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Not peer-reviewed version

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Posted Date: 17 April 2026

doi: 10.20944/preprints202604.1301.v1

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Article

On the Observer-Frame Interpretation of Radial Coordinates in Gravitational Potentials

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Abstract

The discrepancy between galactic rotation curves and visible baryonic mass persists despite empirical scaling relations like the Radial Acceleration Relation (RAR) and Baryonic Tully-Fisher Relation (BTFR). We explore a phenomenological framework where this discrepancy arises from the geometric misinterpretation of observables. Inspired by Painlevé-Gullstrand coordinates, we model the vacuum as a radially infalling compliant medium that induces an apparent compression of radial coordinates for distant observers, the "Mezzi effect". Assuming Newtonian dynamics govern an undistorted "true frame", we developed a discrete shell reconstruction method parameterized by a single universal compliance constant, tested against photometric and kinematic data from 175 late type galaxies in the SPARC database. This single parameter model yields universal scaling relations of $\Sigma_{\text{true}}/\Sigma_{\text{obs}} \propto (R_{\text{true}}/R_{\text{obs}})^{-0.5}$ and $M_{\text{obs}}/M_{\text{true}} \propto r_{\text{obs}}/r_{\text{true}}$. And reproduces observed rotation curves (RMS residual ~ 34 km/s). The geometric projection recovers the empirical RAR and shifts the BTFR slope from ~ 2.8 in the true frame to ~ 3.7 in the observer frame, and eliminating the normalization offset. Furthermore, The Mezzi scale factor ζ governs mass and lensing corrections via distinct power laws: $M_{\text{true}}/M_{\text{obs}} \propto \zeta^{-1.84}$ and $\alpha_{\text{true}}/\alpha_{\text{obs}} \propto \zeta^{-1.26}$, revealing that geometric scaling affects dynamical mass more strongly than lensing mass. These results indicate that geometric projection effects may offer a viable phenomenological explanation for galactic dynamics while remaining consistent with both Newtonian gravity and weak field general relativity. For reproducibility, the code used for this analysis is publicly available at <https://github.com/Brahim-Benaissa/Zeta>.

Keywords: Mezzi effect; radial acceleration relation; Baryonic Tully-Fisher Relation; surface mass density

1. Introduction

Early observations, beginning with Zwicky (Zwicky, 1933) and later reinforced by Rubin et al. (Rubin et al., 1980), revealed a mismatch between the visible matter in galaxies and the mass required to explain their observed motions. In the prevailing cosmological framework, this discrepancy is interpreted as evidence for non-baryonic matter, which has been remarkably successful in explaining phenomena such as the formation of large scale structure, anisotropies in the cosmic microwave background, and the evolution of cosmic structure (Adame et al., 2025; Aghanim et al., 2020; Grimm et al., 2025; Jung et al., 2024). Despite these successes, challenges remain on galactic scales, including variations in rotation curve profiles and the cusp core problem (De Blok, 2010; Matos et al., 2024; Nelson & Williams, 2024; Oman et al., 2015). Although simulations incorporating baryonic feedback have made progress in alleviating some of these discrepancies, it is still unclear whether they fully resolve them, leaving open the possibility that additional or alternative physical explanations may be required to better understand galaxy dynamics (Acevedo et al., 2024; Crain & van de Voort, 2023; Schaller et al., 2025).

Two observational relations exhibit remarkably tight correlations that any viable dynamical framework must accommodate. The Radial Acceleration Relation (RAR) describes a precise coupling between the observed dynamical acceleration and that predicted from the baryonic mass distribution

alone (Lelli et al., 2017; McGaugh et al., 2016). And the Baryonic Tully-Fisher Relation (BTFR) establishes a power law scaling between baryonic mass and asymptotic rotation velocity with slope ≈ 4 (McGaugh, 2005). These correlations persist across multiple orders of magnitude in galaxy mass and surface brightness, suggesting underlying organizational principles that serve as critical benchmarks for theoretical research. Whether these relations reflect fundamental physics, emergent phenomena, or observational selection effects remains an active subject of investigation (Gogate et al., 2023; Ravi et al., 2025; Vukcevic et al., 2024).

Current approaches typically fall into modifying the matter content or modifying the gravitational action (Boehm, 2024). Although Λ CDM successfully explains cosmological observations, it requires galaxy specific halo parameter adjustments to reproduce detailed kinematic structure (Katz et al., 2017). MOND on the other hand, reproduces galactic scaling relations by the introduction of a characteristic acceleration scale $a_0 \approx 1.2 \times 10^{-10} \text{ m s}^{-2}$ (Milgrom, 1983; Famaey & McGaugh, 2012; Sanders & McGaugh, 2002), this achieves a remarkable phenomenological success on rotation curves (Milgrom, 2024); however, it still faces theoretical challenges due to its differences from general relativity (Famaey et al., 2024; Kelleher & Lelli, 2024; Nagesh et al., 2024). These considerations motivate investigating whether mass discrepancy can arise from the geometric assumptions underlying the interpretation of astrophysical observables (Harte, 2019; Kumar Aluri et al., 2023; Tizfahm et al., 2025; Zhang et al., 2026).

Standard analyses of galactic structure typically assume that observed angular measurements correspond directly to physical radial distances within a static spacetime background. We investigate the consequences of modifying that assumption. Inspired by the kinematic structure of Painlevé-Gullstrand metrics (Fischer & Visser, 2003), in which gravitational fields can be imagined as an effective radial flow (Abellán et al., 2025; Gaur & Visser, 2022). This paper builds on the Mezzi effect, first proposed in (Benaissa, 2025), studied using an inverse problem approach. It suggests that errors in how geometry is interpreted during measurements may contribute to the apparent mass deficits observed in galaxies, causing the underlying geometry of the system, called in this paper “true frame”, to appear compressed from the observer’s frame, and thereby leading to a systematic underestimation of physical scales.

This paper suggests a phenomenological model in which the vacuum behaves as a compliant medium to the gravitational potential gradients, under the assumptions that Newtonian dynamics holds in the true frame, observed velocities are accurately measured, and apparent discrepancies arise from coordinate transformation rather than modified force laws. The radial scaling factor $\zeta(r)$ describes how the observed radial coordinates are transformed into their corresponding true frame coordinates. We validate this framework against the Spitzer Photometry and Accurate Rotation Curves (SPARC) database (Lelli et al., 2016), comprising galaxies with consistent photometric and kinematic data. The analysis addresses four components: (i) reconstruction of true surface density profiles from observed photometry, (ii) kinematic validation through rotation curve, (iii) Consistency tests through surface density scaling and the deflection angle from the gravitational lensing in the weak field, and (iv) test cases examining the emergence of the RAR and BTFR from the Mezzi effect.

2. Galaxy Mass Profiles from SPARC Dataset

SPARC provides a high-quality, well-characterized sample of 175 late type galaxies with homogeneous surface photometry at $3.6 \sim \mu\text{m}$ and high-quality rotation curves. The sample spans a wide dynamic range in stellar mass ($\sim 10^7$ to $10^{11} M_\odot$), effective surface brightness (~ 1 to $10^3 L_\odot/\text{pc}^2$), and rotation velocity (~ 20 to $300 \sim \text{km}\cdot\text{s}^{-1}$) (Lelli et al., 2016).

The database includes two primary data tables relevant to this study. The global properties, including distances, total luminosities $L_{[3.6]}$, disk scale lengths R_{disk} , and effective radii R_{eff} . The Mass Models provides radially resolved kinematic data, including the observed circular velocity V_{obs} and the velocity contributions of the gas V_{gas} , disk V_{disk} , and bulge V_{bul} .

2.1. Baryonic Mass Profiles

The baryonic mass profiles $M_{\text{bar}}(r' < r)$ in this study are based on the integration of the stellar mass distribution derived from photometry with the gas mass distribution derived from kinematics. The total baryonic mass at a given radius r is defined as the sum of the enclosed disk, bulge, and gas masses:

$$M_{\text{bar}}(< r) = M_{\text{disk}}(< r) + M_{\text{bulge}}(< r) + M_{\text{gas}}(< r) \quad (1)$$

For the stellar disk, we assume an exponential surface density profile (Freeman, 1970). The cumulative mass as a function of radius is calculated as:

$$M_{\text{disk}}(< r) = M_{\text{disk}}^{\text{tot}} \left[1 - \left(1 + \frac{r}{R_{\text{disk}}} \right) \exp\left(-\frac{r}{R_{\text{disk}}}\right) \right] \quad (2)$$

where $[R_{\text{disk}}]$ is the disk scale length provided in the SPARC sample. The total disk mass $[M_{\text{disk}}^{\text{tot}}]$ is derived from the luminosity of the disk ($L_{\text{disk}} = L_{[3.6]} - L_{\text{bulge}}$) and the stellar mass-to-light ratio.

For the stellar bulge, we assume a Hernquist density profile (Hernquist, 1990), to approximate the mass distribution in galaxy centers. The cumulative mass is given by:

$$M_{\text{bulge}}(< r) = M_{\text{bulge}}^{\text{tot}} \left(\frac{r}{r+a} \right)^2 \quad (3)$$

Where a is the scale radius related to the effective radius R_{eff} by $[a \approx \frac{R_{\text{eff}}}{1.8153}]$

The gas distribution is inferred from the kinematics, assuming spherical symmetry for the mass calculation (Levy et al., 2018). We calculate the enclosed gas mass from the observed gas velocity contribution $[V_{\text{gas}}]$ using the centripetal force equation:

$$M_{\text{gas}}(< r) = \frac{V_{\text{gas}}^2 r}{G} \quad (4)$$

We adopt the standard SPARC definition where $[V_{\text{gas}}]$ includes a factor of 1.33 to account for the contribution of Helium (Lelli et al., 2016; Liu, 2026).

For the stellar mass-to-light ratios (Y_*), we adopt the SPARC values provided for the disk and bulge (Lelli et al., 2016; Schombert et al., 2022). We treat these values as fixed parameters and do not tune or adjust them to fit the rotation curves. To ensure that our derived mass profiles are independent of assumptions regarding the dark matter halo or the Mezzi effect, preserving the objectivity of the subsequent test (Mistele et al., 2022).

This approach is adopted to reduce, as much as possible, the influence of implicit theoretical assumptions inherent in the dynamical mass estimates and baryonic velocity parameters $[V_{\text{bar}}]$

and $[V_{\text{disk}} + V_{\text{bul}}]$ provided in the database. By deriving the gas mass from $[V_{\text{gas}}]$, we acquire the gas mass distribution, without relying on surface density integrations of HI maps. (Yu et al., 2020), which are not explicitly available within the currently available dataset.

3. The Mezzi Effect

3.1. The Non Local Metric

In contrast to the assumption that space structure has a static metric (Hogg, 1999), in which observational distances correspond directly to true distances. We propose a formulation where the vacuum behaves as a dynamic medium, exhibiting radial inflow toward gravitational potential wells (Hamilton & Lisle, 2008). Within this framework, galactic geometry is not characterized by a fixed curvature, but by the evolving kinematic state of the metric itself (Barcelo et al., 2011).

The metric evolution is proposed to be non local, arising from the flow accumulation along the signal trajectory between source and observer. The observed geometry of distant objects is not an

intrinsic property of the space at emission or observation. Rather, a distorted observation of the true geometry arises as photons accumulate the signature of metric evolution between source and observer.

We adopt the Painlevé-Gullstrand description of spacetime, which interprets gravitational fields not as static deformations of a rigid background, but as a spatial velocity field (Gullstrand, 1922; Painlevé, 1921). In general relativity, these coordinates encode the gravitational field through a spatial flow.

$$v(r) = -\sqrt{\frac{2GM(r)}{r}} \quad (5)$$

Here $M(r)$ denotes the enclosed mass within radius r , and the negative sign indicates inward flow toward the galactic center. In this study, this velocity represents the free fall speed of the metric itself, the rate at which space streams toward the gravitational potential (Chakrabarti, 2024).

3.2. 3.2 Kinematic Accumulation Along the Line of Sight

Light emitted from radius r within a galaxy and received by a distant observer at distance D must traverse this flowing vacuum. The first assumption of this proposition is that this light carries geometric distortion information of the observed galaxy, determined not solely by the local flow velocity at the emission point, but by the cumulative kinematic drag accumulated along the entire line of sight (Benisty & Mota, 2025; da Silva Barbosa et al., 2025).

To quantify this accumulation, we construct a dimensionless path integral of the form

$$A(r) = \int_r^D \frac{v(r')}{c \cdot r'} dr' \quad (6)$$

$v(r')/(c \cdot r')$ represents the local contribution to the distortion per unit fractional radial distance. Substituting the velocity field from Eq. 5 yields the explicit form:

$$A(r) = -\frac{\sqrt{2G}}{c} \int_r^D \frac{\sqrt{M(r')}}{r'^{3/2}} dr' \quad (7)$$

This integral represents the total normalized velocity potential traversed by the photon as it climbs out of the galactic potential well. Since the flow is inward, the accumulated potential $A(r)$ is negative, growing more negative toward the center where the flow is strongest. The kinematic compression factor encoding this accumulated flow is obtained through:

$$K(r) = \exp[A(r)] \quad (8)$$

This factor quantifies the radial compression of the coordinate grid that would result if the accumulated kinematic flow translated directly into geometric distortion.

3.3. Vacuum Compliance and the Mezzi Scale Factor

The second assumption of this proposition is that kinematic accumulation does not dictate observed geometry directly. Instead, the vacuum possesses a property, compliance, that determines the efficiency by which the flow induced stress translates into geometric distortion. We define the compliance of the vacuum, denoted C , as the dimensionless constant governing the apparent distortion measured by a distant observer:

$$\sigma(r) = K(r) - 1 \quad (9)$$

$$\varepsilon(r) = \ln[\zeta(r)] \quad (10)$$

Here $\zeta(r)$ is the Mezzi scale factor, a dimensionless factor between $[0,1]$, that relates observed lengths from an asymptotically flat region, the observer frame, to the lengths that would be inferred by

a hypothetical observer colocated with the source, the true frame. The stress term $(K - 1)$ represents the deviation of the accumulated flow metric from static space. The compliance C relates these quantities:

$$\varepsilon(r) = C \cdot \sigma(r) \quad (11)$$

Thus, the Mezzi scale factor is defined by:

$$\zeta(r) = \frac{1}{\exp[-C \cdot (K(r) - 1)]} \quad (12)$$

The parameter C is a dimensionless coupling constant characterizing the response of space geometry to kinematic history. Its value constitutes an intrinsic property of the vacuum, independent of the specific mass distribution or scale of the gravitating system. The term compliance formalizes the proposition that measurements in space are not invariant, but depend upon the accumulated kinematic history of the geometric information. The Mezzi effect, the apparent compression of galactic morphologies, arises as a frame dependent measurement artifact.

3.4. Geometry According to the Inertial Stress and the Rheology Analogy

The proposed framework has an analogy within the context of continuum mechanics, specifically drawing parallels to inertial stress and history dependent material response (Coussot, 2025). In fluid dynamics, inertial stress is typically associated with the acceleration of a fluid element. However, in viscoelastic materials, the stress state often depends on the entire history of deformation rather than only on the instantaneous kinematic state (Srinivasan et al., 2020). This proposition has a similar logic.

Although explicit temporal acceleration terms, such as $\partial v / \partial t$, are absent, the spatial dependence of the velocity field $v(r)$ encodes convective inertia, corresponding to the acceleration experienced by an electromagnetic wave traversing a non uniform velocity field (J. F. Price, 2006). In this sense, the accumulation integral $A(r)$ acts similar to a proxy for the integrated inertial influence along the photon trajectory (Maxey, 1990), thereby representing a path dependent history of the underlying dynamical environment (Lambert & Makri, 2012).

This history effect is captured by the kinematic compression factor $K(r) = \exp[A(r)]$. The deviation from static space $\sigma(r) = K(r) - 1$, is interpreted as a history induced stress. This construction bears a resemblance to integral constitutive equations in viscoelasticity, where the current stress is determined by the integral of past deformation rates (Mitsoulis & Hatzikiriakos, 2023). In this analogy, the vacuum behaves not as a passive empty stage but as a medium with apparent rheological memory, where the history dependence emerges from accumulated propagation effects.

In solid mechanics, the compliance defines how much strain results from a given stress. In this framework C determines how the accumulated kinematic effects translate into observable geometric deformation. The relationship $\varepsilon(r) = C \cdot \sigma(r)$ is a linear relationship between the internal history dependent stress and the external observed strain (Mittelstedt, 2023).

Finally, the Mezzi scale factor $\zeta(r)$ completes the physical picture by serving as the observable transfer function. Defined by $\zeta(r) = 1 / \exp[-C \cdot (K(r) - 1)]$, it rescales observed distances relative to the locally defined true distances. Consequently, the framework establishes this causal chain: the vacuum's flow creates a history dependent inertial stress (σ), the vacuum's compliance (C) dictates the susceptibility to this stress, and the Mezzi factor (ζ) operates as an effective scale factor, a measurement transfer function that rescales inferred distances based on the integrated propagation history. It represents the observational signature of the vacuum's kinematic structure, not a compression of the space manifold itself.

3.5. Methodological Assumptions and Boundary Conditions

The mathematical formulation developed in this paper rests upon three critical simplifying assumptions that bound its domain of applicability and interpretive scope.

Spherical Symmetry: The integrals and velocity profiles presented in Equations (1) through (3) are constructed under the assumption of spherical symmetry. In this model, the enclosed mass $M(r)$ is treated as the mass contained within a sphere of radius r , rather than a cylindrical shell. This standard approximation simplifies the path integral accumulation into a one dimensional radial problem (Cardone et al., 2011).

Isolation of the Source: The inward flow of the vacuum is assumed to be driven exclusively by the gravitational potential of the galaxy under observation (Senatore, 2015). In calculating the velocity field $v(r)$ and the subsequent accumulation potential $A(r)$, the framework treats the target galaxy as an isolated system. The contribution of the surrounding structure or nearby neighbors to the local flow potential is neglected. Consequently, the stress accumulated is strictly a function of the target galaxy's mass distribution.

The Observer in Flat Space: A critical boundary condition for the integration is the state of the observer (Takizawa et al., 2020). The upper limit of the integral in Eq.6 implies an observer situated at a distance D , we use the specific distance between the observed galaxy and Earth, which differs for each galaxy. However, we assume that the observer resides in a region that is effectively flat. The total accumulated potential $A(r)$ therefore represents the full differential in flow conditions between the emission point, deep within the galactic potential, and the observer that is in the asymptotically flat region at distance D .

The scaling factor: $\zeta(r)$ is a scale factor that corrects the observed geometry (Giddings & Sloth, 2011; Suyama et al., 2020). The transformation required to infer what a local observer would measure. It is not a reversal of physical compression (Dvorsky, 2025; Vavryčuk, 2022).

3.6. The Universal Compliance Parameter

A central prediction of the flowing vacuum framework is that the compliance C represents a fundamental constant of nature, a dimensionless property of the vacuum. Consequently, C must be universal across all scales and environments, reducing the geometric distortion of space to a single parameter.

To test this hypothesis, we performed a global fitting exercise to determine C using the full SPARC galaxy sample, incorporating multiple observational diagnostics described in the following sections. Importantly, C is not optimized for individual galaxies.

The best fit value is: $C = 377$

We adopt this value as a fixed, universal constant in all calculations. The theory's ability to reproduce the observed scaling relations across the full range of galaxy masses and geometries in the SPARC sample, using this single parameter, constitute the primary empirical support for the framework.

4. Geometry of the True Frame

4.1. Mezzi Scale Factor for the SPARC Sample

To recover the true geometry of a galaxy from observed data, we relate the radial coordinate in the true frame r_{true} to the observed frame coordinate r_{obs} using the Mezzi scale factor. The value of $\zeta(r)$ characterizes the intensity of the geometric distortion: a value approaching $\zeta(r) \rightarrow 1$ indicates a flat, undistorted metric where observed lengths correspond to true lengths, while a value approaching $\zeta(r) \rightarrow 0$ indicates extreme geometric compression.

$$r_{obs} \approx \frac{r_{true}}{\exp[\mathcal{C} \cdot (K(r_{obs}) - 1)]} \quad (13)$$

This rescaling of radial distances has immediate consequences for the inferred physical properties of galaxies, specifically the scale radius $R_{s,obs}$ and surface density Σ . Under the influence of the Mezzi effect, the observed scale radius represents a compressed projection of the true scale radius. This implies that the physical disks of galaxies are more extended than standard photometry suggests. Furthermore, since surface density is defined as mass per unit area and area scales with the square of

the length, the true surface density should be lower than Σ_{true} the observed Σ_{obs} . The true physical extent of galactic disks is larger than standard photometric measurements suggest (Benaissa, 2025; Flynn & Cannaliato, 2025). As a consequence, galaxies are physically larger and more diffuse structures than their observed photometric profiles suggest.

$$R_{s,\text{true}} = \frac{R_{s,\text{obs}}}{\zeta(R_{s,\text{obs}})} > R_{s,\text{obs}} \quad (14)$$

We calculate the Mezzi scale factor, $\zeta(r)$, relating the observed length, $l_{\text{obs}}(r)$, to the true physical length, $l_{\text{true}}(r)$, at a given radial coordinate:

$$l_{\text{obs}}(r) = \zeta(r) \cdot l_{\text{true}}(r) \quad (15)$$

The profile of the Mezzi scale factor $\zeta(r)$ follows a distinct radial dependence that mirrors the mass distribution of the host galaxy. It is anticipated to reach its minimum value near the galactic center, and to asymptotically approach 1 toward the outer regions. This trend is coupled to the radial gravitational potential. Consequently, the geometry of the scaling effect acts as a tracer for the underlying space geometrical curvature, aligning the observed distortion directly with the physical structure of the galaxy. Figure 1 presents the Mezzi profiles for the SPARC sample.

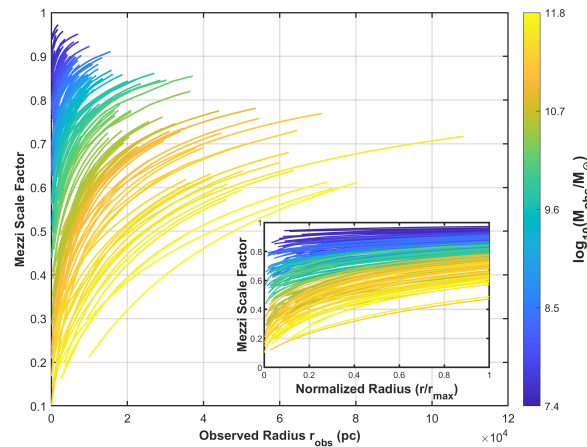


Figure 1. The Mezzi Scale Factor ζ as a function of observed radii for the SPARC galaxy sample. Curves are color coded by total observed stellar mass $\log_{10}(M_{\text{obs}}/M_{\odot})$. The main panel shows ζ versus absolute radius, while the inset displays it versus normalized radius.

A strong correlation emerges between galaxy mass and the Mezzi scale factor profile. Low-mass galaxies with $\log M_{\text{obs}} \lesssim 9.0$ maintain Mezzi profile unity ($\text{mean} \approx 0.90$) with small radial variation, their spatial coordinates remain close to Euclidean.

Intermediate mass galaxies ($9.0 \lesssim \log M_{\text{obs}} \lesssim 10.5$) reveal deeper Mezzi profiles between 0.6 and 0.9, with noticeable radial gradients.

Massive galaxies ($\log M_{\text{obs}} \gtrsim 10.5$) exhibit substantial mezzi effect, with values at the center of the galaxy falling as low as 0.10. The extreme case UGC11914 demonstrates a 70% reduction in coordinate scale from center 0.120 to edge 0.473.

All galaxies display monotonically decreasing radial profiles from edge to center, consistent with theoretical expectations. The profile shapes transition systematically from flat gradients in small galaxies, with center-edge ratios between 0.90 and 0.95. The Mezzi profiles has moderate declines in intermediate mass galaxies (0.70 to 0.85) and steep drops in massive galaxies (0.30 to 0.60)

The mass dependence suggests the Mezzi scale factor encodes gravitational field strength, as deeper potential wells produce stronger deviations from flat space geometry. The systematic radial decline exhibits similarity to the gravitational time dilation effects described by general relativistic frameworks.

4.2. Mass Reconstruction via Geometric Projection

In an axisymmetric system, the annular area element $dA = 2\pi r dr$ transforms according to the Jacobian of the coordinate transformation. Consider a discrete annular shell bounded by radii r_j and r_{j+1} . The observed and true areas of this shell are, respectively:

$$\Delta A_{\text{obs},j} = \pi (r_{\text{obs},j+1}^2 - r_{\text{obs},j}^2) \quad (16)$$

$$\Delta A_{\text{true},j} = \pi (r_{\text{true},j+1}^2 - r_{\text{true},j}^2) \quad (17)$$

The area ratio $R_j \equiv \Delta A_{\text{true},j} / \Delta A_{\text{obs},j}$ quantifies the geometric distortion of the j -th shell. For a Mezzi scale factor with $\zeta < 1$, the shells expand in true coordinates ($R_j > 1$), causing the enclosed mass to be distributed over larger physical areas (Chemin et al., 2016; Fukushima, 2016; S. Price et al., 2022).

The reconstruction of the true mass profile utilizes a local approximation of density invariance. We posit that the observed surface density Σ_{obs} provides the correct mass density for the local shell, but the coordinate grid is compressed, mapping the true extent onto smaller observed radii. To reconstruct the mass contained within the expanded true geometry, we project the observed local density onto the true metric.

Consequently, the mass element for the expanded true shell is approximated by applying the observed surface density to the larger true geometry:

$$\Delta M_{\text{true},j} \approx \Sigma_{\text{obs},j} \cdot \Delta A_{\text{true},j} \quad (18)$$

Substituting the area ratio $R_j = \Delta A_{\text{true},j} / \Delta A_{\text{obs},j}$, this can be expressed as:

$$\Delta M_{\text{true},j} = \Sigma_{\text{obs},j} \cdot (R_j \cdot \Delta A_{\text{obs},j}) = \Delta M_{\text{obs},j} \cdot R_j \quad (19)$$

Since $R_j > 1$ for the Mezzi effect, the true shell mass exceeds the observed shell mass. In this manner, this study suggests that the true total mass of the galaxy exceeds the estimation derived from the compressed coordinate frame.

4.3. 4.3 Cumulative Mass Reconstruction

The true cumulative mass profile is reconstructed by summing the redistributed shell masses (Cheng et al., 2024; Hua et al., 2025):

$$M_{\text{true}}(< r_{\text{true},j}) = M_1 + \sum_{j=1}^{j-1} \Delta M_{\text{true},j} \quad (20)$$

Where $M_1 = M_{\text{obs}}(\langle r_{\text{obs},1} \rangle)$ is the innermost enclosed mass, treated as a central boundary condition. The assumption that the central region is unaffected by geometric distortion ensures $M_{\text{true}}(\langle r_{\text{true},1} \rangle) = M_{\text{obs}}(\langle r_{\text{obs},1} \rangle)$

The mass ratio M_{ratio} quantifies the net geometric bias accumulated across the entire profile:

$$M_{\text{ratio}} = \frac{M_{\text{total,true}}}{M_{\text{total,obs}}} = \frac{M_{\text{true}}(\langle r_{\text{true},N} \rangle)}{M_{\text{obs}}(\langle r_{\text{obs},N} \rangle)} \quad (21)$$

For the Mezzi effect with $\zeta < 1$, the expansion of outer shells yields $R_j > 1$ throughout the profile, resulting in $M_{\text{ratio}} > 1$. The true total mass exceeds the observed estimate.

Practical implementation employs a discrete shell algorithm:

1. Discretize the observed profile into $N - 1$ annular shells between N radii
2. Compute observed surface density: $\Sigma_{\text{obs},j} = \Delta M_{\text{obs},j} / \Delta A_{\text{obs},j}$
3. Transform shell boundaries: $r_{\text{true},j} = r_{\text{obs},j} / \zeta(r_{\text{obs},j})$
4. Compute true shell areas and area ratios R_j

5. Redistribute mass: $\Delta M_{\text{true},j} = \Sigma_{\text{obs},j} \cdot \Delta A_{\text{true},j}$
6. Reconstruct cumulative profile via Eq. 20

4.4. Surface Density of the True Frame

To evaluate the surface density scaling results, we group the sample into three mass regimes: low mass galaxies ($7.0 \leq \log M_* < 9.0$), intermediate mass galaxies ($9.0 \leq \log M_* < 10.5$), and high mass galaxies ($10.5 \leq \log M_* < 12.0$). We examine the distributions of the ratios between the true frame scale radii and surface densities and their corresponding observed values for each mass bin. In particular, we examine whether the Mezzi scale factor introduces systematic variations in the inferred scale radius and central density across stellar mass within or across mass regimes.

Figure 2 presents a comparative analysis of the geometric correction factors arranged by galactic mass regime. The visualization displays the distribution of the scale radius ratio $R_s^{\text{true}} / R_s^{\text{obs}}$ and the central surface density ratio $\Sigma_0^{\text{true}} / \Sigma_0^{\text{obs}}$ for Low-Mass, Intermediate, and high mass galaxies.

The figure immediately highlights mass dependence in the magnitude of geometric distortion. In the low mass regime, the radius ratio distribution is tightly clustered near 1, $\text{Mean} \approx 1.14$, indicating that low mass galaxies reside in a near Euclidean geometry where observed coordinates closely approximate true physical distances. As mass increases, the distributions shift dramatically: high mass galaxies exhibit a radius ratio distribution centered high above unity, $\text{Mean} \approx 3.5$, with a long tail extending toward extreme values.

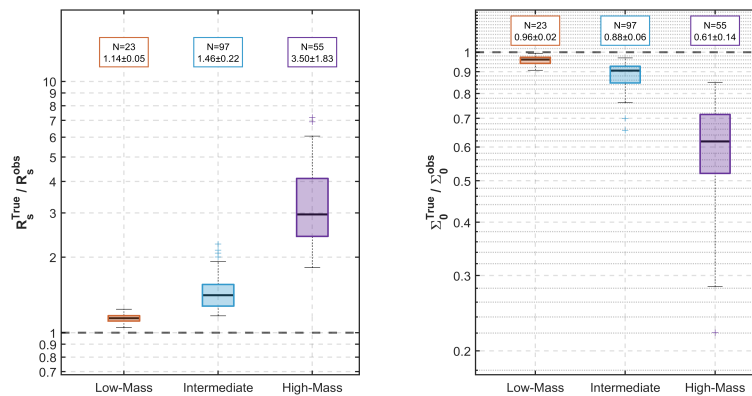


Figure 2. Boxplots show the ratio of Mezzi corrected to observed scale radius ($R_{s,\text{true}}/R_{s,\text{obs}}$ left panel) and central density ($\Sigma_{0,\text{true}}/\Sigma_{0,\text{obs}}$ right panel) across three stellar mass regimes: low mass galaxies (orange), intermediate mass galaxies (blue), and high mass galaxies (purple). The horizontal dashed line denotes no change.

The density ratio distributions demonstrate an inverse trend. The low mass regime shows densities near the observed values, $\text{Mean} \approx 0.96$, while the high mass regime shows a pronounced suppression, $\text{Mean} \approx 0.61$. Visually, the separation between the box plots for different mass regimes confirms that the distortion is not uniform across the galaxy population but scales systematically with the depth of the gravitational potential.

A key test of the Mezzi framework is whether the geometric corrections used to recover intrinsic galaxy parameters obey a systematic scaling relation. If the distortion field is governed by consistent physical principles, then the correction factors for scale radius and surface density should display a predictable covariation that is independent of galaxy mass. Figure 3 illustrates this test by plotting the logarithmic ratio of scale radius against the logarithmic ratio of surface density. The sample is highlighted by stellar mass to assess.

The results confirm a strong empirical correlation, $R^2 = 0.85$, between the magnitude of size underestimation and the magnitude of density overestimation for galaxies. The regression reveals a slope of approximately -0.5, corresponding to a square root scaling, that governs the relationship between the geometric correction required for the scale radius, R_{ratio} , and the correction required for the surface density Σ_{ratio} :

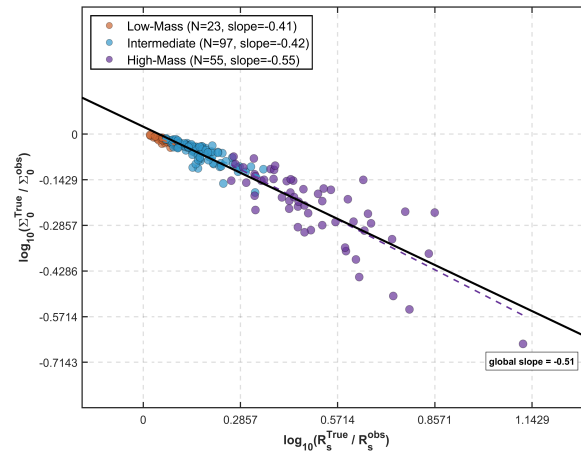


Figure 3. Scatter plot of $\log_{10}(R_{s,\text{True}}/R_{s,\text{obs}})$ versus $\log_{10}(\Sigma_0^{\text{True}}/\Sigma_0^{\text{obs}})$ for the SPARC sample. Points are color coded by the stellar mass regime: low mass (orange), intermediate mass galaxies (blue), and high mass (purple). The solid line shows the global linear regression.

$$\frac{\Sigma_{\text{obs}}}{\Sigma_{\text{true}}} \propto \left(\frac{r_{\text{obs}}}{r_{\text{true}}} \right)^{-1/2} \quad (22)$$

While the surface density at the characteristic scale radius follows the power law with exponent -0.5, this local relationship does not propagate linearly to the mass profile. The Mezzi scale factor $\zeta(r)$ exhibits radial variation, as shown in Figure 1. The geometric distortion is non constant across the galaxy profile. The effective area scaling is suppressed because the outer regions, where most of the mass resides, experience less coordinate distortion than the inner regions. This decouples the mass ratio from the linear geometric expectation, yielding a measured mass to radius relationship that approaches linearity, Figure 4 examines this scaling relation.

$$\frac{M_{\text{obs}}}{M_{\text{true}}} \propto \frac{r_{\text{obs}}}{r_{\text{true}}} \quad (23)$$

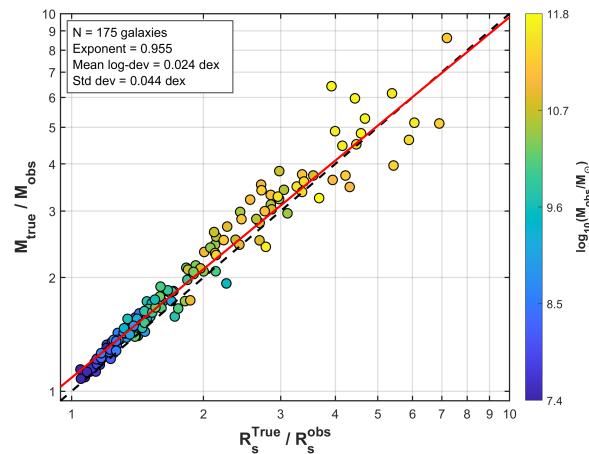


Figure 4. Mass scaling in the true and observed frames ($M_{\text{true}}/M_{\text{obs}}$) compared the scale radii scaling in the true and observed frames $R_{s,\text{True}}/R_{s,\text{obs}}$. Points are color-coded by observed stellar mass, ranging from low mass (dark blue) to high mass (yellow) galaxies. The dashed line shows the 1:1 reference relation, and the solid red line indicates the best fit power law regression with slope.

The figure shows a near unity slope of the mass-radius scaling relation. Providing support for the linear proportionality between mass and radius scaling of Eq 23. The different radial dependence of the Mezzi scale factor $\zeta(r)$ resulted in mass and radius scaling that remain consistent over three orders of magnitude in stellar mass.

4.5. Integrity of the Central Surface Density

We perform a rank order preservation test by directly correlating the observed central surface density $\Sigma_{0,\text{obs}}$ against the Mezzi scaled true central surface density $\Sigma_{0,\text{true}}$ (McGaugh et al., 1995; Zwaan et al., 1995). Figure 5 presents a scatter plot of the log transformed central surface densities; color coded by stellar mass regime. The visualization tests the hypothesis that a galaxy with a higher observed surface density than its peer should maintain a higher true surface density after correction.

The results validate the rank order preservation hypothesis. Pearson correlation analysis yields a coefficient of ~ 1 , indicating that the Mezzi correction is monotonic. A galaxy with a higher observed surface density than its peer maintains a higher true surface density after correction.

However, the preservation of hierarchy manifests differently across mass regimes. The Low Mass regime exhibits near perfect rank preservation, with a regression slope of 0.98 and negligible scatter $\sigma = 0.01\text{dex}$. In contrast, the High-Mass regime shows increased scatter $\sigma = 0.12\text{dex}$ and a significant deviation from the 1:1 line. While the correlation remains strong, the flatter slope of 0.83 suggests that the distortion field compresses the range of densities for massive systems.

Freeman's Law hypothesizes that the central surface density of disk galaxies is constant. This universality is often challenged by the significant scatter in surface densities across galaxies of large mass (Freeman, 1970; Impey & Bothun, 1997).

The slope of 0.83 in the High Mass regime suggests that the variance in the true frame is smaller than in the observed frame. Thus, the scatter seen in high mass galaxies can be interpreted as an observation distortion, demonstrating that the true densities of massive galaxies are more universal than their observed values suggest (Courteau et al., 2007; Shen et al., 2003). Thus, Freeman's Law appears as a manifestation of a deeper intrinsic uniformity. This study suggests that the constant surface density is a fundamental anchor point of galactic structure.

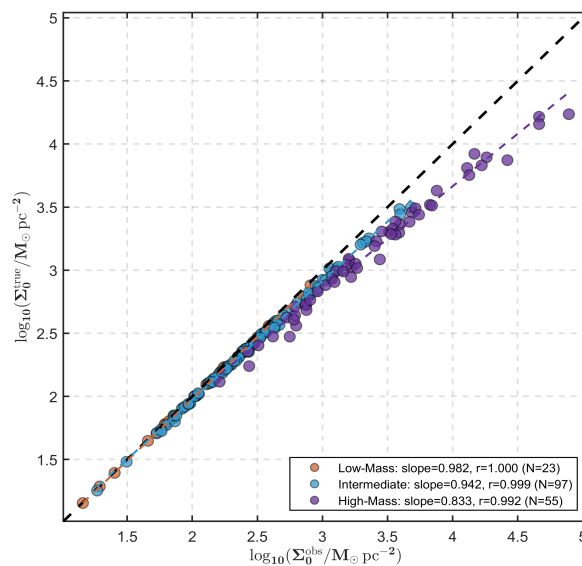


Figure 5. Rank order preservation test for central surface density. Points are color coded by the stellar mass regime: low mass (orange), intermediate mass (blue), and high mass (purple). The horizontal dashed line represents the 1:1 correspondence. Dashed colored lines indicate the linear regression fit for each mass regime.

4.6. Total Mass in True Frame

Figure 6 shows the observed baryonic mass (M_{obs}) and the Mezzi scaled mass (M_{true}) for the SPARC sample. On the left side, corresponding to low mass correction, the observed and true mass markers are close. Such as D564-8 and UGC04483, which exhibit correction ratios of 1.09 and 1.13, respectively. For slightly larger galaxies like UGCA444, the ratio remains modest at 1.22. This indicates that low mass galaxies experience minimal geometric compression, their observed masses are not far from their true masses.

On the right side, the vertical gap widens. The larger galaxies exhibit extreme corrections; for instance, UGC11914 and UGC02953 display mass ratios of 8.62 and 8.26, respectively. Similarly, NGC5985 and UGC12506 show ratios exceeding 5. For these galaxies, the observed masses capture only a small fraction of their true masses, often less than 20% of the true mass. The color mapping transitions from blue to red tones with increasing correction magnitude, directly correlating with the depth of the gravitational potential. Figure 7 shows the total mass scaling against the mean Mezzi scale factor.

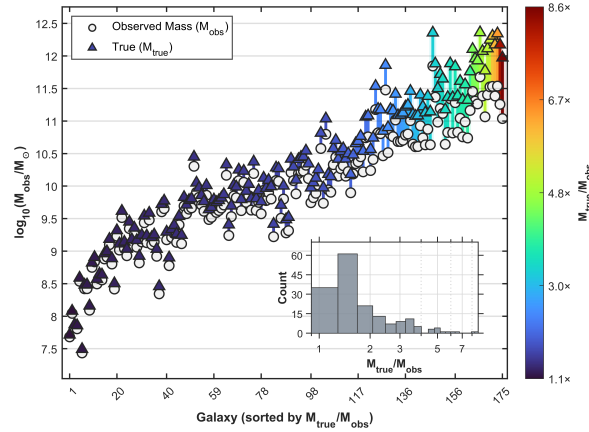


Figure 6. Comparison of observed and true total masses. Main panel: observed masses (circles) and Mezzi scaled masses (triangles), with galaxies sorted by ascending correction factor M_{true}/M_{obs} and connecting lines showing the Color encoded mass ratios for each galaxy. Inset: distribution of mass ratios across the sample.

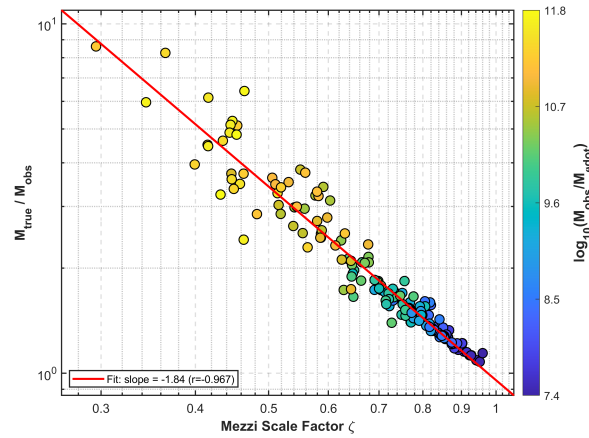


Figure 7. Total Mass Ratio vs Mezzi Scale Factor, Scatter plot of the total mass correction factor against the Mezzi scale factor ζ for all galaxies, color coded by its observed stellar mass. The red solid line shows the best-fit power-law relation $M_{True}^{tot}/M_{obs}^{tot} \propto \zeta^\beta$.

The total mass ratio has a power law correlation with the Mezzi scale factor, $M_{True}^{tot}/M_{obs}^{tot} \propto \zeta^{-1.84}$. This slope near -2 is consistent with an approximate inverse square geometric scaling, yet it deviates slightly from the naive expectation of ζ^{-2} , possibly due to spherical assumptions. The high correlation coefficient indicates that this scaling is robust across three orders of magnitude in stellar mass, Supporting the interpretation that radial variations in $\zeta(r)$, of stronger distortions in the inner regions and weaker effects in the mass-dominant outer regions, that lead to a predictable correction to the mass profile that is independent of the total mass. Besides the the local surface density relation in Eq. 22; and the mass-radius proportionality, in Eq. 23, the proposed approach implies $M_{obs}^{tot} \propto \zeta^2 M_{true}^{tot}$.

5. Deflection Angle

5.1. Weak Field Gravitational Deflection in the True Frame

The gravitational deflection angle α experienced by an Electromagnetic wave grazing a mass distribution scales linearly with the enclosed mass and inversely with the impact parameter, reflecting the curvature of spacetime induced by the mass (Prat & Bacon, 2025). Within the Mezzi framework, the coordinates used in standard analysis are geometrically compressed projections of the true physical coordinates. Consequently, in the weak field approximation, the standard calculation of deflection angles underestimates the true gravitational lensing potential of the galaxy. To recover the deflection angle, α_{true} , we consider the true geometry.

While the Mezzi effect predicts larger galactic masses, it also increases the size. As a result, the ratio M/R , and therefore the gravitational potential $\Phi = -G M / R$, remains very small. This keeps the system within the weak-field regime, where $|\Phi| / c^2 \ll 1$, allowing the use of a linearized description of gravity (Chowdhuri et al., 2023; Prat & Bacon, 2025). The true deflection angle in the weak field approximation is given by the standard formula:

$$\alpha_{\text{true}}(r_{\text{true}}) = \frac{4G}{c^2} \frac{M_{\text{true}}(< r_{\text{true}})}{r_{\text{true}}} \quad (24)$$

The deflection angle is calculated in the observed frame using the observed mass profile as a baseline.

$$\alpha_{\text{obs}}(r_{\text{obs}}) = \frac{4G}{c^2} \frac{M_{\text{obs}}(< r_{\text{obs}})}{r_{\text{obs}}} \quad (25)$$

Figure 8 compares the maximum deflection angle calculated in the observed frame ($\alpha_{\text{max, obs}}$) and the maximum deflection angle calculated in the true frame ($\alpha_{\text{max, true}}$) for the SPARC sample. The dashed 1:1 line represents the null hypothesis where standard observations would accurately predict α .

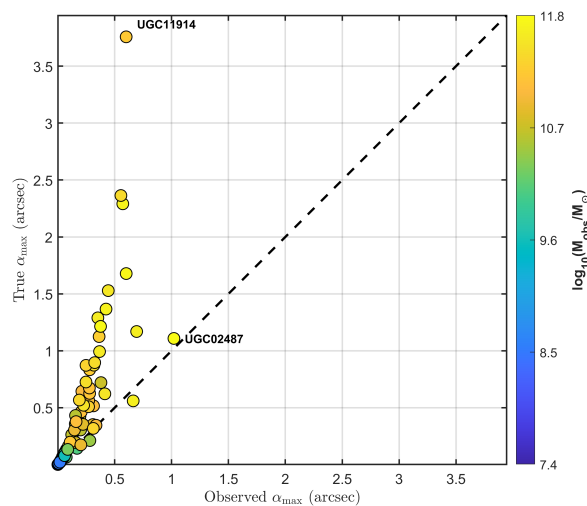


Figure 8. Comparison of observed and true maximum deflection angles. Scatter plot of $\alpha_{\text{max, obs}}$ versus $\alpha_{\text{max, true}}$, with points colored by observed stellar mass. The dashed line denotes the 1:1 relation. Two extreme cases are labeled.

The results reveal a systematic correction, indicating that the true frame lensing strength is consistently underestimated in observation. The results reveal a distinct mass dependent hierarchy in the deviation. Low-mass galaxies, in blue, cluster near the 1:1 line, suggesting their shallow gravitational potentials induce minimal geometric compression. In contrast, high mass galaxies, in yellow, exhibit significant vertical displacement, with true frame deflection angles substantially exceeding their observed values.

To discuss the severity of the distortion, we group the galaxies into three regimes based on their mass enhancement factor ($M_{\text{ratio}} = M_{\text{true}} / M_{\text{obs}}$):

Low Correction ($M_{ratio} < 1.3$): 36 galaxies, 20.6% of the sample, these galaxies show minimal geometric distortion. For example, D564-8, $M_{ratio} = 1.09$, exhibits a deflection ratio $\alpha_{ratio} = 1.03$, and UGC04483, $M_{ratio} = 1.13$, shows $\alpha_{ratio} = 1.07$. These values hover near unity, confirming that small galaxies reside in a near Euclidean spatial regime where the Mezzi effect is negligible.

Moderate Correction ($1.3 \leq M_{ratio} < 2.5$): 94 galaxies fall into this intermediate category. They demonstrate a consistent larger deflection, with α_{true} typically 1.2 to 1.7 larger than observed estimates. A typical example is NGC4183, where a mass ratio of 2.15 results in a α_{ratio} of 1.59.

High Correction ($M_{ratio} \geq 2.5$): 45 galaxies exhibit extreme deviations. For example, UGC11914 with $M_{ratio} = 8.62$, yields $\alpha_{ratio} = 6.25$, and NGC5985, with $M_{ratio} = 5.96$, yields $\alpha_{ratio} = 4.01$.

While the dominant trend is enhancement, a distinct subset of intermediate mass galaxies exhibits a deflection ratio near or even below unity $\alpha_{ratio} \leq 1$. Notable examples include NGC4138 ($M_{ratio} = 1.73$, $\alpha_{ratio} = 0.75$) and UGC00634 ($M_{ratio} = 1.40$, $\alpha_{ratio} = 0.88$). Others in the high mass regime, such as the UGC02487, despite falling into the High Correction category with a substantial mass ratio $M_{ratio} = 3.25$, it exhibits a deflection ratio $\alpha_{ratio} = 1.09$, that is nearly unity.

Possible interpretation is that the geometric rescaling spreads the mass over a large physical volume that the surface density drops sharply, effectively diluting the lensing signal. UGC02487 serves as a critical counter example, demonstrating that for extremely massive, compact systems, the Mezzi correction can redistribute mass in a way that neutralizes lensing enhancement, distinguishing it from the high scaling behavior seen in galaxies like UGC11914. Figure 9 shows the scaling of the lensing deflection ratio against the average mezzi scale factor.

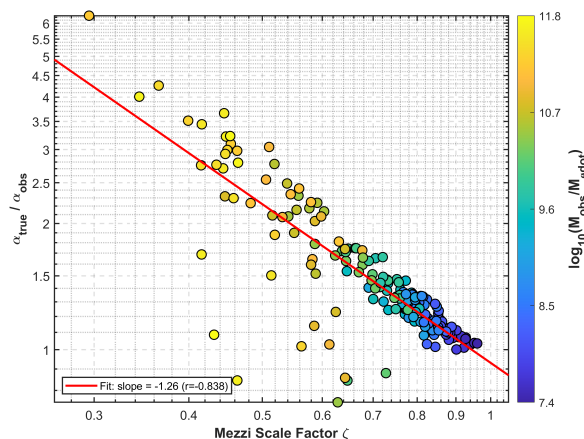


Figure 9. Deflection Angle Ratio vs Mezzi Scale Factor, Scatter plot of the gravitational deflection angle correction factor ($\alpha_{true} / \alpha_{obs}$) against the Mezzi scale factor ζ for all galaxies, color coded by its observed stellar mass. The red solid line shows the best-fit power-law relation $\alpha_{true} / \alpha_{obs} \propto \zeta^{-1.26}$.

This slope is shallower than the mass scaling slope. $\alpha_{true} / \alpha_{obs} \propto \zeta^{-1.26}$. This suggests that the proposed framework affects dynamics and lensing differently. The radial variation of the scale factor $\zeta(r)$, which induces stronger distortions in the inner regions and weaker effects in the mass-dominant outer regions, redistributes the corrected mass across a larger physical volume, effectively diluting the surface density that governs lensing efficiency. Consequently, although the true mass increases substantially for highly distorted systems, this mass is spread over proportionally larger radii, partially offsetting the enhancement in the deflection angle. The correlation coefficient of $r = 0.83$ and higher scatter, demonstrates a slightly weaker relationship compared to the total mass ratio ($r = 0.96$), supporting the idea that lensing deflection angles, being sensitive to the projected mass distribution rather than the total integrated mass, experience a more complex effect from geometric scaling.

5.2. True Frame Deflection vs. Observed Rotation Velocity

Figure 10 presents a cross analysis. The x-axis represents the observed rotation velocity, a proxy for the total gravitational potential depth. The y-axis represents the maximum deflection angle, calculated from the true mass profile.

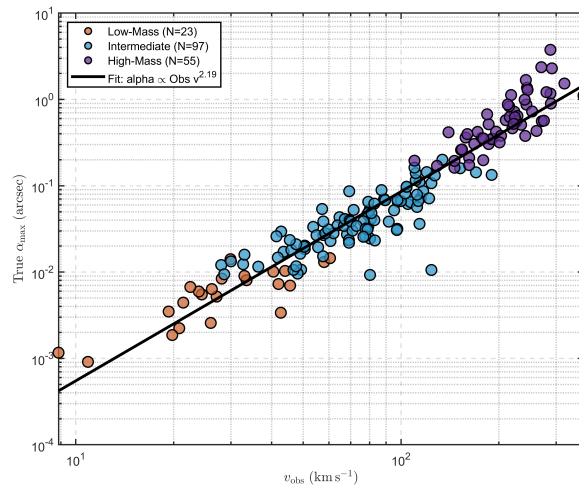


Figure 10. Cross comparison of true deflection and observed velocity. Scatter plot of true maximum deflection angle versus observed rotation velocity, with points colored by stellar mass regime. The solid line shows the power law fit $\alpha_{\max} \propto v_{\text{obs}}^{\beta}$.

This analysis serves as a bridge between the geometric reconstruction of the mass profile, derived from photometry and the Mezzi scale factor scaling, versus the independent dynamical measurement of the gravitational potential. The BTFR establishes a tight correlation between baryonic mass and rotation velocity (Mistele et al., 2024).

Since the deflection angle scales as $\alpha \propto M / R$ and the rotation velocity scales as $v^2 \propto M / R$, a correlation is expected if the reconstructed mass is physically meaningful. A strong correlation in this plot would imply that the scaled lensing recovered by the Mezzi scale factor corresponds to the excess gravitational pull required to sustain the observed rotation speeds.

The resulting slope of 2.19 approximates the theoretical expectation of slope 2.0 derived from fundamental gravitational principles. Supported by a high correlation coefficient of 0.95, indicating that the true frame deflection angle reliably tracks the observed rotation velocity. The figure supports the idea that this scaling law is valid across the mass spectrum. Low mass galaxies adhere to the power law, similarly to the high mass galaxies.

6. Rotation Curves

The missing mass problem is conventionally defined as the discrepancy between the observed circular velocity profile and that predicted by the Newtonian gravitational potential sourced by the observed baryonic mass distribution. We investigate whether the Mezzi scaling of radial coordinates and enclosed mass, derived from the flow accumulation formalism, can reconcile this discrepancy within a purely Newtonian framework.

The analysis proceeds under three operational assumptions regarding the physical system:

Validity of Newtonian Dynamics: We assume that standard Newtonian gravity applies in the true frame. The orbital velocities obey the Keplerian relation, with no modifications to the underlying dynamics.

Accuracy of Observed Velocities: The observed orbital velocities of tracers (stars and gas), v_{obs} , are assumed to be correct measurements of the true orbital speed. The spectroscopic determination of line of sight velocities is unaffected by the geometric distortion of the radial coordinate grid.

Radial Coordinate Misattribution: While the velocity magnitudes are correct, the radii at which they are observed, r_{obs} , are systematically underestimated. The observed rotation curve v_{obs} is therefore a mapping of true velocities onto a compressed spatial coordinate frame.

In the observational framework, the Newtonian prediction for the orbital velocity at an observed radius r_{obs} is calculated using the observed enclosed mass $M_{\text{enc,obs}}(< r_{\text{obs}})$.

$$v_{\text{Newton,obs}}(r_{\text{obs}}) = \sqrt{\frac{G M_{\text{enc,obs}}(< r_{\text{obs}})}{r_{\text{obs}}}} \quad (26)$$

This formulation fails to match the observed rotation curve in the outer regions of galaxies, yielding velocities significantly lower than v_{obs} . This deficit constitutes evidence for dark matter.

In the proposed framework, the galaxy is described by the true coordinates and mass profile derived in Section 4. The Newtonian velocity in the source frame is calculated using the true enclosed mass $M_{\text{enc,obs}}$ and the true radial coordinate r_{obs} :

$$v_{\text{Newton,true}}(r_{\text{true}}) = \sqrt{\frac{G M_{\text{enc,true}}(< r_{\text{true}})}{r_{\text{true}}}} \quad (27)$$

6.1. Newtonian Velocity in the True Frame

We compare the observed rotation curves with the theoretical predictions in both the observed and true frames. If the Mezzi effect correctly describes the geometric distortion, then the true Newtonian velocity should correspond to the observed velocity.

Since the observed velocities are assumed correct, we define the ground truth velocity field as $v_{\text{true}} = v_{\text{obs}}$. The residual errors are quantified for both the observed and mezzi scaled models. The error between the observed velocity and the Newtonian velocity in the true frame is:

$$\text{Residual}(r_{\text{obs}}) = v_{\text{obs}}(r_{\text{obs}}) - v_{\text{Newton,true}}(r_{\text{true}}) \quad (28)$$

$v_{\text{Newton,true}}$ is evaluated at the radius r_{true} . Since v_{obs} is mapped to r_{obs} , we treat the comparison as a direct mapping of velocities: the velocity observed at the compressed radius r_{obs} is compared against the velocity in the true frame in r_{true} .

The results of the orbital velocity are presented in Figure 11. Divided into two panels: linear residuals, in the left Panel, and logarithmic fractional residuals, in the Right Panel. The galaxies are color coded by their observed stellar mass to highlight mass dependent trends in the reconstruction fidelity.

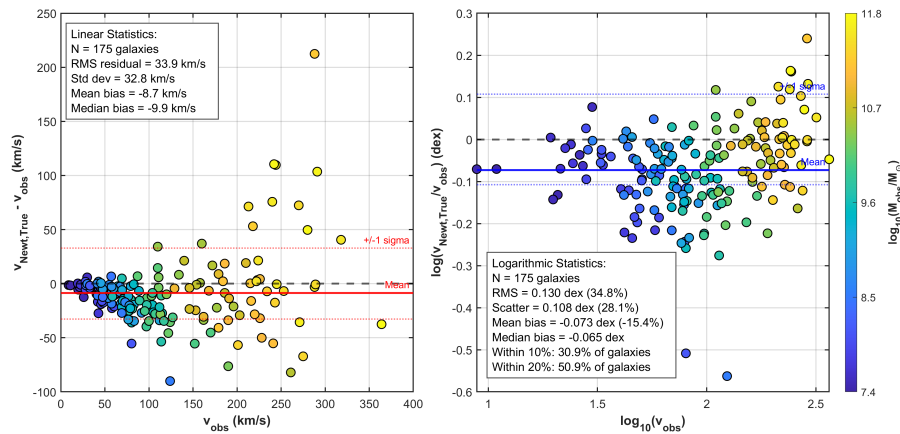


Figure 11. Residuals of the Newtonian velocity in the true frame. Left Panel: linear residuals $v_{\text{Newton,true}} - v_{\text{obs}}$ versus observed velocity. Right Panel: logarithmic residuals $\log_{10}\left(\frac{v_{\text{Newton,true}}}{v_{\text{obs}}}\right)$ versus $\log_{10}(v_{\text{obs}})$. Points are colored by observed stellar mass. Horizontal lines denote zero residual (dashed), mean bias (solid), and $\pm 1\sigma$ scatter (dotted).

The application of the geometric correction results in an improvement in the agreement between the predicted Newtonian velocities and the observed kinematics. The statistical summary for the sample of 175 galaxies revealed that the Root Mean Square residual is 33.88 km/s with a standard deviation of 32.84 km/s. The mean bias is -8.69 km/s, indicating a slight systematic under prediction

of velocities on average. In the Logarithmic Residuals, the RMS residual is 0.130 dex. The mean logarithmic bias is -0.073 dex.

In the Low Mass Galaxies ($\log M < 9.0$), the RMS residual is 11.25 km/s and a bias of -7.56 km/s. In the Intermediate Mass Galaxies ($9.0 \leq \log M < 10.5$), the largest systematic deviation is registered, a consistently negative residual with bias of -16.92 km/s. And in the High Mass Galaxies ($\log M \geq 10.5$), a positive bias of +5.36 km/s and a significantly larger scatter (RMS 50.63 km/s) is registered. The positive bias indicates that the model slightly *over-predicts* the velocities for these galaxies. This over-correction is consistent with the extreme geometric scaling factors ($\zeta \ll 1$) derived for high mass galaxies. The high scatter may be caused by the assumptions of isolation and spherical symmetry, which may not accurately represent these structures (Pontzen et al., 2015).

The results support the hypothesis of the Mezzi effect, that the discrepancy between Newtonian dynamics and observation can be a geometric artifact. The reconstruction successfully brings the vast majority of the sample (86.9%) within a factor of 1.5 of the observed velocity. The remaining residuals, particularly the systematic under prediction in intermediate mass galaxies, point toward model refinement as the next logical step in the development of this framework.

6.2. Systematic Rotation Curve Bias

A statistical distribution of the rotation curves errors is presented in Figure 12. On the Left Panel, the histogram of logarithmic residuals, and in the Right Panel, the cumulative distribution function of the absolute errors. This analysis characterizes the underlying probability distribution of the reconstruction fidelity.

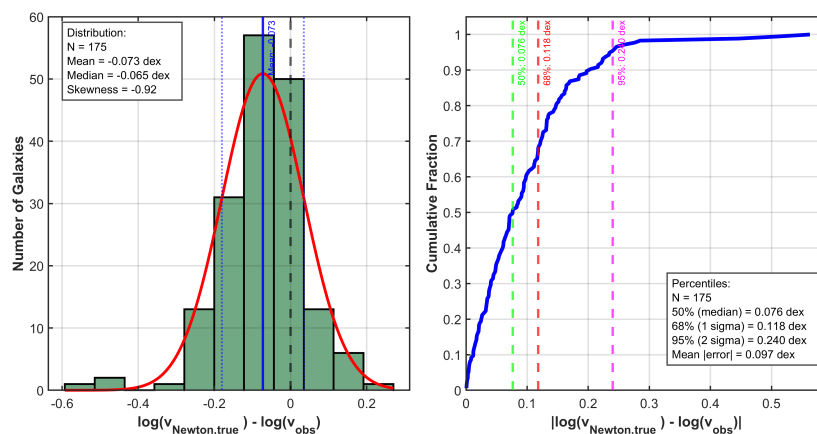


Figure 12. Distribution of Mezzi velocity errors. Left Panel: histogram of logarithmic residuals with Gaussian fit. Right Panel: cumulative distribution of absolute logarithmic errors, with vertical lines marking the 50th, 68th, and 95th percentiles.

The logarithmic residuals $\log_{10}(v_{\text{Newton, true}} / v_{\text{obs}})$ distribution follows a Gaussian profile, as confirmed by a Kolmogorov-Smirnov test with $p = 0.30$. There is no significant departure from normality at the 5% level. The reconstruction errors appear to reflect stochastic scatter, presumably driven by Mezzi scale factor variations, rather than evidence that the model itself is fundamentally wrong.

Nevertheless, the distribution shows systematic deviations from a perfect Gaussian: it exhibits negative skewness of -0.918, and a mean bias of -0.073 dex. The negative skewness indicates an extended tail of galaxies for which the predicted velocities fall substantially below the observed values. This tail consists primarily of intermediate mass galaxies.

The absolute error distribution via the Cumulative Distribution Function (CDF) provides a measure of the typical error expected for a random galaxy in the sample, independent of the averaging effects that can mask outliers in RMS calculations. The percentiles of this distribution define the framework's operational accuracy:

The Median (50th percentile), 0.076 dex, corresponds to a 19.2% fractional error. This establishes that the typical galaxy in the SPARC sample has its rotation curve recovered within $\sim 20\%$ of the observed value.

1-sigma (68th Percentile), 0.118 dex, corresponding to a 31.2% fractional error. This value aligns with the sample standard deviation of 0.108 dex, reinforcing the interpretation that errors follow an approximately Gaussian distribution.

2-sigma (95th Percentile), 0.240 dex, corresponding to a 73.8% fractional error. 89.7% of galaxies have velocities recovered within a factor of 1.6, and 4.6% of the sample (8 galaxies) exceed the 95th percentile threshold, established as statistical outliers.

The analysis identifies a distinct population of outliers where the Mezzi framework fails to recover observed velocities within reasonable tolerance. The most significant cases include NGC 3741, UGCA 281, UGC 05721, and NGC 2915. Many outliers are low surface brightness galaxies, gas rich galaxies like NGC 3741. The failure in these specific cases may indicate limitations of the proposed approach. The assumption that the observed surface density profile scales geometrically may not hold if the gas distribution responds non-linearly to the vacuum flow. Alternatively, the discrepancy may stem from mass profile calculations based on gas velocity. This issue will be addressed in future work.

6.3. Validation in the Baryonic Regime

In the inner regions of galaxies the observed Newtonian dynamics accurately predicts the observed rotation curves. This region serves as a rigorous test for the Mezzi effect, as the geometric correction must not introduce a systematic error where the standard model already holds. To test for such artifact, we segment the rotation curve data into two dynamical regimes, the Inner Region ($r < 0.3 r_{\max}$), and the Outer Region ($r \geq 0.3 r_{\max}$), with filter requiring at least 10 valid data points per galaxy for statistical robustness.

We calculate the RMS and velocity bias for both the naive Newtonian model (Newtonian in the observed frame) and the Mezzi scaled model (Newtonian model in the true frame), the bias is defined as the mean residual, $Bias = \langle v_{\text{obs}} - v_{\text{Newton, true}} \rangle$. Figure 13 presents a comparative analysis of these bias distributions across the SPARC sample. This visualization directly assesses whether the Mezzi framework introduces systematic deviations in the galactic core.

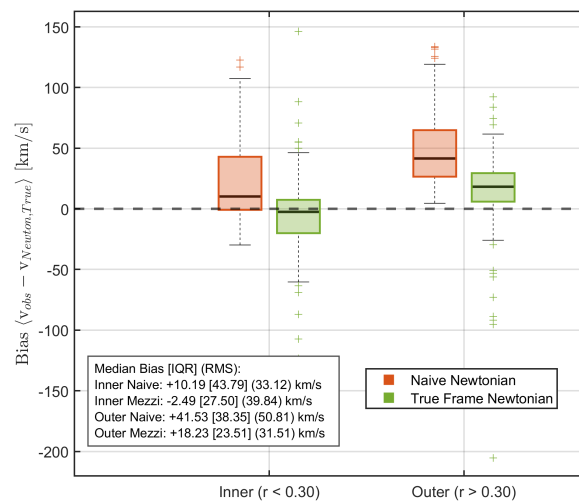


Figure 13. Velocity bias comparison for inner and outer regions. Boxplots show the distribution of velocity residuals $Bias = \langle v_{\text{obs}} - v_{\text{Newton, true}} \rangle$ for naive Newtonian (orange) and Mezzi scaled Newtonian (green) models. Left pair: inner region. Right pair: outer region.

In the outer region, the Mezzi framework demonstrates a clear and consistent improvement across all statistical metrics. The standard Newtonian model shows a large positive bias, with a median error of +41.5 km/s. The Mezzi correction reduces this median bias by 56.1% to +18.2 km/s. Simultaneously, the range of residuals is smaller. The Interquartile Range (IQR), the measure of spread

for the typical galaxy, decreases by 38.7%, from 38.4 km/s to 23.5 km/s, indicating that the central 50% of the sample is significantly better clustered around the observed velocities. The RMS error drops by 38.0%, confirming that the proposed geometric rescaling recovers the observed velocities. With a close alignment between the Mean of +14.1 km/s and Median of +18.2 km/s.

The inner region presents a more complex statistical landscape, revealing a tradeoff between the accuracy of the typical galaxy and the errors of the outliers. For the typical galaxy, the Mezzi correction is effective. The median bias drops by 75.6% from +10.2 km/s to -2.5 km/s. The IQR contracts by 37.2%, from 43.8 km/s to 27.5 km/s. This supports the observation that for the bulk of the sample, the Mezzi framework tightens the fit. Despite the improvement in median performance, the RMS error increases by 20.3%, and the standard deviation rises significantly. The discrepancy arises because the Mean and RMS are sensitive to large errors, whereas the Median and IQR are robust against them. The Naive model's positive mean of +23.8 km/s vs. median of +10.2 km/s indicates galaxies with under predictions. However the Mezzi model's negative mean of -9.4 km/s vs. median of -2.5 km/s indicates galaxies with over predictions.

These results suggest that the suggested framework improves the fit for the majority of galaxies, tightening the distribution and reducing errors in both inner and outer regions. However, there exist outliers, leading to an apparent degradation of the global fit in the inner region.

7. Radial Acceleration Relation

7.1. The Radial Acceleration Relation as a Geometric Transformation Effect

This section tests the hypothesis that the observed mass discrepancies arise from geometric coordinate distortion. We examine whether applying the Mezzi scale factor $\zeta(r)$ to map Newtonian physics in the true frame into the observed frame can reproduce the empirical Radial Acceleration Relation (McGaugh et al., 2016). The analysis considers three cases, each isolating a distinct aspect of the coordinate transformation.

7.1.1. Test 1: Newtonian Consistency in the True Frame

This case serves as the theoretical baseline. Here, M_{true} and r_{true} are the reconstructed true mass and radial coordinates, and $v_{Newton,true}$ is the Newtonian velocity derived from them.

$$\begin{cases} g_{bar}^{(1)} = \frac{GM_{true}}{r_{true}^2} \\ g_{dyn}^{(1)} = \frac{v_{Newton,true}^2}{r_{true}} \end{cases} \quad (29)$$

Since $v_{Newton,true}^2 = GM_{true} / r_{true}$ by construction, mathematically ($g_{dyn}^{(1)} = g_{bar}^{(1)}$), thus near zero scatter is expected. Any deviation would indicate a failure in the mass reconstruction (Desmond et al., 2023).

Figure 14 illustrates the first test reconstruction. The gray points represent the observed data, showing the characteristic deviation from the unity line. The blue points represent acceleration from the Mezzi scaled frame. As expected the blue points collapse onto the unity line across the full dynamic range.

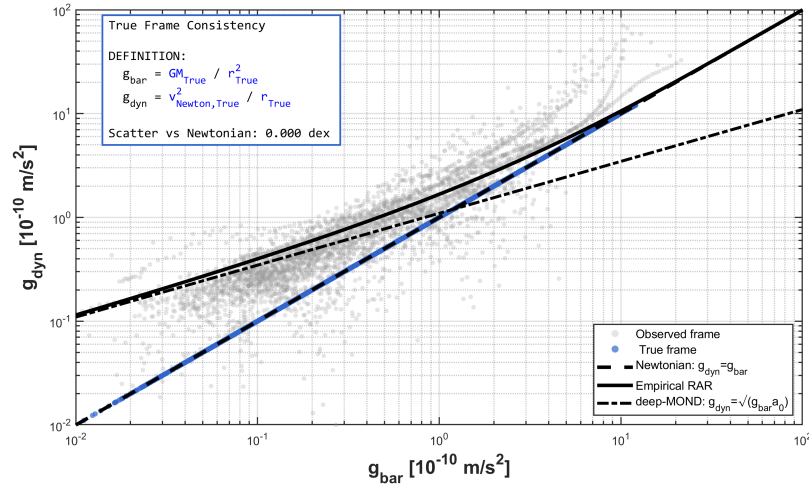


Figure 14. RAR True frame consistency test. Dynamical acceleration g_{dyn} versus baryonic acceleration g_{bar} , with points showing the observed frame (gray) and Mezzi-scaled frame (blue). Reference lines denote the Newtonian expectation (dashed), empirical radial acceleration relation (solid), and deep-MOND limit (dash-dotted). The missing mass problem is absent in the true frame. The deviation seen in the observed accelerations, shown in gray points does not exist.

7.1.2. Test 2: The Geometric Mapping Hypothesis

This case provides the primary test of the Mezzi effect hypothesis. It compares the baryonic acceleration in the true frame against the observed dynamical acceleration. Under the Mezzi framework, we assume the observed velocity v_{obs} correctly measures the true orbital speed, maintaining kinematic integrity, while the observed radius r_{obs} represents the geometrically distorted coordinate.

$$\begin{cases} g_{\text{bar}}^{(2)} = \frac{GM_{\text{true}}}{r_{\text{true}}^2} \\ g_{\text{dyn}}^{(2)} = \frac{v_{\text{obs}}^2}{r_{\text{obs}}} \end{cases} \quad (30)$$

Plotting observed dynamics against true baryonic fields tests whether the geometric scaling factor account $\zeta(r)$ ts for the apparent mass discrepancy. If valid, the data should align with the empirical RAR, supporting the hypothesis that the observed RAR emerged from projecting Newtonian physics through the mezzi effect (Desmond et al., 2024; Mercado et al., 2024). Figure 15 discusses the results of this test.

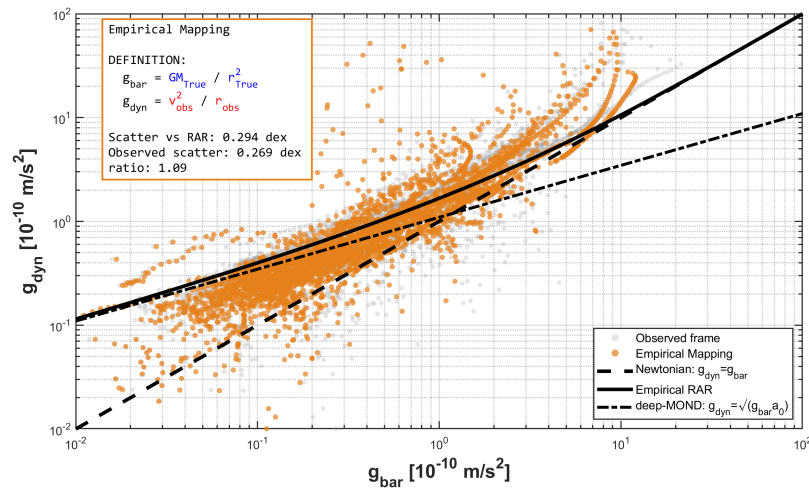


Figure 15. RAR Empirical mapping test. Dynamical acceleration g_{dyn} versus baryonic acceleration g_{bar} , comparing the baryonic accelerations based on Mezzi scaled masses and radii (orange) with dynamical accelerations use observed velocities and radii. Reference lines show the Newtonian expectation (dashed), empirical radial acceleration relation (solid), and deep-MOND limit (dash-dotted).

The results show that the observed RAR arises from projecting Newtonian physics onto the observer's compressed coordinate grid. The critical finding is the statistical equivalence between the Mezzi mapping, shown in Orange points, and the empirical RAR. The Mezzi mapping yields a scatter of 0.294 dex, close to the 0.269 dex scatter in the observed data, shown in Gray points, with a ratio of 1.09. This statistically significant result indicates that the geometric scaling factor $\zeta(r)$, which connects the true mass distribution to observed kinematics, may account for the empirical shape of the RAR. The high scatter in both the Mezzi mapping (0.294 dex) and the observed data (0.269 dex) suggests that uncertainties in mass profile calculations may contribute significantly to the scatter in both cases, highlighting the need for future work to develop more accurate mass profiles.

7.1.3. Test 3: Baryonic Axis Shift Test

This test simulates the error introduced by attributing the true mass distribution to the compressed spatial grid. It isolates the specific geometric error introduced by the static metric assumption. This configuration mimics an observer who correctly calculates the total mass but fails to account for the expanded spatial geometry. It evaluates the true enclosed mass M_{true} at the observed radius r_{obs} . Since $r_{obs} < r_{true}$, evaluating the mass at this smaller radius artificially inflates the baryonic acceleration.

$$\begin{cases} g_{bar}^{(3)} = \frac{GM_{true}}{r_{obs}^2} \\ g_{dyn}^{(3)} = \frac{v_{obs}^2}{r_{obs}} = g_{dyn}^{(2)} \end{cases} \quad (31)$$

This test is expected to yield

$$\frac{g_{bar}^{(3)}}{g_{bar}^{(2)}} = \left(\frac{r_{true}}{r_{obs}} \right)^2 = \frac{1}{\zeta^2} \quad (32)$$

This represents a horizontal shift of Test 2 by factor ζ^2 on the log-log plot. The results is shown in Figure 16.

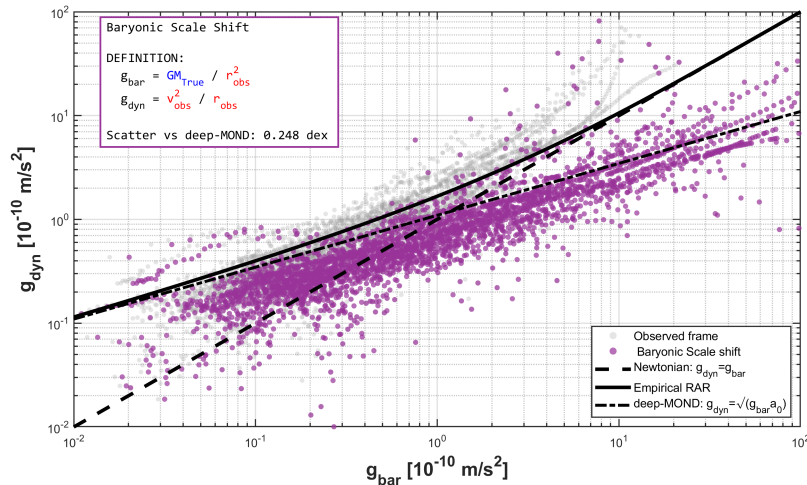


Figure 16. RAR Baryonic-axis shift test. Comparing the Baryonic accelerations based on Mezzi scaled masses and observed radii (purple), with dynamical accelerations based on observed velocities and radii. Reference lines show the Newtonian expectation (dashed), empirical radial acceleration relation (solid), and deep-MOND limit (dash-dotted).

The results show that this specific misalignment produces a scatter of 0.248 dex against the deep-MOND scaling relation $g_{obs} = \sqrt{g_{bar} a_0}$, revealing a possible origin of the Modified Newtonian Dynamics. This figure suggests that the apparent boost in gravitational efficiency, often interpreted as a modification of gravity, can be a direct consequence of underestimating the radius over which the mass acts.

7.2. Residual Diagnostics and Model Limitations

This section analyzes residuals in order to detect systematic dependencies in the Empirical Mapping test. We define the residual Δ_{RAR} as the logarithmic deviation of the observed dynamical acceleration from the empirical RAR prediction, where (g_{emp} is the acceleration defined by the empirical RAR fit (McGaugh et al., 2016) for a given observed baryonic acceleration.

$$\Delta_{RAR} = \log_{10}(g_{\text{dyn}}^{\text{obs}}) - \log_{10}(g_{\text{emp}}) \quad (33)$$

We look for a radial systematic error by plotting the residuals against the observed radial coordinate r_{obs} , color coded by the galaxy's observed total baryonic mass. This diagnostic as any radial trend in the residuals would indicate a failure of the Mezzi scale factor (Pontzen et al., 2015; Stiskalek et al., 2026). Furthermore, checking for mass dependence explores whether the geometric scaling behaves universally across the galaxy population.

The statistical analysis of the residuals for the Empirical Mapping test reveals a nuanced portrait of the framework's performance. While the global metrics confirm the mathematical viability of the geometric hypothesis, the analysis exposes systematic dependencies that delineate the boundaries of the model's current approximation. Figure 17 reveals a strong radial gradient.

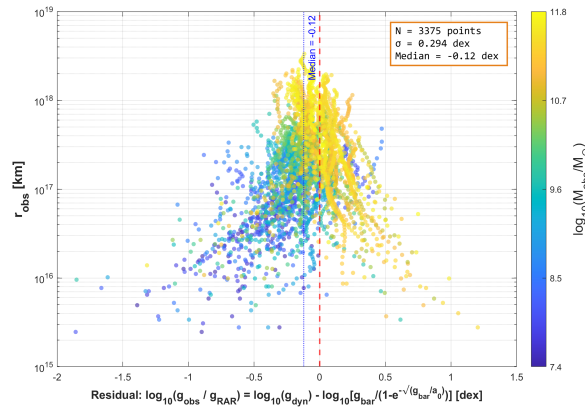


Figure 17. Residuals versus observed radius for the empirical mapping case. Scatter plot of logarithmic residuals $\log_{10}(g_{\text{obs}} / g_{\text{emp}})$ versus observed radius r_{obs} , with points colored by galaxy stellar mass. Vertical dashed red line denotes zero residual and the dotted blue denote the median residual.

In the outer Regions ($r > 20$ kpc) the residuals converge toward zero, median ≈ -0.08 dex, with a tight scatter, $\sigma \approx 0.11$ dex. This is the region traditionally dominated by the dark matter halo. The performance of the Mezzi mapping here validates the framework's core tenet: the apparent mass discrepancy in the outer disk can be a geometric artifact.

The inner Regions ($r < 5$ kpc) exhibits a significant negative bias, median ≈ 0.49 dex, and a catastrophic increase in scatter, $\sigma \approx 0.72$ dex. This indicates that observed dynamics in the core substantially deviate from the scaling relation. However, this radial variation can be caused by a failure of the spherical symmetry and isolation assumption (Pontzen et al., 2015; Stiskalek et al., 2026), future work may investigate this deviation.

The mass based analysis reveals a scaling transition across the galaxy population: residuals shift from negative to positive with increasing mass. Low mass galaxies show a consistent negative bias, median ≈ -0.28 dex, while high mass galaxies display a slight positive bias with median $+0.05$ dex.

7.3. Radial Evolution of the Acceleration Scale

To examine the radial structure more directly, we analyze how the characteristic acceleration ratio g_{ratio} / a_0 varies with radial distances. We define the parameter g_{ratio} as:

$$g_{\text{ratio}}(r) = \frac{g_{\text{dyn}}^2(r)}{g_{\text{bar}}(r)} \quad (34)$$

This quantity represents the effective acceleration scale needed to reconcile observed dynamics with the baryonic mass distribution. In the context MOND, this ratio serves as a direct diagnostic of the gravitational regime:

In the Newtonian regime, where the observed dynamical acceleration closely tracks the baryonic acceleration such that $g_{\text{dyn}} \approx g_{\text{bar}}$, for typical galaxy, the ratio g_{ratio} far exceeds the MOND characteristic scale a_0 (Milgrom & Sanders, 2003). In another hand, in the deep MOND regime, where the dynamics are dominated by the modified gravitational law $g_{\text{dyn}} \approx \sqrt{g_{\text{bar}} a_0}$, the ratio g_{ratio} converges to the universal constant a_0 .

The Mezzi framework suggests that the transition between these regimes arises not from modifying the force law, but from the progressive increase in the geometric scale factor $\zeta(r)$. Here, the ratio g_{ratio} / a_0 serves as a proxy for the strength of geometric distortion in the observer frame. Figure 18 shows the radial profiles of g_{ratio} / a_0 for the SPARC sample, normalized by each galaxy's maximum observed radius.

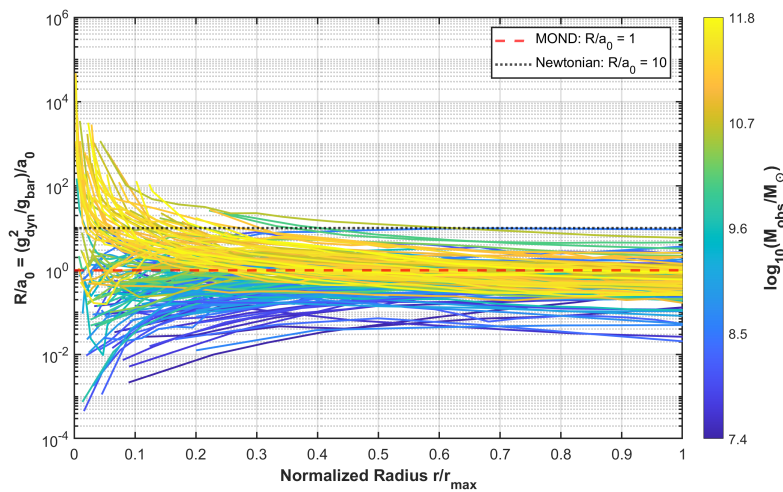


Figure 18. Radial profiles of the Mezzi scaled g_{ratio} / a_0 , plotted against normalized, Curves colored by observed stellar mass. Horizontal reference lines denote the MOND $g_{\text{ratio}} / a_0 = 1$ (dashed red) and the Newtonian limit $g_{\text{ratio}} / a_0 = 10$ (dotted black).

In the Inner Region ($r / r_{\text{max}} < 0.3$), g_{ratio} / a_0 varies dramatically with clear distinction between the low mass and high mass galaxies. In the Outer Region ($r / r_{\text{max}} > 0.3$): The median $g_{\text{ratio}} / a_0 \approx 0.60$, falls below the MOND regime. This suggests that in the outer disk, where the geometry has expanded, the acceleration scale appears nearly universal.

8. Baryonic Tully Fisher Relation

8.1. Geometric Interpretation of the Baryonic Tully Fisher Relation

To examine the geometric origin of the BTFR (Lelli et al., 2017), we consider scaling relations in the true frame and in the observer frame (Vukcevic et al., 2024).

For Newtonian consistency, We correlate the true total mass M_{true} with the true Newtonian velocity v_{true} . $M = M_{\text{true}}(< r_{\text{max,true}})$, $v = v_{\text{Newton}}(r_{\text{true}})$ This case tests the underlying dynamics in the source frame. If the reconstruction is valid, this relation is expected to produce the Newtonian slope of 2.0 ($M \propto v^2$).

For the Empirical Mapping hypothesis, we correlate the true total mass M_{true} with the observed velocity v_{obs} . This maps the true mass distribution to kinematic measurements made in the observer's compressed frame: $M = M_{\text{true}}(< r_{\text{max,true}})$, $v = v_{\text{obs}}(r_{\text{obs}})$. This configuration tests if scaling mass in the true frame with observed velocity is expected to produce the BTFR slope of 4.0 ($M \propto v^4$)

We adopt the standard operational definition of the BTFR, wherein each galaxy is represented by a single data point corresponding to its global baryonic mass M_{tot} and its characteristic outer rotation velocity v_{flat} . The total mass is defined as the enclosed mass at the outermost observed radius:

$M_{\text{tot}} = M(r_{\text{max}})$. The characteristic rotation velocity v_{flat} is calculated as the mean of the observed velocity profile over the asymptotic flat region of the rotation curve. Specifically, we consider velocities in the radial range $r \geq 0.3 r_{\text{max}}$ encompassing the outer 70% of the galactic disk. This definition ensures that the velocity measurement is representative of the flat rotation curve regime.

Data points are retained for radius $r > 1$ kpc, to exclude the bulge-dominated core. The considered galaxies contain at least 10 points passing these criteria. At least 5 valid points in the outer disk ($r \geq 0.3 r_{\text{max}}$) for both observed and true profiles independently. This ensures robust linear regression in the flat rotation curve region for each case comparison.

Figure 19 presents the BTFR scaling relations for the True Frame and Empirical Mapping cases in a side by side comparison. The two panels contrast the recovered scaling exponents against the theoretical Newtonian slope of 2 (green dashed line) and the empirical BTFR slope of 4 (red dashed line).

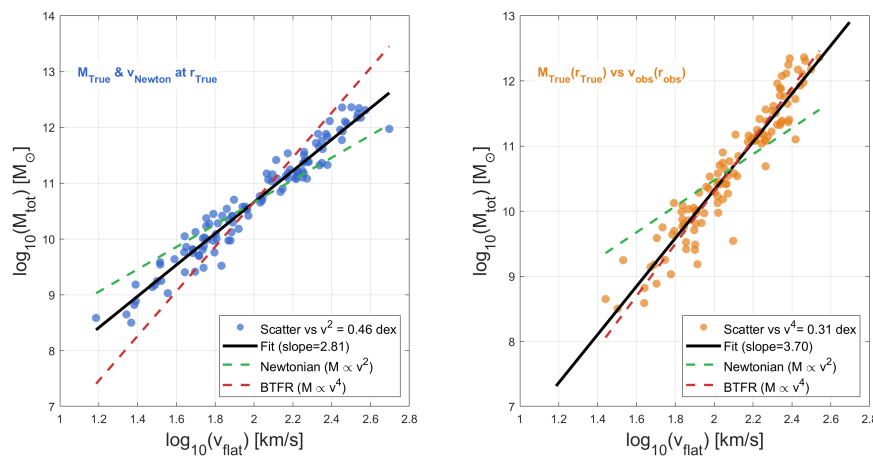


Figure 19. Comparative scaling of total baryonic mass versus rotation velocity for the True Frame (left) and Empirical Mapping (right). The True Frame compared against the Newtonian $M \propto v^2$ slope, in green dashed line, whereas the Empirical Mapping against the observed $M \propto v^4$ relation, in red dashed line. Solid lines indicate the best linear fits, while dashed lines mark the theoretical reference slopes for comparison.

The Empirical Mapping case), which correlates the true mass with the observed velocity, yields a slope of 3.70 (95% CI [2.62, 3.79]). This value lies closest to the BTFR prediction of 4.0 and exhibits the lowest scatter relative to that slope (0.30 dex). This result supports the central hypothesis that the characteristic v^4 relation may emerge when the true mass is mapped onto the observed kinematic frame. The mass recovered by the coordinate transformation scales with velocity to steepen the relation, creating the observed BTFR.

In contrast, the True Frame, which operates entirely within the Mezzi scaled frame, yields a significantly flatter slope of 2.81 (95% CI [2.51, 3.04]). This value represents a shift toward the Newtonian expectation of 2.0, as evidenced by the minimized scatter relative to the v^2 relation (0.351 dex). While the confidence intervals for the two cases overlap, the systematic steepening of the mean slope from 2.81 in the True Frame to 3.70 in the Empirical Mapping supports the hypothesis that the BTFR may not be a fundamental law of gravity, but a signature of geometric distortion.

8.2. BTFR Normalization in the True Frame

(Lelli et al., 2019) demonstrated that the baryonic BTFR is sensitive to the method used to define galaxy rotation velocity. Using different velocity measures, such as the maximum rotational velocity, the flat part of the rotation curve, or the velocity at a specific radius, can systematically shift the BTFR, altering both its slope and zero point. This underscores that the choice of velocity definition directly influences inferred galaxy angular momentum and comparisons across studies.

In this test we consider the BTFR definition written as:

$$\log M = 4 \log v + \beta \quad (35)$$

M is the total baryonic mass and v is the asymptotic rotation velocity (McGaugh, 2012). Observationally, the slope is set to 4. And the intercept β defines the normalization of the relation; researchers study its connection to a combination of factors, including the cosmic baryon fraction, star formation efficiency, gas content, and the effects of feedback processes (Bradford et al., 2016; Gurovich et al., 2010).

In the proposed framework, the observed mass M_{obs} is assumed to be systematically underestimated due to a geometric scaling effect (Fu, 2024; McGaugh et al., 2016). We examine in this section if the normalization offset β reflects a true physical constant or arises from this coordinate compression. To study this, we define the normalization offset β as the vertical residual from a reference BTFR relation with slope 4:

$$\beta = \log_{10} M - 4 \log_{10} v \quad (36)$$

If the observed mass is a geometrically underestimated representation of the true mass, the measured offset β_{obs} should correlate with the degree of geometric distortion. To test this hypothesis, we calculate the normalization offset β for each galaxy in the observed frame, where β_{obs} is computed using the observed mass M_{obs} . And in the true frame, where β_{true} is computed using the true mass M_{true} . To identify secondary dependencies, we divide the sample into three stellar mass bins: Low, Intermediate, and High. And we measure the scatter around the regression line. Figure 20 shows β_{obs} versus the average Mezzi scale factor value ζ , and Figure 21 shows the same relationship in the true frame, plotting β_{true} versus ζ . Lower β values indicate mass deficit, and lower ζ values indicate stronger geometric compression.

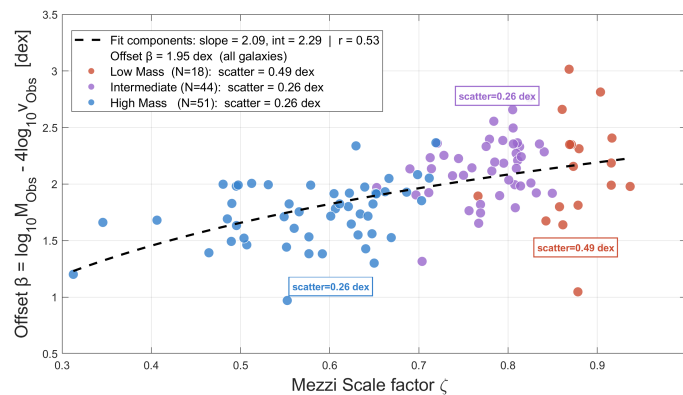


Figure 20. The BTFR normalization offset in the observed frame β_{obs} as a function of the Mezzi scale factor ζ in the observed frame. Points are color coded by stellar mass bin: Low mass galaxies (Orange), Intermediate mass galaxies (Purple), and High mass galaxies (blue). The solid line shows the best fit linear relation.

The results reveal a systematic dependence of the observed BTFR normalization β_{obs} on the Mezzi scale factor ζ . The data show a correlation ($r = 0.53$) with a slope of 2.09. The observed mass BTFR normalization appears to rise with the square of the Mezzi scale factor ζ , consistent with geometric area compression scaling.

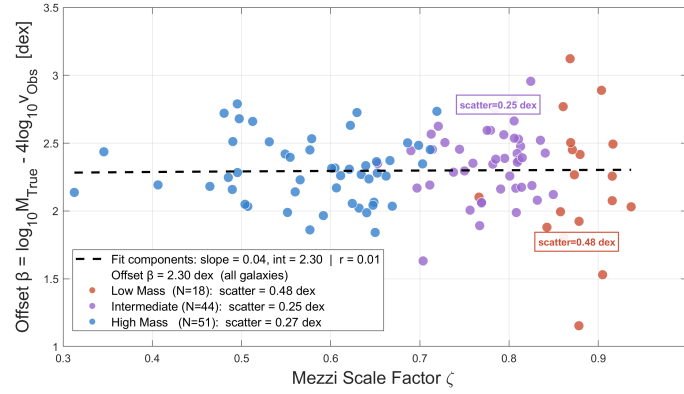


Figure 21. The BTFR normalization offset in the true frame β_{true} as a function of the Mezzi scale factor ζ in the true frame. Points are color coded by stellar mass bin: Low mass galaxies (Orange), Intermediate mass galaxies (Purple), and High mass galaxies (blue). The solid line shows the best fit linear relation.

This figure shows that the strong correlation evident in Figure 20 is effectively eliminated. The slope approaches zero (0.04) and the correlation coefficient disappears ($r = 0.01$), confirming that the normalization offset is no longer dependent on geometric compression. This supports the theoretical prediction that reconstructing the true mass restores a universal BTFR normalization. The scatter of the relation remains consistent across mass bins, indicating that while the geometric correction removes the systematic bias, variability persists.

This framework could help understand both the slope and the fine structure of the BTFR normalization as the Newtonian law viewed through a distorted geometry.

The emergence of a distinct zero slope solution for the $\beta - \zeta$ relation revealed a fundamental underlying structure, which subsequently played a central role in defining the Compliance. The value of C is determined by simultaneously satisfying four constraints: a vanishing slope for the true frame normalization offset, minimized rotation curve residuals, a lensing slope of approximately 2 relative to observed velocities, and a BTFR slope close to 4. This approach is selected because it encompasses the broader geometric and kinematic landscape. Consequently, all results presented in this paper are the outcome of the C value derived from this unified constraint.

8.3. Geometric BTFR in the Observer Frame

Applying Newtonian dynamics in the true frame, under the assumption that observed velocities accurately reflect the true kinematic state $v_{true} = v_{obs}$.

$$v_{obs}^2 = \frac{GM_{true}}{r_{true}} \quad (37)$$

Rearranging produces the scaling of true mass with observed velocity and true radius:

$$M_{true} \propto v_{obs}^2 r_{true} \quad (38)$$

On the other hand, the observer frame photometry is constrained by Freeman's Law (Section 4.5), which assumes a constant characteristic surface density.

$$\Sigma_{obs} \propto \frac{M_{obs}}{r_{obs}^2} \approx \text{const} \quad (39)$$

Defining surface density as mass per unit area, we obtain

$$M_{obs} \propto r_{obs}^2 \quad (40)$$

To connect the true frame dynamics with the observed frame photometry, we invoke the geometric distortion relation derived from the Mezzi scale factor (section 4.4). The integrated mass ratio scales linearly with the radius ratio following Eq. 23.

$$\frac{M_{\text{obs}}}{M_{\text{true}}} \propto \frac{r_{\text{obs}}}{r_{\text{true}}} \Rightarrow M_{\text{obs}} \propto M_{\text{true}} \left(\frac{r_{\text{obs}}}{r_{\text{true}}} \right) \quad (41)$$

Substituting the expression for M_{true} from the dynamical relation into the geometric mapping gives

$$M_{\text{obs}} \propto \left(v_{\text{obs}}^2 r_{\text{true}} \right) \left(\frac{r_{\text{obs}}}{r_{\text{true}}} \right) \Rightarrow M_{\text{obs}} \propto v_{\text{obs}}^2 r_{\text{obs}} \quad (42)$$

Finally, combining this result with the Freeman's Law constraint $r_{\text{obs}} \propto M_{\text{obs}}^{1/2}$ (from Eq 40) eliminates the radius dependence:

$$M_{\text{obs}} \propto v_{\text{obs}}^2 \left(M_{\text{obs}}^{1/2} \right) \Rightarrow M_{\text{obs}}^{1/2} \propto v_{\text{obs}}^2 \Rightarrow \boxed{M_{\text{obs}} \propto v_{\text{obs}}^4} \quad (43)$$

This analysis recovers the BTFR based on the combination of three premises: (i) Newtonian gravity holds in the true frame assumed in section 6.1; (ii) Freeman's Law discussed in section 4.5; and (iii) Scaling law emergent in Section 4.4.

The resulting derivation supports the interpretation that the empirical scaling relation $M_{\text{obs}} \propto v_{\text{obs}}^4$ emerge as consequence of mapping gravitational dynamics from the true frame into the observed photometric frame via a systematic geometric distortion.

9. Implications

This framework proposes that observed mass discrepancies in galaxies arise from geometric coordinate compression, the Mezzi effect, rather than particulate dark matter or modified gravity. In this framework, vacuum flow toward mass concentrations accumulates potential $A(r)$, inducing a scale factor $\zeta(r) < 1$ that makes observed radii appear smaller than true physical scales.

The implications that follow are derived from this mechanism and are designed to expose the framework to empirical falsification: they specify quantitative correlations between kinematic observables and geometric parameters. For this framework to remain physically viable, these predictions must be validated by independent observations.

9.1. The Inclination Dependent Dark Matter Fraction

The inferred dark matter fraction should correlate systematically with the observed inclination of the galaxy. Edge-on galaxies should yield the highest inferred dark matter fractions, while face-on galaxies of the same should yield lower fractions.

The framework assumes spherical symmetry for the vacuum flow, however real galaxies are flattened disks. The geometric compression acts primarily within the radial plane rather than the vertical axis. In edge-on galaxies, the observer integrates through the plane of maximum geometric compression, maximizing the apparent mass discrepancy. In face-on systems, the line of sight aligns with the axis of minimal compression, reducing the observed distortion.

In standard dynamical modeling, inclination serves exclusively as a geometric deprojection parameter to convert line of sight velocities to rotation speeds. Inclination uncertainties are well documented sources of systematic error in mass modeling, particularly for low mass galaxies (Downing & Oman, 2023). Halo orientation affects the inferred mass profiles not only within halos but also on large scales, implying that orientation is a non negligible hidden variable in cosmological measurements (Sharma et al., 2025). MOND introduces acceleration dependent modifications to dynamics that are natural to the system and not dependent on viewing angle, although observational measurements of those accelerations can still be viewing angle dependent (Harvey-Hawes & Galoppo, 2025)

9.2. The Cluster Dark Matter Deficit

Galaxies residing in dense cluster environments should exhibit a systematic deviation in inferred dark matter compared to isolated field galaxies of the same baryonic mass and morphology.

In a cluster, the external gravitational potential of the host disrupts the symmetry of the accumulation integral. The proposed vacuum flow toward a satellite galaxy is skewed or captured by the cluster's deeper potential well. This suppresses the accumulation of flow potential within the observed galaxy, affecting its geometric compression factor and making the galaxy appear larger and dynamically lighter than its isolated counterpart.

Simulation results show that low mass galaxies in high tidal environments tend to be more extended and relatively dark matter poor at fixed stellar mass, with many satellites occupying a dark matter poor and extended region of parameter space, however, the correlation between dark matter content and tidal perturbation is weak (Mercado et al., 2025). Observational claims of dark matter deficient galaxies such as NGC 1052-DF2 and DF4 have motivated theoretical work, though their properties remain debated. Simulations show that massive dark matter deficient satellite galaxies can naturally arise in cluster environments through tidal stripping during repeated pericentric passages, making them rare but expected (Contreras-Santos et al., 2024).

9.3. Distinct Redshift Evolution of the Missing Mass

The apparent dark matter fraction should correlate tightly with the compactness of the baryonic distribution, evolving as galaxies grow larger over cosmic time. High redshift galaxies ($z > 2$), being more compact, should exhibit a different missing mass fraction than local galaxies of the same mass. The suggested Mezzi scale factor depends on the mass concentration. Since the metric distortion is a direct response to the baryonic potential, the missing mass signal evolves with the physical expansion and diffusion of the stellar disk.

High redshift star forming galaxies observed with KMOS-based surveys show lower dark matter fractions and baryon dominated inner regions, with some massive systems exhibiting declining outer rotation curves, in contrast to the typically flat rotation curves of local spirals (Genzel et al., 2020; Tiley et al., 2019). (Sharma et al., 2025) analyzed 263 disk galaxies and found that the dark matter fraction decreases with increasing redshift. They also report correlations between dark matter content and baryonic properties such as surface density, while maintaining that galaxies remain on average dark matter dominated within their effective radii.

9.4. The Absence of Dark Matter in Globular Clusters

Similarly to galaxies, compact clusters are not expected to have significant geometric compression compared to diffused clusters. As a result, globular clusters should contain little relative mass discrepancy.

Kinematic studies of globular clusters generally find that their internal dynamics can be fully explained by their stellar populations, with no requirement for a substantial dark matter component (Carlberg & Grillmair, 2021). Observed velocity dispersion profiles and mass to light ratios in globular clusters are generally consistent with little or no dark matter. Any dark component, if present, is constrained to be subdominant relative to the stellar mass, typically $\lesssim 10 - 20\%$ within uncertainties (Garani et al., 2023).

In MOND, the external field effect (EFE) can push globular cluster dynamics toward the Newtonian regime. However, Simulations of NGC 2419 show that even when including the EFE, a Newtonian dynamics provides a significantly better fit to the observed surface brightness and velocity dispersion profiles than MOND models. This suggests that MOND with EFE struggles to reproduce the dynamics of this particular cluster (Derakhshani, 2014).

9.5. The Nature of the Core-Cusp Problem

The core is an artifact of coordinate geometry. Unlike CDM, which requires bursty feedback to flatten cusps, the core should arise naturally in the true frame.

CDM simulations universally predict cuspy inner density profiles, while observations of low surface brightness and dwarf galaxies consistently favor flat cores. High resolution velocity fields have significantly reduced the impact of systematic observational biases as explanations for the

discrepancy (De Blok, 2010). The prevailing baryonic solution involves fluctuations in the central gravitational potential driven by supernova feedback, which can transfer energy to dark matter and produce cores. However, it remains unclear whether this mechanism is effective in the lowest mass galaxies (Baumschlager et al., 2025; Weinberg et al., 2015). Recent observational studies suggest that the diversity and mass dependence of inner dark matter halo profiles may require either complex baryonic processes or potential modifications to the standard cold dark matter paradigm, although current results do not uniquely favor either explanation (Hayashi et al., 2026).

9.6. The Shape of the Lensing Profile

Strong gravitational lensing models should be able to fit the observed Einstein radii and radial profiles using a purely baryonic mass distribution in the true frame, that is physically larger and more diffuse than in the observed frame, without requiring a dense NFW dark matter cusp.

The suggested framework expands the scale length of the mass distribution. The dark matter halo inferred in standard models is actually the compressed physical extension of the baryonic disk. Consequently, the lensing concentration parameter should correlate with the Mezzi scale factor, eliminating the need for an over concentrated halo in massive ellipticals.

(Grillo et al., 2009) analyze SLACS lenses and find that the total mass within the Einstein radius correlates strongly with stellar mass estimates and exceeds the stellar component, implying a dark matter fraction of order $\sim 30\%$ within an average Einstein radius of ~ 4 kpc. Strong lensing (Shajib et al., 2024) shows that massive early type galaxies have nearly isothermal total mass density profiles within their inner regions. Detailed lens modeling of observables such as Einstein rings and magnification ratios generally requires contributions from both baryonic and dark components. MOND and its relativistic extensions can reproduce some gravitational lensing observations, but they encounter significant challenges, particularly for strong lensing in galaxy clusters and complex systems, often requiring additional unseen mass (Famaey et al., 2024; Famaey & McGaugh, 2012).

10. Conclusions

This study explored the hypothesis that the dynamical discrepancies observed in galaxies, traditionally attributed to non-baryonic dark matter, may arise from a geometric misinterpretation of observed geometry. We proposed a framework in which the vacuum acts as a dynamic medium, flowing radially toward gravitational potential wells. By introducing the concept of vacuum compliance, quantifying how this flow accumulates along the line of sight, we suggest the apparent compression of radial coordinates as measured by distant observers.

To test this hypothesis, we applied the framework to a sample of 175 galaxies from the SPARC database. The analysis focused on four primary areas:

The universal compliance constant $C \approx 377$ emerges as a single parameter, resulting in consistent reconstruction of kinematic and geometric scaling. This distinguishes the model from approaches requiring galaxy-specific tuning, and positions the framework in the theoretical position of: preserving Newtonian dynamics and general relativistic weak field lensing while reinterpreting the coordinate basis of astrophysical measurements.

The proposed discrete shell reconstruction approach recovers the true geometry of the true frame from observed surface density. The analysis reveals a universal scaling relation: The true-to-observed surface density ratio scales with the radius ratio as $\Sigma_{\text{true}} / \Sigma_{\text{obs}} \propto (R_{\text{true}} / R_{\text{obs}})^{-0.5}$, yielding a mass scaling relation $M_{\text{obs}} / M_{\text{true}} \propto r_{\text{obs}} / r_{\text{true}}$. This emerges empirically across all mass regimes, serving as a characteristic signature that may distinguish this geometric distortion mechanism from alternatives.

The rotation curve analysis shows that a geometric coordinate transformation can reproduce Newtonian dynamics without altering the law of gravity. This result depends on key assumptions: that Newtonian dynamics are valid in the true frame, that observed velocities are accurate, and that any discrepancies arise from misattributing radial coordinates. The method yields a statistically significant improvement: in a sample of 175 galaxies, 86.9% reproduce observed velocities within a 50% tolerance. The model also achieves an RMS residual of 33.88 km/s, and the logarithmic errors are consistent

with a Gaussian distribution (Kolmogorov–Smirnov test, $p = 0.30$). Validation in the inner regions ($r < 0.3 r_{\text{max}}$) confirms that the framework preserves established dynamics where observer frame models already perform well. The median bias improves from 10.2 km/s to -2.5 km/s. However, persistent outlier-driven discrepancies indicate that incorporating more accurate flow geometry may be necessary to achieve better agreement in future model development.

The gravitational deflection angles test reveal a systematic mass dependent underestimation in standard observations. Low mass galaxies show negligible correction, while high mass galaxies exhibit significant enhancement, up to 6x. There exist counter examples where extreme radial expansion suppresses lensing despite mass enhancement such as galaxy UGC02487. The overall gravitational lensing scales with the Mezzi scale factor by $\alpha_{\text{true}}/\alpha_{\text{obs}} \propto \zeta^{-1.26}$, compared to the total mass scaling relation of $M_{\text{true}}/M_{\text{obs}} \propto \zeta^{-1.84}$, revealing that geometric scaling affects dynamical mass more strongly than lensing mass across multiple galaxies.

To test the hypothesis that the RAR emerges as a geometric projection effect, we present three distinct analyses. First, true frame accelerations remain consistent with Newtonian dynamics, demonstrating that the mass reconstruction methodology preserves baseline physical fidelity. Second, the central geometric mapping hypothesis, which correlates true frame baryonic acceleration with observed dynamical acceleration, reproduces the empirical RAR shape, with scatter of 0.294 dex, statistically equivalent to observed data, with ratio 1.09, indicating that the radial scale factor. Third, isolating radial coordinate compression alone reveals MOND-like behavior, suggesting that the scaling relations typically attributed to modified gravity may arise from geometric distortion.

To test if the BTFR can be interpreted as a geometric projection effect, we conduct a systematic progression in the scaling slope in the true frame and in the observer frame. In the true frame, the scaling slope is 2.81 (95% CI: 2.48–3.03), shifting toward the Newtonian expectation of 2.0. In the observer frame, the resulting scaling slope is 3.70, which closely approximates the recognized empirical BTFR. This supports the geometric interpretation of the scaling relation. Furthermore, the BTFR normalization offset β exhibits predicted systematic variation: in the observed frame, β_{obs} correlates strongly with compression factor ζ ($r = -0.526$, slope -2.087), indicating apparent mass deficit scales quadratically with geometric distortion; after coordinate correction, this correlation vanishes ($r = -0.012$, slope -0.038), restoring universal normalization. The quantitative agreement between predicted and observed slope reduction are consistent with an origin in observer frame geometric projection.

However, the significant scatter and consistent residual patterns found in this study indicate that the model still needs improvement. In particular, it requires more accurate mass profiles, as well as a more developed field extension that explicitly accounts for the underlying vacuum flow dynamics.

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