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Article

Understanding the Role of Nuclear Elementary Charge and Its Potential in Estimating Nuclear Binding Energy and Strong Coupling Constant

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Abstract: Considering our 4G model of final unification, there exists a nuclear elementary charge of magnitude $2.9464e$ and strong coupling constant, α_s , is the squared ratio of ordinary elementary charge to nuclear elementary charge. Nuclear elementary charge is having many applications in nuclear physics and other branches of physics like particle physics, super conductivity, condensed matter physics and unified physics. By refining the general nuclear binding energy formula, there is a possibility for defining a set of binding energy coefficients assumed to be linked with the strong coupling constant in the form of $k[(2.95e)^2/4\pi\epsilon_0(1.25\text{ fm})] \cong (k/\alpha_s)[e^2/4\pi\epsilon_0(1.25\text{ fm})]$ where $k = (1.0 \text{ to } 2.5)$. By minimizing the error bars in the estimated binding energy, there is a chance for fixing the value of the strong coupling constant. Based on volume, surface, coulomb, asymmetry, pairing and congruent terms, it seems possible to fit the binding energy of isotopes of $Z=1$ to 137 and needs fine tuning for heavy isotopes of light proton numbers. Another interesting point is that, surface, coulomb and asymmetry energy coefficients can be considered as variable coefficients. Coulomb energy point of view, it is noticed that, increasing number of neutrons help in minimizing the nuclear radii against the coulomb repulsion. With reference to protons, neutrons and the coulomb energy, nuclear radii, $R_{(Z,N)} \cong \sqrt{1 - [(N-Z)/A]^2} A^{1/3} \times 1.25\text{ fm} \cong (\sqrt[3]{Z} + \sqrt[3]{Z^2 N}) \times 0.79\text{ fm}$.

Keywords: 4G model of final unification; nuclear binding energy; nuclear radii; 6 term semi empirical mass formula; strong and electroweak mass formula; variable energy coefficients;

1. Introduction

With reference to our 4G model of final unification, in our recent publications [1–12], we have developed a new formula for estimating nuclear binding energy [13–24] in terms of strong and electroweak interactions [25,26]. Our formula constitutes 4 simple terms and only one energy coefficient of magnitude 10.1 MeV. First term is a volume term, second term seems to be a representation of free nucleons associated with electroweak interaction, third term is a radial term and fourth one is an asymmetry term about the mean stable mass number. Considering this kind of approach, nuclear structure can be understood in terms of strong and weak interactions and complicated concepts like cold nuclear fusion can be understood in a theoretical approach positively.

In this short paper, we have presented a revised form of semi empirical mass formula (SEMF) with surface, coulomb and asymmetric energy coefficients as variables. This can be considered as a hybrid form of available SEMF. With a single set of energy coefficients, it seems to work for $Z=1$ to 137 and needs fine tuning for heavy isotopes of light proton numbers. In this context, we would like to emphasize the point that, strong coupling constant plays a vital role in fitting the nuclear binding energy coefficients. Clearly speaking, strong coupling can be inferred from nuclear binding energy coefficients. Considering our strong and electroweak mass formula and considering the proposed

(revised) 6 term semi empirical mass formula, there is a chance to identify the strong coupling constant as a fundamental building block of atomic nuclei.

2. Three Assumptions of 4G Model of Final Unification

Following our 4G model of final unification [1–12]

- 1) There exists a characteristic electroweak fermion of rest energy, $M_{wf}c^2 \cong 584.725 \text{ GeV}$. It can be considered as the zygote of all elementary particles.
- 2) There exists a nuclear elementary charge in such a way that, $\left(\frac{e}{e_n}\right)^2 \cong \alpha_s \cong 0.1152 = \text{Strong}$ coupling constant and $e_n \cong 2.9464e$.
- 3) Each atomic interaction is associated with a characteristic large gravitational coupling constant. Their fitted magnitudes are,

$$G_e \cong \text{Electromagnetic gravitational constant} \cong 2.374335 \times 10^{37} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2}$$

$$G_n \cong \text{Nuclear gravitational constant} \cong 3.329561 \times 10^{28} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2}$$

$$G_w \cong \text{Electroweak gravitational constant} \cong 2.909745 \times 10^{22} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2}$$

It may be noted that,

- 1) Weak interaction point of view [25,26], following our assumptions, Fermi's weak coupling constant can be fitted with the following relations.

$$\left. \begin{aligned} G_F &\cong \left(\frac{m_e}{m_p}\right)^2 \hbar c R_0^2 \cong G_w M_{wf}^2 R_w^2 \cong 1.44021 \times 10^{-62} \text{ J.m}^3 \\ \text{where, } \left\{ \begin{aligned} R_0 &\cong \frac{2G_n m_p}{c^2} \cong 1.24 \times 10^{-15} \text{ m} \\ R_w &\cong \frac{2G_w M_{wf}}{c^2} \cong 6.75 \times 10^{-19} \text{ m} \end{aligned} \right. \end{aligned} \right\} \quad (1)$$

- 2) In a unified approach, most important point to be noted is that [10],

$$\hbar c \cong G_w M_{wf}^2 \quad (2)$$

Clearly speaking, based on the electroweak interaction, the well believed quantum constant $\hbar c$ seems to have a deep inner meaning. Following this kind of relation, there is a possibility to understand the integral nature of quantum mechanics with a relation of the form,

$$n^2 \hbar \cong \frac{G_w (n M_{wf})^2}{c} \text{ where } n = 1, 2, 3, \dots \text{ It needs further study with reference to EPR argument [27–}$$

30] and [1,31]. String theory [32–36] can be made practical with reference to the three atomic gravitational constants associated with weak, strong and electromagnetic interaction gravitational constants. See Table 1. and Table 2. for sample string tensions and energies without any coupling constants.

Table 1. Charge dependent string tensions and string energies.

S.No	Interaction	String Tension	String energy
1	Weak	$\frac{c^4}{4G_w} \cong 6.94 \times 10^{10} \text{ N}$	$\sqrt{\frac{e^2}{4\pi\epsilon_0} \left(\frac{c^4}{4G_w} \right)} \cong 24.975 \text{ GeV}$
2	Strong	$\frac{c^4}{4G_n} \cong 6.065 \times 10^4 \text{ N}$	$\sqrt{\frac{e_n^2}{4\pi\epsilon_0} \left(\frac{c^4}{4G_n} \right)} \cong 68.79 \text{ MeV}$
3	Electromagnetic	$\frac{c^4}{4G_e} \cong 8.505 \times 10^{-5} \text{ N}$	$\sqrt{\frac{e^2}{4\pi\epsilon_0} \left(\frac{c^4}{4G_e} \right)} \cong 874.3 \text{ eV}$

Table 2. Quantum string tensions and string energies.

S.No	Interaction	String Tension	String energy
1	Weak	$\frac{c^4}{4G_w} \cong 6.94 \times 10^{10} \text{ N}$	$\sqrt{\hbar c \left(\frac{c^4}{4G_w} \right)} \cong 292.36 \text{ GeV}$
2	Strong	$\frac{c^4}{4G_n} \cong 6.065 \times 10^4 \text{ N}$	$\sqrt{\hbar c \left(\frac{c^4}{4G_n} \right)} \cong 273.3 \text{ MeV}$
3	Electromagnetic	$\frac{c^4}{4G_e} \cong 8.505 \times 10^{-5} \text{ N}$	$\sqrt{\hbar c \left(\frac{c^4}{4G_e} \right)} \cong 10234.77 \text{ eV}$

2.1. Understanding the Electroweak Coefficient and Nuclear Stability

Our basic idea is that, all the nucleons are not participating in the nuclear binding energy scheme and non-participating nucleons can be called as ‘Free nucleons’. These free nucleons revolve round the nuclear core. Each free nucleon reduces the nuclear binding energy by 10.1 MeV. Protons and neutrons jointly play a crucial role in fixing the number of free nucleons. Electroweak interaction is having a key role in understanding free nucleons and nuclear stability against beta decay. In this context, we noticed that,

$$\frac{m_p}{M_{wf}} \cong \left(\frac{\sqrt{(m_\pi c^2)^0 (m_\pi c^2)^\pm}}{\sqrt{(m_z c^2)^0 (m_w c^2)^\pm}} \right) \cong \left(\frac{\sqrt{134.98 \times 139.57} \text{ MeV}}{\sqrt{91187.6 \times 80379.0} \text{ MeV}} \right) \cong 0.0016032$$

(3)

Here ratio of rest mass of proton to the assumed electroweak fermion is equal to the ratio of mean mass of pions to the mean mass of electroweak bosons. Based on this unique and concrete observation, we are very confident to say that, strong and weak interactions play a vital role exploring the secrets of nuclear structure. Based on this electroweak coefficient 0.0016, stability corresponding to nuclear beta decay can be understood with the following relation.

$$\left. \begin{aligned} A_s &\cong 2Z + 0.0016(2Z)^2 \cong 2Z + 0.0064Z^2 \\ \rightarrow \frac{A_s - 2Z}{4Z^2} &\cong 0.0016 \end{aligned} \right\}$$

(4A)

One can find a similar relation in the literature [16]. This relation can be well tested for Z=21 to 92. For example,

$$\frac{45 - (2 \times 21)}{4(21)^2} \cong 0.00170; \quad \frac{63 - (2 \times 29)}{4(29)^2} \cong 0.00149; \quad \frac{89 - (2 \times 39)}{4(39)^2} \cong 0.00181;$$

$$\frac{109 - (2 \times 47)}{4(47)^2} \cong 0.0017; \quad \frac{169 - (2 \times 69)}{4(69)^2} \cong 0.00163; \quad \frac{238 - (2 \times 92)}{4(92)^2} \cong 0.001595;$$

This is one best practical and quantitative application of our proposed electroweak fermion and bosons. Following this relation and based on various semi empirical mass formulae [13–21], by knowing any stable mass number, its corresponding proton number can be estimated with,

$$Z \cong \frac{A_s}{1 + \sqrt{1 + 0.0064A_s}} \cong \frac{A_s}{2 + 0.0153A_s^{2/3}} \quad (4B)$$

$$\text{where } \frac{a_c}{2a_{asy}} \cong \frac{0.71 \text{ MeV}}{2 \times 23.21 \text{ MeV}} \cong \frac{0.6615 \text{ MeV}}{2 \times 21.6091 \text{ MeV}} \cong 0.0153$$

Considering this relation, we are working on understanding stable super heavy elements.

2.2. Revised Strong and Electroweak Mass Formula

Baaed on the liquid drop model, we present our revised strong and electroweak mass formula

[1]. It needs a review for accuracy and we are working in this new direction. For $Z=6$ to 125,

$$BE \cong (A - A_{free} - A_{radial} - A_{asym})(B_0 \cong 10.1 \text{ MeV})$$

$$\cong \left\{ A - \left\{ \frac{1}{2} + \left[\Upsilon_Z \left[\left(Z + Z^{2/3} + Z^{1/3} \right)^2 + \left(N + N^{2/3} + N^{1/3} \right)^2 + \left(\frac{Z}{N} (Z^2) \right) \right] \right] \right\} \right. \\ \left. - A^{1/3} - \left(\beta \times \frac{(A_s - A)^2}{A_s} \right) \right\} 10.1 \text{ MeV}$$

(5)

where, $A \times 10.1 \text{ MeV}$ represents the volume term

$A_{free} \times 10.1 \text{ MeV}$ represents the electroweak term

$A_{radial} \times 10.1 \text{ MeV}$ represents the radial term

$A_{asym} \times 10.1 \text{ MeV}$ represents the asymmetry term

$$\beta = 1 - \left(\frac{N - Z}{A} \right)^2$$

$$\Upsilon_Z \cong \Upsilon_0 [1 + (\Upsilon_0 Z)] \cong 0.000935 [1 + 0.000935 Z]$$

$$\text{where } \Upsilon_0 \cong \sqrt{\frac{e}{e_n}} (0.001605) \cong 0.000935$$

$$A_s \cong 2Z + 0.0064Z^2$$

\cong Light house like (mean) stable mass number of Z .

$$B_0 \cong - \left(\frac{1}{\sqrt{\alpha_s}} \right) \frac{e^2}{8\pi\epsilon_0 (\hbar/m_p c)} \cong - \frac{e_n^2}{8\pi\epsilon_0 (G_n m_p / c^2)} \cong -10.1 \text{ MeV}$$

$$\text{where } \begin{cases} \alpha_s = \text{Strong coupling constant} \cong 0.115 \text{ to } 0.12 \\ \hbar/m_p c = \text{Reduced Compton wavelength of proton} \\ G_n m_p / c^2 \cong 0.62 \times 10^{-15} \text{ m} \end{cases}$$

For evaluating the effectiveness of relation (5), we consider the following advanced relation as a reference [19].

$$BE \cong \left\{ \left[1 + \left(\frac{4k_v}{A^2} \right) |T_z| (|T_z| + 1) \right] a_v * A \right\} + \left\{ \left[1 + \left(\frac{4k_s}{A^2} \right) |T_z| (|T_z| + 1) \right] a_s * A^{\frac{2}{3}} \right\} \\ + \left\{ a_c * \left(\frac{Z^2}{A^{1/3}} \right) \right\} + \left\{ f_p * \frac{Z^2}{A} \right\} + E_p \quad (6)$$

where, $T_z \cong 3\text{rd component of isospin} = \frac{1}{2}(Z - N)$

$$\left\{ \begin{array}{l} a_v = -15.4963 \text{ MeV}, a_s = 17.7937 \text{ MeV} \\ k_v = -1.8232, k_s = -2.2593 \\ a_c = 0.7093 \text{ MeV}, f_p = -1.2739 \text{ MeV} \\ d_n = 4.6919 \text{ MeV}, d_p = 4.7230 \text{ MeV} \\ d_{np} = -6.4920 \text{ MeV} \end{array} \right\} \text{ and } \left\{ \begin{array}{l} \text{for } (Z, N) \text{ Odd, } E_p \cong \frac{d_n}{N^{1/3}} + \frac{d_p}{Z^{1/3}} + \frac{d_{np}}{A^{2/3}} \\ \text{for } (\text{Odd } Z, \text{ Even } N), E_p \cong \frac{d_p}{Z^{1/3}} \\ \text{for } (\text{Even } Z, \text{ Odd } N), E_p \cong \frac{d_n}{N^{1/3}} \\ \text{for } (\text{Even } Z, \text{ Even } N), E_p \cong 0 \end{array} \right\}$$

3. Revised Semi Empirical Mass Formula Having 6 Terms

With reference to relation (6) and considering surface, Coulomb and asymmetry energy coefficients as variables, we present our revised 6 term semi empirical mass formula.

Volume energy coefficient, $a_v \cong 16.0 \text{ MeV}$

Surface energy coefficient, $a_s \cong \left[1 - \left(\frac{N-Z}{A} \right)^2 \right] 19.4 \text{ MeV}$

Coulombic energy coefficient, $a_c \cong \left\{ 1 - \left(\frac{N-Z}{A} \right)^2 \right\}^{-x} \times 0.71 \text{ MeV}$ where $x \cong 0.75 - \frac{Z}{2A}$

Asymmetry energy coefficient, $a_{asy} \cong \left[1 - \frac{1}{A} \right] \times 24.5 \text{ MeV}$

Pairing energy coefficient, $a_p \cong 10.0 \text{ MeV}$

Congruent energy coefficient, $a_{cg} \cong 10.0 \text{ MeV}$

$$BE \cong \left[16.0 \times A \right] - \left[\beta \times 19.4 \times A^{2/3} \right] - \left[\frac{0.71 \times Z^2}{\beta^x A^{1/3}} \right] - \left\{ \left[\left(1 - \frac{1}{A} \right) 24.5 \right] \frac{(A-2Z)^2}{A} \right\} \pm \left[\frac{10}{\sqrt{A}} \right] + \left[10 \times \exp \left(-4.2 \frac{|N-Z|}{A} \right) \right] \quad (7)$$

It seems to work from $Z=1$ to 137 without any difficulty and needs fine tuning for very heavy isotopes of very light proton numbers. See the following Table 3 for the binding energy curves prepared for magic numbers [37], $Z=2, 8, 20, 28, 50, 82, 100$ and 114. Red curve is our fit and green curve is the reference binding energy curve [19]. See the attached nuclear binding energy data table (supplementary material) starting from $Z=1$ to 137 where lower and upper mass numbers are $(2Z)-1$ and $(3.5Z)+3$ respectively. For our crude approximation, with reference to relation (6), for a wide range of protons and neutrons, relation (7) is having a root mean square deviation of 3.25 MeV.

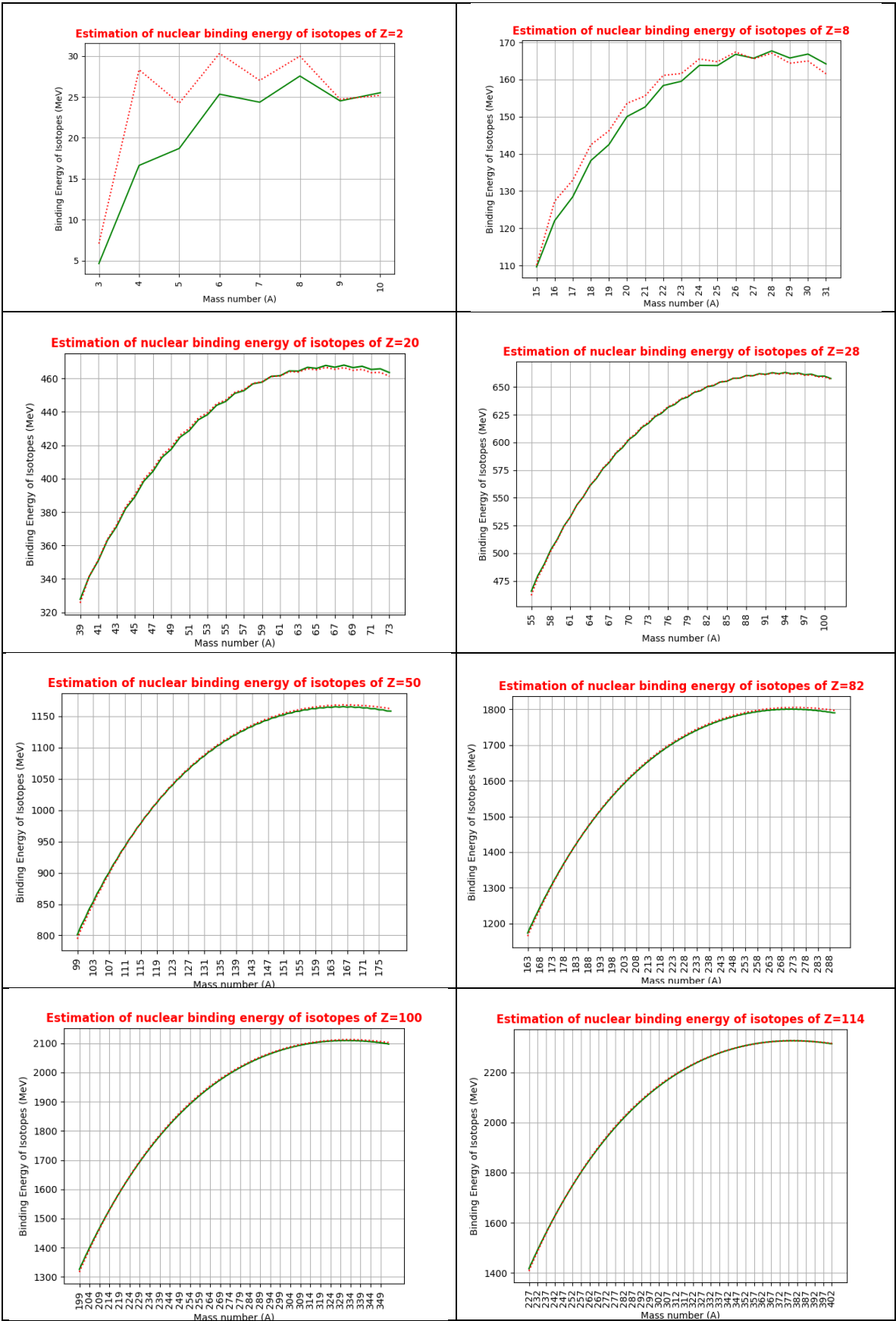
4. Short Discussion on the Energy Coefficients of the Revised Semi Empirical Mass Formula

Let $e_n \equiv$ Nuclear elementary charge $\cong 2.9464 e$.

$R_{pp} \cong \frac{G_n m_p}{c^2} \cong 0.61965 \text{ fm} \cong$ Characteristic physical radius of proton

where, $\begin{cases} G_n \cong \text{Nuclear gravitatio nal constant} \\ m_p \cong \text{Rest mass of proton} \end{cases}$

Table 3. Fitting the binding energy curves for Z=2,8,20,28,50,82,100 and 114 based on relations (6) & (7).



Nuclear potential associated with e_n and R_{pp} can be expressed as,

$$PE \cong -\frac{e_n^2}{4\pi\epsilon_0 R_{pp}} \cong \left(\frac{1}{\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 R_{pp}} \cong -20.174 \text{ MeV} \quad (8)$$

where, $\alpha_s \cong$ Strong coupling constant

With reference to both nucleons and considering a characteristic potential energy as,

$$(PE)_{np} \cong -\frac{2e_n^2}{4\pi\epsilon_0 R_{pp}} \cong \left(\frac{1}{\alpha_s}\right) \frac{2e^2}{4\pi\epsilon_0 R_{pp}} \cong -40.35 \text{ MeV} \quad (9)$$

Based on this energy unit and beta decay, we consider the following relations for formulation of the energy coefficient. It needs further study.

$$\text{Volume energy coefficient, } a_v \cong \left(\frac{m_e}{m_n - m_p}\right) \left(\frac{2e_n^2}{4\pi\epsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right) \left(\frac{1}{2.531}\right) \frac{2e^2}{4\pi\epsilon_0 R_{pp}} \cong 15.94 \text{ MeV}$$

$$2) \text{ Asymmetry energy coefficient, } a_{asy} \cong \left(1 - \frac{1}{2.531}\right) \left(\frac{2e_n^2}{4\pi\epsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right) \left(1 - \frac{1}{2.531}\right) \frac{2e^2}{4\pi\epsilon_0 R_{pp}} \cong 24.4 \text{ MeV}$$

$$3) \text{ Surface energy coefficient, } a_s \cong \frac{2a_v a_{asy}}{a_v + a_{asy}} \cong 19.3 \text{ MeV}$$

$$4) \text{ Pairing energy coefficient, } a_p \cong \frac{e_n^2}{8\pi\epsilon_0 R_{pp}} \cong \left(\frac{1}{\alpha_s}\right) \frac{e^2}{8\pi\epsilon_0 R_{pp}} \cong 10.1 \text{ MeV}$$

$$5) \text{ Congruence energy coefficient, } a_{cg} \cong \frac{e_n^2}{8\pi\epsilon_0 R_{pp}} \cong \left(\frac{1}{\alpha_s}\right) \frac{e^2}{8\pi\epsilon_0 R_{pp}} \cong 10.1 \text{ MeV}$$

$$6) \text{ Coulombic energy coefficient, } a_c \cong 0.7 \text{ MeV}$$

Based on these energy coefficients, to a very good approximation, starting from $Z=1$ to 137,

$$BE \cong a_v A - \beta a_s A^{2/3} - a_c \frac{Z^2}{\beta^x A^{1/3}} - a_{asy} \frac{(A - 2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} + a_{cg} \exp\left(-4.2 \frac{|N - Z|}{A}\right)$$

$$\text{where } \beta \cong 1 - \left(\frac{N - Z}{A}\right)^2 \text{ and } x \cong \left[0.75 - \left(\frac{Z}{2A}\right)\right] \cong (0.5 \text{ to } 0.6) \rightarrow \text{Needs further study.}$$

(10)

Points of interest are:

1) Characteristic nuclear radii linked with coulomb energy and nuclear binding energy [4,17] can be expressed as,

$$R_{(Z,N)} \cong \left[1 - \left(\frac{N - Z}{A}\right)^2\right]^x A^{1/3} \times R_0$$

$$\text{where } \begin{cases} R_0 \cong (1.24 \pm 0.01) \text{ fermi} \\ x \cong 0.75 - \frac{Z}{2A} \approx (0.5 \text{ to } 0.6) \end{cases} \quad (11)$$

Considering $(A^{1/3}R_0)$, increasing neutron number, reduces the Coulomb repulsion between protons and thus helps in minimizing the radius by a factor

$$\left[1 - \left(\frac{N-Z}{A}\right)^2\right]^x \text{ where } x \cong 0.75 - \frac{Z}{2A}.$$

2) Considering the Fermi gas model of the nucleus [39,40],

a) Characteristic Fermi energy can be expressed as

$$E_F \cong \left(\frac{2e_n^2}{4\pi\epsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right) \frac{2e^2}{4\pi\epsilon_0 R_{pp}} \cong \left(\frac{4}{\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 R_0} \quad (12)$$

b) Volume and asymmetry energy coefficients can be expressed as,

$$a_v \cong \left(1 - \frac{3}{5}\right) E_F \cong \left(1 - \frac{3}{5}\right) \left(\frac{2e_n^2}{4\pi\epsilon_0 R_{pp}}\right) \cong \left(\frac{4}{5\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 R_{pp}} \quad (13)$$

$$\cong \left(\frac{8}{5\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 R_0} \cong \frac{(8\sqrt{3})e^2}{4\pi\epsilon_0 R_0} \text{ where } \left(\frac{1}{5\alpha_s}\right) \approx \sqrt{3}$$

$$a_{asy} \cong \frac{3}{5} \left(\frac{2e_n^2}{4\pi\epsilon_0 R_{pp}}\right) \cong \frac{3}{5} E_F \cong \left(\frac{6}{5\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 R_{pp}} \cong \left(\frac{12}{5\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 R_0} \cong \frac{12\sqrt{3}e^2}{4\pi\epsilon_0 R_0} \quad (14)$$

$$(a_{asy}, a_v) \cong \left(\frac{10 \pm 2}{5\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 R_0} \cong \frac{(10 \pm 2)\sqrt{3}e^2}{4\pi\epsilon_0 R_0}$$

c) Surface energy coefficient can be expressed as,

$$a_s \cong \frac{2a_v a_{asy}}{(a_v + a_{asy})} \cong \left(\frac{9.6}{5\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 R_0} \cong \frac{9.6\sqrt{3}e^2}{4\pi\epsilon_0 R_0} \quad (15)$$

d) Congruent and Pairing energy coefficients can be expressed as

$$a_{cg} \cong a_p \cong \frac{E_F}{4} \cong \left(\frac{e_n^2}{8\pi\epsilon_0 R_{pp}}\right) \cong \left(\frac{1}{\alpha_s}\right) \frac{e^2}{8\pi\epsilon_0 R_{pp}} \cong \left(\frac{2}{\alpha_s}\right) \frac{e^2}{8\pi\epsilon_0 R_0} \cong \left(\frac{1}{\alpha_s}\right) \frac{e^2}{4\pi\epsilon_0 R_0} \quad (16)$$

See Table 4 for a rough estimation of our 6 term binding energy coefficients.

Table 4. Approximate energy coefficients of the revised 6 term semi empirical mass formula.

a_v MeV	a_s MeV	a_c MeV	a_{asy} MeV	a_p MeV	a_{cg} MeV
15.94	19.3	0.70	24.4	10.1	10.1
16.0	19.4	0.71	24.5	10.0	10.0

Proceeding further, binding energy point of view, considering a radius value close to the the root mean square radius of proton or neutron [41,42], for light, medium and heavy atomic nuclides, above nuclear radii can be approximated with the following relation. It needs further study.

$$R_{(Z,N)} \cong \left[1 - \left(\frac{N-Z}{A}\right)^2\right]^x A^{1/3} \times 1.25 \text{ fm} \approx \left[\sqrt[3]{Z} + \sqrt[9]{Z^2 N}\right] \times 0.79 \text{ fm} \quad (17)$$

Clearly speaking, Coulomb energy term can be re-written as,

$$E_{cou} \cong -\frac{3Z^2e^2}{20\pi\epsilon_0 R_{(Z,N)}} \quad (18)$$

Binding energy expression can be re-written as,

$$BE \cong 16A - 19.4\beta A^{2/3} - \frac{1.09365 \times Z^2}{(\sqrt[3]{Z} + \sqrt[9]{Z^2N})} - \left[\left(1 - \frac{1}{A}\right) \times 24.5 \frac{(A-2Z)^2}{A} \right] \pm \frac{10}{\sqrt{A}} + 10 \times \exp\left(-4.2 \frac{|N-Z|}{A}\right) \quad (19A)$$

where $\beta \cong 1 - \left(\frac{N-Z}{A}\right)^2$ and $\frac{3Z^2e^2}{20\pi\epsilon_0 R_{(Z,N)}} \cong \frac{3Z^2e^2}{20\pi\epsilon_0} \left[(\sqrt[3]{Z} + \sqrt[9]{Z^2N}) \times 0.79 \text{ fm} \right]^{-1}$

$$BE \cong 16A - 19.4\beta A^{2/3} - \frac{1.1 \times Z^2}{(\sqrt[3]{Z} + \sqrt[9]{Z^2N})} - \left[\left(1 - \frac{1}{A}\right) \times 24.5 \frac{(A-2Z)^2}{A} \right] \pm \frac{10}{\sqrt{A}} + 10 \times \exp\left(-4.2 \frac{|N-Z|}{A}\right) \quad (19B)$$

where $\beta \cong 1 - \left(\frac{N-Z}{A}\right)^2$

Following relations (6) and (19), it can be understood that, radii of higher isotopes of higher proton numbers, seem to have a lower radii than the expected radii. See the following Table 5 for the binding energy curves prepared for Z=28,50,82 and 114. Based on the energy coefficients proposed in relation (7), and if $R_0 \cong 1.25 \text{ fm}$, strong coupling constant [38] can be estimated as,

$$\alpha_s \cong \frac{8e^2}{20\pi\epsilon_0 R_0 a_v} \cong 0.1151973 \quad \text{where } a_v \cong 16.0 \text{ MeV}$$

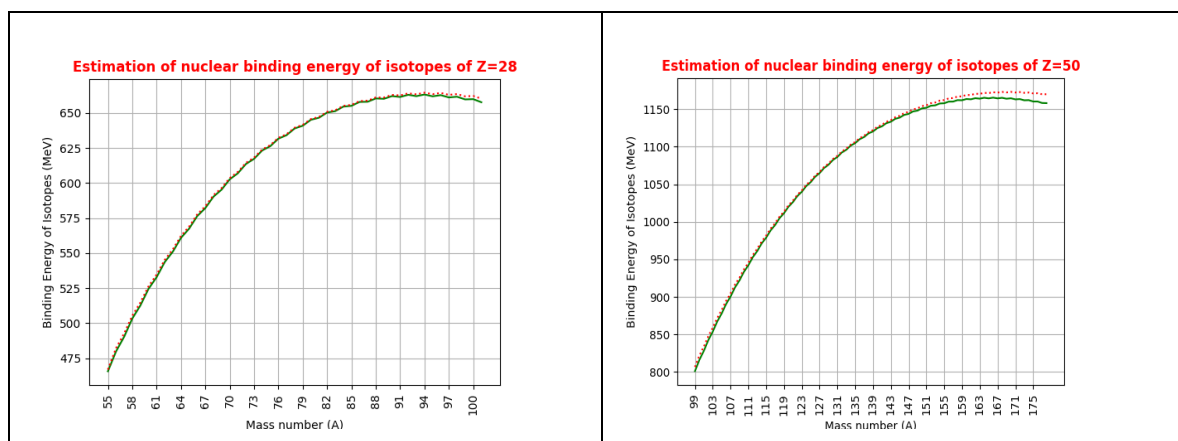
$$\alpha_s \cong \frac{12e^2}{20\pi\epsilon_0 R_0 a_{asy}} \cong 0.1128463 \quad \text{where } a_{asy} \cong 24.5 \text{ MeV}$$

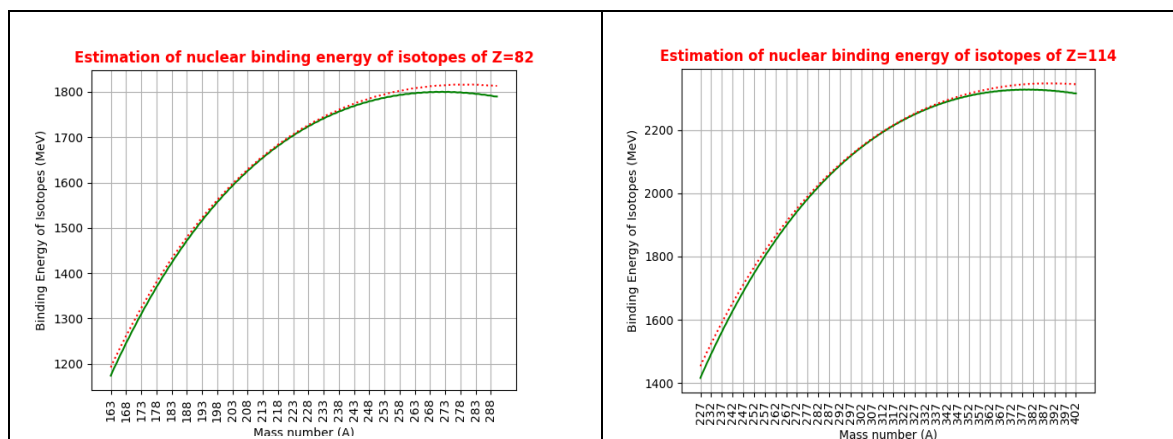
$$\alpha_s \cong \frac{9.6e^2}{20\pi\epsilon_0 R_0 a_s} \cong 0.11401 \quad \text{where } a_s \cong 19.4 \text{ MeV}$$

$$\alpha_s \cong \frac{e^2}{4\pi\epsilon_0 R_0 a_{cg}} \cong \frac{e^2}{4\pi\epsilon_0 R_0 a_p} \cong 0.1151973 \quad \text{where } a_{cg} \cong a_p \cong 10.0 \text{ MeV}$$

$$\alpha_s \cong \frac{1}{5\sqrt{3}} \cong 0.11547$$

Table 5. Fitting the binding energy curves for Z=28,50,82 and 114 based on relations (6) & (19A).





In a simplified and unified approach, approximately, it is possible to infer that,

$$\left. \begin{aligned} (a_v, a_{asy}) &\approx \left(\frac{2 \mp 0.4}{\alpha_s} \right) \left(\frac{e^2}{4\pi\epsilon_0 R_0} \right) \\ \frac{2a_v a_{asy}}{(a_v + a_{asy})} &\geq a_s \leq \sqrt{a_v a_{asy}} \\ a_{cg} \approx a_p &\approx \left(\frac{1}{\alpha_s} \right) \frac{e^2}{4\pi\epsilon_0 R_0} \\ a_c &\approx \left(\frac{3}{5} \right) \frac{e^2}{4\pi\epsilon_0 R_0} \end{aligned} \right\} \quad (20)$$

Interesting point to be noted is that, the expression $\left(\frac{2 \mp 0.40}{\alpha_s} \right)$ represents the ratio of major nuclear binding energy coefficients to the basic nuclear potential.

5. Conclusion

Understanding nuclear binding energy with various physical terms and various coefficients is not a new point. With a single set of energy coefficients, our proposal helps in estimating the binding energy of all atomic nuclides starting from $Z=1$ to 137. Here, we would like to emphasize the point that, 4G model of the strong coupling constant (α_s) plays a vital role in understanding the origin of the binding energy coefficients and further study certainly helps in connecting high and low energy branches of nuclear physics and particle physics. Interesting point to be noted is that, all the binding energy coefficients, can be expressed as, $k \left[(2.95e)^2 / 4\pi\epsilon_0 (1.25 \text{ fm}) \right] \equiv (k/\alpha_s) \left[e^2 / 4\pi\epsilon_0 (1.25 \text{ fm}) \right]$ where $k = (1.0 \text{ to } 2.5)$.

Data availability statement: The data that support the findings of this study are openly available.

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References

1. Seshavatharam U.V.S and Lakshminarayana S. Radius, surface area and volume dependent electroweak term and isospin dependent asymmetry term of the strong and electroweak mass formula. *Int. J. Phys. Appl.* 2025;7(1):122-134.
2. Seshavatharam U.V.S and Lakshminarayana S. Understanding the Origins of Quark Charges, Quantum of Magnetic Flux, Planck's Radiation Constant and Celestial Magnetic Moments with the 4G Model of Nuclear Charge. *Current Physics.* 1, e090524229812, 122-147, 2024.
3. Seshavatharam U.V.S and Lakshminarayana S. Exploring condensed matter physics with refined electroweak term of the strong and electroweak mass formula. *World Scientific News.*193(2) 105-13, 2024.
4. Seshavatharam U. V. S, Gunavardhana Naidu T and Lakshminarayana S. Nuclear evidences for confirming the physical existence of 585 GeV weak fermion and galactic observations of TeV radiation. *International Journal of Advanced Astronomy.* 13(1):1-17, 2025.
5. Seshavatharam U. V. S., Gunavardhana Naidu T and Lakshminarayana S. 2022. To confirm the existence of heavy weak fermion of rest energy 585 GeV. *AIP Conf. Proc.* 2451 p 020003
6. Seshavatharam U V S and Lakshminarayana S. 4G model of final unification – A brief report *Journal of Physics: Conference Series* 2197 p 012029, 2022.
7. Seshavatharam U. V. S and Lakshminarayana S., H. K. Cherop and K. M. Khanna, Three Unified Nuclear Binding Energy Formulae. *World Scientific News*, 163, 30-77, 2022.
8. Seshavatharam U.V.S and Lakshminarayana S., On the Combined Role of Strong and Electroweak Interactions in Understanding Nuclear Binding Energy Scheme. *Mapana Journal of Sciences*, 20(1), 1-18, 2021.
9. Seshavatharam U.V.S and Lakshminarayana S., Strong and Weak Interactions in Ghahramany's Integrated Nuclear Binding Energy Formula. *World Scientific News*, 161, 111-129, 2021.
10. Seshavatharam U.V.S and Lakshminarayana S. Is reduced Planck's constant - an outcome of electroweak gravity? *Mapana Journal of Sciences.* 19(1), 1,-13, 2020.
11. Seshavatharam U.V.S and Lakshminarayana S. A very brief review on strong and electroweak mass formula pertaining to 4G model of final unification. *Proceedings of the DAE Symp. on Nucl. Phys.* 67,1173, 2023.
12. Seshavatharam U.V.S and Lakshminarayana S. Computing unified atomic mass unit and Avogadro number with various nuclear binding energy formulae coded in Python. *Int. J. Chem. Stud.* 2025;13(1):24-30.
13. Bethe H. A. Thomas-Fermi Theory of Nuclei. *Phys. Rev.*, 167(4), 879-907, 1968.
14. Myers W. D. and Swiatecki W. J. Nuclear Properties According to the Thomas-Fermi Model. *LBL-36557 Rev. UC-413*, 1995.
15. Myers W. D. and Swiatecki W. J. Table of nuclear masses according to the 1994 Thomas-Fermi model. *United States: N. p.*, 1994. Web.
16. P.R. Chowdhury, C. Samanta, D.N. Basu, Modified Bethe–Weizsacker mass formula with isotonic shift and new driplines. *Mod. Phys. Lett. A* 20, 1605–1618, 2005.
17. G. Royer, On the coefficients of the liquid drop model mass formulae and nuclear radii. *Nuclear Physics A*, 807, 3–4, 105-118, 2008.
18. Djelloul Benzaid, Salaheddine Bentriddi, Abdelkader Kerraci, Naima Amrani. Bethe–Weizsacker semiempirical mass formula coefficients 2019 update based on AME2016. *NUCL. SCI. TECH.* 31:9, 2020.
19. Gao, Z.P., Wang, YJ., Lü, HL. et al., Machine learning the nuclear mass. *NUCL. SCI. TECH.* 32, 109, 2021.
20. Peng Guo, et. al. (DRHBc Mass Table Collaboration), Nuclear mass table in deformed relativistic Hartree-Bogoliubov theory in continuum, II: Even-Z nuclei. *Atomic Data and Nuclear Data Tables* 158 (2024) 101661
21. Cht. Mavrodiev S, Deliyergiyev M.A. Modification of the nuclear landscape in the inverse problem framework using the generalized Bethe-Weizsäcker mass formula. *Int. J. Mod. Phys. E* 27: 1850015, 2018.
22. Ghahramany N, Gharaati, S., & Ghanaatian, M. New approach to nuclear binding energy in integrated nuclear model. *Journal of Theoretical and Applied Physics*, 6(1), 3, 2012.
23. Ghahramany N, Sh Gharaati, Ghanaatian M, Hora H. New scheme of nuclide and nuclear binding energy from quark-like model. *Iranian Journal of Science & Technology A3*, 201-208, 2011.

24. N. Ghahramany M. Ghanaatian and M. Hooshmand. Quark-Gluon Plasma Model and Origin of Magic Numbers. Iranian Physical Journal, 1-2, 35-38, 2007.
25. C. Grojean. Higgs Physics. Proceedings of the 2015 CERN–Latin-American School of High-Energy Physics, 143-157, 2016, CERN-2016-005 (CERN, Geneva, 2016)
26. Ahmed Abokhalil. The Higgs Mechanism and Higgs Boson: Unveiling the Symmetry of the Universe. arXiv:2306.01019v2 [hep-ph]
27. A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete?. Phys. Rev. 47, 777, 1935.
28. N. Bohr. Can quantum mechanical description of physical reality be considered complete? Phys. Rev. 480, 696, 1935,
29. J.S. Bell. On the Einstein Podolsky Rosen paradox. Physics 1, 195, 1964.
30. J. Bell, On the problem of hidden variables in quantum mechanics. Reviews of Modern Physics. 38, 3, 447, 1966.
31. Seshavatharam U.V.S, Lakshminarayana S. EPR argument and mystery of the reduced Planck's constant. Algebras, Groups, and Geometries. 36(4), 801-822, 2020.
32. Sunil Mukhi. String theory: a perspective over the last 25 years. Class. Quantum Grav. 28 153001, 2011.
33. Sachdev, Subir. Strange and stringy. Scientific American. 308 (44): 44–51, 2013.
34. Blumenhagen R., Lüst D., Theisen S. Basic Concepts of String Theory. Theoretical and Mathematical Physics Springer Heidelberg, Germany, 2013.
35. Arnab Priya Saha and Aninda Sinha Phys. Field Theory Expansions of String Theory Amplitudes. Rev. Lett. 132, 221601, 2024.
36. Seshavatharam U.V.S. and Lakshminarayana S. On the compactification and reformation of string theory with three large atomic gravitational constants. International Journal of Physical Research, 9(1), 42-48, 2021.
37. M. Ismail¹, W. M. Seif, and A. Abdurrahman. Relative stability and magic numbers of nuclei deduced from behavior of cluster emission half-lives. Phys. Rev. C 94, 024316.
38. D d'Enterria et al. The strong coupling constant: state of the art and the decade ahead. J. Phys. G: Nucl. Part. Phys. 51 090501, 2024.
39. Maruhn, J.A., Reinhard, P.G., Suraud, E. The Fermi-Gas Model. In: Simple Models of Many-Fermion Systems. Springer, Berlin, Heidelberg. 2010.
40. Chapter 7. Bulk Nuclear Properties and Nuclear Matter. <https://www.physics.umd.edu/courses/Phys741/xji/chapter7.pdf>
41. H. Gao M. Vanderhaeghen, The proton charge radius. Rev. Mod. Phys. 94, 015002, 2022.
42. G.A. Miller. Defining the proton radius: a unified treatment. Phys. Rev. C 99, 035202, 2019.

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