = \mathsf{p} - 1$ and comparing this with

$$\exp\left(-i2\pi\frac{E\tau}{h}\right)$$

shows that, when E and τ are phase labels in $\mathbb{Z}_{\mathsf{p}-1}$, h is represented by the full phase-cycle order:

$$h = \mathsf{p} - 1 \equiv -1.$$

Thus, h converts frequency measured relative to the chosen time scale into frequency measured relative to the complete phase cycle. It is not a separate modular unit domain.

Using angular frequency gives $\omega = 2\pi f$ and $E = \hbar\omega$. Because h and 2π are represented by the same complete-cycle count, the angular phase-normalized value is

$$\hbar = \frac{h}{2\pi} \equiv 1.$$

This is a statement about phase-count normalization, not about the SI numerical value of $\hbar$. It says that $\hbar$ is the unit conversion for angular phase count.

Proposition 3 (Energy-frequency domain equivalence)**.** *In the phase-normalized interface, energy and frequency occupy the same modular unit domain:*

$$[E] = [f] = [T]^{-1}.$$

Proposition 4 (Action neutrality)**.** *By the forced energy-frequency domain equivalence, the energy-time product has neutral domain:*

$$[E][T] = 1.$$

Consequently, h and $\hbar$ are domain-neutral in the phase-normalized interface:

$$[h] = [\hbar] = 1.$$

This proposition is not a claim about the SI numerical values of h or $\hbar$. It says that both are domain-neutral phase conversions. The equality $\hbar = 1$ above is representational: in angular phase units, one radian-normalized phase count is the unit. This is still not the finite quarter-turn label i_{t}. The roles are different. In conventional units, h fixes the energy scale assigned to one complete phase cycle, while $\hbar$ fixes the corresponding angular phase scale. The quarter-turn label is supplied by i_{t}. Thus the phase factor has two components:

$$h, \hbar : \text{ conventional energy-action units } \longrightarrow \text{ phase count},$$

and

$$i_{\mathrm{t}} : \text{ phase count } \longrightarrow \text{ framed-complex quarter-turn label.}$$

This separation is useful because it explains why energy can be meaningful without spatial propagation. A frame-localized shell-vector pattern may carry energy through an internal temporal recurrence rate in a chosen frame. In classical relativity, the same energy domain can be read through spacetime motion, stress-energy, or gravitational coupling. The domain is the same; the frame-readable carrier expression differs.

The phase-count language also touches the informational tradition in quantum foundations. In Wheeler's "it from bit" formulation, physical facts are tied to finite distinctions, while Landauer stresses that information requires physical embodiment [23,24]. Here the use is narrower: phase count is a finite physical-interface quantity, not a claim that the full FRC shell is a digital computation.

4.4. Speed and the Cardinal Shell Bound

The speed domain is the relation between the space-like meridian chart and the time-like latitude chart.

Definition 11 (Speed domain)**.** *The speed domain is*

$$[v] := [L][T]^{-1}.$$

This domain statement is independent of any metric or geodesic construction. Geometrically, one may interpret speed as a comparison between the spatial meridian chart and the temporal phase-step chart, but the present paper only uses the domain relation $[v] = [L][T]^{-1}$. The temporal scale $[T]$ is an admissible scale assignment, so the interface expression where speed is defined uses a nonzero temporal increment and $[T]^{-1}$ exists. The speed-bound domain is the frame-invariant comparison

$$[c] = [L][T]^{-1}.$$

Its numerical value depends on the chosen $[L]$ and $[T]$ units, but the domain relation does not.

The framed shell also gives a native finite packet count for this speed-bound relation. Since $\mathsf{p} = 4\mathsf{t} + 1$, the multiplicative phase cycle has order $4\mathsf{t}$. Its cardinal quarter-turns are

$$1, \quad e_{\mathsf{t}}^{\mathsf{t}}, \quad e_{\mathsf{t}}^{2\mathsf{t}}, \quad e_{\mathsf{t}}^{3\mathsf{t}}, \quad e_{\mathsf{t}}^{4\mathsf{t}} = 1.$$

An elementary cardinal packet tick is the transition from one cardinal packet element to the next. In this normalization, a quarter-turn contains exactly t fine phase-generator steps.

Definition 12 (Shell-native packet units)**.** *In shell-native packet units, one temporal unit is one cardinal packet tick, and the speed-bound packet count is the maximal meridian advance count* t *during one such tick.*

Proposition 5 (Shell-native packet count of the speed bound)**.** *In shell-native packet units, the numerical packet count of the speed bound is*

$$c = \mathsf{t} = \frac{\mathsf{p} - 1}{4}.$$

Proof. One cardinal packet tick is one quarter-turn of the phase cycle. Since the phase cycle has order $4\mathsf{t}$, one quarter-turn contains t fine phase-generator steps. In shell-native packet units, the maximal meridian advance during this tick is the quarter period t. Hence, the maximal meridian advance per packet tick is $c = \mathsf{t}$. □

The equality $c = \mathsf{t}$ is a shell-native packet normalization. A different observer choice of $[L]$ and $[T]$ changes the numerical representation of a speed in the usual way, but it does not change the speed-bound domain. The invariant statement is

$$[c] = [L][T]^{-1}.$$

This domain role is compatible with the Lorentzian extension layer of FRC [20], where causal structure is expressed by the quadratic-extension and square-class construction. It is important to note however, that the Lorentzian construction in [20] refers to a transition between two subsequent instantaneous shells in the shell temporal succession, making c an explicit time-like element external to the instant shell. Here, we explore the structure of the complete symmetry space of all space-time frames of reference, making c internal to the shell. The present paper records only the dimensional interface: a speed-bound relation compares the additive meridian scale with the multiplicative phase-step scale.

4.5. Mass, Mechanics, and Gravitational Coupling

With the energy and speed domains fixed, the basic mechanical domains follow by algebra. The first important consequence is the domain of mass.

Proposition 6 (Mass domain)**.** *Using the dimensional identity*

$$[m] = [E][v]^{-2},$$

together with

$$[E] = [T]^{-1}, \qquad [v] = [L][T]^{-1},$$

one obtains

$$[m] = [L]^{-2}[T].$$

Proof. Substitute the energy and speed domains:

$$[m] = [T]^{-1}\left([L][T]^{-1}\right)^{-2}.$$

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Since
$$([L][T]^{-1})^{-2} = [L]^{-2}[T]^2,$$
we obtain
$$[m] = [T]^{-1}[L]^{-2}[T]^2 = [L]^{-2}[T].$$
□

Thus, mass is not primitive in this domain system. It is a coupling domain between the temporal phase chart and the inverse square of the additive spatial chart. This statement is a domain identity. It does not by itself assert a microscopic mechanism for mass; it fixes the shell-domain in which mass quantities live.

The remaining elementary mechanical domains follow similarly. We write $[a_{\mathrm{acc}}]$ for acceleration to avoid confusing it with the affine shift parameter.

Proposition 7 (Basic mechanical domains)**.** *In the FRC-native* $[L]$, $[T]$ *domain system,*
$$[a_{\mathrm{acc}}] = [L][T]^{-2},$$
$$[F] = [L]^{-1}[T]^{-1},$$
$$[p_{\mathrm{mom}}] = [L]^{-1},$$
$$[S_{\mathrm{action}}] = 1,$$
$$[P_{\mathrm{power}}] = [T]^{-2},$$
and
$$[P_{\mathrm{pressure}}] = [L]^{-3}[T]^{-1}.$$

Proof. Acceleration is velocity per time:
$$[a_{\mathrm{acc}}] = [v][T]^{-1} = [L][T]^{-1}[T]^{-1} = [L][T]^{-2}.$$
Force is mass times acceleration:
$$[F] = [m][a_{\mathrm{acc}}] = ([L]^{-2}[T])([L][T]^{-2}) = [L]^{-1}[T]^{-1}.$$
Momentum is mass times velocity:
$$[p_{\mathrm{mom}}] = [m][v] = ([L]^{-2}[T])([L][T]^{-1}) = [L]^{-1}.$$
Action is energy times time:
$$[S_{\mathrm{action}}] = [E][T] = [T]^{-1}[T] = 1.$$
Power is energy per time:
$$[P_{\mathrm{power}}] = [E][T]^{-1} = [T]^{-2}.$$
Pressure is force per area:
$$[P_{\mathrm{pressure}}] = [F][L]^{-2} = [L]^{-1}[T]^{-1}[L]^{-2} = [L]^{-3}[T]^{-1}.$$
□

The action result is especially important. It is consistent with the neutral-domain status of h and $\hbar$: action, phase-cycle scale conversion, and angular-phase scale conversion all live in the neutral phase-conversion domain.

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4.6. Gravitational Coupling

Newton's constant is a particularly informative test case because it translates mass-coupling into geometric length and time scales.

Proposition 8 (Domain of Newton's gravitational constant). *From Newton's law*

$$F = \frac{Gm_1 m_2}{r^2},$$

the gravitational constant has FRC-native domain

$$[G] = [L]^5[T]^{-3}.$$

Proof. Newton's law gives

$$[G] = [F]\frac{[r]^2}{[m]^2}.$$

Using $[r] = [L]$, $[F] = [L]^{-1}[T]^{-1}$, and $[m] = [L]^{-2}[T]$, we get

$$[G] = ([L]^{-1}[T]^{-1})\frac{[L]^2}{([L]^{-2}[T])^2}.$$

Since

$$([L]^{-2}[T])^2 = [L]^{-4}[T]^2,$$

we obtain

$$[G] = [L]^{-1}[T]^{-1}[L]^2[L]^4[T]^{-2} = [L]^5[T]^{-3}.$$

□

Table 1 summarizes the conventional mechanical domains used above and their native FRC assignments.

Table 1. Conventional mechanical domains and their FRC-native domain assignments.

Conventional domain	Meaning	FRC-native domain
$[M]$	mass	$[L]^{-2}[T]$
$[E]$	energy	$[T]^{-1}$
$[S]$	action	1
$[G]$	gravitational coupling	$[L]^5[T]^{-3}$

Several geometric consequences follow immediately.

Corollary 1 (Geometric conversion domains). *The gravitational coupling domain satisfies*

$$\left[\frac{G}{c^3}\right] = [L]^2,$$

$$\left[\frac{Gm}{c^2}\right] = [L],$$

$$\left[\frac{Gm}{c^3}\right] = [T].$$

Moreover, if ρ_m is mass density and ρ_E is energy density, then

$$[G\rho_m] = [T]^{-2},$$

and

$$\left[\frac{G}{c^4}\rho_E\right] = [L]^{-2}.$$

Proof. Since $[c] = [L][T]^{-1}$, we have $[c]^3 = [L]^3[T]^{-3}$. Hence

$$\left[\frac{G}{c^3}\right] = [L]^5[T]^{-3}[L]^{-3}[T]^3 = [L]^2.$$

For a mass m,

$$\left[\frac{Gm}{c^2}\right] = [L]^5[T]^{-3}[L]^{-2}[T][L]^{-2}[T]^2 = [L],$$

and

$$\left[\frac{Gm}{c^3}\right] = [L]^5[T]^{-3}[L]^{-2}[T][L]^{-3}[T]^3 = [T].$$

Mass density has domain

$$[\rho_m] = [m][L]^{-3} = [L]^{-5}[T],$$

so

$$[G\rho_m] = [L]^5[T]^{-3}[L]^{-5}[T] = [T]^{-2}.$$

Energy density has domain

$$[\rho_E] = [E][L]^{-3} = [L]^{-3}[T]^{-1}.$$

Finally,

$$\left[\frac{G}{c^4}\right] = [L]^5[T]^{-3}[L]^{-4}[T]^4 = [L][T],$$

and therefore

$$\left[\frac{G}{c^4}\rho_E\right] = [L][T][L]^{-3}[T]^{-1} = [L]^{-2}.$$

□

The last expression has the domain of curvature. Thus, the FRC-native domain algebra reproduces the standard dimensional role of the Einstein coupling: energy density is converted into curvature by G/c^4. The result also shows that G/c^3 has area domain, matching the dimensional structure of the Planck area when h and $\hbar$ are neutral.

5. Finite Periodicity and Examples

The finite domain system is modular. Since domain labels lie in

$$\mathcal{D}_\mathsf{p} = \mathbb{Z}_\mathsf{p} \times \mathbb{Z}_{\mathsf{p}-1},$$

one has

$$[L]^{r+\mathsf{p}}[T]^s = [L]^r[T]^s,$$

and

$$[L]^r[T]^{s+\mathsf{p}-1} = [L]^r[T]^s.$$

This is not a defect. It is the global finite-periodic structure of the shell. Ordinary nonperiodic exponent bookkeeping is recovered as a local comparison inside a bounded exponent horizon.

Theorem 5 (Local recovery of classical exponent bookkeeping)**.** *Let $H \geq 0$ be an exponent horizon. Suppose*

$$\mathsf{p} - 1 > 2H.$$

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Then the finite modular label system $\mathcal{D}_{\mathsf{p}}$ distinguishes all conventional exponent pairs (r,s) satisfying

$$|r| \leq H, \qquad |s| \leq H.$$

Proof. Suppose two conventional exponent pairs (r,s) and (r',s'), each with components bounded in absolute value by H, have the same image in $\mathcal{D}_{\mathsf{p}}$. Then $r - r'$ is cyclically zero modulo p, and $s - s'$ is cyclically zero modulo $\mathsf{p} - 1$. But

$$|r - r'| \leq 2H, \qquad |s - s'| \leq 2H.$$

Since $\mathsf{p} - 1 > 2H$, also $\mathsf{p} > 2H$. Therefore the only multiples of p or $\mathsf{p} - 1$ in this range are zero. Hence $r = r'$ and $s = s'$. □

Thus, conventional dimensional analysis is the bounded-horizon comparison of the finite modular shell-domain structure.

Example 1 (The shell $\mathsf{p} = 13$)**.** *For* $\mathsf{p} = 13$*, one has*

$$\mathsf{p} - 1 = 12, \qquad \mathsf{t} = 3.$$

The domain labels lie in $\mathbb{Z}_{13} \times \mathbb{Z}_{12}$. If the exponent horizon is $H = 5$, then

$$12 > 10 = 2H,$$

so all conventional exponent pairs with entries between -5 and 5 are distinguished exactly by the finite modular labels.

Example 2 (Kinetic energy consistency)**.** *The domain identity $E \sim mv^2$ is checked by*

$$[m][v]^2 = ([L]^{-2}[T])([L]^2[T]^{-2}) = [T]^{-1} = [E].$$

Thus, the usual mechanical expression for energy is compatible with the forced energy-frequency domain equivalence.

Example 3 (Squaring changes the modular domain)**.** *Let $Q = q[L]$. Then*

$$Q^2 = q^2[L]^2.$$

The expression $Q + Q^2$ is not a homogeneous physical-interface sum, because the first term lies in the length fibre and the second term lies in the area fibre. In FRC this is a modular-domain obstruction: multiplication changes the domain label, and addition remains valid only inside one fixed domain fibre.

Example 4 (Phase exponent neutrality)**.** *The phase exponent in the quantum expression is neutral:*

$$[E][T][\hbar]^{-1} = [T]^{-1}[T]1^{-1} = 1.$$

The factor $\hbar$ converts conventional angular-action units into phase count, while the quarter-turn label is carried by i_{t}.

Example 5 (Schwarzschild length domain)**.** *The expression*

$$\frac{Gm}{c^2}$$

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has domain

$$[L].$$

This is the dimensional structure behind the Schwarzschild radius. In the FRC-native domain reading, G converts the mass-coupling domain into a geometric length domain.

Example 6 (Gravitational frequency domain)**.** *The expression*

$$\frac{Gm}{r^3}$$

has domain

$$\frac{[L]^5[T]^{-3}[L]^{-2}[T]}{[L]^3} = [T]^{-2}.$$

Thus, gravitational mass distributed over a spatial scale generates a squared temporal recurrence domain, as in orbital-frequency expressions.

6. Discussion and Conclusion

The construction above treats dimensional analysis as a finite shell-domain calculus. The additive meridian and multiplicative latitude charts of a framed FRC shell are unit-free coordinate actions. Once spatial and temporal scale assignments are fixed, these actions generate a modular domain algebra

$$\mathcal{A}_{\mathsf{p}} = \bigoplus_{(r,s)\in\mathcal{D}_{\mathsf{p}}} \mathbb{F}_{\mathsf{p}}[L]^r[T]^s, \qquad \mathcal{D}_{\mathsf{p}} = \mathbb{Z}_{\mathsf{p}} \times \mathbb{Z}_{\mathsf{p}-1}.$$

Physical homogeneity is fibrewise addition in this algebra. Physical invariance is neutrality of the total domain label.

Several consequences are immediate. First, energy is forced into the same modular domain as temporal recurrence frequency. Since $E = hf$, h converts frequency measured relative to the chosen time scale into frequency measured relative to the complete phase cycle. No distinctive unit-domain separation between energy and frequency can be defined in the phase-normalized interface:

$$[E] = [T]^{-1}.$$

This does not require spatial propagation. In a quantum expression, energy can appear as local phase cycling. The conversion from conventional energy-action units to phase count is carried by h for complete-cycle frequency and by $\hbar = h/(2\pi)$ for angular frequency. In the finite phase-count chart, $h = \mathsf{p} - 1$ and 2π is represented by the same complete-cycle order, so $\hbar = 1$ in angular phase units. The framed-complex phase direction is carried by the finite quarter-turn label i_{t}. These are separate roles: $h, \hbar :$ energy-action units $\to$ phase count, while $i_{\mathsf{t}} :$ phase count $\to$ quarter-turn label. This separation removes the need to regard the imaginary unit of quantum notation as an additional physical dimension. In a symmetry-complete FRC shell, the relevant quarter-turn is already an internal field residue.

Second, mass is not primitive in the $[L]$, $[T]$-domain system. It is derived from energy and speed:

$$[m] = [E][v]^{-2} = [L]^{-2}[T].$$

This identifies mass as a coupling domain between the temporal phase chart and the inverse square of the additive spatial chart. This statement is purely dimensional at the level proved here, but it gives a precise algebraic location for mass inside the shell-domain system.

Third, gravitational coupling becomes a geometric conversion domain:

$$[G] = [L]^5[T]^{-3}.$$

Consequently, $[G/c^3] = [L]^2$, $[Gm/c^2] = [L]$, and $[(G/c^4)\rho_E] = [L]^{-2}$. These identities reproduce the dimensional structure of Planck area, Schwarzschild length, and Einstein curvature coupling when h and $\hbar$ are neutral as phase-count scale conversions.

The natural-constant system is therefore interpreted as domain conversion data. The constant c converts temporal scale to spatial scale at the domain level. In shell-native packet units the corresponding speed-bound count is $c = \mathsf{t} = (\mathsf{p}-1)/4$; under observer choices of $[L]$ and $[T]$, its numerical representation changes with the chosen units, while the speed-bound domain itself remains invariant. The Planck constants h and $\hbar$ convert conventional energy-action units to phase count; h is represented by the complete phase-cycle order $\mathsf{p}-1$, while $\hbar = h/(2\pi) = 1$ in angular phase units. They do not add a modular unit domain. The constant G converts mass-coupling into geometric length, time, and curvature scales. The thermodynamic constant k_B converts temperature scale into energy scale; the classification of the temperature domain within FRC is left for later work.

The two-generator notation is the classical-dimensional interface. In the underlying FRC shell, $[L]$ and $[T]$ are two scale assignments of one framed finite reference-frame structure: 1 supplies the additive meridian-scale assignment, and e_{t} supplies the multiplicative or exponential phase-scale assignment. This is used here as a reading of the domain algebra; a canonical map between the two charts is left to later work.

The coefficient records a coordinate or phase count inside its assigned modular domain; by itself it is not a physical measurement.

The dimensional algebra constructed here is the physical-interface projection of one finite reference-frame shell. The direct-sum fibres record admissible comparison domains. Neutral-domain expressions record quantities whose domain profile is invariant under the chosen frame reading. Physical content is expressed by quantities, invariant profiles, and shell-vector patterns over the coordinate support.

The scope of this paper is intentionally limited. It proves the modular $[L],[T]$-domain algebra and derives the domains of basic mechanics and gravitational coupling. It does not claim a complete classification of all physical unit classes. In particular, electromagnetic and thermodynamic units require separate treatment. Likewise, the finite phase propagation law corresponding to full quantum dynamics is not developed here. The present paper only isolates the dimensional mechanism: E shares the temporal recurrence domain by $E = hf$, h is the complete-cycle phase count, $\hbar$ is the angular phase-count unit, and i_{t} is the finite quarter-turn label.

Within that scope, the main result is compact: physical unit readings are modular domains of the FRC reference-frame shell. The usual rule that unlike units cannot be added is the direct-sum homogeneity condition. The usual rule that products and ratios change dimension is the modular domain multiplication law. Dimensionless physical invariants are neutral-domain monomials. Classical dimensional analysis is recovered locally inside any bounded exponent horizon, while the finite shell retains its global modular structure.

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