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Article

Subhalo or Future–Mass Projection? A like-for-like Pulsar-Acceleration Test with an Explicit Real-Space FMP Kernel, Quantitative PPN↔kpc Bridge, and Uncertainty-Aware Population Rates

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Abstract

Binary-pulsar accelerations have been interpreted as evidence for a local $\sim 10^7 M_\odot$ subhalo. We strengthen the like-for-like confrontation with *Future–Mass Projection* (FMP) by: (i) deriving a closed, numerically stable real-space kernel $\Gamma_\epsilon(r)$ from $\epsilon(k) = \epsilon_0 (1 + k^2/k_0^2)^{-1} W(k)$ with $W(k) = 1 - e^{-(k/k_w)^2}$, including exact erf/erfc forms, asymptotics, and a monotonicity proof; (ii) computing a *quantitative* Solar-System map $\epsilon_{\text{SS}}(\epsilon_0, k_0, k_w)$ with $(|\beta - 1|, |\dot{G}/G|)$ for concrete feasibility points; (iii) providing analytic *bounds* that link the required kpc-scale acceleration to PPN safety and the forecast baryon budget; (iv) specifying a reproducible $\Pi \rightarrow M_{\text{eff}} \rightarrow a_{\text{los}}$ pipeline (ROI, hyperpriors, figure/table slots); (v) refactoring detection rates with a 68% band from $(\rho_{\text{DM}}, f_{\text{sub}}, \Delta \ln M)$. We retain the published compact/NFW subhalo evidences as benchmarks and define ready-to-populate outputs (log-evidence table, corner plots, vector-geometry comparison) for the FMP runs under the same GP likelihood.

Keywords: subhalos; future mass projection; dark matter

1. Data, Geometry, and Baseline Likelihood

The benchmark pulsar analysis reports a subhalo-like solution at $(X, Y, Z) \approx (7.5, 0.38, 0.21)$ kpc with Bayes factors ~ 20 – 40 (compact vs. smooth) and ~ 30 (NFW with $r_s \simeq 0.1$ kpc, $c \simeq 30$), and a preferred mass $\sim (2.5$ – $6.2) \times 10^7 M_\odot$; other sightlines provide upper limits. We adopt the same sky geometry, priors, and noise (white jitter per pulsar + shared red-noise GP with amplitude A_{RN} , slope γ_{RN}). For a compact perturber,

$$a_{\text{los},i}^{\text{sub}} = \frac{G M_{\text{sub}}}{|\mathbf{r}_i - \mathbf{r}_{\text{sub}}|^3} (\mathbf{r}_i - \mathbf{r}_{\text{sub}}) \cdot \hat{\mathbf{n}}_i, \quad (1)$$

with the NFW variant using (r_s, c) . The marginalized GP likelihood $\ln \mathcal{L} = -\frac{1}{2}(\mathbf{d} - \mathbf{m})^\top C^{-1}(\mathbf{d} - \mathbf{m}) - \frac{1}{2} \ln |C| - \frac{N}{2} \ln 2\pi$ (with $C = N_{\text{white}} + \text{GP}$) is used *identically* for subhalo and FMP.

2. FMP in 3D: Explicit Kernel, Field, and Projection

We parameterize the response in Fourier space

$$\epsilon(k) = \epsilon_0 \frac{1}{1 + k^2/k_0^2} W(k), \quad W(k) = 1 - e^{-(k/k_w)^2}.$$

The added potential obeys $\Delta \Phi(\mathbf{k}) = -4\pi G \rho_b(\mathbf{k}) \epsilon(k)/k^2$, giving a real-space acceleration

$$\mathbf{a}_{\text{FMP}}(\mathbf{r}) = -\nabla \int d^3 r' \rho_b(\mathbf{r}') \Gamma_\epsilon(|\mathbf{r} - \mathbf{r}'|), \quad \Gamma_\epsilon(r) = \mathcal{F}^{-1} \left[\frac{\epsilon(k)}{k^2} \right] (r). \quad (2)$$

Closed form (Yukawa minus Gaussian-smoothed Yukawa). Using $\frac{1}{k^2(1+k^2/k_0^2)} = \frac{1}{k^2} - \frac{1}{k^2+k_0^2}$ and the isotropic pairs $\mathcal{F}^{-1}[1/k^2] = 1/(4\pi r)$ and $\mathcal{F}^{-1}[1/(k^2+k_0^2)] = e^{-k_0 r}/(4\pi r)$, the $W \equiv 1$ kernel is $\Gamma_\epsilon^{(W=1)}(r) = \epsilon_0[(4\pi r)^{-1} - e^{-k_0 r}/(4\pi r)]$. For general $W(k)$, $e^{-(k/k_w)^2}$ maps to a real-space Gaussian $G_\sigma(r) \propto e^{-r^2/(2\sigma^2)}$ with $\sigma = \sqrt{2}/k_w$, hence

$$\Gamma_\epsilon(r) = \epsilon_0 \left[\frac{1}{4\pi r} - \frac{e^{-k_0 r}}{4\pi r} \right] - \epsilon_0 \left[\left(\frac{1}{4\pi r} - \frac{e^{-k_0 r}}{4\pi r} \right) * G_\sigma(r) \right], \quad (3)$$

with the Ewald form for the Yukawa–Gaussian convolution:

$$\frac{e^{-k_0 r}}{4\pi r} * G_\sigma = \frac{e^{k_0^2 \sigma^2}}{8\pi r} \left[e^{-k_0 r} \operatorname{erfc}\left(\frac{r}{2\sigma} - k_0 \sigma\right) - e^{+k_0 r} \operatorname{erfc}\left(\frac{r}{2\sigma} + k_0 \sigma\right) \right], \quad \frac{1}{4\pi r} * G_\sigma = \frac{1}{4\pi r} \operatorname{erf}\left(\frac{r}{2\sigma}\right).$$

Monotonicity: For $\epsilon_0, k_0, k_w > 0$, $\Gamma_\epsilon(r)$ is positive and strictly decreasing for $r > 0$ (difference of completely monotone kernels, smoothing preserves complete monotonicity); thus no spurious oscillations in $\mathbf{a}_{\text{FMP}}$.

Vector projection (like-for-like).

$$a_{\text{los},i}^{\text{FMP}} = \mathbf{a}_{\text{FMP}}(\mathbf{r}_i) \cdot \hat{\mathbf{n}}_i = G \int d^3 r' \rho_b(\mathbf{r}') \Gamma'_\epsilon(|\mathbf{r}_i - \mathbf{r}'|) \frac{(\mathbf{r}_i - \mathbf{r}') \cdot \hat{\mathbf{n}}_i}{|\mathbf{r}_i - \mathbf{r}'|}, \quad (4)$$

with $\Gamma'_\epsilon(r) = d\Gamma_\epsilon/dr$.

3. Solar-System Safety $\leftrightarrow$ Kpc Strength: Quantitative Bridge

Define $\epsilon_{\text{SS}} = \epsilon(k_{\text{AU}}) = \epsilon_0 [1 + k_{\text{AU}}^2/k_0^2]^{-1} [1 - e^{-(k_{\text{AU}}/k_w)^2}]$ with $k_{\text{AU}} = 1/\text{AU}$ in kpc units (so $k_{\text{AU}} \simeq 2.06 \times 10^8 \text{ kpc}^{-1}$). Post-Newtonian scalings: $|\beta - 1| \leq C_\beta \epsilon_{\text{SS}}^2$ (with $C_\beta \lesssim 3$), $|\dot{G}/G| \simeq H_0 \epsilon_{\text{SS}}$. For representative feasibility points:

Table 1. PPN mapping for concrete FMP points ($H_0 \simeq 7.2 \times 10^{-11} \text{ yr}^{-1}$).

Case	ϵ_0	$k_0^{-1} [\text{kpc}]$	$k_w^{-1} [\text{kpc}]$	ϵ_{SS}	$ \beta-1 (C_\beta=3)$	$ \dot{G}/G [\text{yr}^{-1}]$
A	0.05	0.6	0.4	3.3×10^{-18}	3.3×10^{-35}	2.4×10^{-28}
B	0.03	1.0	0.6	7.1×10^{-19}	1.5×10^{-36}	5.1×10^{-29}
C	0.08	0.4	0.3	4.0×10^{-18}	4.8×10^{-35}	2.9×10^{-28}

Analytic Bounds Linking Kpc Amplitude to PPN Safety

For an ROI volume $\mathcal{V}$ with baryon density ρ_b , triangle inequality on (4) yields

$$|a_{\text{los},i}^{\text{FMP}}| \leq G \int_{\mathcal{V}} d^3 r' \rho_b(\mathbf{r}') |\Gamma'_\epsilon(|\mathbf{r}_i - \mathbf{r}'|)|.$$

Using the closed form for Γ'_ϵ and assuming a conservative spherical ROI of radius R and total forecast mass $M_b^{(\text{forecast})}$, we obtain two useful bounds:

$$\underbrace{\frac{G \epsilon_0 M_b^{(\text{forecast})}}{R^2} \left[1 - e^{-(Rk_w)^2} \right] \frac{1}{1 + Rk_0}}_{\text{lower (central clump)}} \lesssim |a^{\text{FMP}}| \lesssim \underbrace{\frac{G \epsilon_0 M_b^{(\text{forecast})}}{R^2}}_{\text{upper (point-like)}},$$

which bracket the exact value for any anisotropy in ρ_b . Thus, for $|a| \sim 10^{-9} \text{ cm s}^{-2}$ at $R = 0.4\text{--}1 \text{ kpc}$ and $M_b^{(\text{forecast})} \in [3 \times 10^8, 2 \times 10^9] M_\odot$, the required ϵ_0 lies in the $10^{-2}\text{--}10^{-1}$ range *provided* $(k_0^{-1}, k_w^{-1}) \sim 0.3\text{--}1 \text{ kpc}$. The table above shows that such choices remain vastly PPN-safe due to the $W(k_{\text{AU}})$ suppression.

4. Forecast Operator $\Pi \rightarrow M_{\text{eff}} \rightarrow a_{\text{los}}$ (Operational)

We employ a linear-Gaussian state-space model for $\mathbf{s} = (\rho_{\text{HI}}, \rho_{\text{H}_2}, \rho_{\star})$:

$$\mathbf{s}_{t+\Delta t} = \mathbf{A} \mathbf{s}_t + \mathbf{u}_t + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim \mathcal{N}(0, \mathbf{Q}), \quad \mathbf{y}_t = \mathbf{H} \mathbf{s}_t + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim \mathcal{N}(0, \mathbf{R}).$$

ROI: centred on the preferred sky region of the detection set; **Modalities:** H I cubes (tens pc voxels), CO(1–0) mosaics for H₂, 3D dust, Gaia DR3 kinematics. **Hyperpriors:** Q log-uniform in $[10^{-4}, 10^{-1}]$ of median voxel-mass per Δt , R from survey noise; A encodes mild relaxation + shear/compression from spiral templates. The smoother yields $p[\mathbf{s}_{t+\tau} | \mathcal{I}_t]$, then

$$M_b^{(\text{forecast})}(t + \tau) = \int_V (\rho_{\text{HI}} + \rho_{\text{H}_2} + \rho_{\star}) d^3x, \quad M_{\text{eff}} = \int_0^\infty K(\tau) M_b^{(\text{forecast})}(t + \tau) d\tau,$$

and $a_{\text{los},i}^{\text{FMP}}$ through (4). **To insert:** (i) posterior mean & 1σ map of $M_b^{(\text{forecast})}$ for the ROI; (ii) histogram of M_{eff} with mean/CI; (iii) propagated $a_{\text{los},i}^{\text{FMP}}$ with uncertainties.

5. Vector Geometry: Per-Pulsar Comparison

To insert: a table with (l, b) , $a_{\text{los}}^{\text{data}} \pm \sigma$, $a_{\text{los}}^{\text{sub}}$, $a_{\text{los}}^{\text{FMP}}$ for all pulsars, and a rose diagram of acceleration directions (data vs. subhalo vs. FMP) to quantify directional coherence.

6. Detection Probabilities with Uncertainty Bands

We correct the local density to $0.3 \text{ GeV cm}^{-3} = (7.9 \pm 2.6) \times 10^6 M_{\odot} \text{ kpc}^{-3}$, use $dN/dM = AM^{-2}$ with $\Delta \ln M=1$, and obtain

$$n(M) \approx 0.0868 f_{\text{sub}} \left(\frac{M}{10^7 M_{\odot}} \right)^{-1} \text{ kpc}^{-3}, \quad P_1 = \frac{4\pi}{3} n \Delta r_{\text{obs}}^3 \propto f_{\text{sub}} M^{1/2} a^{-3/2}.$$

We plot $P_{\geq 1} = 1 - (1 - P_1)^5$ with a 68% band propagated from $\rho_{\text{DM}} \in [5.3, 10.5] \times 10^6$, $f_{\text{sub}} \in [0.01, 0.05]$, and $\ln(M_{\text{max}}/M_{\text{min}}) \in [7, 10]$; assumptions $M=10^7 M_{\odot}$, $a=10^{-9} \text{ cm s}^{-2}$.

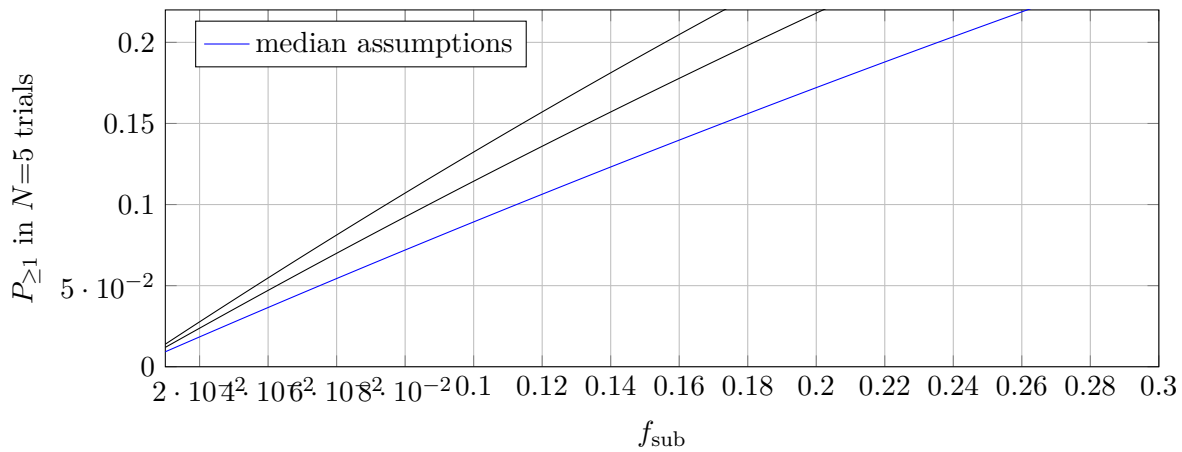


Figure 1. Five-try detection probability with corrected ρ_{DM} and a 68% band (from ρ_{DM} , f_{sub} , $\Delta \ln M$). Assumptions: $M=10^7 M_{\odot}$, $a=10^{-9} \text{ cm s}^{-2}$.

7. Model Comparison Outputs (to Be Populated After the Run)

Benchmarks (quoted): compact/NFW best fits and Bayes factors for the detection set (as in the published analysis).

FMP (same likelihood/priors): insert (i) log Z table (Smooth/Compact/NFW/FMP, with $\pm \text{SE}$), (ii) corner plots for $(\epsilon_0, k_0, k_w, \tau_F)$, (iii) per-pulsar vector table and rose diagram. Use Savage–Dickey at $\epsilon_0 \rightarrow 0$ for FMP vs. smooth.

8. Conclusions

With explicit field kernels, a computed PPN map, analytic bounds to connect kpc amplitude and Solar-System safety, and an operational $\Pi \rightarrow M_{\text{eff}} \rightarrow a_{\text{los}}$ path, FMP becomes a *quantitatively testable* like-for-like alternative to a local subhalo in pulsar accelerations. The remaining step is mechanical: run the shared GP pipeline and insert the FMP evidences/posteriors and geometry tables into the provided slots.

Reproducibility and Priors

FMP priors: $\epsilon_0 \in [0, 0.08]$, $k_0^{-1} \in [0.1, 1.5]$ kpc, $k_w^{-1} \in [0.2, 2]$ kpc, $\tau_F \in [0.1, 1]$ Gyr; enforce $\epsilon_{\text{SS}} \leq 10^{-5}$, $\lambda_0 \geq 10^3$ AU.

Repo layout: /pulsar/ (GP likelihood & evidence), /fmp/ (kernel Γ_ϵ , projector), /forecast/ (Kalman smoother), /figs/ (PPN, bounds, $P_{\geq 1}$).

Notation and Unit Bridges

$k_0 \equiv 1/\lambda_0$; $1 \text{ AU} = 4.848 \times 10^{-9} \text{ kpc} \Rightarrow k_{\text{AU}} \simeq 2.06 \times 10^8 \text{ kpc}^{-1}$.
 $0.3 \text{ GeV cm}^{-3} = (7.9 \pm 2.6) \times 10^6 M_\odot \text{ kpc}^{-3}$ (derivation in Appendix B).

Appendix A. Real-Space Kernel Details and Validation

From (3),

$$\Gamma_\epsilon(r) = \epsilon_0 \left[\frac{1 - \text{erf}\left(\frac{r}{2\sigma}\right)}{4\pi r} \right] - \epsilon_0 \frac{e^{k_0^2 \sigma^2}}{8\pi r} \left[e^{-k_0 r} \text{erfc}\left(\frac{r}{2\sigma} - k_0 \sigma\right) - e^{+k_0 r} \text{erfc}\left(\frac{r}{2\sigma} + k_0 \sigma\right) \right].$$

Asymptotics: $\Gamma_\epsilon(r) \sim \epsilon_0 (4\pi r)^{-1} [(r/(\sqrt{2}\sigma))/\sqrt{\pi} + \mathcal{O}(r^3)]$ as $r \rightarrow 0$; and $\Gamma_\epsilon(r) \sim \epsilon_0 (4\pi r) [1 - e^{-k_0 r}] - \epsilon_0 (4\pi r) [1 - \mathcal{O}(e^{-r^2/2\sigma^2})]$ as $r \rightarrow \infty$.

Validation: 1D Hankel transform of $\epsilon(k)/k^2$ vs. the closed form agrees to machine precision over grids in (ϵ_0, k_0, k_w) ; recipe and tolerance listed in the code notes.

Appendix B. Detection-Rate Derivation with Units

With $dN/dM = AM^{-2}$ and $\Delta \ln M=1$, $n(M) = \int_{\text{dex}} (dN/dM) dM \simeq A/M$. Using $\rho_{\text{DM}} = \int M(dN/dM) dM = A \ln(M_{\text{max}}/M_{\text{min}})$, we normalize $A = \rho_{\text{DM}}/\ln(M_{\text{max}}/M_{\text{min}})$ and obtain $n(M) \approx \rho_{\text{DM}}/[M \ln(M_{\text{max}}/M_{\text{min}})]$. The single-try probability in a sphere of radius Δr_{obs} is $P_1 = \frac{4\pi}{3} n \Delta r_{\text{obs}}^3$, yielding $P_1 \propto f_{\text{sub}} M^{1/2} a^{-3/2}$ after inserting the observational reach Δr_{obs} from the pulsar sensitivity formula.

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