Article

Not peer-reviewed version

Identification of Decentralised Control Systems

Nikolay Karabutov *

Posted Date: 23 December 2024

doi: 10.20944/preprints202412.1808.v1

Keywords: identification; decentralised system; adaptation; nonlinearity; quadratic condition; Lyapunov vector function; identifiability; exponential dissipation; excitation constancy; S-synchronizability

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

Article

Identification of Decentralised Control Systems

Nikolay Karabutov

MIREA – Russian Technological University, Moscow, 119454 Russia; nik.karabutov@gmail.com

Abstract. The identification problem of decentralised control systems (DS) is considered. Analysis shows that this problem has not been given sufficient attention. The complexity of systems and a priori uncertainty require the development of approaches and methods. DS parametric identifiability (PI) requires a solution. We propose the approach to the PI assessment based on the fulfilment of the constant excitation condition and consider relationships in the subsystems. PI conditions are got and algorithms for parametric and signal adaptive identification are received. We consider DS with non-linearities satisfying the quadratic condition. The exponential dissipativity of the identification system is proved using Lyapunov vector functions. The influence of interrelations considers on properties of parameter estimates. Examples are given. A method is proposed for the construction of adaptive algorithms under functional constraints.

Keywords: identification; decentralised system; adaptation; nonlinearity; quadratic condition; Lyapunov vector function; identifiability; exponential dissipation; excitation constancy; S-synchronizability

---

1. Introduction

Decentralised control systems (DCS) are widely used to solve various tasks. Ensuring the DS stability and quality is the principal goal of control. The system works under incomplete a priori information. Thus, an adaptive robust DCS with a reference model is proposed for interconnected time-delayed systems in [1]. the system asymptotic stability is proved. A similar problem of stabilising the DS output using feedback is considered in [2]. Control laws are based on the application of nonlinear damping, adaptive state observer and Lyapunov functions. Various variants of the adaptive control problem under uncertainty are studied in [3, 4]. In [5], a design method is proposed for adaptive decentralised regulators based on an identifier and a reference model. Recurrent neural networks [6] are used to control large-scale systems under uncertainty. Algorithms are used to control a flat robot with two degrees of freedom.

Robust DS control of a nonlinear multidimensional object is proposed in [7]. The system identification is based on the frequency approach. The model approach [8] recommends for the control unknown large-scale DS. In [9], correlation analysis and the least squares method were used to identify DS. Correlation analysis and the least squares method [9] are the basis for the DS identification. Stochastic procedures for the DS identification are proposed in [10, 11]. The identification of DS with feedback [12] is based on the analysis of transient characteristics. Adaptive control of nonlinear large-scale systems (LSS) with limited perturbations is considered in [13]. The asymptotic tracking issue for LSS based on nonlinear output feedback considers in [14, 15]. Adaptive algorithms guarantee compensation disturbances.

We see that various identification procedures and methods are used in the DS. Parametric uncertainties are compensated by adjusting the parameters of the adaptive control law. Applied methods of retrospective identification do not always consider the current state of the system. The properties of the proposed algorithms, the system identifiability, and the influence of connections in the system are studied. These difficulties are compensated using multistep identification procedures.

 doi:10.20944/preprints202412.1808.v1

We consider the adaptive identification problem of DS with nonlinearities (NDS) for which the quadratic condition is satisfied (Section 2). Section 3 contains a solution to the parametric identifiability (PI) problem for NDS. We study the influence of the information space on PI. The approach to the synthesis of adaptive identification algorithms based on the second Lyapunov method is proposed. We analyse parametric and signalling algorithms. Properties of the identification system are studied. We proof the exponential dissipativity of the adaptive identification system (AS I).

2. Problem Statement

Consider the system comprising m interconnected subsystems

$$S_i: \begin{cases} \dot{X}_i = A_i X_i + B_i u_i + \sum_{j=1, j\neq i}^{m} \bar{A}_{ij} X_j + F_i(X_i), \\ Y_i = C_i X_i, \end{cases} \quad (1)$$

where $X_i \in \mathbb{R}^{n_i}$, $Y_i \in \mathbb{R}^{q_i}$ are vectors of the state and output of S_i-subsystem, $u_i \in \mathbb{R}$ is a control, $i = \overline{1,m}$, $\sum_{i=1}^{m} n_i = n$. Parameters of the matrices $A_i \in \mathbb{R}^{n_i \times n_i}, B_i \in \mathbb{R}^{n_i}, \bar{A}_{ij} \in \mathbb{R}^{n_i \times n_j}$ are unknown, $C_i \in \mathbb{R}^{q_i \times n_i}$. The matrix $\bar{A}_{ij}$ reflects the mutual influence of the S_j subsystem. $F_i(X_i) \in \mathbb{R}^{n_i}$ consider the nonlinear state of the S_i-subsystem, and A_i is Hurwitz matrix (stable).

Assumption 1. $F_i(X_i)$ belongs to the class

$$\mathscr{N}_F(\pi_1, \pi_2) = \{F(X) \in \mathbb{R}^n : \pi_1 X \le F(X) \le \pi_2 X, F(0) = 0\} \quad (2)$$

and satisfies the quadratic condition

$$(\pi_2 X - F(X))^T (F(X) - \pi_1 X) \ge 0, \quad (3)$$

where $\pi_1 > 0, \pi_2 > 0$ are set numbers.

Information set of measurements for subsystems

$$\mathbb{I}_{o,i} = \{X_i(t), u_i(t), X_j(t), t \in J = [t_0, t_k]\}.$$

Mathematical model for (1)

$$\dot{\hat{X}}_i = K_i(\hat{X}_i - X_i) + \hat{A}_i X_i + \hat{B}_i u_i + \sum_{j=1, j\neq i}^{m} \hat{\bar{A}}_{ij} X_j + \hat{F}_i(X_i), \quad (4)$$

where $K_i \in \mathscr{H}$ is Hurwitz matrix with known parameters (reference model); $\hat{A}_i$, $\hat{B}_i$, $\hat{\bar{A}}_{ij}$ are tuning matrices of corresponding dimensions, $\hat{F}_i$ is a priori given nonlinear vector function.

Problem: find such algorithms for estimating model (4) parameters based on the analysis of the set $\mathbb{I}_{o,i}$ and the fulfilment assumption 1, so

$$\lim_{t\to\infty} \|\hat{X}_i(t) - X_i(t)\| \le \delta_i,$$

where $\delta_i \ge 0$.

3. On identifiability S_i-Subsystem

Identifiability is the basis for estimating S_i-subsystem parameters. It knows that fulfilment the constant excitation (CE) condition for $\mathbb{I}_{o,i}$ guarantees identifying DS parameters.

The CE condition

$$\mathscr{E}\mathscr{C}_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}} : \underline{\alpha}_{u_i} \le u_i^2(t) \le \bar{\alpha}_{u_i} \quad \forall t \in [t_0, t_0 + T], \quad (5)$$

doi:10.20944/preprints202412.1808.v1

where $\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}$ are positive numbers, $T > 0$. Next, condition (5) will be written as $u_i(t) \in \mathscr{EC}_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}}$. If $u_i(t)$ does not have the CE property, then we will write $u_i(t) \notin \mathscr{EC}_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}}$ or $u_i(t) \notin \mathscr{EC}$.

Remark 1. Condition (5) requires modification, considering S-synchronisation for NDS [16]. We write property (5) as:

$$\mathscr{EC}^S_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}} : \left(\underline{\alpha}_{u_i} \leq u_i^2(t) \leq \bar{\alpha}_{u_i}\right) \& \left(\Omega_{u_i}(\omega) \subseteq \Omega_S(\omega)\right),$$

where $\Omega_{u_i}(\omega)$ is the set of frequencies for u_i; $\Omega_S(\omega)$ is the set of acceptable frequencies for u_i, guaranteeing S-synchronizability. Next, we will denote the CE property as $\mathscr{EC}_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}}$, assuming, that it guarantees $\mathscr{EC}^S_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}}$.

To obtain the conditions of identifiability, consider the model (4). The error equation:

$$\dot{E}_i = K_i E_i + \Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\neq i}^{M} \Delta \bar{A}_{ij} X_j + \Delta F_i(X_i), \qquad (6)$$

where $E_i = \hat{X}_i - X_i$; $\Delta A_i = \hat{A}_i - A_i, \Delta B_i = \hat{B}_i - B_i, \Delta \bar{A}_{ij} = \hat{\bar{A}}_{ij} - \bar{A}_{ij}, \Delta F_i = \hat{F}_i - F_i$ are parametric residuals.

Lemma 1. If the nonlinearity $F(X) \in \mathbb{R}^n$, $X \in \mathbb{R}^n$ belongs to the class $\mathscr{N}_F(\pi_1, \pi_2)$ and

$$\pi_1 X \leq F(X) \leq \pi_2 X, \qquad (7)$$

then

$$\|F(X)\|^2 \leq \eta \bar{\alpha}_X, \qquad (8)$$

where $\pi_1 > 0, \pi_2 > 0,$ $\eta = \eta(\pi_1, \pi_2) > 0, \bar{\alpha}_X = \bar{\alpha}_X(X) > 0$.

Lemma 1 proof is presented in Appendix A.

Lemma 2. If Lemma 1 conditions are satisfied, then the estimate is valid for $\Delta F(X)$

$$\Delta F^T \Delta F \leq 2\eta \bar{\alpha}_X + \delta_F,$$

where $\eta = 2\bar{\pi} + \pi^2$, $\bar{\pi} = \pi_1 \pi_2$, $\pi = \pi_1 + \pi_2$, $\delta_F > 0$.

Lemma 2 proof is presented in Appendix B.

Consider the system (6) and Lyapunov function (LF) $V_i(E_i) = 0.5 E_i^T R_i E_i$, where $R_i = R_i^T > 0$ is a positive symmetric matrix. Let $\|\Delta A_i\| = \sqrt{\mathrm{Sp}(\Delta A_i^T \Delta A_i)}$, $\|\Delta \bar{A}_{ij}\| = \sqrt{\mathrm{Sp}(\Delta A_{ij}^T \Delta A_{ij})}$ is

Let $\|\Delta A_i\| = \sqrt{\mathrm{Sp}(\Delta A_i^T \Delta A_i)}$, $\|\Delta \bar{A}_{ij}\| = \sqrt{\mathrm{Sp}(\Delta A_{ij}^T \Delta A_{ij})}$ are matrix norms ΔA_i, $\Delta \bar{A}_{ij}$.

Theorem 1. Let 1) $A_i \in \mathscr{H}$; 2) $X_i(t) \in \mathscr{PE}_{\underline{\alpha}_{X_i}, \bar{\alpha}_{X_i}}$, $X_j(t) \in \mathscr{PE}_{\underline{\alpha}_{X_j}, \bar{\alpha}_{X_j}}$ $u_i(t) \in \mathscr{PE}_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}}$; 3) conditions of Lemma 1, 2 are satisfied for $F_i(X_i)$. Then subsystem (1) is identifiable on the set $\mathbb{I}_{o,i}$ if

$$2\left(\bar{\alpha}_{X_i} \|\Delta A_i\|^2 + \bar{\alpha}_{u_i} \|\Delta B_i\|^2 + \sum_{j=1, j\neq i}^{m} \bar{\alpha}_{X_j} \|\Delta \bar{A}_{ij}\|^2 + 2\eta \bar{\alpha}_{X_i} + \delta_{F_i}\right) \leq \bar{\lambda}_i V_i, \qquad (9)$$

where $\bar{\lambda}_i = \lambda_i - k_i$, $\lambda_i > 0$ *is the minimum eigenvalue of the matrix* Q_i, $k_i > 0$, $K_i R_i + K_i^T R_i = -Q_i$, Q_i *is positive symmetric matrix,* $\eta = 2\bar{\pi} + \pi^2$, $\pi = \pi_1 + \pi_2$, $\bar{\pi} = \pi_1 \pi_2$, $\delta_{F_i} \geq 0$.

Theorem 1 proof is presented in Appendix C.

If Theorem 1conditions are fulfilled, then the subsystem S_i is identifiable on the set $\mathbb{I}_{o,i}$ or $\mathscr{PI}_{X_i}$-identifiable.

Consider the identifiability of the S_i-subsystem on the set

$$\mathbb{I}_{o,Y_i} = \left\{Y_i(t), u_i(t), Y_j(t), t \in J = [t_0, t_k]\right\}.$$

Representation of the S_i-subsystem on $\mathbb{I}_{o,Y_i}$

$$\dot{Y}_i = A_{\#,i}Y_i + B_{\#,i}u_i + \sum_{j=1, j\neq i}^{m} \bar{A}_{\#,ij}Y_j + \tilde{C}_i^{\#} F_i\left(\tilde{C}_i Y_i\right), \qquad (10)$$

where $\tilde{C}_i = \left(C_i^T C_i\right)^{\#} C_i^T$, $A_{\#,i} = \tilde{C}_i^{\#} A_i \tilde{C}_i$, $B_{\#,i} = \tilde{C}_i^{\#} B_i$, $\bar{A}_{\#,ij} = \tilde{C}_i^{\#} \bar{A}_{ij} \tilde{C}_j$, # is a sign of a pseudo-inreversal of the matrix.

The model for (10) has a similar structure. Introduce the error $E_{\#,i} = \hat{Y}_i - Y_i$ and LF $V_{\#,i}\left(E_{\#,i}\right) = 0.5E_{\#,i}^T R_i E_{\#,i}$.

Theorem 2. Let 1) $A_{\#,i} \in \mathscr{H}$; 2) $Y_i(t) \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{Y_i}, \bar{\alpha}_{Y_i}}$, $Y_j(t) \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{Y_j}, \bar{\alpha}_{Y_j}}$ $u_i(t) \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}}$; 3) $F_i\left(X_i\right)$ satisfies conditions of Lemmas 1, 2; 4) the S_i subsystem is observable. Then subsystem (1) is identifiable on the set $\mathbb{I}_{o,Y_i}$ if

$$2\left(\bar{\alpha}_{Y_i}\left\|\Delta A_{\#,i}\right\|^2 + \bar{\alpha}_{u_i}\left\|\Delta B_{\#,i}\right\|^2 + \sum_{j=1, j\neq i}^{m} \bar{\alpha}_{Y_j}\left\|\Delta\bar{A}_{\#,ij}\right\|^2 + 2\eta\bar{\alpha}_{Y_i} + \delta_{F_i}\right) \leq \bar{\lambda}_{\#,i} V_{\#,i},$$

where $\Delta A_{\#,i} = \hat{A}_{\#,i} - A_{\#,i}, \Delta B_{\#,i} = \hat{B}_{\#,i} - B_{\#,i}, \Delta\bar{A}_{\#,ij} = \hat{\bar{A}}_{\#,ij} - \bar{A}_{\#,ij}, \Delta F_i = \hat{F}_i - F_i$, $\bar{\lambda}_{\#,i} > 0$.

The proof of Theorem 2 coincides with the proof of Theorem 1

4. Synthesis of Adaptation Algorithms

Consider LF $V_i\left(E_i\right) = 0.5E_i^T R_i E_i$ and

$$\dot{V}_i = -E_i^T Q_i E_i + E_i^T R_i\left(\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\neq i}^{m} \Delta\bar{A}_{ij} X_j + \Delta F_i\left(X_i\right)\right).$$

We require that the functional constraint be satisfied for all $\forall t \geq t_0$

$$\dot{V}_i \leq -\chi\left(\Delta A_i, \Delta\tilde{A}_i, \Delta F_i\right),$$

where

$$\chi\left(\Delta A_i, \Delta\bar{A}_i, \Delta B_i, \Delta F_i\right) = 0.5\left(\varphi_{A_i}(t)\left\|\Delta A_i(t)\right\|^2 + \varphi_{\bar{A}_i}(t)\left\|\Delta\bar{A}_i(t)\right\|^2 + \varphi_{B_i}(t)\left\|\Delta B_i\right\|^2 + \varphi_{F_i}(t)\left\|\Delta F_i(t)\right\|^2\right),$$

$\varphi_{A_i}(t)$, $\varphi_{\bar{A}_i}(t)$, $\varphi_{B_i}(t)$, $\varphi_{F_i}(t)$ are limited non-negative functions. Then:

$$\dot{V}_i = -E_i^T Q_i E_i + \chi\left(\Delta A_i, \Delta\bar{A}_i, \Delta F_i\right) + E_i^T R_i\left(\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\neq i}^{m} \Delta\bar{A}_{ij} X_j + \Delta F_i\left(X_i\right)\right) \qquad (11)$$

From (11), we obtain adaptive algorithms

$$\Delta\dot{A}_i = -\Gamma_{A_i}\left(\varphi_{A_i}(t)\Delta A_i + E_i^T R_i X_i\right),$$

$$\Delta\dot{\bar{A}}_{ij} = -\Gamma_{\bar{A}_{ij}}\left(\varphi_{\bar{A}_{ij}}(t)\Delta\bar{A}_{ij} + E_i^T R_i X_j\right),$$

$$\Delta\dot{B}_i = -\Gamma_{B_i}\left(\varphi_{B_{ij}}(t)\Delta B_i + E_i^T R_i u_i\right), \qquad (12)$$

where Γ_{A_i}, $\Gamma_{\bar{A}_{ij}}$, Γ_{B_i} are diagonal matrices of corresponding dimensions with positive diagonal elements, ensuring the stability of adaptation processes.

4.1. Parametric Algorithm for F_i

Parametric and signal algorithms can be used to evaluating F_i. Consider the parametric approach [17].

Assumption 2. The function $F_i(X_i)$ is given on the set

$$F_i \in \mathbb{F}_{F_i} = \left\{F_i \in \mathscr{N}_F\left(\pi_1, \pi_2\right) : F_i(X_i) = \tilde{F}_i^T(X_i, N_{i,1})N_{i,2},\right.$$
$$\left. N_i = \left[N_{i,1}^T, N_{i,2}^T\right]^T,\ N_i \in \mathbb{N}_a\right\}, \qquad (13)$$

 doi:10.20944/preprints202412.1808.v1

where $\mathbb{N}_{i,a} = \left\{ N_i \in \mathbb{R}^n : \underline{N_i} \le N_i \le \overline{N}_i \right\}$ is a posteriori generated parametric domain for F_i; $\underline{N}, \overline{N}$ are vector boundaries for N, understood as $\underline{n}_i \in \underline{N}_i, \overline{n}_i \in \overline{N}_i$; $N_{i,1}$ is a priori set vector of nonlinearity parameters, $N_{i,2}$ is a priori, an unknown set of parameters, which we consider as a vector to be evaluated. Some elements of $\underline{N}_i, \overline{N}_i$ may be unknown. The $\tilde{F}_i\left(X_i, N_{i,1}\right)$ structure is formed a priori considering the known vector $N_{i,1}$.

As follows from (13), the estimation for function $F_i\left(X_i\right)$ is defined in the form:

$$\hat{F}_i(X_i) = \tilde{F}_i^T\left(X_i, \hat{N}_{i,1}\right)\hat{N}_{i,2}, \qquad (14)$$

where $\hat{N}_{i,1} \in \mathbb{R}^{n_{i,1}}$ is a priori estimation of known parameters, $\hat{N}_{i,2} \in \mathbb{R}^{n_{i,2}}$ is the vector of tuning parameters.

We believe $\mathbb{N}_i = \mathbb{N}_{i,1} \cup \mathbb{N}_{i,2}$. The set $\mathbb{N}_{i,1} \subset \mathbb{R}^{n_{i,1}}$ $\left(N_{i,1} \in \mathbb{N}_{i,1}\right)$ contains elements that are not available for adjusting. We get estimates of elements $N_{i,2} \in \mathbb{N}_{i,2} \subset \mathbb{R}^{n_{i,2}}$ at the identification stage. The matrix $\tilde{F}_i\left(y, \hat{N}_{i,1}\right)$ is formed at the stage of structural synthesis (analysis) of the system. Representation (14) is a consequence of the proposed parametric concept for $F_i\left(X_i\right)$.

Remark 2. The vector $\hat{N}_{i,1}$ can be adjusted iteratively based on the coercion algorithm [17].

As $\Delta F = \tilde{F}_i^T\left(y, \hat{N}_{i,1}\right)\hat{N}_{i,2} - F_i(X_i)$, then we get an adaptive algorithm for $\hat{N}_{i,2}$ from the condition $\dot{V}_i \le 0$

$$\dot{\hat{N}}_{i,2} = -\Gamma_{F_i}\left(\varphi_{F_i}\hat{N}_{i,2} + \tilde{F}_i^T R_i E_i\left(X_i, \hat{N}_{i,1}\right)\right), \qquad (15)$$

where Γ_{F_i} is a diagonal matrix with positive diagonal elements. Designate the system (6), (11), (15) as AS_{AF_i}.

4.2. Signal Algorithm

Consider the model

$$\dot{\hat{X}}_i = K_i\left(\hat{X}_i - X_i\right) + \hat{A}_i X_i + \hat{B}_i u_i + \sum_{j=1, j\ne i}^{m} \hat{\bar{A}}_{ij} X_j + U_i \qquad (16)$$

and the equation for the error

$$\dot{E}_i = K_i E_i + \Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\ne i}^{m} \Delta\bar{A}_{ij} X_j + U_i\left(X_i\right) - F_i\left(X_i\right). \qquad (17)$$

Then

$$\dot{V}_i = -E_i^T Q_i E_i + E_i^T R_i\left(\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\ne i}^{m} \Delta\bar{A}_{ij} X_j + U_i - F_i\right). \qquad (18)$$

Choose the algorithm for U_i in the form:

$$U_i = -D_i R_i E_i, \qquad (19)$$

where $D_i \in \mathbb{R}^{n_i \times n_i}$ is a diagonal matrix with positive diagonal elements. As $F_i\left(X_i\right) \in \mathscr{N}_F\left(\pi_1, \pi_2\right)$, then Lemma 1 is valid for $F_i\left(X_i\right)$.

Apply the approach [18]. Select matrix D_i elements from the condition $\left\|D_i\right\| \ge d_i \ge \eta\bar{\alpha}_{X_i}$. Then:

$$\dot{V}_i = -E_i^T Q_i E_i + E_i^T R_i\left(\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\ne i}^{m} \Delta\bar{A}_{ij} X_j - D_i R_i E_i - F_i\right). \qquad (20)$$

As $E_i^T R_i D_i R_i E_i \ge 2\eta\underline{\alpha}_{X_i}\underline{\lambda}_{R_i} V_i$ then

$$\dot{V}_i \le -\sigma V_i + E_i^T R_i\left(\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\ne i}^{m} \Delta\tilde{A}_{ij} X_j\right), \qquad (21)$$

where $\underline{\lambda}_{R_i}$ is the smallest eigenvalue of the matrix R_i, $\sigma = \lambda_{Q_i} + 2\eta\bar{\alpha}_{X_i}\underline{\lambda}_{R_i}$. We apply the approach outlined in the proof of Theorem 1, and get

$$\dot{V}_i \le -\sigma V_i + 2\left(\bar{\alpha}_{X_i}\left\|\Delta A_i\right\|^2 + \bar{\alpha}_{u_i}\left\|\Delta B_i\right\|^2 + \sum_{j=1, j\neq i}^{m}\bar{\alpha}_{X_j}\left\|\Delta\bar{A}_{ij}\right\|^2\right).$$

If

$$2\left(\bar{\alpha}_{X_i}\left\|\Delta A_i\right\|^2 + \bar{\alpha}_{u_i}\left\|\Delta B_i\right\|^2 + \sum_{j=1, j\neq i}^{m}\bar{\alpha}_{X_j}\left\|\Delta\tilde{A}_{ij}\right\|^2 + \eta\bar{\alpha}_{K_i}\right) \le \sigma V_i,$$

then the system (16) is parametrically identifiable on the set $\{u_i(t), X_i(t), X_j(t)\}$ on the algorithms (12), (19) class, if $\left(u_i(t), X_i(t), X_j(t)\right) \in \mathscr{EC}$.

Designate the system (6), (12), (19) as AS_{AS_i}.

5. Functional Restriction and Synthesis of Adaptive Algorithms

The described synthesis method of adaptive algorithms (AA) is typical for the adaptive identification. Another approach is based on accounting of the limitations that are imposed on ASI. This approach requires some knowledge and does not always provide workable algorithms. We propose the method based on consideration of requirements for ASI. Show the algorithm synthesis method using the example of the ΔA_i matrix.

Consider of LF

$$V_i^{\Delta}\left(E_i, \Delta A_i, \Delta\dot{A}_i\right) = 0.5E_i^T R_i E_i + 0.5\mathrm{Sp}\left(\Delta A_i^T \Delta\dot{A}_i\right).$$

Let

$$\dot{V}_i^{\Delta} \le -\chi_{\Delta} = -\alpha_{\Delta}\mathrm{Sp}\left(\Delta A_i^T \Delta A_i\right), \quad \alpha_{\Delta} \ge 0. \qquad (22)$$

Denote $\eta_{\Delta} = \dot{V}_i^{\Delta} + \chi_{\Delta}$ and obtain:

$$\eta_{\Delta} = -E_i^T Q_i E_i + E_i^T R_i \Delta A_i X_i + \mathrm{Sp}\left(\Delta\dot{A}_i^T \Delta\dot{A}_i\right) + \mathrm{Sp}\left(\Delta A_i^T \Delta\ddot{A}_i\right) + \alpha_{\Delta}\mathrm{Sp}\left(\Delta A_i^T \Delta A_i\right).$$

Adaptive algorithm for ΔA_i

$$\Delta\ddot{A}_i = -\Delta\dot{A}_i - \alpha_{\Delta}\Delta A_i - \Gamma_{A_i} E_i R_i X_i^T. \qquad (23)$$

Let $\Delta A_i = Z_1$. Then:

$$S_{AA}: \begin{cases} \dot{Z}_1 = Z_2, \\ \dot{Z}_2 = -\alpha_{\Delta} Z_1 - Z_2 - \Gamma_{A_i} E_i R_i X_i^T. \end{cases} \qquad (24)$$

So, if the functional restriction $\chi_{\Delta} \ge 0$ is imposed on ASI, then AA is described by the system S_{AA}. The S_{AA}-algorithm use is associated with difficulties of application. Therefore, using the S_{AA} requires their modification.

Let $\chi_{e,\Delta} = \alpha_{e,\Delta}\mathrm{Sp}\left(\Delta A_i^T \mathcal{D}\left(|E|\right)\Delta\dot{A}_i\right)$ and $\eta_{e,\Delta} = \dot{V}_i + \chi_{e,\Delta}$. Then $\mathscr{A}_{\mathcal{M}}$-algorithm is presented as:

$$\Delta\dot{A}_i = -\Gamma_{A_i} E_i R_i X_i^T - \alpha_{e,\Delta}\Gamma_{\dot{A}_i} D\left(|E_i|\right)\Delta\dot{A}_i \qquad (25)$$

or

$$\Delta\dot{A}_i = -M^{-1}\Gamma_{A_i} E_i R_i X_i^T, \qquad (26)$$

where $M = I_{n\times n} + \alpha_{e,\Delta}\Gamma_{\dot{A}_i}\mathcal{D}\left(|E|\right)$, $\mathcal{D}\left(|E|\right)$ – диагональная матрица от вектора $|E|$, $I_{n\times n}$ – единична матрица, $\Gamma_{\dot{A}_i} = \Gamma_{\dot{A}_i}^T > 0$.

We describe $\mathscr{A}_{\mathcal{M}}$-algorithm (25) as:

$$\Delta\dot{A}_i(t) = -\Gamma_{A_i} E_i(t) R_i X_i^T(t) - \omega_e \Gamma_{\dot{A}_i}\mathcal{D}\left(|E|\right)\left(\Delta A_i(t) - \Delta A_i(t-\tau)\right), \qquad (27)$$

where $\omega_{e,\Delta} = \tilde{\alpha}_{e,\Delta}\tau^{-1}$. It is difficult to evaluate the properties of the algorithm (27). If the matrix D2 is unique, then

$$\Delta\dot{A}_i(t) = -\Gamma_{A_i} E_i(t) R_i X_i^T(t) - \omega_e \Gamma_{\dot{A}_i}\left(\Delta A_i(t) - \Delta A_i(t-\tau)\right). \quad (27a)$$

Convergence conditions of estimates for algorithm (27a) at $\omega_{e,\Delta} = 1$ and the matrix Γ_{A_i} is diagonal.

Theorem 3. Let 1) $A_{\#,i} \in \mathscr{H}$; 2) $X_i(t) \in \mathscr{PE}_{\underline{\alpha}_{X_i},\bar{\alpha}_{X_i}}$, $X_j(t) \in \mathscr{PE}_{\underline{\alpha}_{X_j},\bar{\alpha}_{X_j}}$ $u_i(t) \in \mathscr{EC}_{\underline{\alpha}_{u_i},\bar{\alpha}_{u_i}}$; 3) $F_i(X_i)$ satisfies conditions of Lemmas 1, 2; 4) $V_{\Delta,i} = \mathrm{Sp}\left(\Delta A_i^T \Gamma_{\dot{A}_i}^{-1} \Delta A_i(t)\right)$; 5) there exists $\upsilon > 0$ such that

$$\mathrm{Sp}\left(\Delta A_i^T \Gamma_{\dot{A}_i}^{-1} \Gamma_{A_i} E_i R_i X_i^T\right) = \upsilon\left[\left(\mathrm{Sp}\left(\Delta A_i^T \Gamma_{\dot{A}_i}^{-1} \Gamma_{A_i} \Delta A_i\right) + E_i^T R_i^2 E_i \|X_i\|^2\right)\right]$$

is valid at. Then algorithm (27a) estimates are bounded if

$$\|\Delta A_i(t-\tau)\|^2 \le \eta V_{\Delta,i} - \upsilon \frac{2\sigma_e}{3\sigma_\Delta} V_i \quad \forall t > t_0,$$

where $\eta = \underline{\lambda}_{\Gamma_{\dot{A}_i}}\left(1 + \frac{3}{4}\upsilon\sigma_\Delta\right)$, $\sigma_e = \underline{\alpha}_{X_i} 2\underline{\lambda}_{R_i}$ $\sigma_\Delta = \underline{\lambda}_{\Gamma_{\dot{A}_i}} \underline{\lambda}_{\Gamma_{A_i}}$, $\underline{\lambda}_{\Gamma_{\dot{A}_i}}$ *is the minimum eigenvalue of the matrix* $\Gamma_{\dot{A}_i}$

Theorem 3 proof is presented in Appendix D.

Remark 3. Algorithm (27) is a differential equation with an aftereffect. The equation (27) discrete analogues are proposed by various authors for regression models. They are based on the intuition of the researcher.

6. Properties of Adaptive System

6.1. System AS_{AF_i}

Consider systems AS_{AF_i}, AS_{AF_j} и LF $V_i(E_i) = 0.5E_i^T R_i E_i$,

$$V_{\Delta,i} = 0.5 \cdot \mathrm{Sp}\left(\Delta A_i^T \Gamma_i^{-1} \Delta A_i\right) + 0.5\sum_{j=1}^{m} \mathrm{Sp}\left(\Delta\bar{A}_{ij}^T \Gamma_{ij}^{-1} \Delta\bar{A}_{ij}\right) + 0.5\Delta B_i^T \Gamma_i^{-1} \Delta B_i + 0.5\Delta N_{i,2}^T \Gamma_{N_{i,2}}^{-1} \Delta N_{N_{i,2}}, \quad (28)$$

where $\mathrm{Sp}(\cdot)$ is the spur of matrix. We believe that the interference matrix $\bar{A}_{ij}$ ensures the stability of the S_i-subsystem.

Theorem 4. Let (i) Lyapunov functions $V_i(t)$, $V_{\Delta,i}(t)$, admit an infinitesimal upper limit; (ii) $A_i \in \mathscr{H}$; (iii) $\bar{A}_{ij}$ ensures the stability of the subsystem S_i; (iv) $X_i(t) \in \mathscr{EC}_{\underline{\alpha}_{X_i},\bar{\alpha}_{X_i}}$, $X_j(t) \in \mathscr{EC}_{\underline{\alpha}_{X_j},\bar{\alpha}_{X_j}}$ $u_i(t) \in \mathscr{EC}_{\underline{\alpha}_{u_i},\bar{\alpha}_{u_i}}$; (v) $F_i \in \mathbb{F}_{F_i}$; (vi) the system of inequalities

$$\begin{bmatrix}\dot{V}_i \\ \dot{V}_{\Delta,i}\end{bmatrix} \le \underbrace{\begin{bmatrix} -\mu_i & \frac{2}{\mu_i}\kappa_i \\ \vartheta_{\chi\alpha,i}\rho_i & -\beta_{\lambda\chi,i}\end{bmatrix}}_{A_{W_i}} \begin{bmatrix} V \\ V_{\Delta,i}\end{bmatrix} + \underbrace{\begin{bmatrix}\frac{2}{\mu_i}\bar{\upsilon}_{\tilde{F}_i} \\ 0.5\delta_{N,i}\end{bmatrix}}_{L_i} \quad (29)$$

is valid for the Lyapunov vector function $W_i = \left[V_i, V_{\Delta,i}\right]^T$, where $\mu_i, \kappa_i, \vartheta_{\chi\alpha,i}\rho_i, \beta_{\lambda\chi,i}, \bar{\upsilon}_{\tilde{F}_i}, \delta_{N,i}$ are positive numbers depending on the subsystem S_i parameters and set $\mathbb{I}_{o,i}$ properties; (vi) the upper solution for W_i satisfies the system of equation $\dot{S}_{W_i} = A_{W_i} S_{W_i} + L_i$ if

$$w_\rho(t) \le s_\rho(t) \quad \forall(t \ge t_0) \,\&\, \left(w_\rho(t_0) \le s_\rho(t_0)\right),$$

$w_\rho \in W_i$, $s_\rho \in S_{W_i}$, $\rho = e, i; \Delta, i$. *Then the* $AS_{F,i}$*-system is exponentially dissipative with the estimate:*

$$W_i(t) \le e^{A_{W_i}(t-t_0)} S_{W_i}(t_0) + \int_{t_0}^{t} e^{A_{W_i}(t-\tau)} L_i \, d\tau, \quad (30)$$

if

$$\mu_i^2 \beta_{\lambda\chi,i} \ge 2\kappa_i \vartheta_{\chi\alpha,i}\rho_i. \quad (31)$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

As follows from (30), the $AS_{F,i}$ limiting properties are determined by vector L_i elements. If the vector $\hat{N}_{i,1}$ structure and parameters are known, then the $AS_{F,i}$-subsystem is exponentially stable if $\tilde{F}_i \in \mathscr{EC}$.

Theorem 4 proof is presented in Appendix E.

Consider the system $AS_{F,i,j}$ with subsystems $AS_{F,i}$ and $AS_{F,j}$. We have the system of inequalities for $AS_{F,i,j}$

$$\begin{bmatrix} \dot{W}_i \\ \dot{W}_j \end{bmatrix} \leq \begin{bmatrix} A_{W_i} & 0 \\ 0 & A_{W_j} \end{bmatrix} \begin{bmatrix} W_i \\ W_j \end{bmatrix} + \begin{bmatrix} L_i \\ L_j \end{bmatrix}, \qquad (32)$$

where A_{W_i} and L_i have the form (29). Exponential dissipative conditions

$\mu_i^2 \beta_{\lambda\chi,i} \geq 2\kappa_i \vartheta_{\chi\alpha,i} \rho_i$, $\mu_i^2 \beta_{\lambda\chi,j} \geq 2\kappa_j \vartheta_{\chi\alpha,j} \rho_j$.

6.2. System AS_{AS_i}

Consider LF $V_i(t)$ and

$$V_{\Delta_S,i} = 0.5 \cdot \mathrm{Sp}\left(\Delta A_i^T \Gamma_i^{-1} \Delta A_i\right) + 0.5 \sum_{j=1}^{m} \mathrm{Sp}\left(\Delta \bar{A}_{ij}^T \Gamma_{ij}^{-1} \Delta \bar{A}_{ij}\right) + 0.5 \Delta B_i^T \Gamma_i^{-1} \Delta B_i +$$

$$+0.5 \int_{t_0}^{t} \Delta_{U_i} F_i^T(\tau) \Delta_{U_i} F_i(\tau) d\tau,$$

where $\Delta_{U_i} F_i = U_i - F_i$.

Theorem 5. Let (i) Lyapunov functions $V_i(t)$ and $V_{\Delta_S,i}(t)$ to have an infinitesimal upper limit; (ii) $A_i \in \mathscr{H}$; (iii) $\bar{A}_{ij}$ ensures the stability of the subsystem S_i; (iv) $F_i(X_i) \in \mathscr{N}_F(\pi_1, \pi_2)$; (v) $u_i(t) \in \mathscr{EC}_{\underline{\alpha}_{u_i}, \bar{\alpha}_{u_i}}$, $X_i(t) \in \mathscr{EC}_{\underline{\alpha}_{X_i}, \bar{\alpha}_{X_i}}$, $X_j(t) \in \mathscr{EC}_{\underline{\alpha}_{X_j}, \bar{\alpha}_{X_j}}$; (v) exist $v_i > 0$ such that the condition $-F_i^T \Delta_{U_i} F_i = -v_i\left(\|F_i\|^2 + \|\Delta_{U_i} F_i\|^2\right)$ satisfy at $t >> t_0$ in some area of the origin; (vi) the system of inequalities

$$\underbrace{\begin{bmatrix} \dot{V}_i \\ \dot{V}_{\Delta_S,i} \end{bmatrix}}_{\dot{W}_{S_i}} \leq \underbrace{\begin{bmatrix} -\mu_i & \frac{2}{\mu_i}\kappa_{\Delta_S,i} \\ \vartheta_{\Delta_S,i}\rho_i & -\beta_{\Delta_S,i} \end{bmatrix}}_{A_{WS_i}} \underbrace{\begin{bmatrix} V_i \\ V_{\Delta_S,i} \end{bmatrix}}_{W_{S_i}} + \underbrace{\begin{bmatrix} 0 \\ \sqrt{0.125 v_i \eta_i \bar{\alpha}_{X_i}} \end{bmatrix}}_{L_{S,i}} \qquad (33)$$

is valid for the Lyapunov vector function $W_{S,i} = \left[V_i, V_{\Delta_S,i}\right]^T$, where $\mu_i, \kappa_{\Delta_S,i}, \vartheta_{\Delta,i}, \rho_i, \beta_{\Delta,i}, v_i, \eta_i$ are positive numbers depending on the subsystem S_i parameters and set $\mathbb{I}_{o,i}$ properties; (vii) the upper solution for W_i satisfies the system of equation $\dot{S}_{W_i} = A_{W_i} S_{W_i} + L_i$ if

$w_\rho(t) \leq s_\rho(t)$ $\forall\left(t \geq t_0\right) \& \left(w_\rho(t_0) \leq s_\rho(t_0)\right)$,

$\rho = e, i; \Delta_{S,i}$ for elements W_{S_i}, $w_\rho \in W_{S_i}$, $s_\rho \in S_{W_{S,i}}$. Then the AS_{AS_i}-system is exponentially dissipative with an estimate

$$W_{S_i}(t) \leq e^{A_{WS_i}(t-t_0)} S_{W_{S_i}}(t_0) + \int_{t_0}^{t} e^{A_{WS_i}(t-\sigma)} L_{S,i} d\sigma, \qquad (34)$$

if $\mu_i^2 \beta_{\Delta,i} \geq 2\vartheta_{\Delta,i} \rho_i \kappa_{\Delta_S,i}$.

As follows from Theorem 4, the AS_{AS_i}-system application gives biased estimates for the parameters of the S_i-subsystem.

Theorem 5 proof is presented in Appendix F.

Remark 4. Signalling algorithms (SA) are widely used in adaptive control systems (see review [19]). The rationale SA is based on ensuring on non-positivity derivative LF. This is a feature of using

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

quadratic LF, which does not fully reflect the specifics of the processes in the system AS_{AS_i}. The Lyapunov function $V_{\Delta_S,i}$ proposed in the paper allows to prove the properties of the adaptive system.

Remark 5. Algorithm (19) is a compensating control. Therefore, the term "signal adaptation" reflects only the gain factor in (19). In identification systems, the SA use depends on the quality requirements of the identification system.

Remark 6. The analysis of the properties of algorithms (12) with $\varphi_i = 0$ is based on the results got in [20].

7. Example

Consider the system

$$S_1 : \begin{cases} \begin{bmatrix} \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_{21} & -a_{22} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{a}_1 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ b_1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ c_1 \end{bmatrix} f_1(x_{11}), \\ y_1 = x_{11}, \end{cases}$$
$$S_2 : \begin{cases} \dot{x}_2 = -a_2 x_2 + \bar{a}_2 x_{11} + b_2 u_2 + c_2 f_1(x_{11}), \\ y_2 = x_2, \end{cases} \qquad (35)$$

where $X_1 = [x_{11}\ x_{12}]^T$, y_1 is the state vector and output of the subsystem S_1; u_1 is input (control)); $f_1(x_{11}) = \mathrm{sat}(x_{11})$ is saturation function; $f_2(x_2)=\mathrm{sign}(x_2)$ is sign function; y_2 is subsystem S_2 output. System parameters (35): $b_1 = 1$, $a_{21} = 2$, $a_{22} = 3$, $\bar{a}_1 = 1.5$, $b_1 = 1$, $c_1 = 1$, $a_2 = 1.25$, $\bar{a}_2 = 0.2$, $b_2 = 1$, $c_2 = 0.25$. Inputs $u_i(t)$ are sinusoidal.

Since the variable x_{12} is not measured, the subsystem S_1 is converted to a form where only observable variables are used [20]. Subsystem S_1 has the form in the input-output space:

$$\dot{y}_1 = -\alpha_1 y_1 + \alpha_2 p_{y_1} + \beta_{12} p_{x_2} + b_1 p_{u_1} + c_1 p_{f_1}, \qquad (36)$$

where α_1, α_2, β_{12}, b_1, c_1 is unknown coefficients; $\mu > 0$,

$$\begin{aligned} \dot{p}_{y_1} &= -\mu p_{y_1} + y_1, \ \dot{p}_{x_2} = -\mu p_{x_2} + x_2, \\ \dot{p}_{u_1} &= -\mu p_{u_1} + u_1, \ \dot{p}_{f_1} = -\mu p_{f_1} + f_1. \end{aligned} \qquad (37)$$

We present the phase portrait for S1 in Fig. 1. Processes in S_1 are nonlinear. There is a relationship between y_1 and y_2 (the determination coefficient is 75%). This reflects in properties of the subsystem S_1 (see Fig. 1). In particular, y_2 effects on S-synchronizability and parameter estimation. Apply the approach [16] and get that the S_1 subsystem is structurally identifiable.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

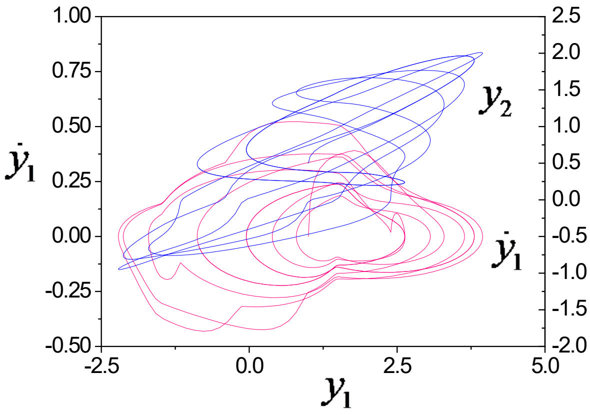

Figure 1. Phase portrait of subsystem S_1.

Models for subsystems S_1 и S_2

$$\dot{\hat{y}}_1 = -k_1 e_1 + \hat{a}_{11} y_1 + \hat{a}_{12} p_{y_1} + \hat{\beta}_{12} p_{x_2} + \hat{b}_1 p_{u_1} + \hat{c}_1 p_{f_1}, \quad (38)$$

$$\dot{\hat{y}}_2 = -k_2 e_2 + \hat{a}_2 y_2 + \hat{\bar{a}}_2 y_1 + \hat{b}_2 u_2 + \hat{c}_1 f_2, \quad (39)$$

where k_1, k_2 are a priori set positive numbers (reference model); $e_1 = \hat{y}_1 - y_1$, $e_2 = \hat{y}_2 - y_2$ are identification errors; $\hat{a}_i, \hat{\bar{a}}_i, \hat{b}_i, \hat{c}_i$ are tuning parameters.

Apply algorithms (12) with $\varphi_i = 0$:

$$\dot{\hat{a}}_{11} = -\gamma_{a_{11}} e_1 y_1,\ \dot{\hat{a}}_{12} = -\gamma_{a_{12}} e_1 p_{y_1},\ \dot{\hat{\beta}}_{12} = -\gamma_{\beta_{12}} e_1 p_{x_2},$$
$$\dot{\hat{b}}_1 = -\gamma_{b_1} e_1 p_{u_1},\ \dot{\hat{c}}_1 = -\gamma_{c_1} e_1 p_{f_1}, \quad (40)$$

$$\dot{\hat{a}}_2 = -\gamma_{a_2} e_2 y_2, \dot{\hat{\bar{a}}}_2 = -\gamma_{\bar{a}_2} e_2 y_1,$$
$$\dot{\hat{b}}_2 = -\gamma_{b_2} e_2 u_1,\ \dot{\hat{c}}_2 = -\gamma_{c_2} e_2 f_2, \quad (41)$$

where $\gamma_{a_{ij}} > 0, \gamma_{\bar{a}_{ij}} > 0,\ \gamma_{\beta_{ij}} > 0, \gamma_{b_i} > 0, \gamma_{c_i} > 0$ are gain factors of the adaptation subsystem.

Figure 2 shows tuning parameters of the model (38) for S_1.

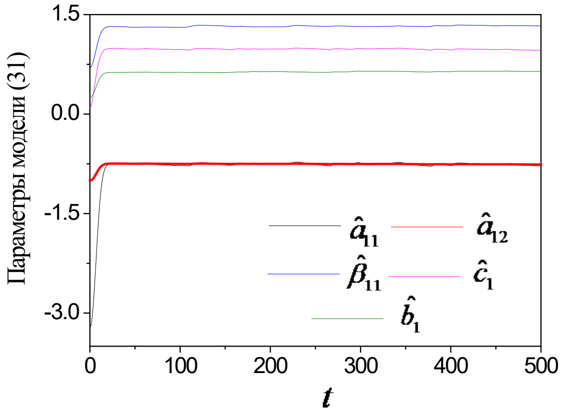

Figure 2. Tuning model (31) parameters.

 doi:10.20944/preprints202412.1808.v1

Show the adequacy estimation of the of models (31), (32) in Fig. 3.

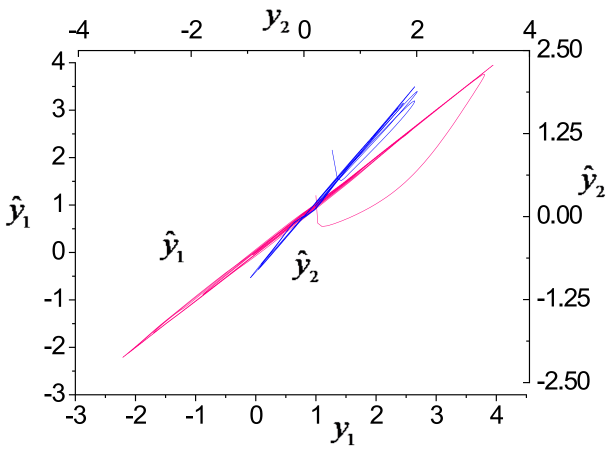

Figure 3. Adequacy of models (38), (39).

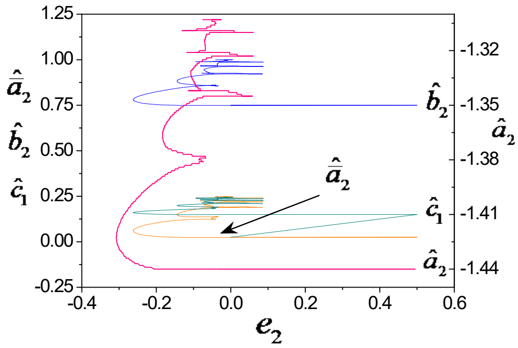

Figure 4. Tuning model (39) parameters.

Fig. 4 shows the tuning process dynamics of the model (32) parameters depending on e_2. We see that the processes in the adaptive identification system (ASI) for S_2 are nonlinear. The tuning process is more regular in the ASI for the S_1 subsystem.

Models (38) and (39) adaptation processes have different speeds (Fig. 5).

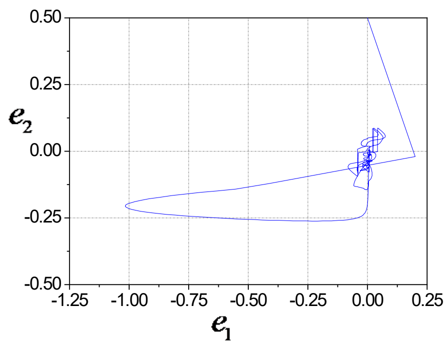

Figure 5. ASI phase portrait in error space.

Consider an ASI with signal adaptation for S_2. Apply the model

$$\dot{\hat{y}}_2 = -k_2 e_2 + \hat{a}_2 y_2 + \hat{\bar{a}}_2 y_1 + \hat{b}_2 u_2 + u_{s,2}, \tag{39a}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

where $u_{s,2} = -d_2 e_2 x_2$.

We show results for the ASI in Fig. 6-9. Show the adaptation of the model parameters (38) and the adequacy of the models in the output space in Fig. 6, 7. Fig. 8 reflects the dynamics in ASI and the change in SA as e_2 function for the subsystem S_2.

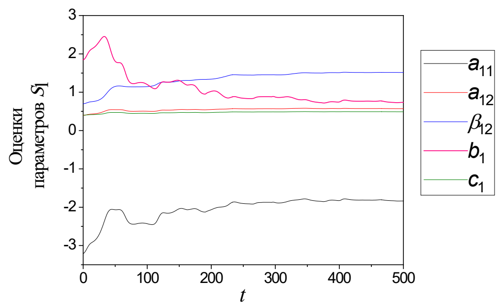

Figure 6. Tuning model (38) parameters.

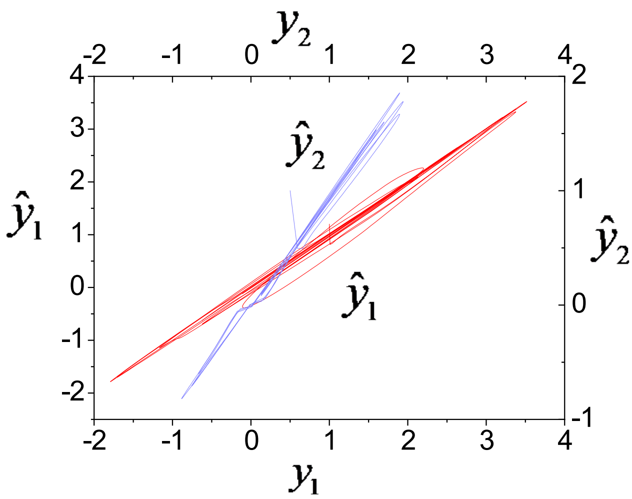

Figure 7. Adequacy of models (38), (39a).

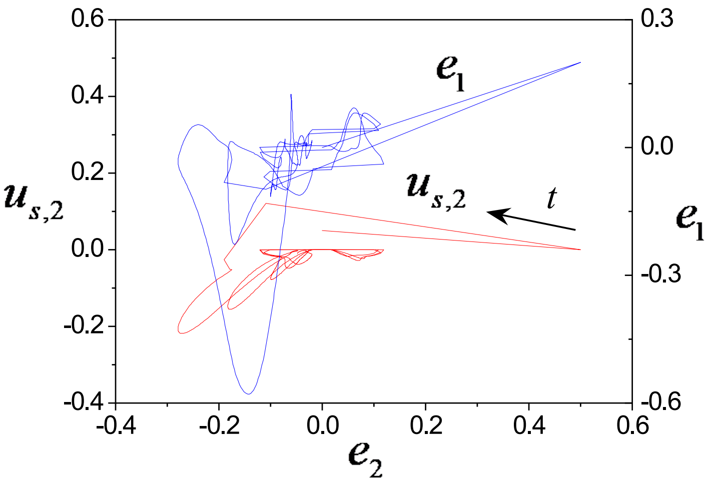

Figure 8. Phase portraits of the ASI and the output of the signal adaptation.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

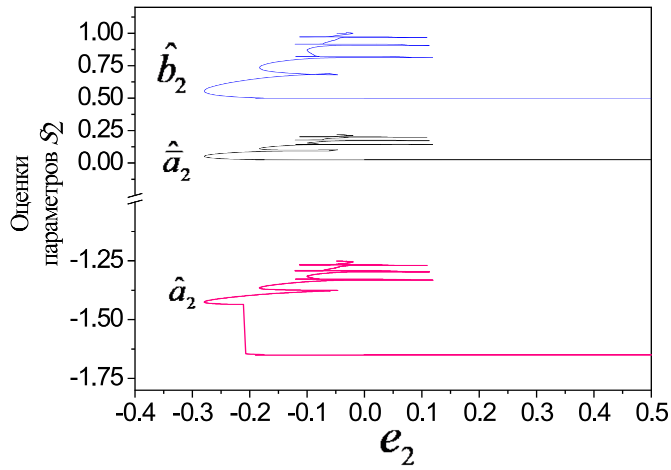

Figure 9. Tuning model (39a) parameters.

We see that the S_2-subsystem output with SA effects on the ASI adaptation of the S_1-subsystem. Therefore, ASI with SA should be applied considering the quality requirements of the identification process. Despite the compensating properties of CA, CA can lead to more complicated processes in ASI.

8. Conclusions

We consider a class of nonlinear decentralised control systems for which the quadratic condition is valid. The problem of identifiability S1-subsystem of DS is studied. We note the constant excitation condition role in the parametric identifiable analysis of decentralised systems. Quadratic estimates are got for the nonlinear part of the S_i-subsystem. Parametric identifiability conditions are got. Algorithms of parametric and signal adaptation are synthesised, and identification system properties are studied. The exponential dissipativity of the adaptive identification system is proved. We present the simulation results confirming the proposed approach efficiency. Appendix contains proof results.

Appendix A. Lemma 1 Proof

As $\pi_1 X \le F(X) \le \pi_2 X$, then

$$\left(\pi_2 X - F(X)\right)^T \left(F(X) - \pi_1 X\right) \ge 0, \tag{A1}$$

where $\pi_1 > 0, \pi_2 > 0$.

After simple transformations, we get

$$\begin{aligned} \chi &= \pi_2 X^T F(X) - \pi_1\pi_2 X^T X - F^T(X)F(X) + \pi_1 X^T F(X) = \\ &= \pi X^T F(X) - \pi_1\pi_2 X^T X - F^T(X)F(X) \ge 0, \end{aligned} \tag{A2}$$

where $\pi = \pi_1 + \pi_2$. Let $X(t) \in \mathscr{E}_{\underline{\alpha}_X, \overline{\alpha}_X}$. Denote $X^T X = \|X\|^2$, $\underline{\alpha}_X \le \|X\|^2 \le \overline{\alpha}_X$, $\overline{\pi} = \pi_1\pi_2$, where $\underline{\alpha}_X, \overline{\alpha}_X$ are positive numbers.

Transform χ to the form

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

$$\chi = \bar{\pi} X^T X - \left(\left[0.5 F^T(X) F(X) + \frac{\left(2\sqrt{0.5}\pi X^T F(X)\right)}{2\sqrt{0.5}} \pm \frac{1}{4 \cdot 0.5} \pi^2 X^T X \right] \right) -$$
$$-0.5 F^T(X) F(X) \geq 0$$

or

$$\chi = \bar{\pi} X^T X - \left[\sqrt{0.5 \|F(X)\|} + \frac{1}{2\sqrt{0.5}} \pi \|X\| \right]^2 -$$
$$-0.5 F^T(X) F(X) + \frac{1}{4 \cdot 0.5} \pi^2 X^T X \geq 0.$$

Then

$$0.5 F^T(X) F(X) \leq \left(\bar{\pi} + 0.5\pi^2\right) \|X\|^2 - \left[\sqrt{0.5 \|F(X)\|} + \frac{1}{2\sqrt{0.5}} \pi \|X\| \right]^2$$
$$\Downarrow$$
$$F^T(X) F(X) \leq 2\bar{\pi} X^T X + \pi^2 X^T X = \left(2\bar{\pi} + \pi^2\right) \|X\|^2.$$

$$F^T(X) F(X) \leq \left(\underbrace{2\bar{\pi} + \pi^2}_{\eta} \right) X^T X = \eta X^T X \leq \eta \bar{\alpha}_X \quad \blacksquare$$

Appendix B. Lemma 2 Proof

As $\hat{F} = F - \Delta F$, then we obtain from (7): $\left(\pi_2 X - \hat{F}\right)^T \left(\hat{F} - \pi_1 X\right) \geq 0$. Transform this inequality to the form

$$\left(f_2 + \Delta F\right)^T \left(f_1 - \Delta F\right) \geq 0, \qquad \text{(B1)}$$

where $f_1 = F - \pi_1 X$, $f_2 = \pi_2 X - F$. Then

$$\begin{aligned} & f_2^T f_1 - f_2^T \Delta F + \Delta F^T f_1 - \Delta F^T \Delta F, \\ & f_2^T f_1 - f_{12} \Delta F - \Delta F^T \Delta F \geq 0, \end{aligned} \qquad \text{(B2)}$$

where $f_{12} = f_2 - f_1$.

Transform (B2)

$$f_2^T f_1 - \left(\sqrt{0.5 \Delta F^T \Delta F} + \sqrt{0.5 f_{12}^T f_{12}} \right)^2 - 0.5 \Delta F^T \Delta F + 0.5 f_{12}^T f_{12} \geq 0,$$
$$0.5 \Delta F^T \Delta F \leq f_2^T f_1 + 0.5 f_{12}^T f_{12}.$$

As $f_2^T f_1 \leq \eta \bar{\alpha}_X$ (see Lemma 1), then:

$$0.5 \Delta F^T \Delta F \leq \eta \bar{\alpha}_X + 0.5 f_{12}^T f_{12} \Rightarrow \Delta F^T \Delta F \leq 2\eta \bar{\alpha}_X + f_{12}^T f_{12}.$$

Obtain for $f_{12}^T f_{12}$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

$$f_{12}^T f_{12} = \left(\pi X - 2F\right)^T \left(\pi X - 2F\right) = \pi^2 X^T X - 4\pi X^T F + 4F^T F,$$
$$f_{12}^T f_{12} = \left\|\pi X - 2F\right\|^2 \le \delta_F,$$

where $\delta_F \ge 0$. So

$$\Delta F^T \Delta F = \left\|\Delta F\right\|^2 \le 2\eta \bar{\alpha}_X + \delta_F. \blacksquare$$

Appendix C. Theorem 1 Proof

Derivative V_i

$$\dot{V}_i = -E_i^T Q_i E_i + E_i^T R_i \left(\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\neq i}^{m} \Delta \bar{A}_{ij} X_j + \Delta F_i\left(X_i\right) \right), \tag{C1}$$

where $K_i R_i + K_i^T R_i = -Q_i$, $Q_i = Q_i^T > 0$. Transform (C1)

$$\begin{aligned} &\dot{V}_i \underbrace{-E_i^T Q_i E_i}_{-\lambda V_i} + \underbrace{\left|E_i^T R_i\right|}_{\sqrt{k_i V_i}} \left| \Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\neq i}^{m} \Delta \bar{A}_{ij} X_j + \Delta F_i\left(X_i\right) \right| \le \\ &\le -\lambda_i V_i + k_i V_i + 0.5 \left(\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\neq i}^{m} \Delta \bar{A}_{ij} X_j + \Delta F_i\left(X_i\right) \right)^2, \end{aligned} \tag{C2}$$

where $\lambda_i > 0$ is the minimum eigenvalue of the matrix Q_i.

We apply the Cauchy-Bunyakovsky-Schwarz inequality and Titus lemma to the last term in (C2)

$$\begin{aligned} &0.5\left(\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\neq i}^{m} \Delta \bar{A}_{ij} X_j + \Delta F_i\left(X_i\right) \right)^2 \le \\ &\le 2\left(\left\|\Delta A_i\right\|^2 \left\|X_i\right\|^2 + \left\|\Delta B_i\right\|^2 \left|u_i\right|^2 + \sum_{j=1, j\neq i}^{m} \left\|\Delta \bar{A}_{ij}\right\|^2 \left\|X_j\right\|^2 + \left\|\Delta F_i\left(X_i\right)\right\|^2 \right). \end{aligned} \tag{C3}$$

Since the conditions of Theorem 1 are fulfilled, then

$$\dot{V}_i \le -\bar{\lambda}_i V_i + 2\left(\bar{\alpha}_{X_i} \left\|\Delta A_i\right\|^2 + \bar{\alpha}_{u_i} \left\|\Delta B_i\right\|^2 + \sum_{j=1, j\neq i}^{m} \bar{\alpha}_{X_j} \left\|\Delta \bar{A}_{ij}\right\|^2 + \left\|\Delta F_i\left(X_i\right)\right\|^2 \right), \tag{C4}$$

where $\bar{\lambda}_i = \lambda_i - k_i$.

Apply lemmas 1, 2. Then

$$\Delta F_i^T \Delta F_i = \left\|\Delta F_i\right\|^2 \le 2\eta \bar{\alpha}_{X_i} + \delta_i, \tag{C5}$$

where $\eta = 2\bar{\pi} + \pi^2$, $\pi = \pi_1 + \pi_2$, $\bar{\pi} = \pi_1 \pi_2$, $\delta_i \ge 0$.

Get estimation for (C4)

$$\dot{V}_i \le -\bar{\lambda}_i V_i + 2\left(\bar{\alpha}_{X_i} \left\|\Delta A_i\right\|^2 + \bar{\alpha}_{u_i} \left\|\Delta B_i\right\|^2 + \sum_{j=1, j\neq i}^{m} \bar{\alpha}_{X_j} \left\|\Delta \bar{A}_{ij}\right\|^2 + 2\eta \bar{\alpha}_{X_i} + \delta_i \right). \tag{C6}$$

As follows from (C6), if state variables have the property CE and

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

$$2\left(\bar{\alpha}_{X_i}\left\|\Delta A_i\right\|^2+\bar{\alpha}_{u_i}\left\|\Delta B_i\right\|^2+\sum_{j=1,j\neq i}^{m}\bar{\alpha}_{X_j}\left\|\Delta\bar{A}_{ij}\right\|^2+2\eta\bar{\alpha}_{X_i}+\delta_i\right)\leq\bar{\lambda}_i V_i,$$

then subsystem (1) is identifiable on the set $\mathbb{I}_{o,i}$ or $\mathscr{PI}_{X_i}$ - identifiable. ■

Appendix D. Theorem 3 Proof

Consider the LF $V_i=0.5\mathrm{Sp}\left(\Delta E_i^T R_i \Delta E_i\right)$ and $V_{\Delta,i}=0.5\mathrm{Sp}\left(\Delta A_i^T\Gamma_{\dot{A}_i}^{-1}\Delta A_i\right)$.

$$\dot{V}_i=-E_i^T Q_i E_i+E_i^T R\left(\Delta A_i X_i+\Delta B_i u_i+\sum_{j=1,j\neq i}^{M}\Delta\bar{A}_{ij}X_j+\Delta F_i\left(X_i\right)\right)\leq$$
$$\leq -E_i^T Q_i E_i+\left\|E_i^T R_i\right\|\left\|\Delta A_i X_i+\Delta B_i u_i+\sum_{j=1,j\neq i}^{M}\Delta\bar{A}_{ij}X_j+\Delta F_i\left(X_i\right)\right\|.$$

The derivative of V_i has the form

$$\dot{V}_i=-E_i^T Q_i E_i+E_i^T R\left(\Delta A_i X_i+\Delta B_i u_i+\sum_{j=1,j\neq i}^{M}\Delta\bar{A}_{ij}X_j+\Delta F_i\left(X_i\right)\right)\leq$$
$$\leq -E_i^T Q_i E_i+\left\|E_i^T R_i\right\|\left\|\Delta A_i X_i+\Delta B_i u_i+\sum_{j=1,j\neq i}^{M}\Delta\bar{A}_{ij}X_j+\Delta F_i\left(X_i\right)\right\|,$$

and after simple transformations (see Appendix E)

$$\dot{V}_i\leq-\mu_i V_i+\frac{2}{\mu_i}\kappa_i V_{\Delta,i}+\frac{2}{\mu_i}\upsilon_{\tilde{F}_i}.$$

For $\dot{V}_{\Delta,i}$ we have

$$\dot{V}_{\Delta,i}=\Delta A_i^T\Gamma_{\dot{A}_i}^{-1}\left(-\Gamma_{\dot{A}_i}\Delta A_i(t)-\Gamma_{A_i}E_i R_i X_i^T+\Gamma_{\dot{A}_i}\Delta A_i(t-\tau)\right)$$

or

$$\dot{V}_{\mathcal{M},i}=-\mathrm{Sp}\left(\Delta A_i^T\Delta A_i(t)\right)-\mathrm{Sp}\left(\Delta A_i^T\Gamma_{\dot{A}_i}^{-1}\Gamma_{A_i}E_i R_i X_i^T\right)+\mathrm{Sp}\left(\Delta A_i^T\Gamma_{\dot{A}_i}^{-1}\Gamma_{\dot{A}_i}\Delta A_i(t-\tau)\right). \quad \text{(D1)}$$

Let $E_i^T R_i^2 E_i\geq\underline{\lambda}_{R_i}E_i^T R_i E_i=2\underline{\lambda}_{R_i}V_i$,

$$2\underline{\lambda}_{\Gamma_{\dot{A}_i}}V_{\Delta,i}\leq\mathrm{Sp}\left(\Delta A_i^T\Delta A_i(t)\right)\leq2\bar{\lambda}_{\Gamma_{\dot{A}_i}}V_{\Delta,i} \quad \text{и} \quad \left|\mathrm{Sp}\left(\Delta A_i^T(t)\Delta A_i(t-\tau)\right)\right|\leq0.5\left[\left\|\Delta A_i\right\|^2+\left\|\Delta A_i(t-\tau)\right\|^2\right],$$

where $\mathrm{Sp}\left(\Delta A_i^T(t)\Delta A_i(t)\right)\geq2\underline{\lambda}_{\Gamma_{\dot{A}_i}}V_{\Delta,i}$, $\underline{\lambda}_{\Gamma_{\dot{A}_i}}$ is the minimum eigenvalue of the matrix $\Gamma_{\dot{A}_i}$.

Then (D1)

$$\begin{aligned}\dot{V}_{\Delta,i}&\leq-2\underline{\lambda}_{\Gamma_{\dot{A}_i}}V_{\Delta,i}-\mathrm{Sp}\left(\Delta A_i^T\Gamma_{\dot{A}_i}^{-1}\Gamma_{A_i}E_i R_i X_i^T\right)+\left|\mathrm{Sp}\left(\Delta A_i^T\Delta A_i(t-\tau)\right)\right|\leq\\&\leq-2\underline{\lambda}_{\Gamma_{\dot{A}_i}}V_{\Delta,i}-\mathrm{Sp}\left(\Delta A_i^T\Gamma_{\dot{A}_i}^{-1}\Gamma_{A_i}E_i R_i X_i^T\right)+0.5\left[\left\|\Delta A_i\right\|^2+\left\|\Delta A_i(t-\tau)\right\|^2\right].\end{aligned} \quad \text{(D2)}$$

Apply condition 5) of Theorem 3 and get

$$\dot{V}_{\Delta,i}\leq-\underline{\lambda}_{\Gamma_{\dot{A}_i}}V_{\Delta,i}+0.5\left\|\Delta A_i(t-\tau)\right\|^2-\upsilon\left[\left(\mathrm{Sp}\left(\Delta A_i^T\Gamma_{\dot{A}_i}^{-1}\Gamma_{A_i}\Delta A_i\right)+\left(E_i^T R_i^2 E_i\right)\left\|X_i\right\|^2\right)\right], \quad \text{(D3)}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

or

$$\dot{V}_{\Delta,i} \le -\underline{\lambda}_{\Gamma_{A_i}} V_{\Delta,i} + 0.5\left\|\Delta A_i(t-\tau)\right\|^2 - \upsilon \underbrace{\underline{\lambda}_{\Gamma_{A_i}} \underline{\lambda}_{\Gamma_{A_i}}}_{\sigma_\Delta} \left\|\Delta A_i\right\|^2 - \upsilon \underbrace{\underline{\alpha}_{X_i} 2\underline{\lambda}_{R_i}}_{\sigma_e} V_i$$

As

$$-\upsilon\sigma_\Delta \left\|\Delta A_i\right\|^2 - \upsilon\sigma_e V_i \le -\frac{3}{4}\upsilon\sigma_\Delta \left\|\Delta A_i\right\|^2 + \upsilon\sqrt{\sigma_e V_i} \le -\frac{3}{8}\upsilon\sigma_\Delta \left\|\Delta A_i\right\|^2 + \upsilon\frac{2\sigma_e}{3\sigma_\Delta}V_i,$$

then

$$\dot{V}_{\Delta,i} \le -\eta V_{\Delta,i} + \left\|\Delta A_i(t-\tau)\right\|^2 + \upsilon\frac{2\sigma_e}{3\sigma_\Delta}V_i, \qquad \text{(D4)}$$

where $\eta = \underline{\lambda}_{\Gamma_{A_i}}\left(1 + \frac{3}{4}\upsilon\sigma_\Delta\right)$.

The system will be stable if the functional limitation is fulfilled

$$\left\|\Delta A_i(t-\tau)\right\|^2 \le \eta V_{M,i} - \upsilon\frac{2\sigma_e}{3\sigma_\Delta}V_i. \blacksquare$$

Appendix E. Theorem 4 Proof

Derivative $V_i = E_i^T P_i E_i$

$$\begin{aligned}\dot{V}_i &= -E_i^T Q_i E_i + E_i^T R\left(\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1,j\ne i}^{M} \Delta\bar{A}_{ij} X_j + \Delta F_i(X_i)\right) \le \\ &\le -E_i^T Q_i E_i + \left\|E_i^T R_i\right\| \left\|\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1,j\ne i}^{M} \Delta\bar{A}_{ij} X_j + \Delta F_i(X_i)\right\|.\end{aligned} \qquad \text{(E1)}$$

Apply the inequality

$$-az^2 + bz \le -\frac{-az^2}{2} + \frac{b^2}{2a}, \quad a>0,\ b\ge 0,\ z\ge 0. \qquad \text{(E2)}$$

As

$$\Delta F_i = \tilde{F}_i^T \Delta N_{i,2} + \Delta\tilde{F}_i^T N_{i,2} = \tilde{F}_i^T \Delta N_{i,2} + D_{\Delta F_i},$$

then apply the conditions of Theorem 3 and get

$$\begin{aligned}\dot{V}_i \le -\mu_i V_i + \frac{2}{\mu_i}\Bigg(&\bar{\alpha}_{X_i}\left\|\Delta A_i\right\|^2 + \bar{\alpha}_{u_i}\left\|\Delta B_i\right\|^2 + \sum_{j=1,j\ne i}^{m} \bar{\alpha}_{X_j}\left\|\Delta\bar{A}_{ji}\right\|^2 + \\ &+ \left\|\tilde{F}_i^T\left(X_i,\hat{N}_{i,1}\right)\Delta N_{i,2} + D_{\Delta F_i}\right\|^2\Bigg).\end{aligned}, \qquad \text{(E3)}$$

where $E_i^T Q_i E_i = \mu_i E_i^T R_i E_i$, $\left\|\Delta A_i\right\|^2 = \mathrm{Sp}\left(\Delta A_i^T \Delta A_i\right)$. $\left\|D_{\Delta\tilde{F}_i}\right\|^2 \le \bar{\upsilon}_{\Delta\tilde{F}_i}$ follows from the system construction, $\bar{\upsilon}_{\Delta\tilde{F}_i} \ge 0$. Obtain for $\left\|\Delta A_i\right\|^2$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

$$\left\|\Delta A_i\right\|^2 = \mathrm{Sp}\left(\Delta A_i^T \Gamma_{A_i}^{-1} \Gamma_{A_i} \Delta A_i\right) \le \bar{\lambda}_{\Gamma_{A_i}} \mathrm{Sp}\left(\Delta A_i^T \Gamma_{A_i}^{-1} \Delta A_i\right), \quad \text{(E4)}$$

where $\bar{\lambda}_{\Gamma_{A_i}}$ is the maximum eigenvalue of the matrix Γ_{A_i}. Estimates for $\left\|\Delta B_i\right\|^2$, $\left\|\Delta \bar{A}_{ji}\right\|^2$ are obtained similarly. For $\tilde{F}_i^T \Delta N_{i,2}$ in (E3), we have

$$\left\|\tilde{F}_i^T \Delta N_{i,2}\right\|^2 \le \left\|\tilde{F}_i\right\|^2 \left\|\Delta N_{i,2}\right\|^2 \le \vartheta_{\tilde{F}_i} \Delta N_{i,2}^T \Gamma_{i,2}^{-1} \Gamma_{i,2} \Delta N_{i,2} \le \vartheta_{\tilde{F}_i} \bar{\lambda}_{\Gamma_{i,2}} \Delta N_{i,2}^T \Gamma_{i,2}^{-1} \Delta N_{i,2},$$

where $\bar{\lambda}_{\Gamma_{i,2}}$ is the maximum eigenvalue of the matrix $\Gamma_{i,2}$, and $\tilde{F}$ is limited in construction, i.e. $\left\|\tilde{F}_i\right\|^2 \le \vartheta_{\tilde{F}_i}$. Therefore, (E3)

$$\dot{V}_i \le -\mu_i V_i + \frac{2}{\mu_i} \kappa_i V_{\Delta,i} + \frac{2}{\mu_i} \bar{\upsilon}_{\tilde{F}_i}, \quad \text{(E5)}$$

where $\kappa_i = \max\left(\bar{\alpha}_{X_i} \bar{\lambda}_{\Gamma_{A_i}}, \bar{\alpha}_{u_i} \bar{\lambda}_{\Gamma_{B_i}}, \bar{\alpha}_{X_j} \bar{\lambda}_{\Gamma_{\bar{A}_{ij}}}, \bar{\alpha}_{\tilde{F}_i} \bar{\lambda}_{\Gamma_{N_{i,2}}}\right)$.

$\dot{V}_{\Delta,i}$ haves the form

$$\begin{aligned} \dot{V}_{\Delta,i} = &-\mathrm{Sp}\left(\Delta A_i^T \left(\varphi_{A_i} \Delta A_i + R_i E_i X_i^T\right)\right) - \sum_{j=1}^{m} \mathrm{Sp}\left(\Delta \bar{A}_{ij}^T \left(\varphi_{\tilde{A}_{ij}} \Delta \tilde{A}_{ij} + R_i E_i X_j^T\right)\right) - \\ &-\Delta B_i^T \left(\varphi_{B_{ij}} \Delta B_i + R_i E_i u_i\right) - \Delta N_{i,2}^T \left(\varphi_{F_i} \Delta N_{i,2} + \tilde{F}_i R_i E_i\right) = \omega_{1,i} + \omega_{2,i}. \end{aligned} \quad \text{(E6)}$$

Consider first components in (E6), i.e. $r_1 = \omega_{1,i(1)} + \omega_{2,i(1)}$:

$$\begin{aligned} r_1 = &-\varphi_{A_i} \mathrm{Sp}\left(\Delta A_i^T \Delta A_i\right) - \mathrm{Sp}\left(\Delta A_i^T R_i E_i X_i^T\right) = -0.5\varphi_{A_i} \mathrm{Sp}\left(\Delta A_i^T \Delta A_i\right) - \\ &-\left(0.5\varphi_{A_i} \mathrm{Sp}\left(\Delta A_i^T \Delta A_i\right) + 2\frac{\sqrt{0.5\varphi_{A_i}}}{2\sqrt{0.5\varphi_{A_i}}} \mathrm{Sp}\left(\Delta A_i^T R_i E_i X_i^T\right) \pm \frac{1}{4 \cdot 0.5\varphi_{A_i}} X_i^T X_i E_i^T R_i R_i E_i\right) \le \\ &\le -0.5\varphi_{A_i} \mathrm{Sp}\left(\Delta A_i^T \Delta A_i\right) + \frac{1}{4 \cdot 0.5\varphi_{A_i}} X_i^T X_i E_i^T R_i R_i E_i. \end{aligned} \quad \text{(E7)}$$

Using the transformations performed above for $\dot{V}_i$, we obtain

$$\begin{aligned} r_1 &\le -0.5\varphi_{A_i} \mathrm{Sp}\left(\Delta A_i^T \Delta A_i\right) + \frac{1}{4 \cdot 0.5\varphi_{A_i}} X_i^T X_i E_i^T R_i R_i E_i \le \\ &\le -0.5\underline{\lambda}_{A_i} \underline{\chi}_{A_i} \mathrm{Sp}\left(\Delta A_i^T \Gamma_{A_i}^{-1} \Delta A_i\right) + \underline{\chi}_{A_i}^{-1} \bar{\alpha}_{X_i} \rho_i V_i, \end{aligned} \quad \text{(E8)}$$

where $\varphi_{A_i} \ge \underline{\chi}_{A_i}$, $\left\|R_i\right\| \le \rho_i$, $\varphi_{A_i}^{-1} \le \underline{\chi}_{A_i}^{-1}$.

Estimate $r_2 = \omega_{1,i(2)} + \omega_{2,i(2)}$, using the approach for r_1:

$$\begin{aligned} r_2 = &-\sum_{j=1}^{m} \varphi_{\tilde{A}_{ij}} \mathrm{Sp}\left(\Delta \bar{A}_{ij}^T \Delta \tilde{A}_{ij}\right) - \sum_{j=1}^{m} \mathrm{Sp}\left(\Delta \bar{A}_{ij}^T R_i E_i X_j^T\right) = -0.5\sum_{j=1}^{m} \varphi_{\tilde{A}_{ij}} \mathrm{Sp}\left(\Delta \bar{A}_{ij}^T \Delta \tilde{A}_{ij}\right) \le \\ &\le -0.5\sum_{j=1}^{m} \varphi_{\tilde{A}_{ij}} \mathrm{Sp}\left(\Delta \bar{A}_{ij}^T \Delta \tilde{A}_{ij}\right) + 0.5 E_i^T R_i R_i E_i \sum_{j=1}^{m} \varphi_{\tilde{A}_{ij}}^{-1} X_j^T X_j. \end{aligned} \quad \text{(E9)}$$

Let $\underline{\chi}_{\tilde{A}_{ij}} \le \varphi_{\tilde{A}_{ij}}(t) \le \bar{\chi}_{\tilde{A}_{ij}}$. Then:

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

$$r_2 \le -0.5\sum_{j=1}^{m} \underline{\lambda}_{\bar{A}_{ij}} \underline{\chi}_{\bar{A}_{ij}} \operatorname{Sp}\left(\Delta \bar{A}_{ij}^T \Gamma_{\bar{A}_{ij}}^{-1} \Delta \bar{A}_{ij}\right) + \rho_i V_i \sum_{j=1}^{m} \underline{\chi}_{\bar{A}_{ij}}^{-1} \bar{\alpha}_{X_{ij}} . \quad \text{(E10)}$$

Estimate for $r_3 = -\varphi_{B_i} \Delta B_i^T \Delta B_i - \Delta B_i^T R_i E_i u_i$

$$r_3 \le -0.5 \underline{\lambda}_{B_i} \underline{\chi}_{B_i} \Delta B_i^T \Gamma_{B_i}^{-1} \Delta B_i + \underline{\chi}_{B_i}^{-1} \bar{\alpha}_{u_i} \rho_i V_i , \quad \text{(E11)}$$

where $\varphi_{B_i}(t) \ge \underline{\chi}_{B_i}$, $\varphi_{B_i}^{-1}(t) \le \underline{\chi}_{B_i}^{-1}$.

Consider $r_4 = -\varphi_{F_i} \Delta N_{i,2}^T \Delta N_{i,2} - \Delta N_{i,2}^T \tilde{F}_i R_i E_i - N_{i,2}^T \Delta \tilde{F}_i R_i E_i$

$$r_4 \le -0.5 \underline{\lambda}_{N_{i,2}} \underline{\chi}_{F_i} \Delta N_{i,2}^T \Gamma_{N_{i,2}}^{-1} \Delta N_{i,2} + \underline{\chi}_{F_i}^{-1} \bar{\alpha}_{\tilde{F}_i} \rho_i V_i + \left| N_{i,2}^T \Delta \tilde{F}_i R_i E_i \right| \le$$
$$\le -0.5 \underline{\lambda}_{N_{i,2}} \underline{\chi}_{F_i} \Delta N_{i,2}^T \Gamma_{N_{i,2}}^{-1} \Delta N_{i,2} + \underline{\chi}_{F_i}^{-1} \bar{\alpha}_{\tilde{F}_i} \rho_i V_i + 0.5 N_{i,2}^T \Delta \tilde{F}_i \Delta F_i^T N_{i,2} + 0.5 E_i^T R_i R_i E_i ,$$

where $\varphi_{F_i}(t) \ge \underline{\chi}_{F_i}, \left\|\tilde{F}_i\right\|^2 \le \bar{\alpha}_{\tilde{F}_i}$. Considering the designations introduced above and

$$0.5 N_{i,2}^T \Delta \tilde{F}_i \Delta \tilde{F}_i^T N_{i,2} \le \delta_{N,i}, \delta_{N,i} \ge 0 ,$$

we obtain

$$r_4 \le -0.5 \underline{\lambda}_{N_{i,2}} \underline{\chi}_{F_i} \Delta N_{i,2}^T \Gamma_{N_{i,2}}^{-1} \Delta N_{i,2} + \left(\underline{\chi}_{F_i}^{-1} \bar{\alpha}_{\tilde{F}_i} + 1 \right) \rho_i V_i + 0.5 \delta_{N_i} . \quad \text{(E12)}$$

Let

$$\min_{\underline{\lambda}_{A_k} \underline{\chi}_{A_k}} \left(\underline{\lambda}_{A_i} \underline{\chi}_{A_i}, \underline{\chi}_{\bar{A}_{ij}} \underline{\lambda}_{\bar{A}_{ij}}, \underline{\lambda}_{B_i} \underline{\chi}_{B_i}, \underline{\lambda}_{N_{i,2}} \underline{\chi}_{F_i} \right) = \beta_{\lambda\chi,i},$$
$$\max_{\chi_{A_i}^{-1} \bar{\alpha}_{X_i}} \left[\chi_{A_i}^{-1} \bar{\alpha}_{X_i}, \sum_{j=1}^{m} \underline{\chi}_{\bar{A}_{ij}}^{-1} \bar{\alpha}_{X_{ij}}, \underline{\chi}_{B_i}^{-1} \bar{\alpha}_{u_i}, \left(\underline{\chi}_{F_i}^{-1} \bar{\alpha}_{\tilde{F}_i} + 1 \right) \right] \le \vartheta_{\chi\alpha,i} .$$

Then

$$\dot{V}_{\Delta,i} \le -\beta_{\lambda\chi} V_{\Delta,i} + \vartheta_{\chi\alpha,i} \rho_i V_i + 0.5 \delta_{N,i} . \quad \text{(E13)}$$

We obtain a system of inequalities for the $AS_{F,i}$ from (E5) and (E13)

$$\begin{bmatrix} \dot{V}_i \\ \dot{V}_{\Delta,i} \end{bmatrix} \le \underbrace{\begin{bmatrix} -\mu_i & \frac{2}{\mu_i}\kappa_i \\ \vartheta_{\chi\alpha,i}\rho_i & -\beta_{\lambda\chi,i} \end{bmatrix}}_{A_{W_i}} \underbrace{\begin{bmatrix} V \\ V_{\Delta,i} \end{bmatrix}}_{W_i} + \underbrace{\begin{bmatrix} \frac{2}{\mu_i} \upsilon_{\tilde{F}_i} \\ 0.5\delta_{N,i} \end{bmatrix}}_{L_i} \quad \text{(E14)}$$

The $AS_{F,i}$-subsystem is asymptotically stable, if $(-1)^q Dm_q\left(A_{W_i}\right) > 0$, where Dm_q is the qth minor of the matrix A_{W_i}. From these terms, we obtain the exponential dissipation condition: $\mu_i^2 \beta_{\lambda\chi,i} \ge 2\kappa_i \vartheta_{\chi\alpha,i} \rho_i$.

The upper solution for W_i satisfies the comparison system

$$\dot{S}_{W_i} = A_{W_i} S_{W_i} + L_i , \quad \text{(E15)}$$

if $w_\rho(t) \le s_\rho(t) \quad \forall (t \ge t_0) \& (w_\rho(t_0) \le s_\rho(t_0))$, $w_\rho \in W_i$, $s_\rho \in S_{W_i}$, $\rho = e,i;\Delta,i$. Then the $AS_{F,i}$-system is exponentially dissipative with the estimate

$$W_i(t) \le e^{A_{W_i}(t-t_0)} S_{W_i}(t_0) + \int_{t_0}^{t} e^{A_{W_i}(t-\tau)} L_i d\tau . \blacksquare \tag{E16}$$

Appendix F. Theorem 5 Proof

Consider $\dot{V}_i$.

$$\dot{V}_i = -E_i^T Q_i E_i + E_i^T R_i \left(\Delta A_i X_i + \Delta B_i u_i + \sum_{j=1, j\ne i}^{m} \Delta \bar{A}_{ij} X_j + \Delta_{U_i} F_i \right). \tag{F1}$$

Apply the approach from Appendix D and get

$$\dot{V}_i \le -\mu_i V_i + \frac{2}{\mu_i} \left(\left\| \Delta A_i X_i \right\|^2 + \left\| \Delta B_i u_i \right\|^2 + \left\| \sum_{j=1, j\ne i}^{M} \Delta \bar{A}_{ij} X_j \right\|^2 + \left\| \Delta_{U_i} F_i \right\|^2 \right). \tag{F2}$$

where $\left\| \Delta A_i \right\|^2 = \mathrm{Sp}\left(\Delta A_i^T \Delta A_i \right)$, $\left\| \Delta A_{ij} \right\|^2 = \mathrm{Sp}\left(\Delta A_{ij}^T \Delta A_{ij} \right)$.

According to section 4, U_i has the form (19), where $D_i \in \mathbb{R}^{n_i \times n_i}$ is a diagonal matrix with positive elements. We select the matrix D_i from the condition $\left\| D_i \right\| \ge d_i \ge \eta \bar{\alpha}_{X_i}$, and $F_i(X_i) \in \mathscr{N}_F(\pi_1, \pi_2)$ satisfies conditions Lemma 1, 2 conditions. Given the choice U_i, we get $\left\| \Delta_{U_i} F_i \right\|^2 \le \sigma_{F_i}$, where $\sigma_{F_i} \ge 0$. Apply the mean theorem,

$$\left\| \Delta_{U_i} F_i \right\|^2 = \tau^{-1} \underbrace{\int_{t}^{t+\tau} \left\| \Delta_{U_i} F_i(\tau) \right\|^2 d\tau}_{I_{\Delta F_i}} = \tau^{-1} I_{\Delta F_i},$$

where $\tau > 0$, $I_{\Delta F_i} \le \varphi$, $\varphi > 0$ Let $\tau\varphi \triangleq \phi$. Then the estimation for $\dot{V}_i$:

$$\dot{V}_i \le -\mu_i V_i + \frac{2}{\mu_i} \left(\left\| \Delta A_i X_i \right\|^2 + \left\| \Delta B_i u_i \right\|^2 + \left\| \sum_{j=1, j\ne i}^{M} \Delta \bar{A}_{ij} X_j \right\|^2 + \tau^{-1} I_{\Delta F_i} \right). \tag{F3}$$

Using the proof scheme from Appendix 4, we get:

$$\bar{\alpha}_{X_i} \left\| \Delta A_i \right\|^2 + \bar{\alpha}_{u_i} \left\| \Delta B_i \right\|^2 + \sum_{j=1, j\ne i}^{m} \bar{\alpha}_{X_j} \left\| \Delta \bar{A}_{ji} \right\|^2 + \tau^{-1} I_{\Delta F_i} \le \kappa_{\Delta_S,i} V_{\Delta_S,i}, \tag{F4}$$

where $\kappa_{\Delta_S,i} > 0$, and

$$\dot{V}_i \le -\mu_i V_i + \frac{2}{\mu_i} \kappa_{\Delta_S,i} V_{\Delta_S,i}. \tag{F5}$$

Consider

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 23 December 2024 doi:10.20944/preprints202412.1808.v1

$$\dot{V}_{\Delta_S,i} = -\mathrm{Sp}\left(\Delta A_i^T \left(\varphi_{\tilde{A}_{ij}} \Delta A_i + R_i E_i X_j^T\right)\right) - \sum_{j=1}^{m} \mathrm{Sp}\left(\Delta \bar{A}_{ij}^T \left(\varphi_{\tilde{A}_{ij}} \Delta \bar{A}_{ij} + R_i E_i X_j^T\right)\right) - \\ -\Delta B_i^T \left(\varphi_{B_{ij}} \Delta B_i + R_i E_i u_i\right) + \Delta_{U_i} F_i^T \Delta_{U_i} F_i. \quad \text{(F6)}$$

Lemma 3. *Estimate*

$$\Delta_{U_i} F_i^T \Delta_{U_i} F_i \le -\bar{\nu} \left\|\Delta_{U_i} F_i\right\|^2 + 2\theta_i \rho_i d_i V_i + \sqrt{0.125}\nu \eta_i \bar{\alpha}_{X_i}$$

is valid for $\Delta_{U_i} F_i^T \Delta_{U_i} F_i$.

Lemma 3 proof. $\Delta_{U_i} F_i^T \Delta_{U_i} F_i$ has the form

$$\Delta_{U_i} F_i^T \Delta_{U_i} F_i = -E_i^T R_i D_i \Delta_{U_i} F_i - F_i^T \Delta_{U_i} F_i .$$

Estimate the $E_i^T R_i D_i \Delta_{U_i} F_i$. Let in the domain $\mathcal{Q}_V(O)$, the equality holds:

$$-E_i^T R_i D_i \Delta_{U_i} F_i = -\pi_i \left(\left\|E_i^T R_i D_i\right\|^2 + \left\|\Delta_{U_i} F_i\right\|^2\right),$$

where $\pi_i > 0$, $\mathcal{Q}_V = \{0,\ 0^{n_i \times n_i}\} \subset \mathbb{R} \times \mathbb{R}^{n_i \times n_i} \times \mathbb{J}_{0,\infty}$, $0^{n_i \times n_i} \in \mathbb{R}^{n_i \times n_i}$ is zero matrix, $\mathcal{Q}_V$ is some neighbourhood of the point 0, $t \in [0,\infty] = \mathbb{J}_{0,\infty}$.

Then

$$-\pi_i \left(\left\|E_i^T R_i D\right\|^2 + \left\|\Delta_{U_i} F_i\right\|^2\right) = -0.5\pi \left\|\Delta_{U_i} F_i\right\|^2 - \\ -\pi_i \left(0.5\left\|\Delta_{U_i} F_i\right\|^2 + \left\|E_i^T R_i D_i\right\|^2 \pm \frac{2}{2}\sqrt{0.5}\left\|\Delta_{U_i} F_i\right\|\left\|E_i^T R_i D_i\right\|\right) \le \\ \le -0.5\pi_i \left\|\Delta_{U_i} F_i\right\|^2 + 0.5\sqrt{0.5}\pi_i \left\|\Delta_{U_i} F_i\right\|\left\|E_i^T R_i D_i\right\|. \quad \text{(F7)}$$

Apply the inequality (E2) to (E7)

$$-E_i^T R_i D_i \Delta_{U_i} F_i \le -\pi_i \left\|\Delta_{U_i} F_i\right\|^2 + \pi_i \left\|E_i^T R_i D_i\right\|^2 .$$

As $\left\|D_i\right\| \le d_i$, $\left\|R_i\right\| \le \rho_i$, then

$$-E_i^T R_i D_i \Delta_{U_i} F_i \le -\pi_i \left\|\Delta_{U_i} F_i\right\|^2 + 2\pi_i \rho_i d_i V_i . \quad \text{(F8)}$$

$\left\|\Delta_{U_i} F_i\right\|^2 = \tau^{-1} I_{\Delta F_i}$. Therefore, (F8)

$$-E_i^T R_i D_i \Delta_{U_i} F_i \le -\pi_i \tau^{-1} I_{\Delta F_i} + 2\pi_i \rho_i d_i V_i . \quad \text{(F9)}$$

Estimate $-F_i^T \Delta_{U_i} F$. Let $\nu_i > 0$ exist such that:

$$-F_i^T \Delta_{U_i} F_i = -\nu_i \left\|F_i\right\|^2 - \nu_i \left\|\Delta_{U_i} F_i\right\|^2$$

is fulfilled on $t >> t_0$.

Then

$$
\begin{aligned}
&-F_i^T\Delta_{U_i}F_i = -v_i\left\|F_i\right\|^2 - v_i\left\|\Delta_{U_i}F_i\right\|^2 = -0.5v_i\left\|\Delta_{U_i}F_i\right\|^2 - 0.5v_i\left\|\Delta_{U_i}F_i\right\|^2 - v_i\left\|F_i\right\|^2 = \\
&= -0.5v_i\left\|\Delta_{U_i}F_i\right\|^2 - v_i\underbrace{\left(0.5\left\|\Delta_{U_i}F_i\right\|^2 + \left\|F_i\right\|^2 + \frac{2\sqrt{0.5}}{2}F_i^T\Delta_{U_i}F_i\right)}_{>0} + v_i\frac{2\sqrt{0.5}}{2}F_i^T\Delta_{U_i}F \leq \qquad \text{(F10)} \\
&\leq -0.5v_i\left\|\Delta_{U_i}F_i\right\|^2 + \sqrt{0.5}v_iF_i^T\Delta_{U_i}F_i \leq -0.25v_i\left\|\Delta_{U_i}F_i\right\|^2 + \sqrt{0.125}v_i\left\|F_i\right\|^2 .
\end{aligned}
$$

Use (E9) and (E10) and get the desired score for $\Delta_{U_i}F_i^T\Delta_{U_i}F_i$

$$
\Delta_{U_i}F_i^T\Delta_{U_i}F_i \leq -\left(\pi_i + 0.25v_i\right)\left\|\Delta_{U_i}F_i\right\|^2 + 2\pi_i\rho_i d_i V_i + \sqrt{0.125}v_i\eta_i\bar{\alpha}_{X_i}
$$

where $\left\|F_i\right\|^2 \leq \eta_i\bar{\alpha}_{X_i}$. ■

Use the estimates obtained in Appendix D. Introduce notations

$$
\min\left(\underline{\lambda}_{A_i}\underline{\chi}_{A_i}, \underline{\chi}_{\bar{A}_{ij}}\underline{\lambda}_{\bar{A}_{ij}}, \underline{\lambda}_{B_i}\underline{\chi}_{B_i}, \left(\pi_i + 0.25v_i\right)\tau^{-1}\right) = \beta_{\Delta_S,i} ,
$$

$$
\max_{\chi_{A_i}^{-1}\bar{\alpha}_{X_i}}\left[\chi_{A_i}^{-1}\bar{\alpha}_{X_i}, \sum_{j=1}^{m}\underline{\chi}_{\bar{A}_{ij}}^{-1}\bar{\alpha}_{X_{ij}}, \underline{\chi}_{B_i}^{-1}\bar{\alpha}_{u_i}, \pi_i d_i\right] \leq \vartheta_{\Delta_S,i} ,
$$

and apply to Lemma 3. Then (E6)

$$
\dot{V}_{\Delta_S,i} \leq -\beta_{\Delta_{S,i}}V_{\Delta_{S,i}} + \vartheta_{\Delta_{S,i}}\rho_i V_i + \sqrt{0.125}v_i\eta_i\bar{\alpha}_{X_i} , \qquad \text{(F11)}
$$

where $F_i^T\Delta_{U_i}F_i = -v_i\left\|F_i\right\|^2 - v_i\left\|\Delta_{U_i}F_i\right\|^2$, $v_i \geq 0$.

We obtain the system of inequalities for AS_{AS_i}

$$
\underbrace{\begin{bmatrix}\dot{V}_i \\ \dot{V}_{\Delta_{S,i}}\end{bmatrix}}_{\dot{W}_{S_i}} \leq \underbrace{\begin{bmatrix}-\mu_i & \dfrac{2}{\mu_i}\kappa_{\Delta_{S,i}} \\ \vartheta_{\Delta_{S,i}}\rho_i & -\beta_{\Delta_{S,i}}\end{bmatrix}}_{A_{W_{S_i}}}\underbrace{\begin{bmatrix}V_i \\ V_{\Delta_{S,i}}\end{bmatrix}}_{W_{S_i}} + \underbrace{\begin{bmatrix}0 \\ \sqrt{0.125}v_i\eta_i\bar{\alpha}_{X_i}\end{bmatrix}}_{L_{S,i}} ..\text{(E12)}
$$

The exponential dissipation condition for AS_{AS_i}

$$
\mu_i^2\beta_{\Delta_{S,i}} \geq 2\vartheta_{\Delta_{S,i}}\rho_i\kappa_{\Delta_{S,i}} . \qquad \text{(F13)}
$$

If we introduce a comparison system (see appendix D), then we get the estimate for AS_{AS_i}

$$
W_{S_i}(t) \leq e^{A_{W_{S_i}}(t-t_0)}S_{W_{S_i}}(t_0) + \int_{t_0}^{t} e^{A_{W_{S_i}}(t-\sigma)}L_{S,i}d\sigma . ■
$$

References

1. Hua C., Guan X., Shi P. Decentralised robust model reference adaptive control for interconnected time-delay systems. *Proceeding of the 2004 American Control Conference Boston, Massachusetts June 30 - July 2, 2004.* 2004:4285-4289.
2. Yang Q., Zhu M., Jiang T. He J., Yuan J., Han J. Decentralised Robust Adaptive output feedback stabilization for interconnected nonlinear systems with uncertainties. *Journal of control science and engineering*. 2016; article ID 3656578:12.

3. Wu H. Decentralised adaptive robust control of uncertain large-scale non-linear dynamical systems with time-varying delays. *IET Control Theory and Applications*. 2012;6(5):629–640.
4. Fan H., Han L., Wen C., Xu L. Decentralised adaptive output-feedback controller design for stochastic nonlinear interconnected systems. *Automatica*. 2012;48(11):2866-2873.
5. Bronnikov A.M., Bukov V.N. Decentralized adaptive management with identification and model coordination in multicommunicated systems 2010:17. https://www.researchgate.net/publication/273038561_Decentralizovannoe_adaptivnoe_upravlenie_s_identifikaciej_i_modelnoj_koordinaciej_v_mnogosvaznyh_sistemah.
6. Benitez V.H., Sanchez E.N., Loukianov A.G. Decentralised adaptive recurrent neural control structure. *Engineering Applications of Artificial Intelligence*. 2007, 20:1125–1132.
7. Lamara A., Colin G., LANUSSE P., Chamaillard Y., Charlet A. Decentralised robust control-system for a non-square MIMO system, the air-path of a turbocharged Diesel engine. 2012 Workshop on engine and powertrain control, simulation and modelling. *The International Federation of Automatic Control Rueil-Malmaison*, France, October 23-25, 2012:130-137.
8. Nguyen T., Mukhopadhyay S. Identification and optimal control of large-scale systems using selective decentralization. *2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC), Budapest, Hungary*, 2016:000503-000508, doi: 10.1109/SMC.2016.7844289.
9. Gudi R.D., Rawlings J.B., Venkat A., Jabbar N. Identification for decentralised MPC. *IFAC Proceedings Volumes*. 2004, 37(9):299-304.
10. Cho S., Park J.-W., Sim S.-H. Decentralised System Identification Using Stochastic Subspace Identification for Wireless Sensor Networks. *Sensors*. 2015, 15(4):8131-8145. https://doi.org/10.3390/s150408131.
11. Mao X., He J. Decentralised System Identification Method for Large-Scale Networks, 2022 *American Control Conference (ACC), Atlanta, GA, USA,* 2022:5173-5178. doi: 10.23919/ACC53348.2022.9867516.
12. Mei H., Cai W.-J., Xiong Q. Decentralised closed-loop parameter identification for multivariable processes from step responses. *Mathematics and Computers in Simulation. 2005*, 68(2):171-192.
13. Ioannou P.A. Decentralised adaptive control of interconnected systems. *IEEE Transactions on automatic control*, 1986,31(4):291-298.
14. Zhong-Ping Jiang. Decentralised and adaptive nonlinear tracking of large-scale systems via output feedback. *IEEE Transactions on Automatic Control*, 2000, 45(11):2122-2128. doi: 10.1109/9.887638.
15. Li X.-J., Yan G.-H. Adaptive decentralised control for a class of interconnected nonlinear systems via backstepping approach and graph theory. *Automatica*. 2017, 76: 87-95.
16. Karabutov N.N. Introduction to structural identifiability of nonlinear systems. Moscow: URSS/Lenand, 2021.
17. Karabutov N. Structural-parametrial design method of adaptive observers for nonlinear systems. *I.J. Intelligent Systems and Applications*, 2018, 2:1-16. doi: 10.5815/ijisa.2018.02.01.
18. Gromyko V. D., Sankovsky E. A. *Self-adjusting systems with a model*. Moscow: Energy: 1974.
19. Podval'ny S.L., Vasil'ev E.M. Multi-level signal adaptation in non-stationary control systems *Bulletin of Voronezh state technical university*. 2022, 18(5):38-45.
20. Karabutov N.N. On adaptive identification of systems having multiple nonlinearities. *Russ. Technol. J.* 2023;11(5):94–105.