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Unidirectional Pedestrian Circulation: Physical Distancing in Informal Settlements

October 27, 2020

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Keywords: Circulation, informal settlements, COVID-19, coronavirus, physical distancing, social distancing, graph theory, oriented graph, cluster graph, urban planning, architecture, Königsberg, Dharavi, Christopher Alexander, slum, favela.

Comments: Abstract accepted for development by *Buildings and Cities* for the special issue *Urban Systems for Sustainability and Health* on August 11, 2020. Editor in Chief: Richard Lorch. Guest Editors: Jonathon Taylor (University of Tampere) and Philippa Howden-Chapman (University of Otago).

Abstract

The COVID-19 pandemic has resulted in a wide range of spatial interventions to slow down the spread of the virus. The spatial limitations of narrow public circulation spaces within informal settlements, which house over one billion people around the world, make it impossible for pedestrians to practice physical distancing (or social distancing). In this paper, we propose a flexible mathematical method, named the Cluster Lane Method, for turning a planar circulation network of any size or complexity into a network of unidirectional lanes, making physical distancing possible in narrow circulation spaces by limiting face-to-face interactions. New notions and theorems about oriented graphs in graph theory are introduced. The paper ends with a discussion of the potential implementation of this cost-efficient, low-tech, sustainable solution, and with the introduction of a novel unidirectional tactile paving for the visually impaired.

1 Dense urban settlements and pandemics

The transformative role of pandemics and disease in changing the trajectory of architectural thought and practice has been well documented. For example, Harriet Jordan argued how the nineteenth century movement pushing for public parks was a response to the overcrowding and

 doi:10.20944/preprints202010.0598.v1

pollution brought about by industrialization [1]. Similarly, the cholera epidemic led administrators to lay the foundations for the now-ubiquitous sanitary and underground infrastructure. Haussmann operated on what he considered the "sick" city of Paris, creating wide boulevards, aqueducts and a major network of sewers and thus improving the city's hygiene [2]. These responses to disease have had a positive long-term impact on the health and sustainability of cities. However, the implementation of such solutions often requires the demolition and construction of large-scale infrastructures, which can take years or decades to implement.

It is highly likely that the current COVID-19 pandemic will trigger significant paradigm shifts in how urban infrastructure is designed in the long run. Yet, more immediate spatial interventions that are easily deployed and implemented are needed. Physical distancing, defined as "the practice of staying at least 6 feet away from others to avoid catching a airborne diseases such as COVID-19" [4], can be practised in public circulation spaces when enough space is available. According to the Centers for Disease Control and Prevention, "Limiting close face-to-face contact with others is the best way to reduce the spread of [COVID-19]." [5] (The more broad term "social distancing", sometimes used interchangeably with physical distancing, means "staying home and away from others as much as possible to help prevent spread of COVID-19" [4] and will not be used throughout this article.) In fact, urban environments have started to become more friendly to cyclists and pedestrians with the recent implementation of physical distancing guidelines in some of the world's major cities [3] including Berlin, Montreal, Paris and Mexico City. However, physical distancing guidelines in public circulation spaces are only possible in places where there is enough room to expand the spatial layout.

As it was with the cholera epidemic, dense urban settlements become fertile breeding grounds for COVID-19 to thrive. Urban slums and informal settlements, which lack proper medical infrastructure and well-enforced sanitary standards, pose the greatest threat to a successful response, partially due to their narrow public circulation spaces. According to the United Nations, over one billion people lived in slums or informal settlements in 2018. [6]. The UN Economic and Social Council stated that "The pandemic will hit the hardest the more than 1 billion slum dwellers worldwide (...) Urgent response plans are needed to prepare for and respond to outbreaks in informal settlements and slums." [6]. The gravity of this situation is unquestionable.

Dharavi in Mumbai, India, is one of the world's densest slums and fits close to a million people into an area roughly two-thirds the size of Central Park. (See Figure 1.) It contains a labyrinthine circulation network with some lanes barely wide enough to outstretch both of one's arms, let alone follow the recommended physical distancing standards. Even though underneath the apparent chaos in Dharavi is a thriving informal economy with an estimated worth of 1 billion USD, [7] it's not uncommon to find a small industrial factory next to a daycare or to see streets dotted with open sewers. Unfortunately, most of the strategies deployed in Western countries' major cities are hardly applicable in such dense slums. Physical distancing is practically impossible while automated and touchless technologies are far from being available in a locality that lacks basic sanitary infrastructure. By combining a strict India-wide lockdown and a rigorous proactive testing protocol, Dharavi seemed to be keeping

Figure 1: Satellite view of a small area of the Dharavi slum in Mumbai, India, which has a population density of 354,167 people per square kilometre. (Hindustan Times, 2020). The main circulation arteries between clusters of buildings are visible but numerous narrow lanes are hidden under the roofs. Image from Google Earth.

the COVID-19 outbreak relatively under control by mid-June. [8] However, by mid-July, 40-60% of the population living in the slums of Mumbai were infected with COVID-19, where "67% of households rely on community toilets, soap and clean water are scarce and physical distancing is impossible".[9] Trailing the United States of America, the second and third countries with the most COVID-19 confirmed cases on the 23th of September were India (5,646,010) and Brazil (4,591,364), according to the John Hopkins Coronavirus Ressource Center. [10] An Indian 2011 census identified 64 million people living in city slums nationwide. [11] In 2014, it was reported that 22.3% of Brazil's urban population lived in slums (or favelas), according to the World Bank collection of development indicators. [12] As much less dense settlements have struggled with fresh COVID-19 outbreaks when reopening after lockdown, it is of primary importance to develop low-tech solutions to slow down the transmission of disease in the narrow circulation spaces of the congested urban developments of informal settlements.

This paper examines existing mathematical literature of graph theory and introduces new mathematical structures to suggest a possible urban planning solution to slow down the spread of disease in dense settlements when physical distancing measures are slowly relaxed. Section 2 explores the origins of graph theory dating back to a problem involving movement through an urban fabric, as well as the important role of this branch of mathematics on architecture and urban planning. Section 3 defines basic notions of graph theory. Section 4 introduces new mathematical structures of graph theory (cluster graphs and relative heights of vertices in graphs) and explains the novel Cluster Lane Method. Finally, section 5 discusses the applications of the method to real informal settlements, along with the potential difficulties of this application, and introduces a novel unidirectional tactile paving for the visually impaired.

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2 Urban origins of graph theory

Throughout history, several popular puzzles and problems relating to movement within an urban fabric have led to significant mathematical breakthroughs. One of these notable problems is called "The Seven Bridges of Königsberg". The Pregel River divides part of the historic Prussian city of Königsberg (now Kaliningrad) into two islands - Kneiphof and Lomse. Seven bridges connected these two islands to the city's northern and southern banks. Legend has it that the citizens wondered if it was possible to cross all the bridges in a route without crossing any one of them more than once. Even though no such path could be found, it was only demonstrated to be impossible when brought to the attention of the prolific mathematician Leonhard Euler, who presented his findings to the members of the Petersburg Academy on August 26, 1735 [14].

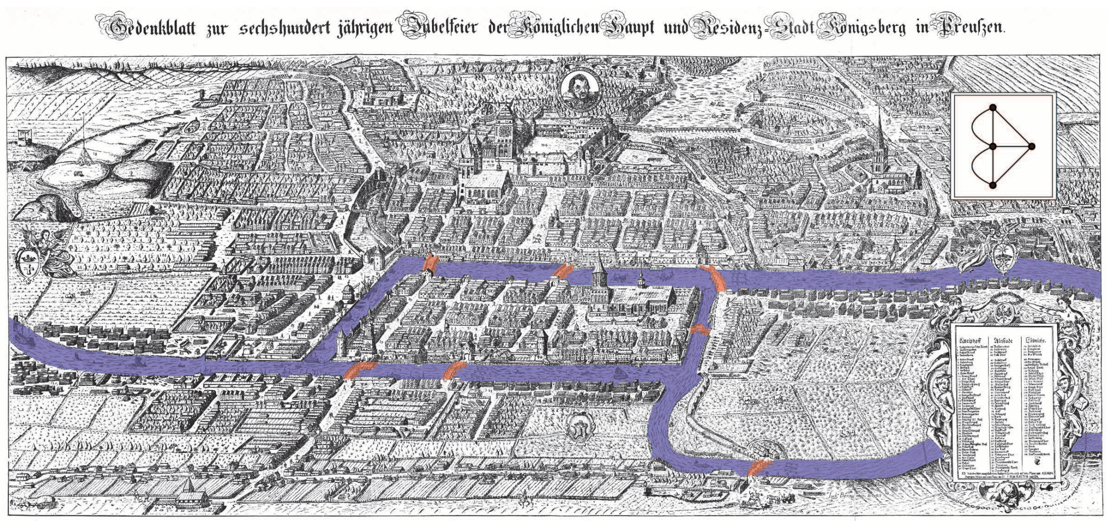

Figure 2: Memorial page of the six-hundredth anniversary of Königsberg from about 1813, according to an engraving by Joachim Bering from 1613. (Image from Wikimedia Commons; {{PD-US-expired}}). The seven bridges (red) and the Pregel river (blue) have been highlighted by the authors for this article. The white square at the top right, added for this article, shows a graph representing the bridges as edges and the islands/banks as vertices.

This seemingly-innocent problem led Euler to lay the foundations of a new branch of mathematics involving the "geometry of position", which is now known as graph theory. This new approach involves simplifying spatial relationships between objects into diagrams of points (vertices) and lines (edges). Insights from the mathematical analysis of these graphs would lead to principles that would be valid for all similar graphs, of any size or complexity - for instance, since more than two of the Königsberg vertices in Figure 2 are connected to an odd number of edges, it is mathematically impossible to go through all of them without crossing any bridge more than once.

Since Euler's solution, graph theory has become an important branch of both pure and applied mathematics. Numerous problems in geometry, topology, among others, have been solved with it, since diverse mathematical structures can be embedded into graphs. Famous theorems have connections to graph theory: The Descartes-Euler Polyhedral Formula, along with the Euler characteristic applied to higher dimensions in algebraic topology and polyhedral combinatorics, relates the number of vertices V, edges E and faces F in convex polyhedra as $V + F - E = 2$ [15]; The Four Colour Theorem, renowned for being the first major theorem to be proved with a computer and for the controversy that came with it, states that "any map in a plane can be coloured using four colors in such a way that regions sharing a common boundary (other than a single point) do not share the same colour". [16].

In architecture and urban planning, graph theory has played an important role which evolved in parallel with the implementation of computation in architectural practice and theory since the 1960s. Theodora Vardouli states that "architects turned to structural abstraction in efforts to purify their inheritance of interwar Modern architecture from stylistic doctrines and empirical conventions. The graph's amenability both to visual depiction and to mathematical analysis furnished it with a strategic position among modern mathematical varieties: graphs made structural abstraction visible and workable. By virtue of this property, graphs proliferated in architectural theory as harbringers of a veritably modern discipline founded on rationality and geared toward ensuring functional efficiency." [17]. Graph theory provided tools to represent the underlying structures of circulation networks, building plans, spatial structures in cities, and much more. Lionel March discusses the educational movement of "new mathematics" during the 1950s which aimed to change the teaching curriculum to include subjects such as graph theory, certain notions of combinatorics and set theory. These tools allowed March and Steadman to study the circulation systems of Frank Lloyd Wright's architectural designs, among others. [18] Thomas Grasl and Athanassios Economou explored possible design implementations of graph theory by generating floor plans of Palladian Villas with an application embedded in a parametric CAD environment [19].

Numerous types of graphs exist and serve as tools for a wide variety of problems. These include weighted graphs, colouring graphs, graph factorizations, and many more, which will not be addressed or explained in this paper. One particular type of graph is explored throughout the following sections: oriented graphs, in which orientations (or directions) are assigned to edges. This can represent circulation networks with both bidirectional and unidirectional lanes. In the context of this paper, the relevance of unidirectional lanes is tied to its ability to limit face-to-face interactions between people. Shopping mall stores and smaller retail environments have deployed similar unidirectional circulation strategies to contain the spread of COVID-19, but the vast complexity of informal settlements like the Dharavi slum require more robust mathematical tools. This paper uses graph theory to conceptualize any informal settlement as a network of points (vertices) and lines (edges) in order to assign orientations to lanes to allow for physical distancing in narrow circulation spaces.

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3 Basic graph theory definitions and Robbins's Theorem

The method we propose in Section 4 uses the graph as a tool to represent circulation networks and to assign orientations to the lanes that make up the network. For this reason, some background knowledge in graph theory is necessary. The following definitions, as well as Robbins's Theorem, are taken from the book *The Fascinating World of Graph Theory* by Benjamin, Chartrand and Zhang. [14] For the sake of this paper, all graphs are finite graphs which have a finite number of vertices and edges.

Definition 1 *A* **graph** *G is a finite nonempty set V of objects called* **vertices** *together with a set E consisting of 2-element subsets of V. Each element of E is called an* **edge** *of G.*

Definition 2 *The numer of vertices in a graph G is called the* **order** *of G and the number of edges in G is its* **size**.

Definition 3 *The number of graph edges meeting at a given vertex v in a graph is called the* **order** *of the vertex v. [20].*

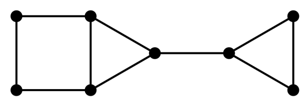

Figure 3: A graph G of order 8 and size 10.

Definition 4 *A graph H is called a* **subgraph** *of a graph G if every vertex and edge of H is a vertex and edge, respectively, of G. Using set notation, this means that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.*

Definition 5 *A* **path** *P in a graph G is a subgraph of G whose distinct vertices can be listed in some order $P = (u = v_0, v_1, \ldots, v_k = v)$, such that v_0v_1, v_1v_2, $\ldots$, $v_{k-1}v_k$ are all edges of P. It can also be called a $u - v$ path. (For the sake of this article, a path consists of the vertices $v_0, v_1, \ldots, v_k$ and the edges $v_0v_1, v_1v_2, \ldots, v_{k-1}v_k$.)*

Definition 6 *A* **cycle** *is a path P that starts and ends with the same vertex.*

Definition 7 *A graph G is* **connected** *if it contains a $u - v$ path for every two vertices u and v of G.*

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Definition 8 *A graph G is* **disconnected** *if it does not contain a $u - v$ path for at least one pair of vertices u and v of G. It can be broken into two or more connected pieces, called* **components***.*

Definition 9 *A* **cycle graph** *or circular graph G is a graph that consists of a single cycle, or, in other words, 3 or more connected vertices of order 2.*

Definition 10 *A* **bridge** *in a connected graph G is an edge whose removal from G results in a disconnected graph.*

Definition 11 *A* **planar graph** *is a connected graph G which can be drawn in the plane so that no two of its edges cross.*

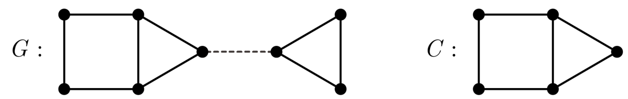

Figure 4: Left: A connected graph G of order 8 and size 10 with one bridge, represented by a dashed line. The removal of this bridge would result in a disconnected graph. Right: a graph H which is a subgraph of G.

The above definitions can be used to represent any given circulation network. For any given planar circulation network, produce a graph G such that the edges correspond to the lanes of the network and the vertices correspond to the intersection between lanes or the ends of each lane. (This is explained in detail in Section 4.) Since the method deals with unidirectional lanes, it is necessary to understand what oriented graphs are, as well as some of their properties.

Definition 12 *An* **oriented graph** *G (also called a* directed graph*) is a graph for which every edge is assigned an orientation (or direction). (See figure 3.)*

According to [14], "If each edge of a graph G is assigned a direction, then the resulting structure is called an *oriented graph* or an *orientation* of G. If u and v are two vertices in an oriented graph, there can be either one or no directed edges between u and v, depending on whether the given graph G contains the edge uv. If this edge is directed from u to v, then the resulting directed edge is denoted by (u, v). A directed edge is also commonly called an *arc*. The arc (u, v) is also represented by $u \to v$ or $v \leftarrow u$. In this case, u is said to be *adjacent to* v, while v is *adjacent from* u." An edge's orientation can be unidirectional or bidirectional. In this article, an oriented edge can be assumed to be unidirectional when not specified if it is unidirectional or bidirectional. Figure 3 shows an oriented graph with strong orientation, for which all edges are unidirectional.

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Definition 13 *An orientation D of G is* **strong** *if, for every two vertices u and v of G, D contains both a directed $u - v$ path and a directed $v - u$ path. (See figure 3.)*

Figure 5: Left: A strong orientation D_1 of a graph. Right: An orientation D_2 of G that is not strong. Arrows are used to represent the assigned orientation of each vertex.

Oriented graphs have been studied for numerous purposes. They can represent abstract structures such as tournaments and winner-loser schemes, information exchanges, cross-references on webpages, and many more. In the context of this article, the orientation of a graph represents something concrete: the orientation assigned to the lanes in a circulation network. The following theorem states which graphs have, or can have, a strong orientation.

Theorem 1 ***(Robbins's Theorem):*** *A graph G has a strong orientation if and only if G is connected and contains no bridges.*

In other words, Robbins' Theorem states that for a connected bridgeless graph G, an orientation D exists such that for every two vertices u and v, D contains both a directed $u - v$ path and a directed $v - u$ path. If the graph G represents a circulation network that is connected and contains no bridges, Robbins's Theorem would state that it is possible to walk from any given location to any other given location within G while respecting the orientation of each lane, for a strong orientation of G. The theorem can be demonstrated using ear decomposition, defined below according to [21].

Definition 14 *An* **ear decomposition** $D = [P_1, P_2, \ldots, P_r]$ *of a graph G is a partition of edge sets of G into an ordered collection of edge-disjoint simple paths $P_1, P_2, \ldots, P_4$ such that P_1 is a simple cycle and P_i, $i \geq 2$, is a path with only its endpoints in common with $P_1 \cup P_2 \cup \cdots \cup P_{i-1}$. Each P_i is called an* **ear**. *(See figure 3.)*

Since P_1 is a cycle, then it has strong orientation. By adding a path P_2 whose endpoints are vertices of P_1, then $P_1 \cup P_2$ has strong orientation. Analogously, for $i = 3, 4, \ldots, r$, we obtain $\bigcup_{i=1}^{r} P_i = G$, which has strong orientation. (See figure 3 for an example with three paths P_i.) This can be used to prove Robbins's Theorem. However, it is important to note that the theorem does not explain how to assign orientations to each edge and in which order, particularly because there are many possible ear decompositions of a graph with a large number of vertices and edges.

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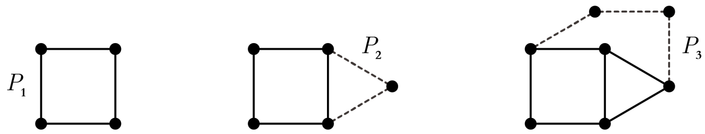

Figure 6: Left: A cycle graph which corresponds to a path P_1, with strong orientation. Middle: P_2 with the addition of a path P_2 composed of one vertex and two edges, such that $P_1 \cup P_2$ has strong orientation. Right: $P_1 \cup P_2$ with the addition of a path P_3 composed of two vertices and three edges, such that $P_1 \cup P_2 \cup P_3 = G$ has strong orientation.

4 The Cluster Lane Method

4.1 Clusters of Dwellings and Circulation Patterns

The Cluster Lane Method is a flexible method which makes it possible to turn any planar circulation network into a circulation network made of unidirectional lanes (with the exception of *bridges*, defined in Section 3) in such a way that any given location can be reached from any other location. This makes it possible to practice physical distancing in narrow circulation spaces by limiting face-to-face interactions between pedestrians. Because of the complexity of circulation systems within slums and informal settlements, there are many different ways of assigning orientations to the lanes, by the means of ear decompositions, for example. Each way of assigning orientations to the lanes makes certain walks shorter, while making others longer.

In *A Pattern Language*, Christopher Alexander discusses the "House Cluster", identified as Pattern 37. He states that "people want to be part of a neighbo[u]rly spatial cluster; contact between people sharing such a cluster is a vital function." [22]. By noticing that most people visit those who live close to their homes, he writes that "the strength of the *spatial* cluster [is] to draw people together into neighbo[u]rly contact." [22]. The Cluster Lane Method aims to adapt to the ways which people live and is inspired by Christopher Alexander's discussion. Because of the density of informal settlements' urban fabric and the lack of public land between dwellings excluding lanes, several elements of the "House Cluster" pattern do not apply to informal settlements. However, the idea that clusters are the "natural focus of neighbo[u]rly interaction" guides the proposed method, since it also applies to informal settlements as lockdown measures are relaxed.

The Cluster Lane Method proposes a unidirectional circulation network which is responsive to the urban fabric of informal settlements and their clusters of dwellings. This is possible thanks to an organizational hierarchy of three types of circulation lanes:

1. Circulation around the boundary of a cluster of dwellings,

2. Circulation within the boundary of a cluster of dwellings,

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3. Circulation between the boundaries of different clusters of dwellings.

This organizational hierarchy of lanes is present throughout the entire circulation network. Two important properties are characteristic of the oriented circulation networks resulting from the Cluster Lane Method: Firstly, people within a cluster of dwellings can move within the cluster without having to leave it (by using the lanes of type 1 and type 2 only). This facilitates short walks and limits long detours. Secondly, people who have to cross one or more clusters of dwellings can go around them without having to enter them (by using the lanes of type 2 and type 3 only). (See Figure 4.1.) This limits unnecessary traffic within clusters of dwellings. Thus, the Cluster Lane Method could reduce the chances of COVID-19 spreading between clusters of dwellings, slowing down contagion within an informal settlement.

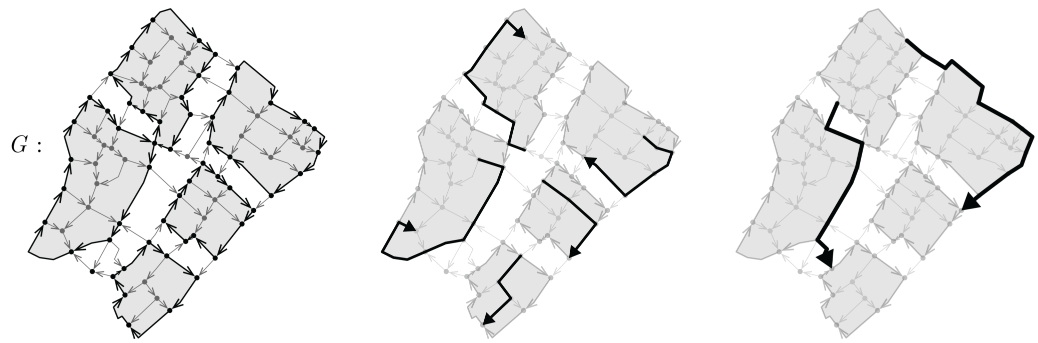

Figure 7: Left: An oriented graph G resulting from the Cluster Lane Method. Each edge corresponds to a lane and the arrow shows the assigned orientation. Selected clusters of dwellings are shown in grey. Middle: Paths within clusters of dwellings using only lanes of types 1 and 2, illustrated by large arrows. Right: Paths starting at the boundary of a cluster of dwellings and ending at a different boundary using only lanes of types 2 and 3, illustrated by large arrows.

In order to introduce the Cluster Lane Method and describe it mathematically, new notions of graph theory must be explained. Subsections 4.2 and 4.3 define and discuss *cluster graphs* and *relative heights* of vertices within a graph, respectively. These notions are used in subsection 4.4 to explain the method as a sequence of steps.

4.2 Cluster Graphs

This subsection introduces the mathematical structures called *cluster graphs*, which can be used to decompose certain planar graphs. Subsection 4.4 explains how they can be applied to the method.

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Definition 15 *A* **cluster graph** *C is a planar bridgeless connected graph. A* **cluster subgraph** *is a cluster graph C which is a subgraph of a given graph G. (See figure 8, left.)*

Definition 16 *For a given planar configuration of a cluster graph C, the* **cluster boundary** *$B(C)$ of C is a subgraph of C and a cycle graph which corresponds, geometrically, to the vertices and edges of C's perimeter.*

Definition 17 *Given a planar configuration of a cluster graph C. The* **interior vertices** *and the* **interior edges** *of C are the edges and vertices of C that are not part of the cluster boundary $B(C)$. The set of interior vertices and interior edges I can be defined as $I = C \setminus B(C)$.*

Definition 18 *Given a planar configuration of a cluster graph C. A* **boundary shortcut** *S of C is the set of edges and vertices of a path that starts at a vertex $v_1 \in B(C)$ and ends at a vertex $v_2 \in B(C)$ of the cluster boundary $B(C)$, for which all other vertices and all edges are interior vertices and interior edges, respectively, of C.*

Definition 19 *Given a circular graph G. For a given planar configuration of G, the* **sense of orientation** *of a strong orientation of G is clockwise if all the edges of G are oriented clockwise around the graph. A counter-clockwise sense of orientation is defined analogously.*

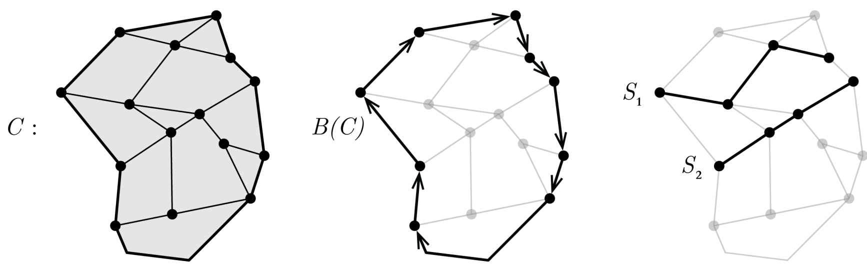

Figure 8: Left: A cluster graph C. Cluster graphs are coloured in grey throughout the article to identify them more easily. Middle: The cluster boundary $B(C)$, composed of black vertices and edges, with a clockwise sense of orientation, being a strong orientation. The interior vertices and interior edges of C, shown in grey. Right: Two boundary shortcuts S_1 and S_2 of C, shown in black.

Two different cluster subgraphs C_1 and C_2 of a cluster boundary C can share cluster boundary edges or vertices, interior edges or vertices, a combination of these, or no edges or vertices at all. The following two theorems show different ways of decomposing cluster graphs into cluster subgraphs. They also describe how senses of orientation can be assigned to the edges of the resulting cluster subgraphs' boundaries.

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Theorem 2 ***(Adjacent Cluster Subgraph Theorem):*** *Given a cluster graph C with a boundary shortcut S. Two cluster subgraphs C_1 and C_2 of C can be defined such that*

$$C_1 \cap C_2 = S \quad \textit{and} \quad C_1 \cup C_2 = C. \tag{4.1}$$

Moreover, a strong orientation of C exists where the cluster boundaries $B(C_1)$ and $B(C_2)$ have opposite senses of orientation.

Proof Let C be a cluster graph with a cluster boundary $B(C)$ and a boundary shortcut S, where $v_1 \in B(C)$ and $v_2 \in B(C)$ are the end vertices of S. Given a vertex $v \in B(C) \setminus \{v_1, v_2\}$, define a connected graph $H_1' \in B(C)$ without any cycles such that $\{v, v_1, v_2\} \in H_1'$ and such that the order of every vertex of H_1' is 2 except for v_1 and v_2, whose orders are 1. Then, define a connected graph $H_2' \in B(C)$ without any cycles such that $H_1' \cup H_2' = B(C)$ and $H_1' \cap H_2' = \{v_1, v_2\}$.

The graphs $H_1 = H_1' \cup S$ and $H_2 = H_2' \cup S$ are both cycle graphs because they are connected graphs and the degree of each of their vertices is 2, since H_1', H_2' and S share no edges and their only vertices with degree 1, for these three graphs, are v_1 and v_2. For a given planar configuration of C, define two cluster subgraphs C_1 and C_2 of C such that $B(C_1) = H_1$ and $B(C_2) = H_2$. Thus, $C_1 \cap C_2 = S$ since $B(C_1) \cap B(C_2) = S$ and $C_1 \cup C_2 = C$ since $B(C) = B(C_1) \cup B(C_2) \setminus \{S \setminus \{v_1, v_2\}\}$.

Moreover, all the edges of S can be assigned an orientation such that it is possible to move from v_1 to v_2 within S. Given this orientation of the edges of S, the edges of H_1' can be assigned an orientation such that it is possible to move from v_2 to v_1 in order for $S \cup H_1' = B(C_1)$ to have a strong orientation. Analogously, the edges of H_2' can be assigned an orientation such that it is possible to move from v_2 to v_1 in order for $S \cup H_2' = B(C_2)$ to have a strong orientation. The senses of orientation of $B(C_1)$ and $B(C_2)$ cannot be equal since this would mean that it is possible to move from v_1 to v_2 within H_1' or within H_2', which contradicts what has been defined above. Thus, the senses of orientation of $B(C_1)$ and $B(C_2)$ must be opposite for $B(C_1) \cup B(C_2)$ to have a strong orientation.

Finally, the interior edges of C_1 and C_2 can be assigned a strong orientation according to Robbins's Theorem (theorem 1), while keeping the assigned senses of orientation of $B(C_1)$ and $B(C_2)$, since both C_1 and C_2 are planar bridgeless connected graphs. In conclusion, a strong orientation of C exists where the cluster boundaries $B(C_1)$ and $B(C_2)$ have opposite senses of orientation. □

Theorem 3 ***(Cluster Subgraph Theorem):*** *Given a cluster graph C with two boundary shortcuts S_1 and S_2 that do not share any edges. Three cluster subgraphs C_1, C_2 and C' of C can be defined such that*

$$C_1 \cap C' = S_1, \quad C_2 \cap C' = S_2 \quad \textit{and} \quad C_1 \cup C_2 \cup C' = C. \tag{4.2}$$

Moreover, a strong orientation of C exists where the cluster boundaries $B(C)$, $B(C_1)$ and $B(C_2)$ all have equal senses of orientation. (See figure 9.)

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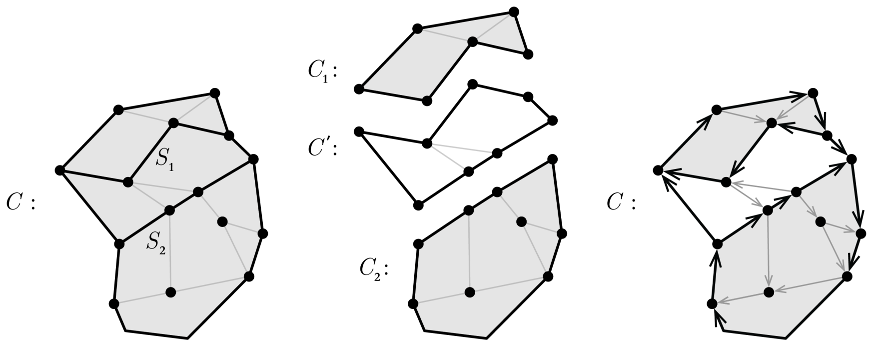

Figure 9: Left: A cluster graph C with two boundary shortcuts S_1 and S_2, shown in bold. Middle: Three cluster subgraphs C_1, C' and C_2 of C such that $C_1 \cap C' = S_1$, $C_2 \cap C' = S_2$ and $C_1 \cup C_2 \cup C' = C$, separated for illustration purposes. Right: A strong orientation of C for which the cluster boundaries $B(C)$, $B(C_1)$ and $B(C_2)$ all have a clockwise sense of orientation.

Proof Let C be a cluster graph with two boundary shortcuts S_1 and S_2 that do not share any edges. According to theorem 4.2, C can be decomposed into two cluster subgraphs C_1 and C_2' such that $C_1 \cap C_2' = S_1$ and $C_1 \cup C_2' = C$, where $S_2 \in C_2'$. Similarly, C_2' can be decomposed into two cluster subgraphs C' and C_2 where $C' \cap C_2 = S_2$ and $C' \cup C_2 = C_2'$. Thus, $C_1 \cap C' = S_1$, $C_2 \cap C' = S_2$ and $C_1 \cup C' \cup C_2 = C$.

Since $B(C)$ is a cycle, assign orientations to the edges of $B(C)$ such that $B(C)$ has a strong orientation with a clockwise or counter-clockwise sense of orientation. Then, since S_1 and S_2 do not share any vertices, the orientation of the edges of one is independent of the orientation of the edges of the other one. Thus, assign an orientation to each edge of S_1 and S_2 such that $B(C_1)$ and $B(C_2)$ have the same sense of orientation as $B(C)$. Therefore, the graph $H = B(C) \cup S_1 \cup S_2$ has strong orientation.

Finally, the interior edges of C_1, C_2 and C' can be assigned a strong orientation according to Robbins's Theorem (theorem 1), while keeping the assigned senses of orientation of $B(C_1)$, $B(C_2)$ and $B(C')$, since C_1, C_2 and C' are planar bridgeless connected graphs. In conclusion, a strong orientation of C exists where the cluster boundaries $B(C)$, $B(C_1)$ and $B(C_2)$ have equal senses of orientation. □

4.3 Relative heights of vertices

Whereas geometrically rigid structures (such as a circulation network within an informal settlement) can be abstracted with the help of graph theory, the following definitions and theorems treat graphs as geometrical objects placed on a plane. Assigning coordinates to each vertex facilitates the definition of the orientations of each lane with the Cluster Lane Method. The definitions in this subsection make use of a Cartesian two-dimensional coordinate space

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defined by an origin $O(0,0)$ and two perpendicular axes X (horizontal) and Y (vertical) which intersect at the origin. By placing a planar graph G on this coordinate system, every vertex v of G can be defined by its X and Y coordinates as $v(x_v, y_v)$, where $X(v) = v_x$ and $Y(v) = v_y$. The Null Alignment Theorem, stated below, is an important element of the Cluster Graph Cascade Theorem, stated further below. These theorems make it possible to assign orientations to the interior edges of a cluster graph in an ordered manner, as explained in section 4.4.

Theorem 4 ***(Null Alignment Theorem) :*** *Given a finite set $V : \{v_1, v_2, \dots\}$ of n non-coincident coplanar points. There is an infinite number of straight lines $L_1, L_2, L_3 \dots$ such that for any L_i, $i = 1, 2, 3, \dots,$ there is only one straight line parallel to L_i passing by each point v_i of V. (See figure 10.)*

Proof Given a finite set $V : \{v_1, v_2, \dots\}$ of n non-coincident coplanar points. Let $S(v_i, v_j)$ with $i \neq j$ be the slope between any two distinct points v_i and v_j of V. For every unordered pair of points of V, compute the slopes $S(v_i, v_j)$ for all unordered combinations of i and j, being $n(n-1)/2 = (n^2 - n)/2$ slopes. By arranging the slopes from smallest to largest and renaming them, we obtain the set of slopes

$$S = \{S_1, S_2, \dots, S_{(n^2-n)/2}\}, \quad \text{where} \quad S_i < S_{i+1} \quad \text{for} \quad i = 1, 2, \dots, (n^2 - n)/2 - 1. \tag{4.3}$$

Given a pair of consecutive slopes $\{S_i, S_{i+1}\}$, a slope S' can be defined as

$$S' = x(S_i) + (1 - x)(S_{i+1}), \quad 1 \leq i \leq (n^2 - n)/2 - 1, \quad \text{with} \quad 0 < x < 1, \tag{4.4}$$

which is different than any value of S, since $S_i < S < S_{i+1}$, for any value of i. Thus, any straight line with slope S' intersects either only one or no points of V, since $S' \notin S$. There are infinite choices for the value of x and a finite number of choices for the value of i. Thus, there are an infinite number of straight lines $L_1, L_2, L_3, \dots$ such that for any L_i, $i = 1, 2, 3, \dots,$ only one straight line parallel to L_i passes by each point of V. □

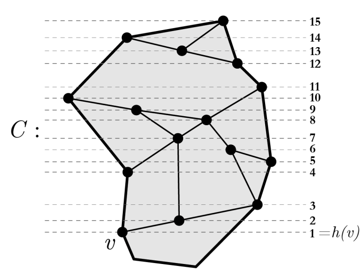

Figure 10: Left: A cluster graph C. Right: A planar configuration of C and a set of parallel straight lines such that each straight line intersects only one vertex of C. Any of these lines can be named L. A relative height $h(v_i)$ is assigned to each vertex v_i of the graph.

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This theorem can be used to generate a two-dimensional coordinate system such that the Y coordinate of every vertex of a set of non-coincident coplanar points V is different. (See Figure 10.) The theorem can be generalized to higher dimensions. The following definitions deal with graphs on a planar configuration for which no two vertices have the same Y coordinate, or relative height.

Definition 20 *Given a cluster graph C with a vertex set $V : \{v_1, v_2, \ldots, v_n\}$ of n vertices such that no two vertices have the same Y coordinate. The* **relative height sequence** *V' is a sequence whose terms are the vertices of V, ordered from the lowest Y coordinate to the highest. It is written*

$$V' : \{v'_1, v'_2, \ldots, v'_n\}, \quad \textit{where} \quad Y(v'_1) < Y(v'_2) < \cdots < Y(v'_n). \tag{4.5}$$

Definition 21 *Given a cluster graph C with a vertex set $V : \{v_1, v_2, \ldots, v_n\}$ of n vertices on a two-dimensional coordinate system such that no two vertices have the same Y coordinate. The* **relative height** *$h(v_i)$ of a vertex v_i corresponds to its rank in the relative height sequence V'.*

For example, given a bridgeless connected planar graph G with a set of vertices $V : \{v_1, v_2, v_3, v_4, v_5\}$ on a two-dimensional coordinate system with Y coordinates $Y(v_1) = 12$, $Y(v_2) = 5$, $Y(v_3) = 2$, $Y(v_4) = 10$ and $Y(v_5) = 15$. The relative height vertex set is the ordered set $V' : \{v_3, v_2, v_4, v_1, v_5\}$. The relative heights of the vertices are $h(v_1) = 4$, $h(v_2) = 2$, $h(v_3) = 1$, $h(v_4) = 3$ and $h(v_5) = 5$. Note: relative heights and relative height sequences could be generalized to higher dimensions, as long as every point has a distinct Y coordinate.

Definition 22 *Given a cluster graph C. The* **in-cluster subgraph** *$I(v)$ of an interior vertex v of C is the set of vertices and edges which belong to all possible paths that start at v and end at a vertex of the cluster boundary $B(C)$ of C, without passing by any other vertices of $B(C)$. (See figure 4.3.)*

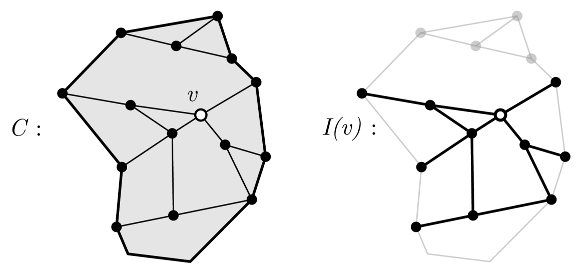

Figure 11: Left: A planar configuration of a cluster graph C with an interior vertex v shown in white. Right: The in-cluster subgraph $I(v)$ of the interior vertex v.

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Definition 23 *Given a bridgeless connected planar graph G with a relative height sequence $V' : \{v'_1, v'_2, \dots, v'_n\}$. The* **maximum adjacent relative height** *of a vertex v'_i, written $A_{max}(v'_i)$, is the maximum relative height of any of the vertices adjacent to v'_i, which can be interior vertices or cluster boundary vertices. The* **minimum adjacent relative height** *of a vertex v'_i, written $A_{min}(v'_i)$, is the minimum relative height of any of the vertices adjacent to v'_i, which can be interior vertices or cluster boundary vertices.*

Definition 24 *Given a cluster graph C with a relative height vertex set $V' : \{v'_1, v'_2, \dots, v'_n\}$. The* **maximum relative boundary height** *of a vertex $v'_i \notin B(C)$, written $B_{max}(v'_i)$, is the maximum relative height of any of the vertices of the in-cluster subgraph $I(v'_i)$ which also belong to the boundary $B(C)$ of C. The* **minimum relative boundary height** *of a vertex $v'_i \notin B(C)$, written $B_{min}(v'_i)$, is the minimum relative height of any of the vertices of the in-cluster subgraph $I(v'_i)$ which also belong to the boundary of C.*

The following theorem uses these notions to assign orientations to the interior edges of a cluster graph C in order for C to have a strong orientation following a certain logic.

Theorem 5 ***(Cluster Graph Cascade Theorem):*** *Given a planar configuration of a cluster graph C with a vertex set $V : \{v_1, v_2, \dots, v_n\}$ and a coordinate system such that $Y(v_i)$ is different for each vertex of C. If each interior vertex v of C satisfies*

$$B_{min}(v) < h(v) < B_{max}(v) \quad \text{and} \quad A_{min}(v) < h(v) < A_{max}(v), \tag{4.6}$$

then a strong orientation of C exists for which each interior edge $v_j v_k$ of C joining v_j and v_k, with $h(v_j) > h(v_k)$, is assigned an orientation going from v_j to v_k, and where the cluster boundary $B(C)$ has a clockwise or counter-clockwise sense of orientation. (See figure 12.)

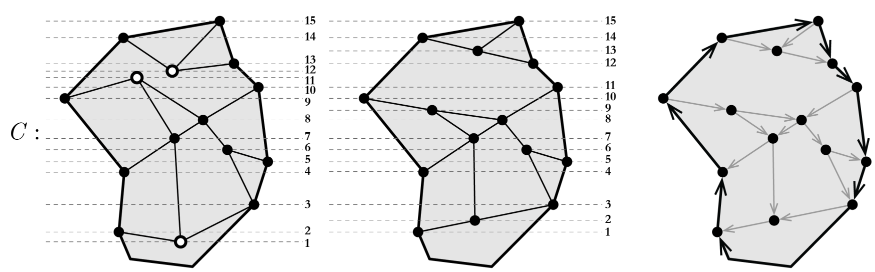

Figure 12: Left: A planar configuration of a cluster graph C. The interior vertices shown in white do not respect equation 4.6. Middle: A planar configuration of C for which all interior vertices respect equation 4.6. Right: A strong orientation of C according to theorem 5.

Proof Given a cluster graph G with a vertex set $V : \{v_1, v_2, \dots, v_n\}$. For any planar configuration of G, let the vertices v_i be represented by non-coincident points $v_1, v_2, \dots, v_n$ on a plane

P. According to theorem 4, there is an infinite number of straight lines $L_1, L_2, L_3, \ldots$ such that for any straight line L_i, $i = 1, 2, 3, \ldots$, there is only one straight line parallel to L_i passing by each point v_i of V. Thus, any of these straight lines L_i can be defined as the X axis of a cartesian plane, with the Y axis being perpendicular to X, such that $Y(v_i)$ is different for each vertex v_i of C.

Let the edges of the cluster boundary $B(C)$ be assigned orientations such that $B(C)$ has a clockwise or counter-clockwise sense of orientation. If, for each interior vertex v of C, it is true that $A_{\min}(v) < h(v) < A_{\max}(v)$, then there can always be an edge oriented from a vertex u with $h(u) > h(v)$ to v, and from v to w with $h(v) > h(w)$, which means that v is not a dead-end. Note that u and w can be interior vertices or boundary vertices. If, for each interior vertex v of C, it is true that $B_{\min}(v) < h(v) < B_{\max}(v)$, then a path P_1, as a sequence of vertices and edges, starting and ending at two vertices of $B(C)$ and passing by v, exists. Since $B(C)$ has strong orientation, then $B(C) \cup P_1$ has strong orientation. Likewise, for all paths $P_1, P_2, P_3, \ldots$, the union $B(C) \bigcup_i P_i = C$ has strong orientation. □

If a planar configuration of a cluster graph C does not respect equation 4.6 for each interior vertex v, then another planar configuration of C that respects equation 4.6 can be found to apply theorem 5, as shown in figure 12.

4.4 The Cluster Lane Method

The Cluster Lane Method's input is a planar circulation network and its output is a unidirectional oriented graph with strong orientation. The method could be described in simple terms as follows: An entire informal settlement, which corresponds to a large cluster of dwellings, is subdivided as desired into as many smaller clusters of dwellings as desired. Given these smaller clusters of dwellings, each lane of the circulation network is assigned an orientation in order to have three types of circulation (around the boundary of each cluster, within the boundary of each cluster and between the boundaries of different clusters), as discussed in section 4.1. The result is a unidirectional circulation system which takes into account the clusters of dwellings of the given informal settlement.

The method, composed of 8 steps, is explained below with the mathematical notions discussed in the previous sections. Figure 13 illustrates these steps progressively. Figure 14 shows the final result, an enlargement of figure 13(i).

1. **(Graph of the circulation network):** For any given planar circulation network, produce a graph G such that the edges correspond to the lanes of the network and the vertices correspond to the intersections between lanes or the ends of each lane.

2. **(Cluster graph of the circulation network):** If G is a planar bridgeless connected graph, then let G be a cluster graph C. If G contains $m \geq 1$ bridges, remove the bridges to produce a set of $m + 1$ cluster graphs. Treat each of these cluster graphs as C for the following steps.

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3. **(Cluster subgraphs and boundaries):** Given a cluster graph C with two boundary shortcuts S_1 and S_2 that do not share any edges. Define three cluster subgraphs C_1, C_2 and C' of C such that

$$C_1 \cap C' = S_1, \quad C_2 \cap C' = S_2 \quad \text{and} \quad C_1 \cap C_2 \cap C' = C. \tag{4.7}$$

According to the Cluster Subgraph Theorem (theorem 3), a strong orientation of C exists where the cluster boundaries $B(C)$, $B(C_1)$ and $B(C_2)$ all have the same sense of orientation. Assign a clockwise sense of orientation to each of these cluster boundaries.

4. **(Cluster subgraphs and boundaries - continuation):** Repeat step 3 as many times as desired to further divide C_1 and C_2 into cluster subgraphs and to assign a clockwise sense of orientation to the boundary of each cluster subgraph. (Note: The decomposition of C into cluster subgraphs should be as responsive as possible to the existing clusters of dwellings and to the needs of the inhabitants of the informal settlement.)

5. **(Interior edges of cluster graphs):** Let $V : \{v_1, v_2, \ldots, v_n\}$ be the vertex set of C. Determine a coordinate system such that $Y(v_i)$ is different for each vertex of C, with the help of the Null Alignment Theorem (theorem 4). For each cluster subgraph C_i, for $i = 1, 2, \ldots, n$, for which every interior vertex v satisfies

$$B_{\min}(v) < h(v) < B_{\max}(v) \quad \text{and} \quad A_{\min}(v) < h(v) < A_{\max}(v), \tag{4.8}$$

assign each interior edge $v_j v_k$ of C_i joining v_j and v_k, with $h(v_j) > h(v_k)$, an orientation going from v_j to v_k. According to the Cluster Graph Cascade Theorem (theorem 5), C_i will have strong orientation since the cluster boundary C_i has a clockwise sense of orientation. If one of more vertices of C_i do not satisfy equation 4.8, find a planar configuration of C_i that does, as shown in figure 12.

6. **(Edges between cluster graphs):** For all other edges of C which are neither boundary edges nor interior edges of a cluster subgraph C_i, for $i = 1, 2, \ldots, n$, which have already been assigned an orientation, assign any orientation such that C has strong orientation. (Note: these orientations should be assigned to make it as easy as possible to move from one cluster subgraph to another.)

7. **(Bidirectional bridges):** Assign a bidirectional orientation to every *bridge* (including dead-ends) of G.

8. **(Bidirectional wide lanes):** Assign a bidirectional orientation to every lane of C that is wide enough to practice physical distancing.

By following these 8 steps, the resulting graph of the circulation network has strong orientation.

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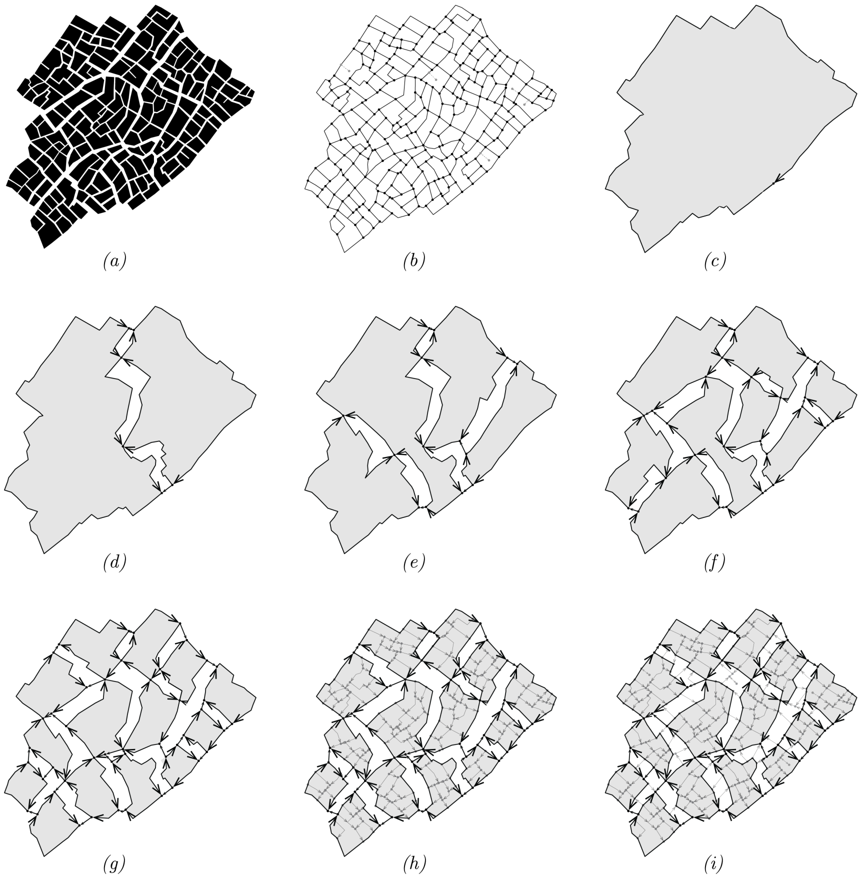

Figure 13: Example of an application of the Cluster Lane Method. Figure (a) shows a plan of a circulation network in which buildings are represented by black polygonal surfaces and in which the white spaces between these black surfaces correspond to the public circulation lanes. Figure (b) corresponds to step 1 and step 2 of the method: a graph G represents the circulation network. In this case, the only *bridges* of G are the dead-ends, shown by grey dashed lines. By removing these *bridges*, G becomes a cluster graph C. Figures (c), (d), (e), (f) and (g) correspond to step 3 and step 4: the cluster graph C is decomposed into cluster subgraphs, which are then decomposed into smaller cluster subgraphs as many times as desired. (Each cluster graph is represented by a grey polygonal surface.) Progressively, a clockwise sense of orientation is assigned to the boundary of each cluster subgraph, shown by arrows. Figure (h) corresponds to step 5: the interior edges of each cluster subgraph are assigned an orientation. Figure (i) corresponds to step 6 and step 7: the remaining edges between cluster subgraphs are assigned an orientation, and the bridges are assigned a bidirectional orientation. Step 8 is not shown in this example but lanes that are wide enough to practice physical distancing should be assigned a bidirectional orientation.

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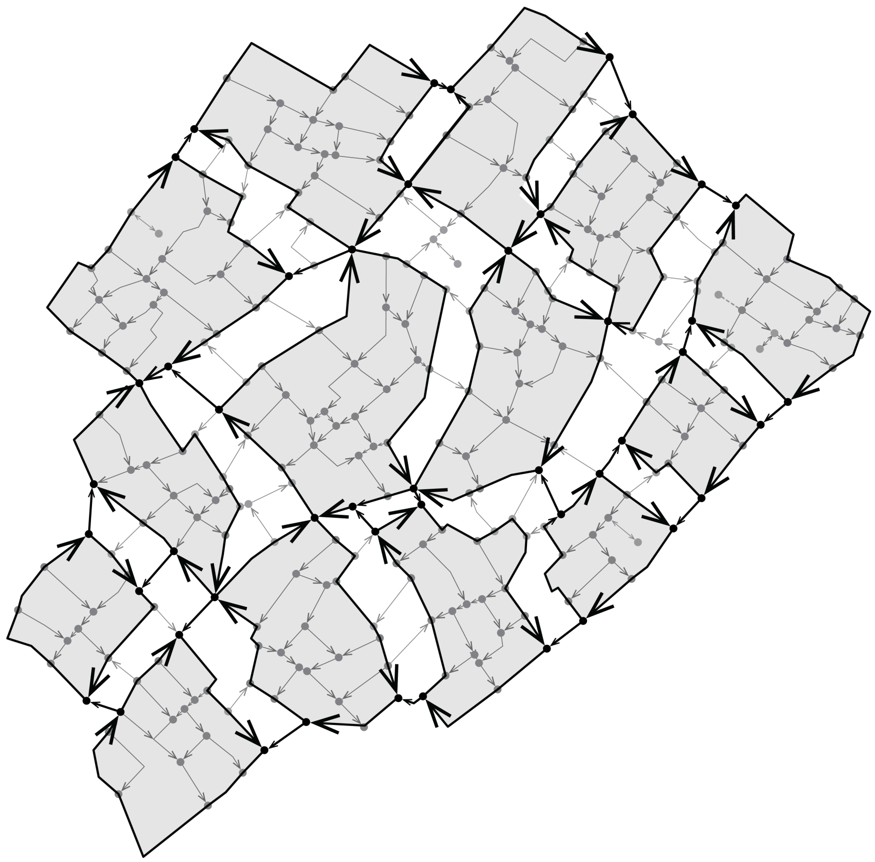

Figure 14: A unidirectional circulation network generated with the Cluster Lane Method. This is a copy of figure 13(i) in a larger format.

5 Applications of the method

5.1 Site-specific implementations, general guidelines and further development of the method

Informal settlements and slums lack the top-down organization inherent in planned urban developments like the Manhattan grid or the Barcelona superblocks. Their organic growth is often dictated by self-building principles responding to local constraints and can result in

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unique spatial organizations. Two of these principles are autonomous growth and continuous development, discussed by McGill University's Minimum Cost Housing Group. [23]. The desire to be in proximity with family, culturally meaningful places, trades, among others, can lead to homes being built in clusters. Coupled with variable plot sizes and forms, this may lead to meandering lanes, non-orthogonal buildings and irregular polygonal building blocks, as well as complex lane intersections. In addition, circulation networks in settlements respond to geography, site conditions, cultural factors, local materials, etc. All of this leads to the creation of circulation networks which could be referred to as "organic" or "free-form", and whose patterns can be hard to discern when looking at a satellite image or a plan. Orangi Town in Karachi (Pakistan), Neza in the State of Mexico (Mexico), Dharavi in Mumbai (India), Kibera in Nairobi (Kenya), Khayelitsha in Cape Town (South Africa) and Rocinha in Rio de Janeiro (Brazil), all present unique spatial qualities. Yet, regardless of the size and complexity of the circulation network, the method can always be applied. The mathematical tools laid out in section 3 and section 4 make it possible to apply the Cluster Lane Method to any connected planar circulation network.

Further research and testing would be necessary to find a site-specific and culture-specific implementation of the Cluster Lane Method. An anthropological study of how people inhabit a particular informal settlement would inform key strategies for implementation - for example, whether a bottom-up or top-down approach would be well received depending on social norms surrounding authority and discipline. Regardless of local variants, it would be important to minimize distances between homes and significant spatial nodes like landmarks, sanitary facilities, marketplaces or public institutions.

The implementation of the Cluster Lane Method would disrupt existing circulation habits, which could lead to non-compliance, confusion or inconvenience on part of the inhabitants. To prevent resentment among the inhabitants, it would be important to involve the community in as many steps of the process as possible. This would help ensure that existing circulation patterns are accommodated in the final unidirectional network while also allowing all parties to arrive at a common set of objectives and priorities. When maps are not available, mapping entire informal settlements would be necessary to apply the method, since as many lanes as possible should be considered when assigning orientations to them. The use of drones or satellite images could prove to be useful, but public circulation networks can often be hidden from aerial views due to informal construction. An alternative could be to map the informal settlement on foot, using a survey team or involving the residents themselves. The latter would help integrate natural circulation patterns such as local desire paths and shortcuts, and it could potentially win the community's support. If a map is produced and if the method is applied, maps with the assigned orientations of lanes should be installed throughout the system and individual maps should be available to the inhabitants of the informal settlement.

By carefully applying the Cluster Lane Method, each one of the three types of circulation lanes discussed in section 4.1 can be defined in a way to best suit the needs and interests of the inhabitants of the informal settlement. Defining the circulation lanes of type 1 (around the boundary of a cluster of dwellings) is very flexible, since cluster graphs can be decomposed into cluster subgraphs in numerous ways. Their sense of orientation can be either all clockwise or

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all counter-clockwise. It would also be possible to use the Adjacent Cluster Subgraph Theorem (theorem 4.2) to decompose cluster graphs into adjacent cluster subgraphs. However, it would not always be possible to obtain a strong orientation for all cluster boundaries this way. Some senses of orientation would have to be counter-clockwise while others are clockwise, which could confuse pedestrians. Defining the orientation of the circulation lanes of type 2 (within the boundary of a cluster of dwellings) can be done as proposed in section 4.4 or by doing ear decompositions in any desired way (as explained in section 3). Moreover, the general direction of the oriented lanes of type 2 can be different from one cluster graph to another. (See figure 5.1). Lastly, defining the orientation of the circulation lanes of type 3 (between the boundaries of different clusters of dwellings) is also very flexible.

C :

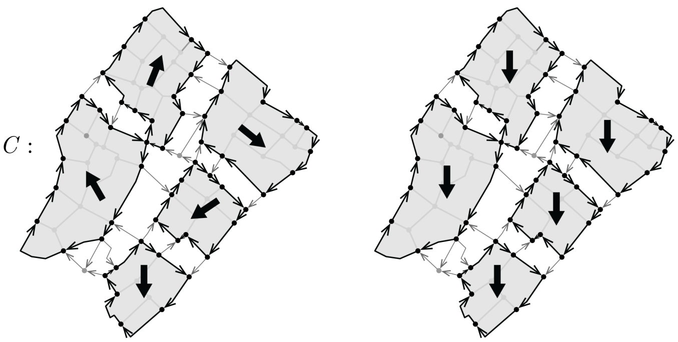

Figure 15: Left: Given a cluster graph C and five cluster subgraphs of C, shown in grey. By assigning a different Y axis (which determines the relative heights of interior vertices) to each cluster subgraph, the general direction of the interior edges can vary from one cluster subgraph to another. This makes the method more adaptable. However, the overall change of direction within cluster graphs could prove to be more confusing to pedestrians. Right: By assigning the same Y axis to all cluster subgraphs, as proposed in section 4.4, the general direction of the interior edges of each graph is the same. This could be more intuitive to pedestrians (for example, if the all interior lanes of a cluster of dwellings are directed toward the south as a general direction, the pedestrians would get used to walking in the general south direction inside the clusters of dwellings).

To apply the method on a large scale, supposing that a map of the desired circulation network is available, a software using the method described in this paper could be created. Algorithms could automate the process and find optimal solutions. Dijkstra's algorithm, the Bellman-Ford algorithm, or similar algorithms dealing with oriented graphs and combinatorics could be used to minimize lengths of the most heavily used paths. Weighted graphs (graphs for which each edge is assigned a weight) could be used to reflect more crucial factors such as distances between lane intersections, the inclination of streets, sunlight at different times of days, etc. It would be important to address less obvious issues, such as the stimulation of the local economy. By creating important circulation arteries through commercial zones within the informal settlement,

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it would allow for both limited disturbance near private residences and controlled access to the public marketplace. As mentioned previously, distances to basic necessities (such as public toilets, water access and waste disposal) should be minimized. Regardless of the algorithms used, it is very important to let the people living in the informal settlement participate in the process to obtain an outcome that suits their movement patterns and make them feel valued.

5.2 Signage system and unidirectional tactile paving for the visually impaired

Once a circulation network has been mapped and the Cluster Lane Method has been used to assign orientations to each lane, this must be clearly communicated to the pedestrians. By installing color-coded signs or by painting arrows on the floor, one could establish a consistent visual communication system that transcends any linguistic divide. An effective signage system would thus make it easier for pedestrians to follow the assigned orientation of each lane. To help people understand the logic of the circulation system as a whole, it would be beneficial to have three different types of arrows or signs associated to the three types of lanes discussed in section 4.1. Arrows could be placed throughout each lane, or simply at the intersections, marking the beginning and end of each lane. This cost-efficient, low-tech solution would be fast to implement, would significantly reduce the number of face-to-face interactions between pedestrians and would allow for physical distancing in narrow public circulation spaces. By including the direction of lanes in **navigation applications** such as Google Maps, minimal routes could be determined for each walk. Eventually, pedestrians could get used to the new unidirectional circulation system.

To ensure universal accessibility and usage of the unidirectional pedestrian circulation, it would be necessary to think of the passageways more holistically. The conditions of public circulation networks in informal settlements, which include irregular surfaces and various obstacles along paths, make it difficult for blind and visually impaired people to navigate through them. The following paragraphs propose a potential solution that makes it possible to apply the Cluster Lane Method while taking the visually impaired into account.

In addition to those who are visually impaired from a young age, numerous informal settlement dwellers develop conditions which deteriorate their sight. The limited access to health facilities and ophthalmologists in these areas result in more serious consequences of eye-related conditions such as cataracts, the leading cause of blindness globally. [25] Despite the implementation of initiatives such as the *National Program for Control of Blindness* in India, cataracts remain a health problem in the urban slums of Delhi [25]. A study in the urban slums of in Nairobi, Kenya, states that "Kibera slum dwellers are in need of comprehensive eye care services offering cataract surgery and low cost spectacles." [26] Similar conditions can be seen in other informal settlements throughout the world. These facts stress the importance of including the visually impaired while applying the method. They should be able to navigate as comfortably and as independently as possible. The following paragraphs describe tactile paving and introduce unidirectional tactile paving.

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Japanese inventor Seiichi Miyake created a revolutionary navigation aid for the blind and the visually impaired. Tactile paving (or Tenji blocks) consists of tactile blocks on pavement which are "intended to alert visually impaired pedestrians of upcoming dangers, like sidwalk curbs and train platform edges". [27] The textures of the blocks can be felt with the feet or with a cane; they can be perceived by the partially sighted since their colour (usually yellow) contrasts with the pavement around it. A decade after the first implementation of tactile paving near the Okayama School for the Blind, the system was popularized by Japan National Railway, since "every (...) platform was modified to include Miyake's invention". [27] Today, Japanese law requires certain buildings and public spaces to install tactile paving, and they can be found in most public circulation spaces in major Japanese cities, like Tokyo. Other cities throughout the word have started to implement tactile paving in particular locations such as train stations and airports, as well as certain public spaces.

To create paths of any desired length, tactile blocks can be placed side by side to create paths of any desired length. According to the International Association of Traffic and Safety Sciences, there are "two types of tactile ground surface indicators: warning blocks that indicate the location of hazards or destination facilities (...) and directional blocks that indicate direction of travel." [28] Warning blocks (or warning tiles) have a blister pattern made up of small domes. (See figure 16, top left.) These blocks usually have 25 or 36 small domes, but different variations exist. Directional blocks (or directional tiles) have long bars that are parallel to the direction of travel. (See figure 16, top middle.) Most commonly, they have four equally spaced bars. Whereas directional blocks are very functional and effective, they do not specify in which direction the path goes and could be said to be *bidirectional* blocks. Two distinct designs of unidirectional blocks are proposed in this paper: *asymmetrical blocks* and *cobblestone blocks*, described below.

An *asymmetrical block* has long bars of different widths that are parallel to the direction of travel. An asymmetrical block has two standard long bars parallel to the direction of travel on the *right* half of the block, and four narrower long bars also parallel to the direction of travel on the *left* half of the block. (See figure 16, top right and bottom.) This allows the users to feel a difference between the left and right sides of the tactile paving with their feet and cane. By informing the users that the standard long bars must always be on the right side while walking on a tactile path in order to respect the assigned direction, the users can internalize this convention to know which direction to follow. Asymmetrical blocks can be mass produced just like any other tactile paving block and they do not require more material than the standard directional blocks. Moreover, asymmetrical blocks can be used as standard directional blocks once that unidirectional circulation is no longer needed, without having to replace them. Implementing unidirectional tactile paving as a general rule in circulation networks with narrow lanes could be a preventive measure for a future outbreak of COVID-19 or any other airborne disease. It would also limit potential collisions between blind or visually impaired people walking in opposite directions on a same path. Lastly, variations of the asymmetrical block design could allow for diverse blocks that could convey different information in addition to a sense of direction. Blocks with different arrangements and widths of bars could produce different sounds when a cane is passed over them, transforming the physical information conveyed by the blocks into audible information which could be recognized by the blind or visually impaired

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pedestrian. For example, a series of long bars of alternating narrow and standard widths could indicate a construction zone.

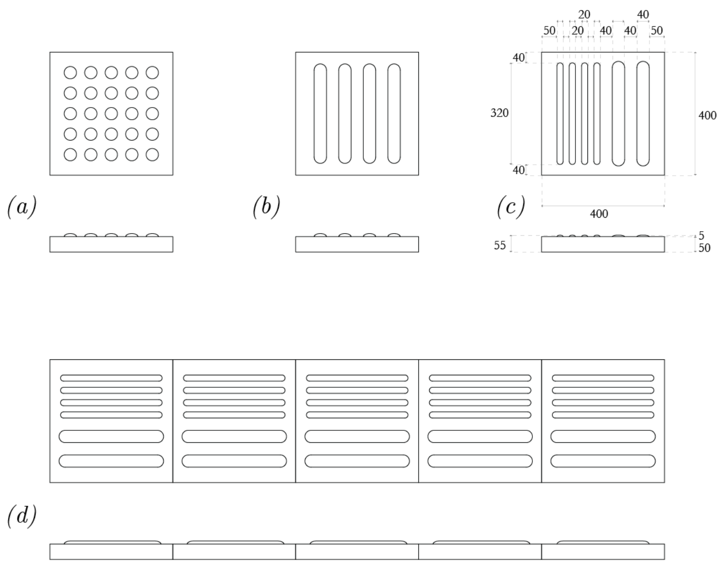

Figure 16: (a) Warning blocks with 25 small domes; top view and elevation. (b) Directional blocks with four long bars; top view and elevation. (c) Asymmetrical blocks with two standard long bars on the right side and four narrow long bars on the left side; top view and elevation. (d) An assembly of five unidirectional blocks; top view and elevation.

A *cobblestone block* has long bars like a standard directional block but, instead of being flat, it consists of two slanted planes with different inclinations sloping in opposite directions. When approaching the block while respecting the assigned direction of unidirectional travel, the pedestrian encounters the less inclined slope; when approaching it from the opposite direction, the pedestrian encounters the more inclined slope. (See figure 17.) These slopes should be inclined enough to be felt with the feet or with a cane but not too inclined to prevent accidents by tripping somebody. Cobblestone blocks are placed between standard directional blocks at regular intervals (every 25 blocks, for example) as a reminder of the direction to follow. It is possible to place consecutive cobblestone blocks one after the other to provide additional information in addition to the direction. For example, in the context of the Cluster Lane Method, a single cobblestone block among standard directional blocks could signal the direction of the lane and identify that it is a lane of type 1, whereas two or three consecutive cobblestone blocks could signal the direction of a lane and identify it as a lane of type 2 or type 3, respectively.

In addition to unidirectional blocks, navigation apps could be an extremely valuable tool for

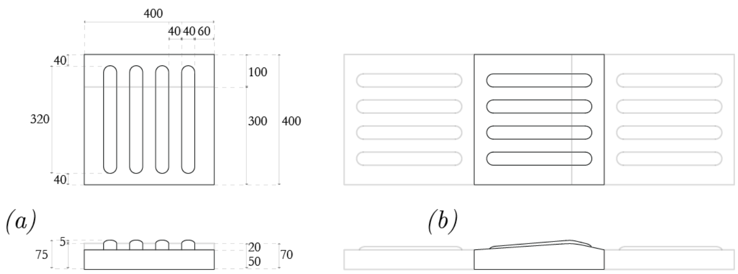

Figure 17: (a) A cobblestone block; top view and elevation. (b) A cobblestone block between two standard directional blocks (lighter lines); top view and elevation.

the blind and the visually impaired in informal settlements. NaviLens (navilens.com) allows its users to scan NaviLens codes placed at strategic locations on the floor, giving them important information about their surroundings. Blindsquare (blindsquare.com) works with a GPS and a smartphone compass, guiding the user through unknown circulation spaces to particular destinations. BlindWiki (https://blind.wiki/) allows blind or partially sighted users to share location-based findings with others, including difficulties and barriers along paths, but also experiences and stories. Also, speakers with unique and identifiable sounds could be placed at the beginning of lanes to specify their orientation.

Unidirectional circulation is not always respected even when proper infrastructure is available. Turnstiles and escalators are meant to enforce unidirectional circulation, yet people can jump over them or walk in the opposite direction. Security checkpoints and shops often compensate for the lack of this type of infrastructure with surveillance and assistance from people in order to ensure unidirectional circulation. If unidirectional tactile paving were to be implemented in an informal settlement, nothing guarantees that it would be used as intended. It is important to acknowledge the fact that the blind and the visually impaired have a significantly harder time while navigating unknown environments. By implementing a unidirectional circulation system with unidirectional tactile paving, this task would become much more difficult since detours would be necessary for most routes. However, navigation applications would minimize the length of routes and unnecessary detours if the lanes of the settlement were mapped, along with their assigned orientations. Regardless of the context of this paper, the proposed unidirectional blocks designs could be implemented elsewhere for other purposes and in different contexts.

For the implementation of the Cluster Lane Method to be more inclusive, people with reduced mobility and using wheelchairs should be taken into account as well. Unfortunately, the existing conditions of public circulation spaces informal settlements are not always adequate for wheelchairs. Irregular surfaces and obstacles make circulation for people with reduced mobility a difficult task. Ideally, certain lanes could be flattened, cleaned, or even paved with the implementation of the method.

 doi:10.20944/preprints202010.0598.v1

5.3 Conclusions

The application of the Cluster Lane Method on a circulation network of an informal settlement could help reduce contagion within public circulation spaces by limiting face-to-face interactions between pedestrians. The flexibility of the method makes it possible to adapt it to any circulation network. By studying the social behaviours within the informal settlement and by including the population in the process, a more adequate site-responsive result could be obtained. Unidirectional tactile paving and other technologies could be used to make the system more inclusive for the blind and visually impaired.

It is uncertain to what degree the implementation of the method in a real informal settlement would reduce transmissions of the virus, and it is uncertain if pedestrians would respect the assigned orientations. The application of the method would in no way reduce most major problems found in informal settlements which facilitate contagion. These include: small dwellings for large numbers of people, lack of basic sanitary infrastructure and many more. However, this paper aims to bring the difficult situation of informal settlements during the pandemic to the readers' attention, and to propose an idea which could potentially be implemented, or which could serve as an inspiration to future ideas to slow down contagion in the narrow circulation networks within informal settlements. By implementing the method at a particular location, the population's response to it could be studied in order to improve the Cluster Lane Method.

Finally, it is important to note that physical distancing would be possible in the public circulation spaces of informal settlement if they were more spacious. By respecting a minimum width for the lanes during the development of an informal settlement, unidirectional circulation would be unnecessary and physical distancing would be possible in all public circulation spaces. Numerous lives could be saved around the globe by slowing down the transmission of COVID-19 in the public circulation spaces of informal settlements.

Acknowledgements

The authors would like to thank Salmaan Craig for his comments on an earlier draft of this paper and for suggesting *Buildings and Cities* as a venue, as well as Annmarie Adams, Theodora Vardouli and David M Covo for their encouragement. Finally, the authors would like to thank Vojtěch Polášek for a fruitful conversation about his personal experience with tactile paving and the potential implementations of unidirectional tactile paving, and for coming up with the name *cobblestone blocks*.

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