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Article

A Version of Exclusively Perturbative Quantum Field Theory

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Abstract: We suggest a version of renormalizable Quantum Field Theory which does not contain non-perturbative effects. This is obtained by the proper use of the boundary conditions in the functional integral of the generating functional of Green functions. It is well known which boundary conditions are applied to the fields of the functional integral to get correct perturbation theory. We propose that these conditions should be used for all fields integrated in the generating functional integral. It is shown that in this case non-perturbative effects are absent. That is we assume that perturbation theory defines the complete generating functional integral. It allows, in particular, to formulate the generating functional integral in a unique way as an exact compact mathematical formula.

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1. Introduction

Most of precision tests of Quantum Field Theory are performed within perturbative approach. The famous example of this type is the anomalous magnetic moment of the electron. The agreement between the Quantum Electrodynamics prediction and the experiment in this case within at least ten decimal points [1] convinces us in the validity of renormalizable Quantum Field Theory.

The known example of applications of non-perturbative effects is the QCD sum rule method [2], where hadronic properties are calculated by means of the so called non-perturbative quark and gluon condensates. As it will be discussed below these condensates can be, in principle, generated within perturbation theory.

Quite recently a solution to the strong CP-problem based on the proper definition of the integration region in the functional integral for the generating functional of Green functions of Quantum Chromodynamics was suggested in the work [3]. This solution defines that the integration measure includes only fields decreasing at infinity, excluding all other fields like instantons [4]. There is the statement [5,6] that instantons solve the known $U(1)$ problem. Thus it seems that they should not be discarded from the theory. But there is also the known solution [7] to the $U(1)$ problem applying the axial anomaly which was suggested before the discovery of instantons.

In the present paper we suggest a version of Quantum Field Theory without non-perturbative contributions and, in particular, demonstrate that the solution [3] to the strong CP problem implies that the corresponding theory is free of non-perturbative effects.

2. Main Part

Let us consider a functional integral for the generating functional of Green functions of some generic theory (this can be QCD or QED or any other theory):

$$Z(J) = \frac{1}{N} \int d\Phi \exp \left(i \int d^4x (L_0(x) + gL_{int}(x) + J_k(x) \cdot \Phi_k(x)) \right), \quad (1)$$

where $d\Phi$ is the integration measure of the functional integral $Z(J)$ including all allowed types of the fields Φ_k of the theory. J_k are sources of these fields. J in $Z(J)$ is the full set of J_k . $L_0(x)$ is the free Lagrangian and $gL_{int}(x)$ is the interaction Lagrangian of the considered theory. g is the coupling

constant of interactions of the theory. N is the standard normalization factor. Generalization to the case of several coupling constants of interactions is completely straightforward.

Of course the generating functional (1) is not completely defined by the defining the Lagrangian $L(x) = L_0(x) + gL_{int}(x)$ only. One should also define which types of the fields Φ_k are allowed in the integration measure. If one allows integration over all possible configurations of the fields Φ_k , then it is not possible to reproduce the perturbative fields propagators of the necessary forms, i.e. with the correct ' $i\epsilon$ ' prescription of the type $1/(k^2 - m^2 + i\epsilon)$.

To obtain the perturbative propagators of the correct forms one should impose on the integrated fields the known boundary conditions. For example, one has the following boundary conditions for the gluon fields in QCD:

$$A_\mu^a(\vec{x}, t \rightarrow \infty) \rightarrow A_{\mu,in}^a(x), \quad (2)$$

$$A_\mu^a(\vec{x}, t \rightarrow \infty) \rightarrow A_{\mu,out}^a(x),$$

where the incoming gluon fields $A_{\mu,in}^a(x)$ contain only the positive frequency part and, in opposite, the outgoing asymptotic gluon fields $A_{\mu,out}^a(x)$ contain the negative frequency part:

$$A_{\mu,in}^a(x) = \frac{1}{(2\pi)^{3/2}} \int d^3k e^{i(\vec{k}\vec{x} - \omega t)} v_\mu^i(k) a_i(k) / \sqrt{2\omega}, \quad (3)$$

$$A_{\mu,out}^a(x) = \frac{1}{(2\pi)^{3/2}} \int d^3k e^{i(\vec{k}\vec{x} + \omega t)} v_\mu^i(k) a_i^*(k) / \sqrt{2\omega}.$$

Here $\omega = \sqrt{k^2}$, $v_\mu^i(k)$ are the polarization vectors of the gluons. In (3) sums over gluon polarizations $i = 1, 2$ are assumed.

These are the known Feynman boundary conditions of emission. They ensure the correct forms of perturbative propagators of the fields of the type $1/(k^2 + i\epsilon)$ with the necessary plus $i\epsilon$ prescription, see [8].

Hence the gluon fields (the quark fields also) in QCD with emission boundary conditions oscillate at time infinities. Making transition to the Euclidean space with the help of the Wick rotation $t \rightarrow ix_4$ one gets that the fields decrease at time infinities. Thus it is easy to see that in the Euclidean space total derivatives in the Lagrangian are zero within perturbation theory. This allows, in particular, to solve [3] the Strong CP problem without involving hypothetical exotic particles like axions; for the review of axions see [9].

One can write perturbative boundary conditions (2) for all fields Φ_i of the considered theory (1) symbolically as follows:

$$\Phi(t \rightarrow \pm\infty) \rightarrow \Phi_{in}^{out}(x). \quad (4)$$

This short notation is used below to formulate the functional integral for the generating functional of Green functions as a uniquely defined compact formula.

If one defines the generating functional with integrations over all possible field configurations, then one should also integrate over the non-perturbative fields like instantons [4] in addition to the perturbative fields with emission boundary conditions. These non-perturbative fields produce only non-perturbative contributions and do not effect the perturbative propagators. But then one comes to uncertainties since there can be, in principle, other non-perturbative solutions like instantons and it is not known in advance how many of them exist.

That is why it seems to be natural to use the boundary conditions of emission (4) for all fields Φ_i over which the integration in the the functional integral (1) proceeds. Then, in particular, all fields decrease in the Euclidean space at the time infinities. Hence total derivatives in the Lagrangian are nullified and this, in particular, solves [3] the strong CP problem. Besides, this definition of the

boundary conditions allows to formulate uniquely the complete generating functional integral of Green functions of the theory as an exact compact mathematical formula:

$$Z(J) = \frac{1}{N} \int_{\Phi(t \rightarrow \pm\infty) \rightarrow \Phi_{in}^{out}} d\Phi \exp \left(i \int d^4x (L_0(x) + gL_{int}(x) + J_k \cdot \Phi_k) \right). \quad (5)$$

It turns out that this expression for the generating functional integral contains only perturbative contributions and does not contain non-perturbative ones. Let us demonstrate it.

One can straightforwardly expand the generating functional (5) in the perturbative series in the coupling constant g at the point $g = -0$ (in the same way as one expands at the point $g = +0$). The expansion is well defined. This is the perturbation theory with the finite coefficients (the counterterms are assumed to be included into the Lagrangian) since integrations in the functional integral (5) proceed only over the fields with perturbative boundary conditions of emission. It excludes the presence of the non-perturbative contributions of the type $\exp(-1/g)$ since otherwise the expansion would produce infinities at the point $g = -0$.

One can also consider an expansion of the functional integral (5) at the point $g = i0$. Again the coefficients of the expansion will be finite on the same reasons as above. This excludes the presence of the non-perturbative terms of the type $\exp(-1/g^2)$ since infinities would be generated otherwise.

More generally, to demonstrate the absence of the non-perturbative terms of the type $\exp(-1/g^x)$, where x is an arbitrary positive number, one can consider an expansion at the point $g = i^{2/x}0$. The expansion is finite. This demonstrates the absence of the non-perturbative contributions in the generating functional (5).

There is the question of the so called non-perturbative vacuum quark and gluon condensates used in the method of the QCD sum rules [2] for calculation of hadronic properties. These condensates have the values

$\langle \bar{q}q \rangle \approx (200 \text{ MeV})^3$ and $\alpha_s \langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle \approx (400 \text{ MeV})^4$ at the normalization point of the order of 1 GeV . Here q is the quark field, $G_{\mu\nu}^a$ is the gluon strength tensor, $\alpha_s = g_s^2/4\pi^2$, where g_s is the strong coupling constant in the QCD Lagrangian.

But, in principle, one can generate these condensates within perturbation theory with massive perturbative propagators after the summations of the complete asymptotic perturbative series for the corresponding vacuum expectation values.

3. Conclusions

We suggest a version of Quantum Field Theory which does not contain non-perturbative effects. This is obtained by the proper, in our opinion, use of the boundary conditions in the functional integral of the generating functional of Green functions. It is well known which boundary conditions are applied to the fields of the functional integral to get correct perturbation theory. These are the boundary conditions which ensure that the fields decrease at the time infinities nullifying all total derivatives in the Lagrangian of the theory. We propose that these conditions should be used for all fields integrated in the generating functional integral. It is shown that in this case the non-perturbative effects are absent. That is we assume that perturbation theory defines the complete generating functional integral. It allows, in particular, to formulate the generating functional integral in a unique way as an exact compact mathematical formula.

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