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Article

Firewalls, Hawking (Tolman) Radiation, and a Tentative Resolution of the Firewall-Mass Problem

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Abstract: It has been theorized that black holes are surrounded by firewalls, although there is not universal agreement concerning this. We first review basic concepts pertaining to Schwarzschild black holes and Hawking radiation. Then we discuss the anticipation of Hawking radiation—albeit from *non*-black holes—initially by R. C. Tolman and shortly thereafter with P. Ehrenfest. We compare evaporation into a vacuum at absolute zero (0 K) of black holes with that of non-black holes, and show that not only black holes but also *non*-black holes evaporate within a *finite* time. The times required for evaporation of black holes and non-black holes are compared. Next, we show that (i) if firewalls exist, they can originate via Hawking radiation at the minimum possible ruler distance (the Planck length) beyond the Schwarzschild horizon, where it has not suffered any gravitational redshift, or, alternatively, suffered maximal gravitational blueshift and (ii) the firewall temperature is on the order of the Planck temperature, *in*dependently of the mass and hence also of the Schwarzschild radius of a Schwarzschild black hole. We then explain the exponential nature of the gravitational frequency shift as a function of the gravitational potential. Next, we consider the firewall-mass problem, and provide an at least *prima facie* tentative resolution thereto based on: (i) the mass of a firewall being canceled by the negative gravitational mass=(negative gravitational energy)$/c^2$ accompanying its formation, (ii) the *un*changed observations of a distant observer upon formation of a firewall, and (iii) Birkhoff's Theorem (actually first discovered by Jørg Tofte Jebsen). We then consider one aspect of thermodynamics in gravitational fields, showing that equilibrium relativistic gravitational temperature gradients can*not* be exploited to violate the Second Law of Thermodynamics.

Keywords: Schwarzschild black holes; Schwarzschild non-black holes; ruler distance; Planck units; Hawking radiation; Tolman radiation; gravitational frequency shift; firewall mass; negative gravitational mass-energy; Birkhoff's Theorem; thermodynamic equilibrium; First Law of Thermodynamics; Second Law of Thermodynamics

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1. Introduction

It has been theorized that black holes are surrounded by firewalls [1–7], although there is not universal agreement concerning this [1–7]. There is a vast literature exploring this topic, of which we have cited only a tiny sample. But the many works cited, and discussed, in our cited Refs. [1–7] could provide at least the beginning of a thorough literature search [1–7].

In Section 2, we review basic concepts pertaining to Schwarzschild black holes and Hawking radiation. In Section 3, we discuss the anticipation of Hawking radiation—albeit from *non*-black holes—initially by R. C. Tolman and shortly thereafter with P. Ehrenfest, and Tolman's [8,9] proof, bolstered with Ehrenfest [10], that *any* gravitator—black hole or *non*-black hole—*must radiate* and hence can*not* be in thermodynamic equilibrium with a surrounding vacuum at (or at least sufficiently close to) absolute zero (0 K), but *must completely evaporate into that vacuum within a finite time*. The times required for evaporation of black holes and non-black holes are compared. (See also Garrod [11].) This has been corroborated by recent research [12]. In Section 4, we show that (i) if firewalls exist, they can originate via Hawking radiation at the minimum possible ruler distance [13] (the Planck length [14–16]) beyond the Schwarzschild horizon, where it has not suffered any gravitational redshift [17,18], or, alternatively, suffered maximal gravitational blueshift [17,18] and (ii) the firewall temperature is on the order of the Planck temperature [19], *in*dependently of the mass and hence also of the Schwarzschild radius of a Schwarzschild black hole. We emphasize and focus on *ruler* distance [13] because, unlike other distance

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measures in General Relativity [13], ruler distance—*uniquely!* [13]—is actual *physical* distance: the distance between two points as measured by rulers laid upon the shortest possible spatial path separating them and hence the *physical* distance separating them [13]. (We employ other distance measures where they are more applicable.) In Section 5, we explain the exponential nature of the gravitational frequency shift as a function of the gravitational potential. In Section 6, we consider the firewall-mass problem [3], and provide an at least *prima facie* tentative resolution thereto based on: (i) the mass of a firewall being canceled by the *negative* gravitational mass [17,18] $=$ (*negative* gravitational energy)$/c^2$ [17,18] accompanying its formation, (ii) the *un*changed observations of a distant observer upon formation of a firewall, and (iii) Birkhoff's Theorem [20–31]—actually first discovered by Jørg Tofte Jebsen [25–28]. We show that the mass of a firewall is *exactly* counterbalanced by the (negative) gravitational mass-energy accompanying its formation. Perhaps this may complement other lines of reasoning [4] disputing massiveness [3] of firewalls. (There is a caveat [28–31][1] with respect to Birkhoff's Theorem [20–31], but it [28–31][1] is *not* relevant with respect to our considerations.[1]) In Section 7, we consider one aspect of thermodynamics in gravitational fields, showing that equilibrium relativistic gravitational temperature gradients can*not* be exploited to violate the Second Law of Thermodynamics. A concluding synopsis is provided in Section 8. Auxiliary topics are discussed in the Notes.

2. Review of Basic Concepts Pertaining to Schwarzschild Black Holes and Hawking Radiation

The Schwarzschild metric of a Schwarzschild black hole of mass M and Schwarzschild radius $r_S = 2GM/c^2$ is [32]

$$\begin{aligned} ds^2 &= \left(1-\frac{2GM}{rc^2}\right)c^2dt^2-\left(1-\frac{2GM}{rc^2}\right)^{-1}dr^2-r^2\left(d\theta^2-\sin^2\theta d\phi^2\right) \\ &= \left(1-\frac{r_S}{r}\right)c^2dt^2-\left(1-\frac{r_S}{r}\right)^{-1}dr^2-r^2\left(d\theta^2-\sin^2\theta d\phi^2\right) \\ &= c^2d\tau^2-dl^2-r^2\left(d\theta^2-\sin^2\theta d\phi^2\right). \end{aligned} \tag{1}$$

Schwarzschild-coordinate radial distance r is *not* radial *ruler* distance but is radial area distance and also radial distance from apparent size [13]. By contrast, in accordance with the *Euclidean* form of the angular term of the Schwarzschild metric [the last term in all three lines of Equation (1)], a spherical shell at r_{shell} has ruler-distance circumference $C_{\text{shell}} = 2\pi r_{\text{shell}}$ and ruler-distance surface area $A_{\text{shell}} = 4\pi r_{\text{shell}}^2$ [13]. At $r \geq r_S$, l—*not* r—is radial *ruler* distance, t is Schwarzschild-coordinate time (proper time measured by a clock at rest at $r \to \infty$) [17,18], and τ is proper time measured by a clock at rest at *any* given $r \geq r_S$ [17,18]. [A clock at rest *at* $r = r_S$—if such a clock can exist—must be constructed entirely of photons (and/or other zero-rest-mass particles)!]

Although not necessary for our derivations, it may be helpful, as an aside, to briefly remark on the following three features of the Schwarzschild metric [Equation (1)]: (i) Setting $ds^2 = 0$ in Equation (1) shows that the *physical* radial velocity of light $V_{\text{phys,light}} = dl/d\tau = c$ at *all* $r \geq r_S$ [32]; but, by contrast, the Schwarzschild-*coordinate* radial velocity of light $V_{\text{coor,light}} = dr/dt = c[1-(r_S/r)]$ decreases monotonically with decreasing r from c at $r \to \infty$ to zero at $r = r_S$ [32]. (ii) We focus on distance [13], especially on *ruler* distance [13], and most especially on *radial ruler* distance [13], *beyond* the Schwarzschild horizon r_S in Schwarzschild spacetime [32]. But it may be interesting to note that, in accordance with the *angular* term of the Schwarzschild metric [the last term in all three lines of Equation (1)] being of the *identical Euclidean* form at *all* $r \geq 0$ [32],[2] *even within* r_S,[2] where a spherical shell can*not* be at rest but *must* be collapsing, while falling through a given $r_{\text{shell}} < r_S$ it has ruler-distance circumference $C_{\text{shell}} = 2\pi r_{\text{shell}}$ and ruler-distance surface area $A_{\text{shell}} = 4\pi r_{\text{shell}}^2$ [32].[2] *Even within* r_S,[2] where r becomes time*like*, r does *not* become time itself:[2] *un*like time itself r still retains these *spatial* geometrical attributes [32].[2] Time itself has *no spatial* geometrical attributes. (iii) Because the gravitational field of a Schwarzschild black hole is purely radial, it seems intuitive that this gravitational field's stretching [33,34][3] of space from the Euclidean [33,34][3] is *purely* in the *vertical*[3]

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radial r direction, and *not at all* in the *horizontal* angular θ and ϕ directions: thus the *identical Euclidean* form of the angular term of the Schwarzschild metric [the last term in all three lines of Equation (1)] at *all* $r \geq 0$ [32]. Indeed, more generally, intuition suggests that stretching [33,34][3] of space from the Euclidean by *any* gravitational field is *purely* in the *vertical* direction [33,34],[3] and *not at all* in any *horizontal* direction [33,34].[3] This intuition augments the immediately preceding Item (ii), and is perhaps most clear in relation to Sakharov's elastic-strain theory of gravity [35–40].[4,5] (Thus, might space be the ether [41,42]?[4,5])

We will be concerned only with *radial* motions of photons in the gravitational fields of Schwarzschild black holes, because only *radial* motions can result in gravitational frequency shifts. In this regard we will be concerned only with the *temporal-radial* part of the Schwarzschild metric [Equation (1)], hence ignoring the angular term thereof (the last term in all three lines thereof).

Hawking radiation, the (at least essentially) blackbody radiation from a Schwarzschild black hole, is most typically construed to have the temperature [16,43–49]

$$T_{\mathrm{H},r\to\infty} = \frac{\hbar c^3}{8\pi GkM} = \frac{\hbar c^3}{8\pi Gk\frac{r_\mathrm{S}c^2}{2G}} = \frac{\hbar c}{4\pi k r_\mathrm{S}}, \tag{2}$$

where k is Boltzmann's constant, M is the mass of the black hole, and $r_\mathrm{S} = 2GM/c^2$ is its Schwarzschild radius [16,43–49]. Note that $T_{\mathrm{H},r\to\infty}$ given by Equation (2) is the temperature of Hawking radiation at a great distance from a Schwarzschild black hole, i.e., at $r \to \infty$, hence after Hawking radiation having suffered the maximum possible gravitational redshift [16–18,43–49]. For all non-primordial black holes, which are all of stellar mass or larger, $T_{\mathrm{H},r\to\infty}$ is extremely low compared to the current temperature $T_\mathrm{CBR} = 2.725\,\mathrm{K}$ [50] of the cosmic background radiation [50]. For sufficiently small primordial black holes [51–61] ($M < \frac{\hbar c^3}{8\pi GkT_\mathrm{CBR}} = 4.50 \times 10^{22}\,\mathrm{kg} \iff r_\mathrm{S} < \frac{\hbar c}{4\pi kT_\mathrm{CBR}} = 6.69 \times 10^{-5}\,\mathrm{m}$), $T_{\mathrm{H},r\to\infty} > T_\mathrm{CBR} = 2.725\,\mathrm{K}$ obtains. But as of this writing, to the best knowledge of the author, no such sufficiently small primordial black holes—indeed, no primordial black holes at all—have been discovered [51–61]. Moreover, while there are rationales according to which primordial black holes might contribute, perhaps significantly, to cold dark matter, there also are both theoretical and observational upper limits on their abundance [51–61] and therefore also on their actual contribution to cold dark matter [51–61]. Hence, while it is possible that they could contribute, perhaps significantly, to cold dark matter, as of this writing, to the best knowledge of the author, it is uncertain whether or not they actually exist [51–61].

Thus far we have considered $T_{\mathrm{H},r\to\infty}$. But $T_{\mathrm{H},r}$ at smaller values of r ($r_S \leq r < \infty$) has been discussed as well [49]. Closer to r_S (at $r_S \leq r < \infty$) [49] than at $r \to \infty$, Hawking radiation has suffered less [17,18,49] gravitational redshift [17,18] and hence has a higher [49] temperature [16,43–49]

$$T_{\mathrm{H},r} = T_{\mathrm{H},r\to\infty}\left(1 - \frac{r_\mathrm{S}}{r}\right)^{-1/2} = \frac{\hbar c^3}{8\pi GkM}\left(1 - \frac{r_\mathrm{S}}{r}\right)^{-1/2} = \frac{\hbar c}{4\pi k r_\mathrm{S}}\left(1 - \frac{r_\mathrm{S}}{r}\right)^{-1/2}. \tag{3}$$

3. Tolman's Anticipation of Hawking Radiation: *All* Gravitators—Black Holes *and Non*-Black Holes—*Must Radiate*

It is *extremely important* to note that Equation (3) is a special case of the more general result derived by Tolman [8,9], bolstered with Ehrenfest [10]. The last two terms of Tolman's Equation (128.6) and the entirety of Tolman's Equation (129.10) in Ref. [9] read:

$$T_\mathrm{T}(g_{tt})^{1/2} = C_\mathrm{T} \iff T_\mathrm{T} = C_\mathrm{T}(g_{tt})^{-1/2}, \tag{4}$$

where (i) C_T is Tolman's constant in Equations (128.6) and (129.10) of Ref. [9], and (ii) T_T is the Tolman gravitational thermodynamic-equilibrium temperature as measured by a local observer, written more explicitly as $T_\mathrm{T}(r,\theta,\phi)$, at a specified location [specified radial area distance and also radial distance from apparent size [13] r and specified direction (θ,ϕ)] from the center of mass of a gravitator (spherically symmetrical or otherwise) where the time-time component of the metric has the value $g_{tt}(r,\theta,\phi)$. [As per the first term of Equation (128.6) in Ref. [9] (this notation is also employed in Refs [8] and [10]), Tolman (with Ehrenfest in Ref. [10]) sets $e^{\nu} = g_{tt}$, but for uniformity in notation we employ only the g_{tt} symbolization. Tolman [8,9] (with Ehrenfest in Ref. [10]) employs the symbols T_0 instead of T_{T} (with the subscript $_0$ designating measurement by a local observer: see p. 489 of Ref. [9]), and C instead of C_{T}. (In Tolman's paper with Ehrenfest [10] const. is employed instead of C.) We employ the subscript $_{\mathrm{T}}$ to refer to Tolman's quantities. We take T_{T} to be the Tolman gravitational thermodynamic-equilibrium temperature as measured by a local observer (the subscript $_0$ omitted for brevity).]

We can choose Tolman's constant C_{T} for a given gravitator to be equal to $C_{\mathrm{T},r\to\infty} = \lim_{r\to\infty} T_{\mathrm{T}}(r,\theta,\phi)[g_{tt}(r,\theta,\phi)]^{1/2} = T_{\mathrm{T},r\to\infty}$, the Tolman gravitational thermodynamic-equilibrium temperature measured by a local observer at radial area distance and also radial distance from apparent size [13] $r \to \infty$ in *any* specified direction (θ,ϕ) from the center of mass of *any* gravitator (spherically symmetrical or otherwise), because spacetime approaches the Minkowskian and hence $g_{tt,r\to\infty} = \lim_{r\to\infty} g_{tt}(r,\theta,\phi) = 1$ at radial area distance and also radial distance from apparent size [13] $r \to \infty$ in *any* specified direction (θ,ϕ) from the center of mass of *any* gravitator (spherically symmetrical or otherwise):

$$C_{\mathrm{T}} = T_{\mathrm{T},r\to\infty}(g_{tt,r\to\infty})^{1/2} = T_{\mathrm{T},r\to\infty} \times 1 = T_{\mathrm{T},r\to\infty}. \tag{5}$$

Hence Equation (4) can be rewritten as:

$$\begin{aligned} &T_{\mathrm{T}}(r,\theta,\phi) = T_{\mathrm{T},r\to\infty}[g_{tt}(r,\theta,\phi)]^{-1/2} \text{ (in general)} \\ &\Longrightarrow T_{\mathrm{T}}(r) = T_{\mathrm{T},r\to\infty}[g_{tt}(r)]^{-1/2} = T_{\mathrm{T},r\to\infty}\left(1 - \frac{r_{\mathrm{S}}}{r}\right)^{-1/2} \text{ (spherical symmetry).} \end{aligned} \tag{6}$$

Tolman presents this general result not only via the last two terms of Equation (128.6) and the entirety of Equation (129.10) in Ref. [9], but also in Ref. [8] via the second equation in the Abstract and Equations (27), (28), (42), (53), and (54). It is also presented in Ref. [10] with Ehrenfest via Equations (2), (3), and (30), and discussed in some detail in Section 7 thereof. [In the weak-field limit ($M \ll r_{\mathrm{S}}c^2/2G \iff r \gg r_{\mathrm{S}} = 2GM/c^2$ given spherical symmetry) this result is presented via Equations (8) and (29) and the last (unnumbered) equation in Section 8 of Ref. [8], and also via Equations (128.4), (128.5), and (128.10) in Ref. [9].] Tolman also evaluates the gradient $d\{\ln[T_{\mathrm{T}}(r)/T_{\mathrm{T},r\to\infty}]\}/dr$ at and near Earth's surface in the last (unnumbered) equation in Section 2 of Ref. [8] and in Equation (128.7) of Ref. [9]. (In Earth's weak gravitational field, this gradient can of course with at most negligible error be construed as being either with respect to radial ruler distance [13] or with respect to radial area distance and/or radial distance from apparent size [13].) We focus on the last two terms of Equation (128.6) in Ref. [9] and of Equation (3) in Ref. [10], on Equation (129.10) in Ref. [9], and on Equation (30) and Section 7 in Ref. [10]. (See also Garrod [11].)

For Schwarzschild black holes, we should expect that $T_{\mathrm{T}} = T_{\mathrm{H}}$. Indeed, letting $T_{\mathrm{T}} \to T_{\mathrm{H}}$, the second line of Equation (6) is identical to Equation (3), taking $T_{\mathrm{T},r\to\infty} = T_{\mathrm{H},r\to\infty}$ as per Equation (2). For $T_{\mathrm{T},r\to\infty}$ of Schwarzschild (spherically-symmetrical, non-rotating) non-black holes, we will give a plausibility argument (albeit not a proof) for a conjecture concerning the value of $T_{\mathrm{T},r\to\infty}$.

We define a Schwarzschild *non*-black hole (of mass M) as a spherically-symmetrical, non-rotating gravitator whose radius r_{NBH} (as per radial area distance/radial distance from apparent size [13]) exceeds the Schwarzschild radius $r_{\mathrm{S}} = 2GM/c^2$. (The subscript $_{\mathrm{NBH}}$ may be omitted for brevity when that will not result in confusion.)

The apparent gravitational acceleration $\mathcal{G}_{\text{BH}}(r)$ towards a Schwarzschild black hole of mass M_{BH}, as measured by dangling a unit mass m at r_{S} from a higher altitude $r > r_{\text{S}}$ (with a massless string), and the limit thereof as $r \to \infty$, are [62]:

$$\mathcal{G}_{\text{BH}}(r) = \frac{c^4}{4GM_{\text{BH}}}\left(1 - \frac{r_{\text{S}}^2}{r^2}\right)^{-1/2} = \frac{GM_{\text{BH}}}{\left(\frac{2GM_{\text{BH}}}{c^2}\right)^2}\left(1 - \frac{r_{\text{S}}^2}{r^2}\right)^{-1/2} = \frac{GM_{\text{BH}}}{r_{\text{S}}^2}\left(1 - \frac{r_{\text{S}}^2}{r^2}\right)^{-1/2}$$
$$\implies \lim_{r\to\infty} \mathcal{G}_{\text{BH}}(r) \equiv \mathcal{G}_{\text{BH}} = \frac{c^4}{4GM_{\text{BH}}} = \frac{GM_{\text{BH}}}{r_{\text{S}}^2}. \tag{7}$$

The limiting value of $\mathcal{G}_{\text{BH}}(r)$ as $r \to \infty$—as per the second line of Equation (7)—is sometimes called the *surface gravity* of a Schwarzschild black hole [62]. *Fortuitously*, *as if* Newtonian theory was adequate for Schwarzschild black holes [62]! *Fortuitously* despite Newtonian theory taking all distance measures to be equivalent—not distinguishing, for example, between ruler distance [13] on the one hand, and area distance and/or distance from apparent size [13] on the other. *Fortuitously* because enormous values of $\mathcal{G}_{\text{BH}}(r)$ if r is only slightly greater than r_{S} suffer gravitational redshift down to Newtonian values in the limit $r \to \infty$. (Note: We employ g to denote components—we focus on the time-time component—of the spacetime metric, G to denote the universal gravitational constant, and $\mathcal{G}$ to denote acceleration due to gravity.) For a Schwarzschild (spherically-symmetrical non-rotating) non-black hole of mass M_{NBH} in the weak-field limit ($M \ll r_{\text{S}}c^2/2G \iff r \gg r_{\text{S}} = 2GM/c^2$)

$$\mathcal{G}_{\text{NBH}} = \frac{GM_{\text{NBH}}}{r_{\text{NBH}}^2}. \tag{8}$$

Based on the fortuitous Newtonian-equivalence of the forms of the second line of Equation (7) on the one hand and Equation (8) on the other, and applying Equation (2), we *prima facie* suggest the following conjecture for $T_{\text{T},r\to\infty}$ of a Schwarzschild non-black hole of the *same mass* M as a Schwarzschild black hole ($M_{\text{NBH}} = M_{\text{BH}} = M$) but with radius (radial area distance and also radial distance from apparent size [13]) from the center to the surface of the Schwarzschild non-black hole) $r_{\text{NBH}} > r_{\text{S}} = 2GM/c^2$:

$$T_{\text{T},r\to\infty} = T_{\text{H},r\to\infty}\frac{\mathcal{G}_{\text{NBH}}}{\mathcal{G}_{\text{BH}}} = \frac{\hbar c^3}{8\pi GkM} \times \frac{\mathcal{G}_{\text{NBH}}}{\mathcal{G}_{\text{BH}}} \text{ for all } r_{\text{NBH}} > r_{\text{S}}$$
$$= T_{\text{H},r\to\infty}\frac{\frac{GM}{r_{\text{NBH}}^2}}{\frac{GM}{r_{\text{S}}^2}} = T_{\text{H},r\to\infty}\left(\frac{r_{\text{S}}}{r_{\text{NBH}}}\right)^2 = \frac{\hbar c^3}{8\pi GkM}\left(\frac{r_{\text{S}}}{r_{\text{NBH}}}\right)^2 \left\{ \begin{array}{c} \text{weak-field limit:} \\ r_{\text{NBH}} \gg r_{\text{S}} \end{array} \right. . \tag{9}$$

We recognize that while the conjecture given by Equation (9) may, at least *prima facie*, seem *plausible*, we have not *proven* it. Nonetheless at least *prima facie* it seems a reasonable conjecture, especially given that, since

$$\lim_{r_{\text{NBH}}\to r_{\text{S}}} T_{\text{T},r\to\infty} = T_{\text{H},r\to\infty} = \frac{\hbar c^3}{8\pi GkM}, \tag{10}$$

at least it is consistent with Equation (2). However, irrespective of the validity (or lack thereof) of this conjecture, two paragraphs hence we will prove the *extremely important point* that $T_{\text{T},r\to\infty}$ *must* be finitely greater than absolute zero (0 K) for *all* non-black holes [as $T_{\text{H},r\to\infty}$ is finitely greater than absolute zero (0 K) for *all* black holes].

In Ref. [8] and in Sections 128 and 129 of Ref. [9], Tolman implies that our Equations (4)–(6) are valid in *any static* spacetime [63]. Together with Ehrenfest [10] this is also implied in Ref. [10]. But given rotation at *constant* angular velocity, a *time-independent* centrifugal potential can be incorporated into the *time-independent* gravitational potential that obtains in *static* spacetime, i.e., into that which obtains neglecting the rotation [63]. This *time-independent* gravitational-centrifugal potential would then of course be a function of θ as well as of r, but at a *given* θ it can still be expressed as a function of r alone. Thus we can construe Equations (4)–(6) to be valid in *any static or stationary* spacetime [63]. Hence these

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results proven by Tolman [8,9], bolstered with Ehrenfest [10], and summarized via our Equations (4)–(6) and the associated discussions, imply that at thermodynamic equilibrium temperature increases downwards[6] in *any static or stationary* gravitational field (the centrifugal contribution in a stationary field construed as incorporated therein given rotation at *constant* angular velocity) [63]. But for simplicity and definiteness we focus on the *static* spacetimes at $r \geq r_S$ of *Schwarzschild*, i.e., spherically-symmetrical *non*-rotating (black hole and non-black-hole) gravitators.

Even more importantly, Tolman [8,9], bolstered with Ehrenfest [10], *furthermore* implies *more than that*, as summarized via our Equations (4)–(6): it is *furthermore* implied [8–10] that $T_{T,r\to\infty}$ *must be finitely higher than absolute zero* (0 K) for *all non*-black holes, as $T_{H,r\to\infty}$ is finitely higher than absolute zero (0 K) for *all* black holes. As per Equations (4)–(6) [most explicitly as per Equation (6)], *in*correctly assuming that $T_{T,r\to\infty} = 0\,\text{K}$ *in*correctly implies that $T_T(r,\theta,\phi) = 0\,\text{K}$ obtains *everywhere*—at least, everywhere that $g_{tt}(r,\theta,\phi) > 0$ or equivalently that $[g_{tt}(r,\theta,\phi)]^{-1/2} < \infty$. In the Schwarzschild (spherically-symmetrical non-rotating) special case, wherein $g_{tt}(r) = 1 - \frac{r_S}{r} \iff [g_{tt}(r)]^{-1/2} = \left(1 - \frac{r_S}{r}\right)^{-1/2}$, for a black hole this *in*correct implication would pertain *everywhere* in the region $r \geq r_S$ except at *exactly* $r = r_S$; and for a non-black hole, whose radius exceeds r_S, this *in*correct implication would pertain *everywhere without exception*. Thus the *correct* implication is that *any* gravitator—black hole or *non*-black hole—*must radiate*: a black hole surrounded by a vacuum colder than $T_{H,r\to\infty}$ and a non-black hole surrounded by a vacuum colder than $T_{T,r\to\infty}$ can*not* be in thermodynamic equilibrium with that vacuum, but *must radiate into that vacuum and completely evaporate into that vacuum within a finite time*! Thus at least the *qualitative* fact that Hawking (Tolman!) radiation emanates from *all* gravitators—not only from black holes but also from *non*-black holes—(even if not also *quantitative* values of $T_{T,r\to\infty}$ and $T_{H,r\to\infty}$ [16,43–49]) was discovered by Tolman [8,9], bolstered with Ehrenfest [10], at least as early as 1930! *Any* gravitator—black hole or *non*-black hole—surrounded by a 0 K vacuum can*not* be at thermodynamic equilibrium unless it is enclosed within an opaque thermally insulating shell [64,65][7] and thereby insulated from that vacuum: otherwise it will *completely* Hawking- (Tolman!-) evaporate into that vacuum within a *finite* time! This has been corroborated by recent research [12].

Black holes evaporate ever more rapidly and get hotter as they lose mass, hence *completely* evaporating into a vacuum at absolute zero (0 K) within a *finite* time $\Delta t_{BH,evap}$. For evaporation of a Schwarzschild black hole into a 0 K vacuum [43–49]:

$$\begin{aligned}
\frac{dM}{dt} &= \frac{1}{c^2}\frac{dE}{dt} = -\frac{1}{c^2}A\sigma T^4_{H,r\to\infty} \\
&= -\frac{1}{c^2}4\pi r_S^2\sigma\left(\frac{\hbar c^3}{8\pi GkM}\right)^4 \\
&= -\frac{1}{c^2}4\pi\left(\frac{2GM}{c^2}\right)^2\sigma\left(\frac{\hbar c^3}{8\pi GkM}\right)^4 \\
&= -\frac{\sigma\hbar^4 c^6}{256\pi^3 G^2 k^4 M^2} \equiv -\frac{C_1}{M^2} \\
&\implies dt = -\frac{M^2}{C_1}dM \\
&\implies \Delta t_{BH,evap} = \int dt = \frac{-\int_{M_{initial}}^0 M^2 dM}{C_1} = \frac{\int_0^{M_{initial}} M^2 dM}{C_1} = \frac{M^3_{initial}}{3C_1} \\
&\doteq 8.4116\times 10^{-17}\left(\frac{M_{initial}}{1\,\text{kg}}\right)^3 \text{s} \doteq 2.0972\times 10^{67}\left(\frac{M_{initial}}{M_\odot}\right)^3 \text{y},
\end{aligned} \tag{11}$$

where the minus signs account for the black hole's mass M *de*creasing during evaporation, $A = 4\pi r_S^2$ is its (decreasing) surface area, $\sigma = 5.670374419 \times 10^{-8}$ W / m^2 K^4 is the Stefan-Boltzmann constant [66],

$$C_1 \equiv \frac{\sigma\hbar^4 c^6}{256\pi^3 G^2 k^4} \doteq 3.9628\times 10^{15}\,\text{kg}^3\,\text{s}^{-1} \doteq 1.5905\times 10^{-68} M_\odot^3\,\text{y}^{-1} \doteq \frac{M_\odot^3}{6.2873\times 10^{67}\,\text{y}}, \tag{12}$$

and $M_\odot \doteq 1.9885 \times 10^{30}$ kg is the mass of the Sun [67]. The dot-equal sign ($\doteq$) means very nearly equal to.

By contrast, *non*-black holes evaporate ever more slowly and get cooler as they lose mass. But the time rate of this slowdown is itself sufficiently slow—they get cooler sufficiently slowly—that they, too, *completely* evaporate into a vacuum at absolute zero (0 K) within a *finite* time $\Delta t_{\text{NBH,evap}}$. For a weak-field ($M_{\text{initial}} \ll r_S c^2/2G \iff r_{\text{initial}} \gg r_S = 2GM/c^2$) Schwarzschild (spherically-symmetrical non-rotating) *non*-black hole, $g_{tt}(r)$ and therefore also $[g_{tt}(r)]^{-1/2}$ is essentially constant at unity, and hence also by Equations (4)–(6) the Tolman [8–10] temperature $T_T(r)$ is essentially constant at $T_{T,r\to\infty}$, as M decreases from M_{initial} to $M_{\text{final}} = 0$ and r decreases from r_{initial} to $r_{\text{final}} = 0$ during the *entire* evaporation process. Moreover, $A = 4\pi r^2 \iff r = (A/\pi)^{1/2}/2$, and, assuming uniform density ρ for simplicity (justified in the weak-field ($r_{\text{NBH}} \gg r_S$) limit because gravity is too weak to significantly compress material with depth), also $M = 4\pi\rho r^3/3 = \rho r A/3 = \rho A^{3/2}/6\pi^{1/2} \iff A = \pi^{1/3}(6M/\rho)^{2/3}$. Hence in the weak-field ($r_{\text{NBH}} \gg r_S$) limit for evaporation of a Schwarzschild (spherically-symmetrical non-rotating) uniform-density non-black hole into a 0 K vacuum:

$$\begin{aligned}
\frac{dM}{dt} &= \frac{1}{c^2}\frac{dE}{dt} = -\frac{\sigma A T^4_{T,r\to\infty}}{c^2} = -\frac{\pi^{1/3}\sigma T^4_{T,r\to\infty}}{c^2}\left(\frac{6M}{\rho}\right)^{2/3} \equiv -C_2 M^{2/3} \\
&\implies dt = -\frac{M^{-2/3}dM}{C_2} \implies \Delta t_{\text{NBH,evap}} = \int dt = \frac{-\int_{M_{\text{initial}}}^{0} M^{-2/3}dM}{C_2} \\
&= \frac{\int^{M_{\text{initial}}}_{0} M^{-2/3}dM}{C_2} = \frac{3M^{1/3}_{\text{initial}}}{C_2}, \qquad (13)
\end{aligned}$$

where the minus signs account for the non-black hole's mass M *de*creasing during evaporation, and

$$C_2 \equiv \frac{\pi^{1/3}(6/\rho)^{2/3}\sigma T^4_{T,r\to\infty}}{c^2} \doteq 3.0511 \times 10^{-24}\rho^{-2/3}T^4_{T,r\to\infty}\,\text{kg}^{1/3}\,\text{s}^{-1}. \qquad (14)$$

$\Delta t_{\text{NBH,evap}}$ is *finite* because although dM/dt decreases with decreasing M, it does so only proportionately to $M^{2/3}$. (In order to render $\Delta t_{\text{NBH,evap}}$ infinite, dM/dt would have to decrease with decreasing M at least proportionately to M itself.) That $\Delta t_{\text{NBH,evap}}$ is *finite* is corroborated by recent research [12].

Equations (11) and (12) yield an exact numerical value for C_1. By contrast (even though the exact numerical values of all factors in C_2 except $T_{T,r\to\infty}$ *are* known) Equations (4)–(6), (13), and (14) do *not* yield enough information to provide an exact (or even less-than-exact) *numerical* value for C_2 and hence also for $T_{T,r\to\infty}$—albeit, as we showed in the eighth paragraph of this Section 3, they *do* yield enough information to prove that $T_{T,r\to\infty}$ *must be finitely higher than absolute zero* (0 K). However, if our conjecture as per Equations (7)–(10) and the associated discussions is correct, then applying our result for $T_{T,r\to\infty}$ in Equation (9) into Equation (13) *does* yield, at least for weak-field ($r_{\text{NBH}} \gg r_S$) uniform-density Schwarzschild (spherically-symmetrical non-rotating) non-black holes, exact numerical values for C_2, hence also for $T_{T,r\to\infty}$, and thence also for $\Delta t_{\text{NBH,evap}}$ as per:

$$\begin{aligned}
\Delta t_{\text{NBH,evap}} &= \frac{3M^{1/3}_{\text{initial}}}{C_2} = \frac{3M^{1/3}_{\text{initial}}}{\frac{\pi^{1/3}(6/\rho)^{2/3}\sigma T^4_{T,r\to\infty}}{c^2}} = \frac{3M^{1/3}_{\text{initial}}c^2}{\pi^{1/3}(6/\rho)^{2/3}\sigma T^4_{T,r\to\infty}} \\
&= \frac{3M^{1/3}_{\text{initial}}c^2}{\pi^{1/3}(6/\rho)^{2/3}\sigma T_{H,r\to\infty}\left(\frac{r_S}{r_{\text{NBH}}}\right)^2} = \frac{3M^{1/3}_{\text{initial}}c^2}{\pi^{1/3}(6/\rho)^{2/3}\sigma \frac{\hbar c^3}{8\pi G k M}\left(\frac{r_S}{r_{\text{NBH}}}\right)^2} \\
&= \frac{24\pi^{2/3}GkM^{4/3}_{\text{initial}}}{\hbar c\sigma}\left(\frac{\rho}{6}\right)^{2/3}\left(\frac{r_{\text{NBH}}}{r_S}\right)^2 \doteq 8.0141\rho^{2/3}M^{4/3}_{\text{initial}}\left(\frac{r_{\text{NBH}}}{r_S}\right)^2\,\text{s}. \qquad (15)
\end{aligned}$$

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Comparing Equations (11) and (13) with the *same* M_{initial} for both a Schwarzschild black hole and a weak-field ($r_{\text{NBH}} \gg r_{\text{S}}$) uniform-density Schwarzschild non-black hole:

$$\frac{\Delta t_{\text{NBH,evap}}}{\Delta t_{\text{BH,evap}}} = \frac{\frac{3M_{\text{initial}}^{1/3}}{C_2}}{\frac{M_{\text{initial}}^3}{3C_1}} = \frac{9C_1}{C_2 M_{\text{initial}}^{8/3}} \equiv \frac{C_3}{M_{\text{initial}}^{8/3}}. \quad (16)$$

Hence, if our conjecture as per Equations (7)–(10) and (15), and the associated discussions, is correct, then applying Equations (7)–(9), for weak-field ($r_{\text{NBH}} \gg r_{\text{S}}$) uniform-density Schwarzschild non-black holes $T_{\text{T},r\to\infty} = T_{\text{H},r\to\infty}(r_{\text{NBH}}/r_{\text{S}})^2$, thereby yielding, at least for weak-field ($r_{\text{NBH}} \gg r_{\text{S}}$) uniform-density Schwarzschild non-black holes, an exact numerical value for C_2 and thence also for C_3:

$$C_2 \doteq 3.0511 \times 10^{-24} \frac{T_{\text{H},r\to\infty}^4}{\rho^{2/3}} \left(\frac{r_{\text{NBH}}}{r_{\text{S}}}\right)^8 \text{kg}^{1/3}\,\text{s}^{-1} \doteq \frac{6.9135 \times 10^{68}}{\rho^{2/3} M_{\text{initial}}^4} \left(\frac{r_{\text{NBH}}}{r_{\text{S}}}\right)^8 \text{kg}^{1/3}\,\text{s}^{-1} \quad (17)$$

and

$$C_3 \equiv \frac{9C_1}{C_2} \doteq 1.1689 \times 10^{40} \frac{\rho^{2/3}}{T_{\text{H},r\to\infty}^4} \left(\frac{r_{\text{S}}}{r_{\text{NBH}}}\right)^8 \text{kg}^{8/3} \doteq 5.1587 \times 10^{-53} \rho^{2/3} M_{\text{initial}}^4 \left(\frac{r_{\text{S}}}{r_{\text{NBH}}}\right)^8 \text{kg}^{8/3}. \quad (18)$$

Whether or not our conjecture is correct, with respect to Equation (16), this qualitative evaluation is valid: If $M_{\text{initial}} < C_3^{3/8}$, $\Delta t_{\text{NBH,evap}} > \Delta t_{\text{BH,evap}}$; if $M_{\text{initial}} = C_3^{3/8}$, $\Delta t_{\text{NBH,evap}} = \Delta t_{\text{BH,evap}}$; if $M_{\text{initial}} > C_3^{3/8}$, $\Delta t_{\text{NBH,evap}} < \Delta t_{\text{BH,evap}}$.

Thus *all* gravitators—*black holes and non-black holes*—are enveloped by atmospheres of equilibrium blackbody radiation. Because both $T_{\text{H},r\to\infty} > 0\,\text{K}$ [16,43–49] and $T_{\text{T},r\to\infty} > 0\,\text{K}$ [8–10], neither a black hole nor a non-black hole can be in thermodynamic equilibrium with a surrounding vacuum at (or at least sufficiently close to) absolute zero (0 K), but *must* radiate into that vacuum and *completely* evaporate into that vacuum within a *finite* time, unless shielded from that vacuum by enclosure within an opaque thermally-insulating shell [64,65].[7]

The Tolman-Hawking evaporation of a Schwarzschild black hole into a vacuum colder than $T_{\text{H},r\to\infty}$ and of a non-black hole into a vacuum colder than $T_{\text{T},r\to\infty}$ is in accordance with the Second Law of Thermodynamics. The entropy of a black hole is large [43–49], but the entropy of the radiation dispersed into a vacuum colder than $T_{\text{H},r\to\infty}$ by its Hawking-evaporation is even larger. The entropy of a non-black hole is not as large as that of a black hole of the same mass, affording even more scope for entropy to increase as it Tolman-evaporates into a vacuum colder than $T_{\text{T},r\to\infty}$.

Tolman was aware of the concept of black holes (even if not of the moniker "black hole"): see, for example, the last paragraph of Section 96 of Ref. [9]. Yet nowhere does this enter into Tolman's [8,9] derivations, bolstered with Ehrenfest [10], that at thermodynamic equilibrium temperature increases downwards[6] in *any* static, or even stationary, gravitational field [63]. Indeed, despite early contemplations of the concept of black holes [68–72], this concept [68–72] (and the moniker "black hole" [68–72]) was not mainstream until the 1960s [68–72]. Hence if Tolman's [8,9] discovery, bolstered with Ehrenfest [10], had borne fruit circa 1930 (or shortly thereafter), it would have (i) initially been construed with respect to *non*-black holes and (ii) dubbed Tolman radiation rather than Hawking radiation: Hawking radiation would then initially have been construed as emanating from *non*-black holes—and dubbed Tolman radiation rather than Hawking radiation!

A caveat pertaining to Hawking-Tolman evaporation in general and to Tolman evaporation of Schwarzschild non-black holes in particular: Our results in this paper in general and in this Section 3 in particular relate, respectively, to gravitators in general and Schwarzschild non-black holes in particular that are *bound solely by their own gravity*. With respect to Schwarzschild non-black holes: any *additional*, *non*-gravitational, e.g., chemical, bonding, is neglected. Any *additional*, *non*-gravitational, e.g., chemical, bonding abetting gravitational bonding must necessarily increase $\Delta t_{\text{NBH,evap}}$, perhaps even rendering $\Delta t_{\text{NBH,evap}} = \infty$ if the temperature is absolute zero (0 K). But this is a *peripheral* issue, *not* the *central*

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issue. The *central* issue relates *only* to *gravitational* bonding: to gravitators in general, and Schwarzschild non-black holes in particular, that are *bound solely by their own gravity*.

At this point, it is worthwhile to note the similarities[8]—owing to the equivalence principle [73,74][8]—between Hawking (Tolman) radiation and Unruh radiation; but also a caveat.[8]

These topics, and related ones, will be further discussed in Sections 4, 5, 6, and 7.

4. All Firewalls Are at the Planck Temperature

In Section 4, it may be helpful to envision a Schwarzschild black hole enclosed concentrically within an opaque thermally-insulating spherical shell at $r_{\text{shell}} \to \infty$ [64,65].[7] Hawking radiation at temperature $T_{\text{H},r\to\infty}$ as per Equation (2) is reradiated and/or reflected downwards[6] from the inner surface of this spherical shell, suffering increasing gravitational blueshift with decreasing r in accordance with Equations (3)–(6) [8–11,16–18,43–49,64,65]. Since thermodynamic equilibrium obtains *perfectly* within the shell [64,65],[7] the caveat "(at least essentially)" can be deleted from the sentence containing Equation (2): radiation within the shell is *exactly* blackbody [64,65]. Indeed, enclosure of *any* radiation—whether emanating from a source or freely existing in space[9]—within an opaque thermally-insulating shell ensures *perfect* thermodynamic equilibrium and hence an *exactly* Planckian blackbody spectrum [64,65] (even if not immediately upon enclosure, then after the relaxation time). For example, if the Sun was so enclosed, the currently *approximately* blackbody radiation [67,70–78] at its photosphere would become *exactly* blackbody [64,65]. Without enclosure within an opaque thermally-insulating shell, radiation *can* be *exactly* blackbody; with enclosure, it *must* be *exactly* blackbody [64,65]. (Of course, *exactly* blackbody radiation incorporates the cutoff of the Planckian blackbody spectrum for wavelengths exceeding the size of an enclosure or cavity [79,80]. But this is not a consideration for our spherical shell, because it is at $r_{\text{shell}} \to \infty$ [64,65].[7]) Moreover, it should be noted that the Planckian form of *any exactly*-blackbody spectrum, and thus its having an *exactly* well-defined temperature, survives gravitational frequency shifting [81]—and also motional Doppler frequency shifting [81], cosmological frequency shifting [81], and any combination of any two or all three types of frequency shifting [81].

Prima facie, by Equation (3), it might seem that arbitrarily close to the Schwarzschild radius r_S (but still at $r > r_S$) $T_{\text{H},r\to r_S} \to \infty$. But this is *not* so. Thus far, we have *not* taken into account that, if at $r > r_S$, it is *not* possible, even in principle (let alone in practice) to be arbitrarily close to r_S, because owing to quantum fluctuations spacetime breaks down as ruler distance [13] on the order of the Planck length [14–16]

$$l_{\text{Planck}} = \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.616255(18) \times 10^{-35}\,\text{m} \tag{19}$$

is approached. (The standard uncertainty is 0.000018×10^{-35} m [16].) Thus, even in principle (let alone in practice), it is *not* possible, if at $r > r_S$, to be any closer to r_S than at minimum radial ruler distance [14–16]

$$(\delta l)_{\text{min}} = l_{\text{Planck}} = \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.616255(18) \times 10^{-35}\,\text{m} \tag{20}$$

beyond r_S.

We now derive $T_{\text{H},r}$ as a function of radial ruler distance [13] δl beyond r_S, which we denote as $T_{\text{H},r_S+\delta l}$. We focus on regions just barely beyond r_S, i.e., where $\delta l \ll r_S$. Also, let $\delta r = r - r_S$ be Schwarzschild-coordinate radial distance [13] (which is also radial area distance and radial distance from apparent size [13]) beyond r_S. We focus on regions just barely beyond r_S, i.e., where, also, $\delta r \ll r_S$. Obviously

$$1 - \frac{r_S}{r} = 1 - \frac{r_S}{r_S + \delta r} = \frac{r_S + \delta r - r_S}{r_S + \delta r} = \frac{\delta r}{r_S + \delta r} \stackrel{\delta r \ll r_S}{=} \frac{\delta r}{r_S}. \tag{21}$$

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Applying Equations (1) and (21) [13],

$$
\begin{aligned}
dl &= \left(1 - \frac{r_S}{r}\right)^{-1/2} dr \\
\implies d(\delta l) &= \left(1 - \frac{r_S}{r}\right)^{-1/2} d(\delta r) \overset{\delta r \ll r_S}{=} \left(\frac{\delta r}{r_S}\right)^{-1/2} d(\delta r) = \left(\frac{r_S}{\delta r}\right)^{1/2} d(\delta r).
\end{aligned} \tag{22}
$$

The last step of Equation (21) and the second-to-last step of Equation (22) are justified because we focus on regions just barely beyond r_S, where $\delta r \ll r_S$. Applying Equation (22), if $\delta r \ll r_S$:

$$
\begin{aligned}
d(\delta l) &\overset{\delta r \ll r_S}{=} \left(\frac{r_S}{\delta r}\right)^{1/2} d(\delta r) \\
\implies \delta l &\overset{\delta r \ll r_S}{=} \int_0^{\delta r} \left(\frac{r_S}{\delta r'}\right)^{1/2} d(\delta r') = 2(r_S \delta r)^{1/2} \implies \delta r \ll \delta l \ll r_S \\
\implies \delta r &\overset{\delta r \ll \delta l \ll r_S}{=} \frac{(\delta l)^2}{4r_S} \\
\implies \left(\frac{r_S}{\delta r}\right)^{1/2} &\overset{\delta r \ll \delta l \ll r_S}{=} \frac{\delta l}{2\delta r} = \frac{\delta l}{2\frac{(\delta l)^2}{4r_S}} = \frac{2r_S}{\delta l}.
\end{aligned} \tag{23}
$$

Hence, applying Equations (2)–(6), (21), (22), and (23), if $\delta r \ll \delta l \ll r_S$:

$$
\begin{aligned}
T_{\mathrm{H},r} = T_{\mathrm{H},r_S+\delta r} &\overset{\delta r \ll \delta l \ll r_S}{=} T_{\mathrm{H},r\to\infty} \left(\frac{r_S}{\delta r}\right)^{1/2} \\
\implies T_{\mathrm{H},r_S+\delta l} &\overset{\delta r \ll \delta l \ll r_S}{=} 2T_{\mathrm{H},r\to\infty} \frac{r_S}{\delta l} = 2\frac{\hbar c}{4\pi k r_S}\frac{r_S}{\delta l} = \frac{\hbar c}{2\pi k \delta l}.
\end{aligned} \tag{24}
$$

As noted in the paragraph containing Equations (19) and (20), even in principle (let alone in practice), δl can be no smaller than $(\delta l)_{\mathrm{min}} = l_{\mathrm{Planck}}$ [14–16]. Thus, minimizing δl at $(\delta l)_{\mathrm{min}} = l_{\mathrm{Planck}}$, by Equations (19), (20), and (24) we obtain

$$
\begin{aligned}
T_{\mathrm{firewall}} &= T_{\mathrm{H},r_S+(\delta l)_{\mathrm{min}}} = T_{\mathrm{H},r_S+l_{\mathrm{Planck}}} = \frac{\hbar c}{2\pi k(\delta l)_{\mathrm{min}}} = \frac{\hbar c}{2\pi k l_{\mathrm{Planck}}} = \frac{\hbar c}{2\pi k\left(\frac{\hbar G}{c^3}\right)^{1/2}} \\
&= \frac{1}{2\pi k}\left(\frac{\hbar c^5}{G}\right)^{1/2} = \frac{1}{2\pi}\left[\frac{1}{k}\left(\frac{\hbar c^5}{G}\right)^{1/2}\right] = \frac{T_{\mathrm{Planck}}}{2\pi},
\end{aligned} \tag{25}
$$

where

$$
T_{\mathrm{Planck}} = \frac{1}{k}\left(\frac{\hbar c^5}{G}\right)^{1/2} = 1.416784(16) \times 10^{32}\,\mathrm{K} \tag{26}
$$

is the Planck temperature [19]. (The standard uncertainty is 0.000016×10^{32} K [19].)

This result is *in*dependent of the mass M and hence also of the Schwarzschild radius $r_S = 2GM/c^2$ of a Schwarzschild black hole. As M and hence also r_S increases, by Equation (2) $T_{\mathrm{H},r\to\infty}$ decreases in inverse proportion. But $r_S/\delta l$ for any given $\delta l \ll r_S$ in general and hence $r_S/(\delta l)_{\mathrm{min}} = r_S/l_{\mathrm{Planck}}$ in particular increases in direct proportion. Hence in accordance with Equations (2) and (24)–(26) these two opposing factors cancel out. Because of quantum fluctuations in the metric at length scales of l_{Planck} [14–16], Equation (25) may be pushing the limit of accuracy of Equation (24), but we should expect Equation (25) to be valid at least in some average sense. Accordingly, perhaps we should not be too adamant about the small numerical factor of $1/2\pi$ in Equation (25), and hence recapitulate Equation (25) as

$$
T_{\mathrm{firewall}} = T_{\mathrm{H},r_S+l_{\mathrm{Planck}}} \approx T_{\mathrm{Planck}}. \tag{27}
$$

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By Equation (24), recapitulated with the help of

$$\frac{\hbar c}{2\pi k} = 0.0003644464403\,\mathrm{m\,K} \tag{28}$$

as

$$T_{\mathrm{H},r_{\mathrm{S}}+\delta l} \overset{\delta r \ll \delta l \ll r_{\mathrm{S}}}{=} \frac{\hbar c}{2\pi k \delta l} = \frac{0.0003644464403\,\mathrm{m\,K}}{\delta l}, \tag{29}$$

$T_{\mathrm{H},r_{\mathrm{S}}+\delta l}$ still has high values in the region $l_{\mathrm{Planck}} \ll \delta l \ll r_{\mathrm{S}}$, hence with quantum fluctuations in the metric of Equation (1) being negligible [14–16]. For example, the temperature of the Sun's core, $1.571 \times 10^{7}\,\mathrm{K}$ [67], is equaled at $\delta l = 2.3198 \times 10^{-11}\,\mathrm{m}$, i.e., only slightly less than typical atomic dimensions; the (effective [67,75–78]) temperature of the Sun's photosphere, 5772 K [67], is equaled at $\delta l = 6.3140 \times 10^{-8}\,\mathrm{m}$, i.e., the dimensions of small microbes; and room temperature, 300 K, is equaled at $\delta l = 1.2148 \times 10^{-6}\,\mathrm{m}$, less than two orders of magnitude below the limit of naked-eye visibility $\approx 10^{-4}\,\mathrm{m}$.

Now let us consider the ruler-distance [13] wavelength of Hawking radiation in the region only slightly beyond r_{S}, i.e., where $\delta r \ll \delta l \ll r_{\mathrm{S}}$. The ruler-distance [13] wavelength $\lambda^{\mathrm{Wien,max}}_{r_{\mathrm{S}}+\delta l}$ of blackbody radiation in general and of Hawking radiation in particular at the Wien's-Displacement-Law maximum with respect to wavelength [66,82] corresponding to temperature T is [66,82]

$$\lambda^{\mathrm{Wien,max}} = \frac{0.002897771955}{T}\,\mathrm{m}\,. \tag{30}$$

(Since we are focusing on wavelength, we employ the Wien's-Displacement-Law maximum with respect to wavelength [66,82] as opposed to that with respect to frequency [66,82].) Hence by Equations (28) and (30) [66,82]:

$$\begin{aligned}
\lambda^{\mathrm{Wien,max}} &= \frac{\frac{0.002897771955}{T}\,\mathrm{m}}{\frac{\hbar c}{2\pi k}}\frac{\hbar c}{2\pi k} = \frac{\frac{0.002897771955}{T}\,\mathrm{m}}{0.0003644464403\,\mathrm{m\,K}}\frac{\hbar c}{2\pi k} \\
&= \frac{0.002897771955\,\mathrm{m\,K}}{0.0003644464403\,\mathrm{m\,K}}\frac{\hbar c}{2\pi k T} = 1.265466416\frac{\hbar c}{kT} \\
&\Longrightarrow \lambda^{\mathrm{Wien,max}}_{r_{\mathrm{S}}+\delta l} \overset{\delta r \ll \delta l \ll r_{\mathrm{S}}}{=} 1.265466416\frac{\hbar c}{kT_{\mathrm{H},r_{\mathrm{S}}+\delta l}} = 1.265466416\frac{\hbar c}{k\frac{\hbar c}{2\pi k \delta l}} \\
&= 1.265466416 \times 2\pi\delta l = 7.951159991\delta l.
\end{aligned} \tag{31}$$

The numerical factor $1.265466416 \times 2\pi = 7.951159991$ is dimensionless and hence is valid in any self-consistent system of units. In the third line of Equation (31) we applied the second line of Equation (24). In accordance with the reasoning concerning quantum fluctuations in the metric in the paragraph ending with Equation (27) [14–16], perhaps we should not be too adamant about the small numerical factor of $1.265466416 \times 2\pi = 7.951159991$ in the last term of Equation (31), and hence recapitulate Equation (31) as

$$\lambda^{\mathrm{Wien,max}}_{r_{\mathrm{S}}+\delta l} \overset{\delta r \ll \delta l \ll r_{\mathrm{S}}}{\approx} \delta l. \tag{32}$$

Thus the ruler-distance [13] wavelength $\lambda^{\mathrm{Wien,max}}_{r_{\mathrm{S}}+\delta l}$ of Hawking radiation in the region $\delta r \ll \delta l \ll r_{\mathrm{S}}$ is on the order of the ruler distance [13] δl itself. In particular, at $(\delta l)_{\mathrm{min}} = l_{\mathrm{Planck}}$ [14–16]

$$\lambda^{\mathrm{Wien,max}}_{r_{\mathrm{S}}+(\delta l)_{\mathrm{min}}} = \lambda^{\mathrm{Wien,max}}_{r_{\mathrm{S}}+l_{\mathrm{Planck}}} \approx l_{\mathrm{Planck}}. \tag{33}$$

Hawking-radiation photons for which Equations (25)–(27) and (33) apply, and consequently for which $T_{\mathrm{firewall}} \approx T_{\mathrm{Planck}}$ and thus $E = h\nu \approx kT_{\mathrm{firewall}} \approx kT_{\mathrm{Planck}} \approx E_{\mathrm{Planck}} = m_{\mathrm{Planck}}c^2$ [14–16,83–86], are *themselves* Planck-mass black holes [14–16,87–89], specifically, Planck-mass geons [87–89], and thereby *themselves* contribute to the breakdown of spacetime as the Planck scale is approached, i.e., as $\delta l \to (\delta l)_{\mathrm{min}} = l_{\mathrm{Planck}}$ [14–16,87–89].

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In accordance with the three immediately preceding paragraphs, and for consistency with Equation (31) keeping the numerical factor 7.951159991 [66,82], by Equations (30) and (31) [66,82]

$$\lambda^{\text{Wien,max}} = \frac{0.002897771955}{T}\,\text{m} \overset{\delta r \ll \delta l \ll r_S}{=} 7.951159991\delta l. \tag{34}$$

Thus Hawking radiation with $\lambda^{\text{Wien,max}}$ corresponding to values of T that are still high occurs in the region $l_{\text{Planck}} \ll \delta l \ll r_S$, hence with quantum fluctuations in the metric of Equation (1) being negligible [14–16]. For example, $\lambda^{\text{Wien,max}}$ corresponding to the temperature of the Sun's core, 1.571×10^7 K [67], is equaled at $\delta l = 2.3198 \times 10^{-11}$ m, i.e., only slightly less than typical atomic dimensions; $\lambda^{\text{Wien,max}}$ corresponding to the (effective [67,75–78]) temperature of the Sun's photosphere, 5772 K [67], is equaled at $\delta l = 6.3140 \times 10^{-8}$ m, i.e., the dimensions of small microbes; and $\lambda^{\text{Wien,max}}$ corresponding to room temperature, 300 K, is equaled at $\delta l = 1.2148 \times 10^{-6}$ m, less than two orders of magnitude below the limit of naked-eye visibility $\approx 10^{-4}$ m.

Of course, the last two lines of Equation (31), and Equations (32) and (34) [let alone Equation (33)], do *not* apply in the region $r \gg r_S$. For, as $r \to \infty$, $\delta l \to r - r_S + (r_S/2)\ln(r/r_S)$ [90], whilst applying Equation (2) and the first two lines of Equation (31) [66,82]:

$$\begin{aligned}\lambda^{\text{Wien,max}}_{r\to\infty} &= 1.265466416\frac{\hbar c}{kT_{\text{H},r\to\infty}} = 1.265466416\frac{\hbar c}{k\frac{\hbar c}{4\pi k r_S}} = 1.265466416 \times 4\pi r_S \\ &= 15.90231998 r_S = \text{ constant.}\end{aligned} \tag{35}$$

The numerical factor $1.265466416 \times 4\pi = 15.90231998$ is dimensionless and hence is valid in any self-consistent system of units.

We have considered Schwarzschild black holes whose *only* energy source is their own Hawking radiation. This may eventually be the case for actual black holes if the Universe expands forever. But in the current Universe, black holes are bathed by photons emanating from $r \gg r_S$—effectively from $r \to \infty$—far more energetic than thermal photons at temperature $T_{\text{H},r\to\infty}$ as per Equation (2): photons from the $T_{\text{CBR}} = 2.725$ K [50] cosmic background radiation [50], from starlight, etc. [50]. Radiation comprised of these far more energetic photons will be blueshifted to $T_{\text{firewall}} \approx T_{\text{Planck}}$ as given by Equations (25)–(27) at $r_S + \delta l$ with $\delta l \gg l_{\text{Planck}}$. But photons corresponding to $T_{\text{firewall}} \approx T_{\text{Planck}}$, i.e., for which $E = h\nu \approx kT_{\text{firewall}} \approx kT_{\text{Planck}} \approx E_{\text{Planck}} = m_{\text{Planck}}c^2$ [14–16,87–89], are *themselves* Planck-mass black holes [14–16,87–89], specifically, Planck-mass geons [87–89], and thereby *themselves* might contribute to the breakdown of spacetime at *this* $r_S + \delta l$, i.e., at *this* $\delta l \gg l_{\text{Planck}}$, *well before* $(\delta l)_{\text{min}} = l_{\text{Planck}}$ is approached [14–16,87–89]. Hence in the current Universe we should consider at least the possibility of the breakdown of spacetime at *this* $r_S + \delta l$, i.e., at *this* $\delta l \gg l_{\text{Planck}}$, *well before* $(\delta l)_{\text{min}} = l_{\text{Planck}}$ is approached [14–16,87–89]. But this is *not* what we mean by a Schwarzschild black hole's firewall. By a Schwarzschild black hole's firewall we mean that which is *intrinsic* to the black hole itself, i.e., owing *solely* to its own Hawking radiation.

5. The Exponential Nature of the Gravitational Frequency Shift

In Section 5, as in Section 4, it may be helpful to envision a Schwarzschild black hole enclosed concentrically within an opaque thermally-insulating spherical shell at $r_{\text{shell}} \to \infty$ [64,65].[7] Hawking radiation at temperature $T_{\text{H},r\to\infty}$ as per Equation (2) is reradiated and/or reflected downwards[6] from the inner surface of this spherical shell, suffering increasing gravitational blueshift with decreasing r in accordance with Equations (3)–(6) [8–11,16–18,43–49,64,65].

Expressed in terms of r, at $r \geq r_S$ the relativistic gravitational scalar potential Φ of a Schwarzschild black hole and its magnitude $|\Phi|$ are [13,17,18]

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$$\Phi = \frac{c^2}{2}\ln\left(1 - \frac{2GM}{rc^2}\right) = \frac{c^2}{2}\ln\left(1 - \frac{r_S}{r}\right)$$
$$\implies |\Phi| = \frac{c^2}{2}\left|\ln\left(1 - \frac{2GM}{rc^2}\right)\right| = \frac{c^2}{2}\left|\ln\left(1 - \frac{r_S}{r}\right)\right|. \tag{36}$$

Applying Equations (2), (3), (21), (22), and (23) [especially Equation (21) and the last two lines of Equation (23)], if $\delta r \ll \delta l \ll r_S$, expressing Φ and $|\Phi|$ in terms of δr and δl [13,17,18]:

$$\Phi \overset{\delta r \ll \delta l \ll r_S}{=} \frac{c^2}{2}\ln\frac{\delta r}{r_S} = \frac{c^2}{2}\ln\frac{(\delta l)^2}{4r_S^2} = \frac{c^2}{2}\ln\left(\frac{\delta l}{2r_S}\right)^2 = c^2\ln\frac{\delta l}{2r_S}$$
$$\implies |\Phi| \overset{\delta r \ll \delta l \ll r_S}{=} \frac{c^2}{2}\ln\frac{r_S}{\delta r} = \frac{c^2}{2}\ln\frac{4r_S^2}{(\delta l)^2} = \frac{c^2}{2}\ln\left(\frac{2r_S}{\delta l}\right)^2 = c^2\ln\frac{2r_S}{\delta l}. \tag{37}$$

It may be interesting to note that corresponding to minimum-definable ruler distance [13] $\delta l_{\text{min}} = l_{\text{Planck}}$ [14–16] beyond r_S

$$|\Phi|_{r_S + (\delta l)_{\text{min}}} = |\Phi|_{r_S + l_{\text{Planck}}} = c^2\ln\frac{2r_S}{\delta l_{\text{min}}} = c^2\ln\frac{2r_S}{l_{\text{Planck}}} = c^2\ln\frac{2r_S}{\left(\frac{\hbar G}{c^3}\right)^{1/2}} = c^2\ln\left[r_S\left(\frac{4c^3}{\hbar G}\right)^{1/2}\right]$$
$$= c^2\ln\left(\frac{4r_S^2c^3}{\hbar G}\right)^{1/2} = \frac{c^2}{2}\ln\frac{4r_S^2c^3}{\hbar G} = \frac{c^2}{2}\ln\frac{Ac^3}{\pi\hbar G} = \frac{c^2}{2}\ln\frac{16GM^2}{\hbar c} = \frac{c^2}{2}\ln\frac{4S}{\pi k}, \tag{38}$$

where A is the surface area of a black hole and S is its entropy [43–49].

We re-emphasize that a relativistic gravitational scalar potential Φ and hence also its magnitude $|\Phi|$ [17,18], and the relation thereof to gravitational potential energy [17,18], are valid concepts for *all* static, and even stationary, spacetimes [63] (not just Schwarzschild spacetime [91–93]). And that the spacetime engendered at $r \geq r_S$ by any Schwarzschild black hole and at *all* $r \geq 0$ by any Schwarzschild non-black hole (we focus on Schwarzschild metrics) is static [91–93], not merely stationary [63].

The blueshift of *any* photon (Hawking/Tolman-radiation photon or otherwise) whose frequency, energy, and mass [83–86] at $r \to \infty$ are $\nu_{r\to\infty}$, $E_{r\to\infty} = h\nu_{r\to\infty}$, and $m_{r\to\infty} = E_{r\to\infty}/c^2 = h\nu_{r\to\infty}/c^2$ [83–86], respectively, upon falling radially inwards from $r \to \infty$, increases *exponentially* rather than merely linearly with decreasing Φ (or, equivalently, with increasing $|\Phi|$) [17,18], in accordance with [17,18]

$$\nu(|\Phi|) = \nu_{r\to\infty}e^{|\Phi|/c^2}$$
$$\implies E(|\Phi|) = h\nu(|\Phi|) = E_{r\to\infty}e^{|\Phi|/c^2} = h\nu_{r\to\infty}e^{|\Phi|/c^2}$$
$$\implies m(|\Phi|) = \frac{E(|\Phi|)}{c^2} = \frac{h\nu(|\Phi|)}{c^2} = m_{r\to\infty}e^{|\Phi|/c^2} = \frac{E_{r\to\infty}e^{|\Phi|/c^2}}{c^2} = \frac{h\nu_{r\to\infty}e^{|\Phi|/c^2}}{c^2}. \tag{39}$$

This obtains because as a photon falls and gets blueshifted its mass [83–86] $m = E/c^2 = h\nu/c^2$ [which of course is solely its (kinetic energy)$/c^2$ [83–86], because a photon's rest mass is zero [83–86]] increases: the photon gets more massive as it falls. Thus as a photon falls through successive ruler-distance [13] increments dl, a Schwarzschild black hole's gravitational field at $r \geq r_S$, a Schwarzschild non-black hole's gravitational field at *all* $r \geq 0$—indeed, the gravitational field $\mathcal{G} = -d\Phi/dl$ in *any* static, or even stationary, spacetime [63,91–93]—does successive increments of (positive) work [92,93]

$$dW = -md\Phi = -m\frac{d\Phi}{dl}dl = m\mathcal{G}dl \tag{40}$$

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not on a fixed mass m but on an *ever-increasing* mass m. [The minus sign in $\mathcal{G} = -d\Phi/dl$ obtains because $\mathcal{G}$ acts *downwards*,[6] i.e., in the direction of *decreasing* l. dW in Equation (40) is positive, because $\mathcal{G}$ is negative, and both $d\Phi$ and dl are negative during infall.] $dW/d\Phi$ and thus the rate of increase of $m = E/c^2 = h\nu/c^2$ with decreasing Φ is proportional to $m = E/c^2 = h\nu/c^2$ itself: consequently the *exponential* form of Equation (39).

Hence also, in accordance with Equations (3), (24), (25), (36), (37), (39), and (40), the temperature T of any Planckian blackbody distribution of photons increases *exponentially* rather than merely linearly with decreasing Φ (or, equivalently, with increasing $|\Phi|$) [16–18,43–49,64,65,81,83–86].

Of course, the same reasoning also applies in reverse: as a photon rises a Schwarzschild black hole's gravitational field at $r > r_S$—indeed, the gravitational field $\mathcal{G} = -d\Phi/dl$ in *any* static, or even stationary, spacetime [63,91–93]—does negative work on, or equivalently receives positive work from, *not* a fixed mass m but an *ever-decreasing* mass m. $dW/d\Phi$ and thus the rate of decrease of $m = E/c^2 = h\nu/c^2$ is proportional to $m = E/c^2 = h\nu/c^2$ itself: consequently as per Equations (39) and (40) a rising photon's mass [83–86] $m = E/c^2 = h\nu/c^2$ decreases *exponentially* rather than merely linearly with increasing Φ (or, equivalently, with decreasing $|\Phi|$) [16–18,43–49,64,65,81,83–86]. Hence also, in accordance with Equations (3), (24), (25), (36), (37), (39), and (40), the temperature T of any Planckian blackbody distribution of photons decreases *exponentially* rather than merely linearly with increasing Φ (or, equivalently, with decreasing $|\Phi|$) [16–18,43–49,64,65,81,83–86].

By contrast, for a slowly radially-moving (slow *physical*—*not* necessarily slow coordinate—radial velocity $V_{\text{phys}} = dl/d\tau \ll c$) nonzero-rest-mass particle, the increase of total mass in free fall (and its decrease in free rise from an upwards[6] flying start) is on a pro rata basis much smaller than for a photon—a linear rather than exponential function of Φ (or $|\Phi|$). This obtains because its (kinetic energy)$/c^2 = V^2/2c^2$ is only a *negligibly small fraction* of its total mass—*not* the *entirety* [83–86] of its total mass as is the case for a photon (or other zero-rest-mass particle) [83–86].

Of course, the First Law of Thermodynamics (energy conservation) always obtains. The kinetic energy that any entity gains (loses) by falling (rising) in a gravitational field is *exactly offset* by the energy of the gravitational field itself becoming more (less) strongly negative. This point will be discussed more thoroughly in Section 6.

In wrapping up Section 5, we note that for static, and even stationary, spacetimes [63], the relativistic gravitational scalar potential Φ is related to the time-time component of the metric in accordance with [94]

$$g_{tt} = e^{2\Phi/c^2} \iff \Phi = \frac{c^2}{2} \ln g_{tt}. \tag{41}$$

Hence in static, and even stationary, spacetimes [63], the substitution $\Phi \to \frac{c^2}{2} \ln g_{tt}$ can be made in Equations (36)–(40).

6. Negative Gravitational Mass-Energy and Birkhoff's Theorem versus Massiveness of Firewalls

Thus far, we have taken for granted that a firewall does not contribute (at a maximum, not more than negligibly) to the mass M of a Schwarzschild black hole. But this has been seriously questioned [3]. It has been averred that this can*not* be even approximately true for any Schwarzschild black hole whose mass M appreciably exceeds the Planck mass $m_{\text{Planck}} = (\hbar c/G)^{1/2}$ [3]—a minimum-possible-mass $M = M_{\min} = m_{\text{Planck}} = (\hbar c/G)^{1/2}$ Schwarzschild black hole—which would Hawking-evaporate on a time scale on the order of the Planck time $t_{\text{Planck}} = \left(\hbar G/c^5\right)^{1/2}$ [3]. And that at best this can just barely be even approximately true *even if* $M = M_{\min} = m_{\text{Planck}} = (\hbar c/G)^{1/2}$ [3]. This is the firewall-mass problem [3].

There is not universal agreement concerning the firewall-mass problem [3]. Counter-arguments resolving this problem have been proposed [4].

In Section 6, we do not make any assumption about what the mass of a firewall might be: small, large, or perhaps annulled to zero (except that it is *not* negative) [3,4]. However, we consider the firewall-mass problem [3], and propose an at least *prima facie* tentative resolution thereto. Our

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tentative resolution is based on: (i) the mass of a firewall (whatever it might be, if not otherwise annulled to zero [4]) being *exactly canceled* by the *negative* gravitational mass [17,18] = (*negative* gravitational energy)$/c^2$ [17,18] accompanying its formation, (ii) the *un*changed observations of a distant observer upon formation of a firewall, and (iii) Birkhoff's Theorem [20–31]—actually first discovered by Jørg Tofte Jebsen [25–28]. This is in addition to, and perhaps may complement, other lines of reasoning [4] disputing massiveness [3] of firewalls. (There is a caveat [28–31][1] with respect to Birkhoff's Theorem [20–31], but it [28–31][1] is *not* relevant with respect to our considerations.[1])

The viewpoint [3] that formation of a firewall imparts a *huge net* increase to the mass of a Schwarzschild black hole [3] seems to overlook the *negative* gravitational mass [17,18] = (*negative* gravitational energy)$/c^2$ [17,18] contribution to the black-hole/firewall system. The *negativity* of gravitational energy [17,18] is the perhaps the central aspect of our tentative resolution of the firewall-mass problem [3]. We hope to show that the *negative* gravitational energy [17,18] accompanying *formation* of a firewall *exactly*—not merely approximately—cancels the firewall mass, so that the mass M of a black hole remains *exactly*—not merely approximately—*un*changed if a firewall forms. Is this, at least *prima facie*, what Ref. [3] overlooks? Reference [3] derives the mass of an *already-extant* firewall of an *already-collapsed* black hole, but seems to overlook the increased negativity of gravitational mass-energy accompanying *formation* of the firewall *during collapse*.

We note that the negative gravitational mass-energy accompanying formation of a firewall should *not* be confused with considerations regarding negative energy states of the firewall *itself* [3]. We do not make any assumption about what the mass of a firewall might be—except that in accordance with the first two paragraphs of the section entitled "Discussion" in Ref. [3], we *always* construe its mass (if not annulled to zero [4]) to be positive—*even if* there exist negative energy states: the squares of both positive and negative numbers are positive: see the term E_F^2 in Equation (10) of Ref. [3]. We show that, whatever the mass of a firewall might be, the *negative* gravitational mass [17,18] = (*negative* gravitational energy)$/c^2$ [17,18] accompanying its formation annuls it (*even if* it is not otherwise annulled [4])—effecting *zero net change* in the mass of a Schwarzschild black hole.

Consider a spherically-symmetrical non-rotating gravitator of mass M but of sufficiently low average density that it is a Schwarzschild *non*-black hole ($r > r_S = 2GM/c^2 \iff M < r_S c^2/2G$), surrounded by a vacuum at absolute zero (0 K). As shown in Section 3, this gravitator will completely *Tolman*-radiation [8–10] evaporate within a *finite* time (see also Garrod [11]), yielding energy $E = Mc^2$ to a distant observer at $r_{obs} \gg r > r_S$. We re-emphasize that this has been corroborated by recent research [12].

Now instead consider another *identical* spherically-symmetrical non-rotating gravitator of mass M. But this time let the structural strength of the gravitator be annulled, so that it gravitationally collapses *radially* to a Schwarzschild black hole. *This* gravitator will then completely *Hawking*-radiation evaporate within a *finite* time, *also* yielding the *same* energy $E = Mc^2$ to a distant observer at $r_{obs} \gg r \gg r_S$. Indeed, this is required not only by the First Law of Thermodynamics (energy conservation), but also by Birkhoff's Theorem [20–31]. (There is a caveat [28–31][1] with respect to Birkhoff's Theorem [20–31], but it [28–31][1] is *not* relevant with respect to our considerations.[1]) For Birkhoff's Theorem [20–31] states that *any* purely radial gravitational collapse (or *any* purely radial dispersion against gravity from a flying start) of a spherically-symmetrical non-rotating gravitator can*not* cause *any* change detectable by a distant observer [not even gravitational waves, because radial collapse (or radial dispersion) does not generate them [20–31]]: Birkhoff's Theorem [20–31] *authorizes no exception* for gravitational collapse of the innermost shell of a gravitator's Tolman-Hawking [8–11] radiation atmosphere to a firewall. This is possible *if and only if* the mass of the gravitator does *not* change during collapse—*even if* a firewall forms. And *this*, in turn, is possible *if and only if* the mass of the firewall is *exactly counterbalanced* by the increased negativity of gravitational mass-energy accompanying its formation.

Thus there *must* be *zero net change* in mass of the gravitator. Any increase in mass—whether due to formation of a firewall and/or otherwise—accompanying collapse *must be exactly counterbalanced by a negative contribution*. Gravitational mass = (gravitational energy)$/c^2$ *is always a negative contribution*

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to mass. And the only possible counterbalancing negative contribution is the gravitational mass-energy of the gravitator becoming *more strongly negative* during collapse. This *must* be true whether or not a firewall forms. If a firewall does *not* form, the increase in mass of the collapsing gravitator's Tolman-Hawking [8–11] radiation atmosphere will be less than if one *does* form—but so will the increase in the negativity of gravitational mass-energy.

It may be helpful to briefly expound on Tolman-Hawking [8–11] radiation atmospheres. Consider a spherically-symmetrical non-rotating entity (Schwarzschild black hole or Schwarzschild non-black hole) enclosed concentrically within an opaque thermally-insulating spherical shell at $r_{\text{shell}} \to \infty$ [64,65].[7] Such an entity is enveloped by a Tolman-Hawking [8–11] radiation atmosphere. Because the entity is enclosed within an opaque thermally-insulating spherical shell, its Tolman-Hawking [8–11] radiation atmosphere is at thermodynamic equilibrium throughout. Hence photons of radiation emanate from *anywhere* in this radiation atmosphere. To be specific, if this entity *is* a black hole, it is equally valid to construe photons emanating either (i) from $r_S + l_{\text{Planck}}$ and then suffering gravitational redshift upon streaming outwards towards the inner surface of our spherical shell at $r \to \infty$ or (ii) from the inner surface of our spherical shell at $r \to \infty$ and then suffering gravitational blueshift upon falling inwards. Thus either we can construe Hawking radiation as suffering maximal gravitational redshift at $r \to \infty$ and no gravitational redshift at $r_S + l_{\text{Planck}}$, or we can construe it as suffering no gravitational blueshift at $r \to \infty$ and maximal gravitational blueshift at $r_S + l_{\text{Planck}}$. This viewpoint is valid because: (a) the *entire region* within our spherical shell is at *thermodynamic equilibrium throughout*. And at thermodynamic equilibrium, the principles of microscopic reversibility and detailed balance obtain [95]: hence it is equally valid to consider microscopic processes occurring in either the "forward" or "reverse" direction [95]. Indeed, at *thermodynamic equilibrium*, which direction (i) or (ii) immediately above is construed as "forward" or "reverse" is arbitrary [95]. (There are caveats [96,97],[10] but they are *not* relevant with respect to our considerations.[10]) (b) Curved spacetime is hot [8–11]. Thus—if the gravitational frequency shift and hence temperature increasing downwards[6] in gravitational fields is taken into account [8–11,81]—it is equally valid to construe Tolman-Hawking [8–11] radiation as emanating from *any* $r > r_S$ [8–11,81,95]. A Tolman-Hawking [8–11] radiation photon of mass $m_{r\to\infty} = E_{r\to\infty}/c^2 = h\nu_{r\to\infty}/c^2 \approx kT_{\text{H},r\to\infty}/c^2$ [83–86] at the inner surface of our spherical shell at $r \to \infty$ does indeed gain mass $m_{\text{Planck}} - m_{r\to\infty}$ during its infall to $r_S + l_{\text{Planck}}$, i.e., to $(\delta l)_{\text{min}} \approx l_{\text{Planck}}$ [14–16,83–86], attaining mass $\approx m_{\text{Planck}} = E_{\text{Planck}}/c^2 = h\nu_{r_S + l_{\text{Planck}}}/c^2 \approx kT_{\text{firewall}}/c^2 \approx kT_{\text{Planck}}/c^2$ after having fallen to $r_S + l_{\text{Planck}}$, i.e., to $(\delta l)_{\text{min}} \approx l_{\text{Planck}}$ [14–16,83–86]. But the increase $m_{\text{Planck}} - m_{r\to\infty}$ in the photon's mass [14–16,83–86] that occurs during its infall is *exactly counterbalanced* by the increased negativity of the gravitational mass-energy [17,18] of the black-hole/photon system [83–86] that, by the First Law of Thermodynamics (energy conservation), *also* occurs during the photon's infall. Thus the *net* contribution to the mass of the black-hole/photon system [83–86] continues to be *only* $m_{r\to\infty}$—it does *not* increase by $m_{\text{Planck}} - m_{r\to\infty}$ to m_{Planck}—*exactly as if the photon had not suffered infall!*

If this is true with respect to any *one* infalling photon, then it must also be true with respect to *all* of the infalling photons combined required to produce a spherical shell of equilibrium blackbody radiation with inner boundary at r_S, of ruler-distance [13] radial thickness l_{Planck}, and at temperature T_{Planck}—i.e., to produce a firewall. Hence at least *prima facie* it seems that a large increase in the mass [3]—indeed *any* increase in mass at all—of the black hole attributable to firewall formation [3] is *exactly canceled out to zero*.

We re-emphasize that the downwards[6] increase in the temperature of Tolman-Hawking [8–11] radiation in the gravitational fields of Schwarzschild gravitators (black holes and non-black holes) is a special case of the general result of relativistic thermodynamics that at thermodynamic equilibrium temperature increases downwards[6] in *any* gravitational field [8–11] (at least, in *any* static, or even stationary, one [63,91–93]). Tolman-Hawking [8–11] radiation should be construed as emanating *not only* from $r_S + l_{\text{Planck}}$—indeed *not only* from *any* $r \geq r_S + l_{\text{Planck}}$—in the gravitational field of a Schwarzschild black hole—but from *anywhere* in *any* gravitational field whatsoever. This was very well conveyed by a seminar given by Dr. James H. Cooke at the Department of Physics at the University of North Texas in the 1980s—and most succinctly expressed by the title of this seminar: "Curved spacetime is hot"—confirming Tolman [8–10] (see also Garrod [11], and recall our Sections 3, 4, and 5). Of course, by "hot" it is meant hotter than absolute zero (0 K)—in even the weakest gravitational fields. Tolman-Hawking [8–11] radiation emanates from *every* location in *any* gravitational field however weak *in general*—not only from black holes, but also from *non*-black holes: Curved spacetime is hot [at least, hotter than absolute zero (0 K)] *in general*. This is *required* for consistency with temperature increasing downwards[6] given thermodynamic equilibrium in *any* gravitational field, however weak [8–11]. In this regard we re-emphasize, as Dr. James H. Cooke pointed out, that not only black holes, but also *non*-black holes, Tolman-Hawking [8–11]) radiate: In this regard, it may at this point be worthwhile to again recall Section 3. Indeed, as we noted in Section 3, when Tolman [8,9], bolstered with Ehrenfest [10], anticipated Hawking radiation (see also Garrod [11]), if that anticipation had borne fruit circa 1930 (or shortly thereafter), it would have (i) initially been construed with respect to *non*-black holes and (ii) dubbed Tolman radiation rather than Hawking radiation!

Generalizing, the free fall of *any* entity in *any* gravitational field can*not* result in *any* change in the mass of the gravitator/entity system, because by the First Law of Thermodynamics (energy conservation) the gain in the falling entity's kinetic energy [via increased frequency if it is a photon, or via increased *physical* downwards[6] velocity $V_{\text{phys}} = dl/d\tau$ (*not* necessarily increased *coordinate* downwards[6] velocity) if it is of nonzero rest mass] *must be exactly counterbalanced* by the gravitational mass-energy [17,18] of the gravitator/entity system becoming more strongly negative. And likewise the free rise (from an upwards[6] flying start) of *any* entity in *any* gravitational field can*not* result in *any* change in the mass of the gravitator/entity system, because by the First Law of Thermodynamics (energy conservation) the loss in the rising entity's kinetic energy [via decreased frequency if it is a photon, or via decreased *physical* upwards[6] velocity $V_{\text{phys}} = dl/d\tau$ (*not* necessarily decreased *coordinate* upwards[6] velocity) if it is of nonzero rest mass] *must be exactly counterbalanced* by the gravitational mass-energy [17,18] of the gravitator/entity system becoming less strongly negative. Furthermore this remains true even if the fall or rise is *not* free but retarded by friction [21], because friction merely thermalizes the entity's kinetic energy within the gravitator/entity system. For example, a landslide on Earth (whether or not retarded by friction) does *not* change Earth's total mass-energy $E_{\text{Earth}} = M_{\text{Earth}}c^2$ (which includes the negative contribution from Earth's gravitational energy), because the kinetic energy of the landslide (whether or not thermalized by friction [21]) is *exactly counterbalanced* by the gravitational mass-energy [17,18] of the Earth/landslide system becoming more strongly negative.

We briefly remark that Earth's negative gravitational mass-energy [17,18] reduces Earth's mass by a fraction on the order of $V_{\text{escape}}^2/c^2 \sim 10^{-9}$, where V_{escape} is the escape velocity from Earth's surface ($\approx 1.1 \times 10^4$ m / s). While this fraction is small in relative terms, in absolute terms it is a substantial negative contribution to Earth's mass, on the order of the mass of an asteroid ~ 10 km in diameter ($\sim 10^{-3}$ of Earth's diameter)—e.g., the K–T boundary asteroid [98] that was the major factor (even if not the only one) that ended the dinosaurs' reign [98].[11]

We close Section 6 with this speculative paragraph. It has been speculated [3] that owing to a firewall perhaps an infalling particle "burns up at the horizon [3]". So we are steered in the direction of asking the following four admittedly speculative questions: (i) Might the particle be saved from falling through the horizon, i.e., through the Schwarzschild radius r_S of a black hole, by burning up? (ii) If so, does this at least *prima facie* seem to suggest the possibility that a collapsing *near*-black hole might be saved from falling through *its own* Schwarzschild radius r_S by beginning to burn up mass as soon as its surface approaches a ruler distance [13] of one Planck length beyond r_S ($r_S + l_{\text{Planck}}$), i.e., that black holes can thus come within a gnat's eyelash of forming, but can*not completely* form? This gnat's eyelash would of course *not* be sufficient to result in any *measurable or observable* astronomical or astrophysical dissimilarity from *completely*-formed black holes. (iii) And, for example, given (ii) immediately above, that as Hawking evaporation of a gnat's-eyelash *near*-black hole proceeds into a vacuum whose temperature is at (or at least sufficiently close to) absolute zero (0 K), its surface always remains a ruler distance [13] of one Planck length beyond r_S, this being maintained until Hawking

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evaporation is complete? (iv) Might this be relevant, for example, with respect to solving the black-hole information paradox? For, if black holes *can* come within a gnat's eyelash of fully forming but can*not* fully form, no information can ever fall into a fully-formed black hole and hence there is no need for it to be retrieved from one. Of course, various (hopefully, at least to some extent, mutually compatible) resolutions of the black-hole information paradox have been proposed [99–108][12]. We note that if black holes can thus come within a gnat's eyelash—but no further—of forming, the *maximum* possible depth $|\Phi|_{\max}$ of their gravitational wells is *finite*. For then, $|\Phi|_{\max} = |\Phi|_{r_S+(\delta l)_{\min}} = |\Phi|_{r_S+l_{\text{Planck}}}$ as per Equations (19) and (38).

7. Relativistic Gravitational Temperature Gradients Can*not* Defy the Second Law of Thermodynamics

We note that equilibrium vertical gravitational temperature gradients that exist [8–11]—indeed that are *required* [8–11]—by relativistic thermodynamics [8–11] can*not* be exploited to violate the Second Law of Thermodynamics.

First, consider a gravitator enclosed concentrically within an opaque thermally-insulating spherical shell.[7] Now consider a heat engine trying to exploit the equilibrium relativistic gravitational temperature gradient, via a hot reservoir at a lower altitude at temperature T_{hot} and a cold reservoir at a higher altitude at temperature T_{cold}.

Macroscopic consideration: Thermodynamic equilibrium [8–11,109,110] exists within the shell, and thermodynamic equilibrium [8–11,109,110] necessarily implies hydrostatic equilibrium [110–115] (but not necessarily vice versa [8–11,109–115]). Owing to hydrostatic equilibrium [110–115] that thermodynamic equilibrium [8–11,109,110] necessarily implies, the weight Eg/c^2 of a parcel of thermal energy E where the gravitational acceleration is g [8–11] *exactly counterbalances* its tendency to flow from higher temperatures at lower altitudes to lower temperatures at higher altitudes, so macroscopically there is staticity and hence no flow of heat that a heat engine can utilize. [Likewise at hydrostatic equilibrium—even without, let alone with, thermodynamic equilibrium, and either relativistically or non-relativistically—the weight mg of a parcel of fluid (gas or liquid) of mass m where the gravitational acceleration is g [8–11] *exactly counterbalances* its tendency to flow from higher pressures at lower altitudes to lower pressures at higher altitudes, so macroscopically there is staticity and hence no flow of fluid that a pneumatic engine can utilize.]

Microscopic consideration: While *macro*scopically thermodynamic equilibrium is, or at least can be construed as, *static*, by contrast, *micro*scopically, thermodynamic equilibrium is *dynamic*. At thermodynamic equilibrium, individual blackbody-radiation photons move up and down in any gravitational field. But: *Even without* our engine trying to convert any heat whatsoever from the hot reservoir into work, the gravitational redshift diminishes the temperature of equilibrium blackbody photons radiated at T_{hot} from the lower altitude of the hot reservoir to T_{cold} upon them reaching the higher altitude of the cold reservoir—thus diminishing the Carnot efficiency $\epsilon_{\text{Carnot}} = 1 - (T_{\text{cold}}/T_{\text{hot}})$ to $\epsilon_{\text{Carnot}} = 1 - (T_{\text{cold}}/T_{\text{cold}}) = 0$. What the gravitational temperature gradient giveth, the gravitational redshift taketh away [8–11]: after the gravitational redshift has taken its cut, there is *nothing* left over to be converted into work [8–11]. (Similarly, in accordance with either relativistic or non-relativistic hydrodynamics and thermodynamics [109–115], even though *micro*scopically at thermodynamic equilibrium individual fluid molecules comprising a gas or liquid move up and down in any gravitational field, gravitational pressure gradients can*not* be exploited by a pneumatic engine: at hydrostatic equilibrium—even without, let alone with, thermodynamic equilibrium [109–115], and either relativistically or non-relativistically—what the gravitational pressure gradient giveth, the weight taketh away [110–115].)

Next, consider a gravitator *not* enclosed concentrically within an opaque thermally-insulating spherical shell, but instead surrounded by a vacuum at (or at least sufficiently close to) absolute zero (0 K). Such a gravitator is *not* at thermodynamic equilibrium. *Any* gravitator's Tolman-Hawking equilibrium blackbody radiation will disperse into the surrounding vacuum. Hence with*out* enclosure

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within such a shell, a heat engine *can* operate—but only at the expense of the increase in entropy owing to dispersal of the radiation into the vacuum. [Similarly, with*out* enclosure within a shell, a gravitator's gas (liquid) atmosphere (hydrosphere) will evaporate into a surrounding vacuum. Hence with*out* enclosure within a shell, a pneumatic engine also *can* operate—but only at the expense of the increase in entropy owing to dispersal of the atmosphere (hydrosphere) into the vacuum.]

Hence both with and without enclosure by an opaque thermally-insulating spherical shell, the Second Law of Thermodynamics is obeyed.

To re-emphasize, thermodynamic equilibrium [8–11,109,110] necessarily implies hydrostatic equilibrium [110–115], but not necessarily vice versa [8–11,109–115]. The terms "hydrostatic equation [110–113]" or "barometric equation [115]" are sometimes employed to denote hydrostatic equilibrium [110–113] but not necessarily thermodynamic equilibrium [8–11,109–115]. Earth's atmosphere and oceans are typically at hydrostatic equilibrium (or at least very nearly so). But, of course, because they are impelled by the large temperature difference between the hot solar disk and the cold rest of the sky, they are not at thermodynamic equilibrium.

8. Conclusion

Following introductory remarks in Section 1, in Section 2 we reviewed basic concepts pertaining to Schwarzschild black holes and Hawking radiation. In Section 3 we discussed the anticipation of Hawking radiation—albeit from *non*-black holes—initially by R. C. Tolman and shortly thereafter with P. Ehrenfest (see also Garrod [11]). This has been corroborated by recent research [12]. This *anticipation* culminates in the *proof* [8–12] that *any* gravitator—black hole or *non*-black hole—*must radiate* and hence can*not* be in thermodynamic equilibrium with a surrounding vacuum at (or at least sufficiently close to) absolute zero (0 K), but *must completely evaporate into that vacuum within a finite time*. The times required for evaporation of black holes and non-black holes were compared. In Section 4, we showed that (i) if firewalls exist, they can originate via Hawking radiation at the minimum possible ruler distance [13] (the Planck length [14–16]) beyond the Schwarzschild horizon, where it has not suffered any gravitational redshift [17,18], or, alternatively, suffered maximal gravitational blueshift and (ii) the firewall temperature is on the order of the Planck temperature [19], *in*dependently of the mass and hence also of the Schwarzschild radius of a Schwarzschild black hole. In Section 5, we explained the exponential nature of the gravitational frequency shift as a function of the gravitational potential. In Section 6, we considered the firewall-mass problem [3], and provided an at least *prima facie* tentative resolution thereto based on: (i) the mass of a firewall being canceled by the *negative* gravitational mass [17,18] $=$ (*negative* gravitational energy)$/c^2$ [17,18] accompanying its formation, (ii) the *un*changed observations of a distant observer upon formation of a firewall, and (iii) Birkhoff's Theorem [20–31]. (The basis upon Birkhoff's Theorem is *un*altered by the caveat [28–31][1] thereto.) We showed that the mass of a firewall is *exactly counterbalanced* by the (negative) gravitational mass-energy accompanying its formation. Perhaps this may complement other lines of reasoning [4] disputing massiveness [3] of firewalls. In Section 7, we showed that equilibrium relativistic gravitational temperature gradients can*not* be exploited to violate the Second Law of Thermodynamics. Auxiliary topics are discussed in the Notes.

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Notes

[1] Based on Birkhoff's Theorem (see Refs. [20–28]), it is usually averred that in General Relativity—as in Newtonian gravitational theory—the gravitational field vanishes and the gravitational potential is negative and *constant* within an evacuated non-rotating spherical shell. This implies that spacetime is Minkowskian within the shell. (See, for example, Ref. [13], Section 12.2B.) However, there is a caveat [28–31]: To the contrary, it has also been averred that, in General Relativity—*un*like in Newtonian gravitational theory—the gravitational field does *not* vanish within an evacuated non-rotating spherical shell, but instead that the field within the shell is directed radially outwards from the center [28–31]. This implies that the gravitational potential within the shell is negative but *not* constant, being least strongly negative at the center of the shell and most strongly negative at the inner surface of the shell. Moreover, contrary to the corresponding usual inference based Birkhoff's Theorem (see Ref. [13], Section 11.2B and Refs. [20–28]), this *further* implies that spacetime is *not* Minkowskian within the shell [28–31]. (The discussion of this caveat in Ref. [28] is in the section entitled "Inside Spherical Shell" under the Talk tab thereof, and is intermediate in viewpoint between the standard interpretation of Birkhoff's Theorem as per Refs. [20–27] and otherwise in Ref. [28] on the one hand, and that as per Refs. [29–31] on the other.) This at least helps to resolve a clock paradox in General Relativity [31]: If the gravitational *field* vanishes, the gravitational potential is negative and *constant*, and hence spacetime is Minkowskian within an evacuated non-rotating spherical shell, how is a clock within the shell to *know* that it is within the shell and thus at a *negative gravitational potential*, and hence that it must tick more slowly than a clock at $r/r_{\text{shell}} \to \infty$ and hence at *zero gravitational potential*? For, like a clock at $r/r_{\text{shell}} \to \infty$, it would then see *zero gravitational field* and hence Minkowski spacetime. And according to General Relativity, a clock, like any other entity, interacts *locally* with a gravitational *field*—*no action at a distance*. A *non*-vanishing gravitational *field* within an evacuated non-rotating spherical shell, which a clock therein can interact with *locally*, thus at least helps to resolve this clock paradox [28–31]: via *local* interaction with a *non*-vanishing gravitational *field* a clock at the center of the shell *knows* that it must tick more slowly than a clock at $r/r_{\text{shell}} \to \infty$ [28–31], and a clock at the inner surface of the shell *knows* that it must tick more slowly yet [28–31]. Nonetheless a non-vanishing gravitational field within an evacuated non-rotating spherical shell does *not* alter any *other* inferences based on Birkhoff's Theorem. If, on the contrary, the gravitational field *does* vanish, the gravitational potential is negative and *constant*, and hence spacetime *is* Minkowskian within an evacuated non-rotating spherical shell, then resolution of this clock paradox would seem to require either (i) *local* interaction of *the clock's gravitational field*—which extends *beyond* the shell—with the *shell's gravitational field* somehow being communicated to the clock itself [30] or (ii) *local* interaction of the clock with the *shell's gravitational potential* [31]. The Aharonov-Bohm-effect counterpart of Option (i) is interpreting the Aharonov-Bohm effect as due to *local* interaction of an electron's *magnetic field* with the *magnetic field* within a tightly-wound solenoid—the electron's *magnetic field* penetrates *into* the solenoid—even though the electron *itself* sees *only* the solenoid's magnetic vector potential and *not* the solenoid's magnetic field [31]. (The electron *must be moving* relative to the solenoid in order for the Aharonov-Bohm-effect to occur and hence *must* generate a *magnetic* field in the reference frame of the solenoid. If the solenoid is tightly wound, the electron's *electric* field can*not* penetrate into it.) The Aharonov-Bohm-effect counterpart of Option (ii) is the standard interpretation of the Aharonov-Bohm effect: *local* interaction of the electron with the solenoid's *magnetic vector potential*, which does *not* vanish *outside* of the solenoid [31]. (Although not related to the topics discussed in this paper, perhaps as a brief aside it should be noted that the Aharonov-Bohm effect is important in both theoretical and experimental investigations of electromagnetic quantum phenomena. See, for example, Imry, Y. In: Fraser, G., editor, Ref. [15]; Chapter 12 (especially Sections 12.4–12.7).

[2] Note, however, that while the *angular* term of the Schwarzschild metric [the last term in all three lines of Equation (1)] is of *identical Euclidean* form at *all* $r \geq 0$, by contrast *radial* ruler distance is $dl = \left(\frac{r_S}{r} - 1\right)^{1/2} dt$ at $r < r_S$, as opposed to $dl = \left(1 - \frac{r_S}{r}\right)^{-1/2} dr$ at $r \geq r_S$. This obtains because the dt and dr terms of Schwarzschild metric [Equation (1)] switch sign as r_S is crossed. See Ref. [13], Sections 11.1 and 12.C–12.1E (especially Sections 12.1D

and 12.1E). Yet also note that in the line immediately following Equation (12.15) in Section 12.1E: At $r < r_S$: r is referred to as a 'time'—quotation marks in the original text—recognizing that while r is time*like*, r is *not* time itself.

[3] The special case discussed in Ref. [34]—the excess (extra-Euclidean) vertical radial ruler distance of $GM/3c^2$ from the center to the surface of a non-rotating sphere of mass M and uniform density (in the weak-field limit, i.e., $M \ll r_S c^2/2G \iff r \gg r_S = 2GM/c^2$)—may help to clarify the vertical stretching of space from the Euclidean by gravity in general. It is a special case of the more general result discussed in Section 11.5 of Ref. [13]. By *vertical* it is of course meant perpendicular to the equipotential surface. The vertical direction does not in general coincide with the geometric center of a gravitator [see Ref. [13], Section 9.6 (especially the last two paragraphs)], but it does so coincide in the special case of a non-rotating spherical gravitator whose density varies at most only radially.

[4] English translations of Ref. [35] are provided in Refs. [36–38]. See also the Editor's Note (Ref. [39]) and Ref. [40], which synopsize and discuss Ref. [35].

[5] Even if the classical vacuum might be construed as nothingness, the quantum-mechanical vacuum—space as it actually exists—certainly can*not*. (See Ref. [13], pp. 418–419 and 480, Section 21.4, and Chapters 43–44; and Refs. [15,16].) If gravity stretches space, can space sustain tension? Since a medium capable of sustaining tension is required for the transmission of transverse waves [by contrast, longitudinal waves, e.g., sound, can travel through any (material, i.e., non-vacuum) medium], and since electromagnetic radiation is comprised of transverse waves, might space be construed as a latter-20th-century and 21st-century interpretation of the ether [sometimes spelled aether (the a is silent)] postulated in 19th-century physics? [See Ref. [13], Chapter 1 (especially Sections 1.6–1.10); Ref. [40], Chapter 1 (especially pp. 8–20), and p. 66; and Ref. [36], pp. 495–496.] The conventional viewpoint is, of course, that electromagnetic waves serve as their own medium—their own ether—via the continual handoff of energy from transverse electric field to transverse magnetic field to transverse electric field ... . See Ref. [41], pp. 450–458 (especially pp. 452–453).

[6] By *downwards* it is of course meant perpendicular to the equipotential surface and towards a gravitator. Downwards is not in general towards the geometric center of a gravitator [see Ref. [13], Section 9.6 (especially the last two paragraphs)], but it is so in the special case of a non-rotating spherical gravitator whose density varies at most only radially. (Of course, upwards is in the opposite direction, i.e., perpendicular to the equipotential surface and away from the gravitator.)

[7] Because matter is not a continuum but is comprised of atoms, our opaque thermally-insulating spherical shell cannot be arbitrarily thin and therefore cannot have an arbitrarily small surface mass density ρ_{shell}. Even to exist at all, it must be at least one atom thick. To be thermally-insulating, it must be opaque, and to be opaque it must be many atoms thick. (Opacity is a necessary but not sufficient condition for thermal insulation.) Hence (ignoring our speculations as per the last paragraph of Section 6) our spherical shell's $M_{\text{shell}}/r_{\text{shell}}$ ratio must be within a finite upper limit if it is not to be a black hole itself and suffer gravitational collapse: we must require the inequality $M_{\text{shell}}/r_{\text{shell}} = 4\pi\rho_{\text{shell}} r^2_{\text{shell}}/r_{\text{shell}} = 4\pi\rho_{\text{shell}} r_{\text{shell}} < c^2/2G \implies r_{\text{shell}} < c^2/8\pi G\rho_{\text{shell}}$. But it certainly is feasible for r_{shell} to greatly exceed $\lambda^{\text{Wien,max}}_{r\to\infty} = 15.90231998 r_S$ [see the paragraph containing Equation (35)] while still meeting this inequality and hence without risk of the shell's gravitational collapse: the strong inequality $\lambda^{\text{Wien,max}}_{r\to\infty} \ll r_{\text{shell}} < c^2/8\pi G\rho_{\text{shell}}$, indeed, even the double strong inequality $\lambda^{\text{Wien,max}}_{r\to\infty} \ll r_{\text{shell}} \ll c^2/8\pi G\rho_{\text{shell}}$, is very easily met. This is sufficient is for $r_{\text{shell}} \to \infty$ to *effectively* obtain for all practical purposes.

[8] Equations (2)–(6), (9), and (10) are in accordance with considerations of Unruh radiation and the equivalence principle (see Refs. [68,69]). An object undergoing acceleration a in Minkowski spacetime experiences Unruh radiation at temperature $T_U = \frac{\hbar a}{2\pi ck}$. Force $f = \frac{mc^4}{4GM}\left(1-\frac{r_S}{r}\right)^{-1/2} = \frac{GMm}{r_S^2}\left(1-\frac{r_S}{r}\right)^{-1/2}$ is required to dangle a mass m at r_S from a higher altitude $r > r_S$ (with a massless string) above a Schwarzschild black hole of mass M: see Ref. [13], Section 12.2 [especially Equation (12.17)]. The corresponding acceleration is $a = \frac{f}{m} = \frac{c^4}{4GM}\left(1-\frac{r_S}{r}\right)^{-1/2} = \frac{GM}{r_S^2}\left(1-\frac{r_S}{r}\right)^{-1/2}$ and hence the corresponding Unruh-radiation temperature is $T_{U,r} = \frac{\hbar a}{2\pi ck} = \frac{\hbar}{2\pi ck} \times \frac{c^4}{4GM}\left(1-\frac{r_S}{r}\right)^{-1/2} = \left(1-\frac{r_S}{r}\right)^{-1/2}\frac{\hbar c^3}{8\pi GkM} = \left(1-\frac{r_S}{r}\right)^{-1/2} T_{H,r\to\infty} = T_{H,r}$. In accordance with the equivalence principle, $T_{U,r}$ is equal to $T_{H,r}$ as per Equation (3). In the limit $r \to \infty$, in accordance with the equivalence principle, $T_{U,r\to\infty} = \frac{\hbar c^3}{8\pi GkM} = T_{H,r\to\infty}$ as per Equation (2). {See Ref. [13], Section 12.2 [especially

Equation (12.17)] and Section 12.6.} But a caveat: It is *important* to note that: Hawking-radiation temperature T_H is by Equations (2) and (3) a function of the gravitational *potential* Φ. By contrast, Unruh-radiation temperature T_U is a function of the motional *acceleration* a in Minkowski spacetime, so *prima facie* the equivalence principle might seem to suggest that it be the *same function* of the magnitude $|\mathcal{G}| = |d\Phi/dl|$ of the gravitational *acceleration*, i.e., the *same function* of the magnitude of the *gradient* of the potential rather than a function of the potential itself (whether of a black hole or a non-black hole). But this *in*correctly implies that T_U need *not* in general be equal to T_H: e.g., at large enough r away from a Schwarzschild black hole or for a sufficiently weak Schwarzschild non-black hole that the Newtonian approximation $a = f/m = GM/r^2$ is valid with negligible error, this *in*correctly implies that $T_{U,r} = \frac{\hbar a}{2\pi ck} = \frac{\hbar GM}{2\pi ckr^2}$, and in the limit $r \to \infty$, that $T_{U,r\to\infty} = 0$—in *dis*agreement with $T_{H,r\to\infty} > 0$ as per Equation (2), $T_{H,r}$ as per Equation (3), and Tolman's [8,9] generalization, bolstered with Ehrenfest [10] (see also Garrod [11]), as per Equations (4)–(6). The *correct* correlation between T_H and T_U is that obtained as per Ref. [13], Section 12.2, especially the paragraph containing Equation (12.17): via dangling a mass at r_S from a higher altitude $r \gg r_S$ (with a massless string). With respect to the *correct* correlation between T_T and T_U, perhaps our plausibility argument as per Equations (7)–(10) and the associated discussions may at least serve as a starting point.

[9] Usually it is assumed that electromagnetic radiation can be tracked to a source, but Maxwell's equations do *not* require this. This was pointed out to me by Dr. James H. Cooke in a private communication in the 1980s.

[10] The concepts of microscopic reversibility and detailed balance require modifications in cases of (i) time-symmetry-violating dynamics and (ii) collisions between *un*symmetrical molecules even given *non*-time-symmetry-violating dynamics. See, for example, Ref. [95] concerning (i) and Ref. [96] concerning (ii). But these modifications do *not* apply with respect to electromagnetic radiation in general and hence with respect to equilibrium blackbody radiation in particular. Hence the analyses provided in Refs. [8–11,64,65] concerning equilibrium blackbody radiation are *completely valid*.

[11] Auxiliary phenomena that might have contributed to the end of the dinosaurs' reign could have included a surge in volcanic activity, the impact of a secondary asteroid if the primary impactor had a satellite, etc.

[12] Reference [104] states that owing to quantum gravitational corrections, Hawking radiation is *not exactly* Planckian, i.e., *not exactly* blackbody, and thus *not exactly* maximum-entropy and hence a carrier of information. But this assumes that a black hole radiates into empty space. What happens if, instead, a black hole is enclosed concentrically by an opaque thermally-insulating spherical shell? *Initially* upon emission from the black hole, Hawking radiation emanating from the black hole would still carry information. But the Hawking radiation emanating from the black hole would then be thermalized to an *exactly* Planckian distribution within the spherical shell. Would its information then be lost? Or would the information be preserved, even if only in latent form, even after thermalization? Also, a few caveats concerning Ref. [104] are quoted in Ref. [105]. References [106] and [107] seem especially pertinent with respect to the last paragraph of Section 6, because they discuss the possibility—at least in principle, even if not in practice with realizable technology—of *experimentally* determining whether the black-hole information paradox is resolved via firewalls, as we discussed qualitatively in the last paragraph of Section 6, or via complementarity, according to which the interior of a black hole and Hawking radiation are not independent, but correlated.

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