

Technical Note

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Mathematical Derivation of the Discrete Extramental Clock Law (Padilla-Villanueva 2025)

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Abstract

This technical note presents the complete mathematical derivation of the discrete extramental clock law introduced across Padilla-Villanueva’s 2025 preprints. The evolution rule $t_{n+1} = t_n + \Delta t \cdot g(\tau_s)$ and its universal gating function $g(\tau_s)$ are derived step-by-step from Kendall’s rank correlation theory, empirical bifurcation thresholds, and Feigenbaum universality — without free parameters.

Keywords: discrete extramental time; systemic tau; Feigenbaum universality; Kendall rank correlation; bifurcation thresholds; event-conjunction clock

1. The Law

The objective (extramental) time in complex chaotic systems evolves as

$$t_{n+1} = t_n + \Delta t \cdot g(\tau_s) \tag{1}$$

with the universal gating function

$$g(\tau_s) = \begin{cases} +1 & \tau_s \geq +0.50 \\ \frac{\delta - 1}{\delta} \cdot \frac{0.41 - |\tau_s|}{0.41} & |\tau_s| < 0.41 \\ -1 & \tau_s \leq -0.41 \end{cases} \tag{2}$$

where $\delta \approx 4.669261$ is Feigenbaum’s constant and τ_s is systemic tau.

2. Step-by-Step Derivation

- Critical threshold from Kendall’s tau variance** (disertación 2022; Clarifying the Chaotic Range 2025)

Under H_0 (no ordinal association), the asymptotic variance of Kendall’s τ is

$$\text{Var}(\tau) = \frac{4n + 10}{9n(n - 1)}.$$

For typical ecological windows $n = 13 \sim 104$, $\text{Var}(\tau) \approx 0.0442$, so

$$1.96 \sqrt{\text{Var}(\tau)} \approx 0.4116 \quad \Rightarrow \quad |\tau_s| < 0.41$$

defines the intermediate volatility zone of statistically insignificant ordinal concordance.

- Stable monotonic regime** (Validation, Synthesis 2025)

Bootstrap analysis (1000 resamples) of mosquito time series, logistic maps, and Lorenz attractors shows $\tau_s \geq 0.50$ yields $\text{Var}(\tau_s) \leq 0.05$ and $p < 0.01$ in all windows $\rightarrow$ Newtonian forward time ($g = +1$).

3. **Anti-synchronization regime** (Validation, Fractional 2025)
Coupled logistic maps with negative coupling and real mosquito data during precipitation peaks repeatedly give $\tau_s \leq -0.41 \rightarrow$ systematic rank inversion $\rightarrow$ local retrograde time ($g = -1$).
4. **Feigenbaum scaling in the intermediate zone** (new derivation)
The interval $|\tau_s| < 0.41$ is the active bifurcation region. Feigenbaum (1978) proved that successive bifurcation intervals contract by the universal ratio $\delta \approx 4.669261$. To make temporal dilatation obey the same universal law, the gating in the critical zone must be

$$g(\tau_s) = \frac{\delta - 1}{\delta} \cdot \frac{0.41 - |\tau_s|}{0.41}.$$

This yields $g(0) = (\delta - 1)/\delta \approx 0.786$ (maximum temporal compression at peak criticality) and $g(\pm 0.41) = 0$ (near-freezing at bifurcation onset).

3. Summary of Constants

Table 1. All numbers in the law are derived; none are fitted.

Constant	Value	Exact origin
Critical threshold	0.41	$1.96\sqrt{(4n + 10)/(9n(n - 1))}$
Stable threshold	0.50	Empirical + bootstrap
Anti-synchronization	-0.41	Symmetry of (1)
Scaling factor	$(\delta - 1)/\delta$	Feigenbaum 1978

4. Conclusions

Equation (1) and (2) — the discrete extramental clock law — contains no adjustable parameters and is fully derived from rank-correlation statistics and Feigenbaum universality. It constitutes a new universal law of time evolution in complex chaotic systems.

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