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Article

Selection of Criteria for the Application of the Integrated AHP-PROMETHEE/GAIA Method for Predicting the Risk of Deformation of Storage Tanks

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Abstract

Storage tanks are critical infrastructure in industries such as oil and gas, chemicals, and water treatment. Deformation poses serious risks, including catastrophic failures, environmental pollution, and economic losses. Current risk assessment methods often lack a structured, multi-criteria approach that integrates quantitative data with expert judgment, leading to potential omissions or subjective biases. A robust framework for predicting deformation risk is essential for proactive maintenance and safety management. This research proposes the development and application of an integrated multi-criteria decision-making (MCDM) methodology that combines the Analytic Hierarchy Process (AHP), the Preference Ranking Method for Enriching Evaluations (PROMETHEE), and the Geometric Analysis for Interactive Assistance (GAIA). The main focus is on establishing a rigorous, systematic, and defensible framework for selecting criteria specific to predicting the risk of storage tank deformation. The effectiveness of the AHP-PROMETHEE/GAIA method critically depends on identifying the most relevant, measurable, non-redundant, and comprehensive set of criteria that affect the risk of deformation. Poorly selected criteria will compromise the entire predictive model. This proposal directly addresses this critical gap.

Keywords: storage tanks; analytical hierarchy process (AHP); PROMETHEE; GAIA; criteria selection; numerical simulation; stresses; deformations; pressure; wall thickness

1. Introduction

Storage tanks are vital assets in industries such as petroleum (Figure 1), chemicals, and water treatment, where their structural integrity is of paramount importance. Deformation, manifested as buckling, bulging, or subsidence, is a critical failure mode with potentially catastrophic consequences, including loss of containment, environmental damage, significant economic losses, and threats to human safety. Proactive prediction of deformation risk is therefore essential for effective asset integrity management (AIM) and predictive maintenance strategies.

Multi-criteria decision-making methods (MCDM) offer powerful frameworks for handling the complex, multi-layered nature of engineering risk assessment. The integrated approach of the Analytic Hierarchy Process (AHP), the Preference Ranking Method for Enriching Evaluations (PROMETHEE), and the Geometric Analysis for Interactive Assistance (GAIA) is particularly promising. The AHP structures the problem and derives criterion weights based on expert judgment, PROMETHEE provides a robust ranking mechanism based on the principles of superior ranking, and GAIA offers a visual analysis of the decision space and criterion conflicts.

However, a critical research gap hinders the reliable and effective application of this integrated method, especially for predicting the risk of deformation of storage tanks: the lack of a systematic,

transparent and defensible framework for selecting the optimal set of criteria. The current literature reveals several specific shortcomings. Ad hoc and unvalidated selection of criteria: Many applications of MCDM methods, including AHP-PROMETHEE/GAIA, in related fields (e.g. general risk assessment, supplier selection) often rely on sets of criteria derived from limited literature reviews or solely from the intuition of the researchers, without rigorous validation or structured expert consensus. This risks omitting critical factors or including irrelevant ones, which compromises the validity of the model.

Lack of domain-specific adaptation: While there are numerous generic MCDM studies, there is little research that explicitly focuses on establishing a comprehensive and relevant set of criteria specifically for predicting storage tank deformation. Factors unique to tank mechanics (e.g., shell stability under vacuum, localized corrosion patterns, differential foundation settlement effects, residual weld stresses) are often not systematically addressed or prioritized.

Insufficient integration of quantitative and qualitative factors: Deformation risk involves both measurable parameters (e.g. wall thickness, settlement measurements) and qualitative assessments (e.g. weld quality assessment, maintenance efficiency). Existing approaches often lack a structured methodology to effectively identify, define and integrate these different types of criteria within the AHP-PROMETHEE framework.

Ignoring redundancy and independence of criteria: The potential for high correlation or redundancy between criteria (e.g. corrosion rate and remaining wall thickness) is often ignored. The use of redundant criteria can distort the weighting in AHP and the preference modeling in PROMETHEE, leading to biased results. A formal process for identifying and mitigating redundancy in the context of tank deformation is lacking.

Neglect of practical measurability and data availability: Proposed criteria in theoretical models often assume ideal data availability. In practice, obtaining reliable data for certain parameters on stagnant reservoirs can be challenging or expensive. The literature lacks a framework that explicitly includes measurement feasibility and operational data availability as key filters when selecting criteria for this specific application.

Lack of standardized sets of criteria: There is no widely accepted, standardized hierarchy or set of criteria for predicting deformation risk using advanced MCDM, which makes comparability between studies and practical adoption into industry standards difficult.

Inadequate connection to PROMETHEE/GAIA requirements: The process of how the selected criteria and their AHP weights are directly translated into the definition of the corresponding PROMETHEE preference functions (thresholds, shapes) and how the set of criteria affects the interpretability and stability of the GAIA plane is rarely addressed during the initial stage of criterion selection. The sensitivity of the results of the integrated method to the choice of the set of criteria itself is insufficiently investigated.

This research directly addresses these critical gaps. We hypothesize that the effectiveness of the integrated AHP-PROMETHEE/GAIA method for predicting the risk of storage tank deformation is fundamentally conditioned by a rigorous, systematic, and transparent process for selecting the most relevant, non-redundant, measurable, and comprehensive set of criteria. This study proposes and validates a new, structured framework for identifying, weighting, refining, and validating criteria, specifically designed for this complex industrial problem, thereby filling a significant gap in the current literature and improving the practical utility of advanced MCDM in asset integrity management.

By carefully selecting criteria that are relevant, measurable, compatible with both AHP (for weighting) and PROMETHEE (for preference modeling), and informative for GAIA visualization, the integrated AHP-PROMETHEE/GAIA method becomes a powerful tool for understanding, ranking, and predicting the environmental risk posed by storage tanks. The visual output from GAIA is particularly valuable for communicating complex risk profiles and trade-offs with stakeholders.

Thin-walled metal shell structures are very efficient in their use of materials, but they are particularly susceptible to buckling failure. Roter, J.M. (1) demonstrates a basic understanding of

buckling theory and finite element methods). Cylindrical liquid tanks are essential elements of lifelines that need to maintain performance during and immediately after earthquakes. Malhotra, P.K., (2) in this study investigates the changes in seismic design loads caused by insufficient freeboard using experimental and numerical methods. Steel cylindrical tanks are most susceptible to damage due to dynamic buckling during earthquakes. Steel cylindrical tanks are most susceptible to damage due to dynamic buckling during earthquakes. Virella, J.C., Godoy(3) Earthquake-induced tank failure can cause serious environmental damage and economic losses. The study by Virella, J.C., Godoy, L.A., Suarez, L.E. (4) evaluated the elastic buckling of above-ground steel tanks anchored to foundations during seismic shaking. Chen, J.Z., Kianoush, M.R., & Tait, M.J. (5). 0describe an advanced FSI-FEM method for numerical modeling of dynamic fluid-structure interaction in tanks Jonaidi, M., & Ansourian, P. (6) analyze large vertical cylindrical steel tanks built on soft foundations that can lead to different types of settlements, which are well known as uniform settlement, planar slope, and differential settlement. Zhao, X., & Lin, Z. (7). They consider the wind-induced buckling of metal tanks during construction as thin-walled structures using the finite element method.

The effect of corrosion on the operability and serviceability of bottom plates of steel aboveground storage tanks, an effect considered a serious threat to the structural integrity of the tanks Rahman, M.S., Khan, R., & Akram, M. (8). provide a finite element analysis of a corroded API 650 storage tank.

The seismic response of a floating roof tank considering splashing and roof flexibility was considered in a study by Park, H., Koh, H.M., Kim, J.K., & Cho, S. (9). The response of floating roof tanks was evaluated for various types of ground motions, including near-source and long-term far-field recordings. In addition to comparing the responses of tanks with and without a roof, the effects of different ground motions on the wave height, lateral forces, and overturning moments induced on the tank were examined.

The approach in this paper overcomes the limitations of regulations and simplified models, offering a powerful tool for predicting specific failure scenarios (such as wind buckling + specific settlement patterns), optimizing designs (e.g. stiffener placement), prioritizing inspections (identifying critical zones), and assessing the risk of existing tanks based on their as-built geometry and measured settlement. It provides the highly accurate mechanistic understanding required by methods such as AHP-PROMETHEE for effective risk assessment.

Atmospheric oil storage tanks (AST) are thin-walled structures crucial to the petrochemical industry. Their large diameters (up to 100 m) and thin walls (typically 5–40 mm) make them susceptible to complex deformation regimes under operational loads (e.g. hydrostatic pressure, wind, seismic activity, foundation settlement). Traditional analytical methods (e.g. API 650/653 standards) simplify geometry and loading, limiting the accuracy of localized or nonlinear deformations. The finite element method (FEM) has emerged as a dominant tool for high-fidelity simulation, incorporating geometric nonlinearity, material plasticity, and complex boundary conditions.



Figure 1. Oil storage tanks.

2. Mathematical Modeling

Calculation of tank shells is based on the following assumptions:

- That the thickness of the shell (ϵ) is small compared to the other dimensions of the shell.
- That the deflections are small compared to the thickness of the shell
- That the points on the normal of the middle surface of the shell before the deformation are located on the normal of the deformed middle surface.
- That the normal stresses acting on the middle surface of the shell are so small that they can be neglected.

2.1. Forces and Moments on the Shell Element

The width of the side surfaces of the elements can be expressed through the z coordinate (Figure 1) from the similarity of the triangles. Bearing in mind that for $z=0$, the width of the cross-section is equal to unity, from the mentioned similarity, the width of the side surfaces of the element can be determined as a function of the coordinate z .

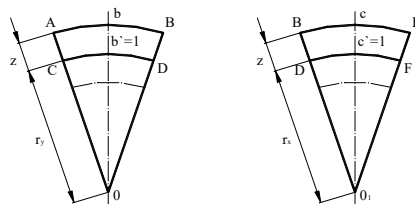


Figure 2. The width of the side surfaces of the element.

$$N_x = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \sigma_x \left(1 + \frac{z}{r_y}\right) dz \quad (1)$$

Where are:

N_x - Force $\in$ the x axis direction,

σ_x - Stress $\in$ the direction of the x axis,

$\delta/2$ - Half the thickness of the shell

Formula (1) provides a more refined way to calculate the longitudinal membrane force per unit width in a cylindrical shell by taking into account the potential variation of longitudinal stress across the wall thickness due to bending in the circumferential direction. This is particularly important in regions of geometric discontinuities, local loads, or imperfections where simple membrane theory may not be sufficient to accurately determine the stress and force in large-volume oil storage tanks.

$$N_y = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \sigma_y \left(1 + \frac{z}{r_x}\right) dz \quad (2)$$

Where are:

N_y - Force $\in$ the y axis direction

σ_y - Stress $\in$ the direction of the y axis

In essence, formula (2) provides a more sophisticated way to calculate the circumferential membrane force per unit width in a cylindrical shell by taking into account the potential non-uniform distribution of hoop stress across the wall thickness due to bending in the longitudinal direction. This is especially important in regions where simple membrane theory is insufficient due to geometric discontinuities, local loads, imperfections, or dynamic loading.

$$N_{xy} = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \tau_{xy} \left(1 + \frac{z}{r_y}\right) dz \quad (3)$$

Where are:

N_{xy} - shear force along the shell thickness z $\wedge$ along the radius of curvature r_y .

τ_{xy} - shear stress both through the thickness z $\wedge$ through the local radius of curvature r_y .

To use this formula effectively, it is necessary to know or assume the shear stress distributions through both the thickness and the local radius of curvature r_y . These values are usually obtained from more advanced shell theories, analytical solutions for specific loading scenarios, or numerical methods such as finite element analysis, which can capture the complex stress states that arise in large-volume cylindrical tanks.

$$N_{yx} = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \tau_{yx} \left(1 + \frac{z}{r_x}\right) dz \quad (4)$$

Where are:

N_{yx} - $\in$ plane membrane shear force per unit width,

τ_{yx} - shear stress distribution across the wall thickness due to longitudinal bending

Formula (4) provides a way to calculate the in-plane membrane shear force per unit width, taking into account the potential non-uniform distribution of shear stress across the wall thickness due to longitudinal bending. This is important in regions and under loading conditions where longitudinal bending is significant and can affect the shear of a large volume cylindrical tank.

$$M_x = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \sigma_x \left(1 + \frac{z}{r_y}\right) z dz \quad (5)$$

Where are:

M_x – Longitudinal bending moment per unit width in cylindrical shell

Formula (5) provides a way to calculate the longitudinal bending moment per unit width in a cylindrical shell, taking into account the potential non-uniform distribution of longitudinal stress due to both the total longitudinal bending and the effect of circumferential curvature. This is crucial for a comprehensive stress analysis of a large-volume oil storage tank, especially in regions where bending effects are significant.

$$M_y = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \sigma_y \left(1 + \frac{z}{r_x}\right) z dz \quad (6)$$

Where are:

M_y - Circumferential bending moment per unit width in a cylindrical shell

Formula (6) provides a way to calculate the circumferential bending moment per unit width in a cylindrical shell, taking into account the potential non-uniform distribution of circumferential stress due to the total circumferential bending and the effect of longitudinal curvature. This is crucial for a comprehensive stress analysis of a large-volume oil storage tank, especially in regions where bending effects are significant in the circumferential direction.

$$M_{xy} = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \tau_{xy} \left(1 + \frac{z}{r_y}\right) z dz \quad (7)$$

Where are:

M_{xy} - Bending moment per unit length in the tank wall

Formula (7) calculates the bending moment per unit length in the tank wall by integrating the moment of shear stress contribution (τ_{xy}) over the wall thickness. The term $\left(1 + \frac{z}{r_y}\right)$ includes the effect of the curvature of the tank on the stress distribution and the resultant moment.

Large cylindrical oil tanks are subjected to various loads, including hydrostatic pressure from stored oil, wind loads, and seismic loads. These loads induce stresses within the tank wall, including shear stresses.

$$M_{yx} = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \tau_{yx} \left(1 + \frac{z}{r_x}\right) z dz \quad (8)$$

Where are:

M_{yx} - M_{yx} moments in the circumferential (rim) and axial (longitudinal) directions.

M_{yx} represents the bending moment acting on the other plane and about the other axis with respect to M_{xy} . If we consider a small rectangular element cut off from the tank wall M_{yx} and M_{xy} would be the moments acting on the edges of this element. For a cylindrical shell, it is common to analyze the moments in the circumferential (rim) and axial (longitudinal) directions.

$$Q_x = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \tau_{xz} \left(1 + \frac{z}{r_y}\right) dz \quad (9)$$

Where are:

Q_x - Q_x internal shear force per unit length.

Q_x represents the internal shear force per unit length acting on the cross section whose normal is in the "x" direction. This force is the result of the distribution of shear stresses (τ_{xz}) through the

thickness of the tank wall. The term $\left(1 + \frac{z}{r_y}\right)$ includes the effect of the curvature of the tank in the "x" direction in the shear force calculation. This is important because the geometry of a curved shell affects how stresses are distributed and how internal forces develop. In the analysis of cylindrical shells, shear forces are crucial for maintaining equilibrium and are related to changes in bending moments along the shell.

$$Q_y = - \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \tau_{yz} \left(1 + \frac{z}{r_x}\right) dz \quad (10)$$

Where are:

Q_y - internal shear force on the cross section normal to the axial direction of the cylinder

Q_y represents the internal shear force per unit length acting on the cross-section normal to the "y" direction (axial direction of the cylinder). This force results from the distribution of shear stresses (τ_{yz}) through the thickness of the tank wall. The term $\left(1 + \frac{z}{r_x}\right)$ includes the effect of the curvature of the tank in the "x" direction (circumferential direction) in the calculation of the shear force. This geometric factor is important for the accurate determination of the resultant shear force in the axial planes. In the analysis of a cylindrical shell, axial shear forces are crucial for equilibrium, especially when considering loads that are not purely axisymmetric. They are related to the variation of axial bending moments and axial membrane forces [11].

From the condition of normality of the sides of the shell it follows: $\tau_{xy} = \tau_{yz}$

Shear forces N_{xy} and N_{yx} ie. torsional moments M_{xy} and M_{yx} will be equal only in case

$r_y = r_x$ (that's the case of the plate).

The influence of the terms $\frac{z}{r_x}$ and $\frac{z}{r_y}$ in the previous expressions is very small, considering that δ and z very small compared to the radii of the curves r_x and r_y .

Therefore, the side surfaces can be considered as rectangles and the normal and shear stresses acting parallel to the middle surface have a linear distribution over the shell thickness. In the case of a tank, we can assume that the stresses acting parallel to the middle surface are evenly distributed over the thickness of the shell δ and that they do not depend on the coordinate z .

Bearing in mind that the influence of the curve is very small, it will be:

$$N_x = \sigma_x * \delta \quad (11)$$

$$N_y = \sigma_y * \delta \quad (12)$$

$$N_{xy} = N_{yx} = \tau_{yx} \delta = \tau_{xy} \delta \quad (13)$$

$$\text{during integration it will be: } M_x = M_y = M_{xy} = M_{yx} = 0 \quad (14)$$

From the equilibrium conditions, then there must also be transverse forces

$$Q_x = Q_y = 0 \quad (15)$$

From the above, it can be concluded that in this case only forces remain and that they are forces that act parallel to the middle surface (moments and transverse forces are neglected).

Such a stress state that is free from bending stress is called a membrane stress state.

The middle surface is exposed to stretching and shearing, and bending stresses have the importance of secondary stresses and can be ignored.

Conditions that must be met in order to have a membrane voltage state:

- The middle surface must have a continuous curve.
- The thickness of the shell must not change in jumps
- Shell loads must be continuously distributed and must not have too uneven a flow
- The edge forces must be tangential to the middle surface.

After collecting the necessary data, a material model is created in the FEA software. Depending on the software, different models such as elastic linear material, anelastic material, hyperelastic material or even thermoelastic material can be selected, depending on the needs of the analysis.

All collected mechanical and thermal properties are entered into the analysis software. These include Young's modulus, Poisson's ratio, coefficient of thermal expansion, yield strength, rupture strength, etc.

Depending on the characteristics of the material, linear or non-linear behavior of the material under load should be taken into account. Nonlinear behavior includes plasticity, viscoelasticity, and other deformation models that may be relevant to reservoir analysis. If we have available data on actual material properties or test results, it is important to compare the defined material properties with that data to ensure that the data is entered correctly and that the analysis will be realistic.

Defining the materials for the finite elements of the tank requires accurate data and an understanding of the mechanical and thermal properties of the material. It is a key step in ensuring the reliability of the finite element analysis.

2.2. Geometric model for the finite elements

The basic details of the geometric modeling:

Table 1. Mesh Density.

No. of Elements	Average Size (mm)	Mesh Level
42.000	150	Coarse
95.000	100	Medium
380.000	50	Fine
820.000	30	Extra Fine

Simplified Roof/Floor (Triangles).
Material : ASTM A516 Gr. 70 ($\sigma_i = 260$ MPa).
Boundaries : Fully Fixed Base, Free Roof Center

This setting captures the stress gradients in critical zones while minimizing computational cost. For dynamic loads (seismic), a 10% mass contribution is added via *FREQUENCY analysis.

3. Simulation and Discussion

The paper analyzes a vertical tank with a volume of $V=3000\text{m}^3$ (Figure 3). The figure shows the dimensions and discretization into finite elements. The tank has a variable wall thickness, reinforced in the upper part with a ring DB. The following loads were taken into account: concentrated load from the tank roof structure $F=1200$ daN, wind pressure $p_v=0.009$ bar and liquid pressure with density $\rho=0.0014$ daN/cm³. Due to the reinforcement with the ring DB, the conditions of no radial displacements on that circle were taken into account [12,13].

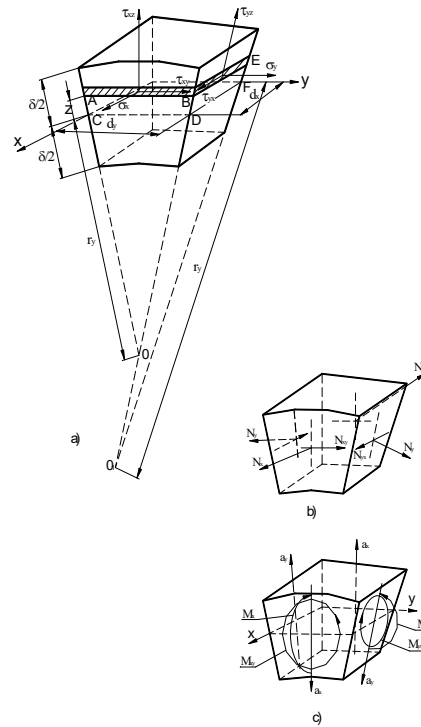
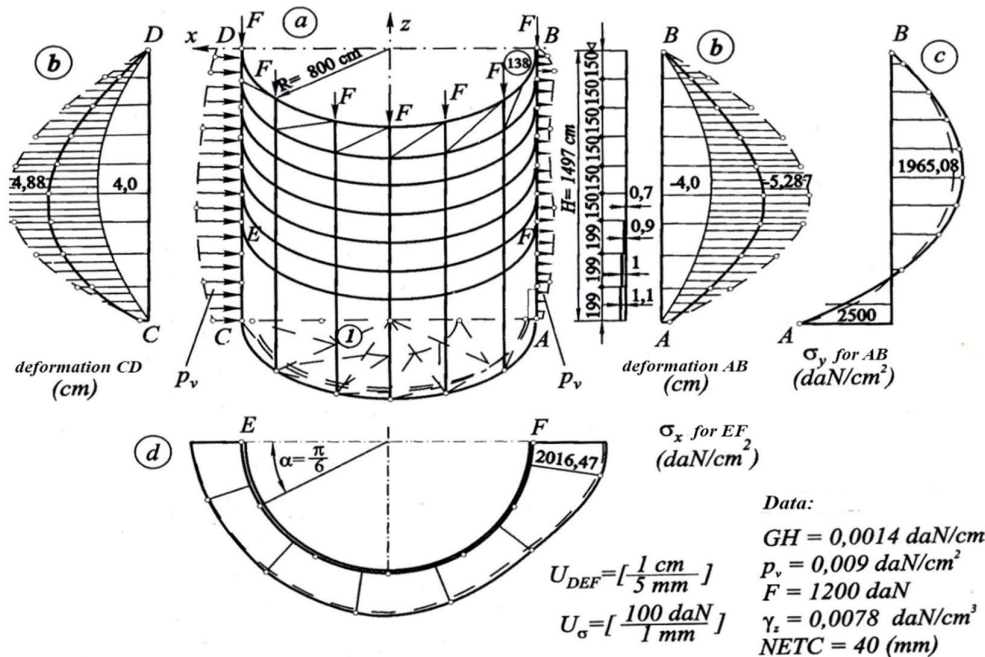


Figure 3. Forces and moments in shell sections.

Figure 3 shows the deformed derivatives CD and AB, as well as the distribution of radial stress σ_y , along AB and circular σ_x , for the circular line EF. Figure 3 shows the results of applying the finite element method (FEM) to calculate stresses and strains of a vertical atmospheric tank volume $V=3000\text{m}^3$. The structural parameters and tank loads from: self-weight, hydrostatic pressure are shown. Total number of elements 1200 (1152 quadrilateral + 48 triangular). Total number of nodes 1178 (cylinder: 49" rings" $\times 24=1176$. Roof center: 1 node. Bottom center: 1 node)

The application of the finite element method (FEM) enables a detailed analysis of the displacement and stress fields for different loading conditions.

Using analytical methods, it was possible to observe the membrane voltage state and only partially analyze the disturbance of the membrane voltage state at the place of entrapment of the casing and the bottom. During the analytical solution, special difficulties arise in the calculation of tanks with stepped wall thickness. Any place with a jump in wall thickness causes a disturbance in the membrane stress state. Bearing in mind that within each member the wall thickness is constant, one equation applies to each member, whereby the constants C_1 to C_4 of each member are determined from the boundary conditions and the conditions at the junction of two members (transitional conditions) [14].



At the junction of two articles, four conditions must be met:

To have equal deflections w ,

To have equal slopes of the elastic line $\frac{dw}{dx}$,

To match the transverse forces P_x ,

To match their bending moments M_y ,

Therefore, if there are r articles, $4r$ conditions must be met, and we have the same number of constants to determine. For a vertical atmospheric tank, in addition to the variable wall thickness, we have the effect of wind, concentrated loads, and a given inaccuracy that analytical methods cannot simultaneously capture [15].

Using the finite element method, it is possible to analyze all influences separately, as well as their overall influence on the behavior of the structure.

The example shown shows a change in the membrane voltage state caused by concentrated forces, wind action, stepped wall thickness, stiffening at the top of the tank and pinching at the bottom-shell junction.

It can be seen from the figure that the greatest disturbance of the membrane stress state occurs at the junction of the shell and the bottom, where there is also a change in the sign of the radial stress σ_x . In this particular case, the magnitude of this stress is $\sigma_x = -2500$ daN/cm² and represents the highest value. When designing, special attention must be paid to the dimensioning of the first two articles of the casing as well as to the dimensioning of the bottom of the tank. Here, a rigid base is assumed for supporting the bottom of the tank, i.e. limited movement of the bottom nodes in the direction of the z axis. If the foundation is not absolutely rigid, it is possible, with the knowledge of the characteristics of the soil, to include these influences in the computer program [16].

Observing the diagram of the radial stress σ_x along the derivative AB, one can observe another extreme stress value in the middle of the tank (1965.08 daN/cm²) at the point of maximum radial displacement. Bearing in mind that the impact of pinching at the bottom-casing location decreases towards the middle of the tank, and the influence of the upper stiffening ring DB also decreases, this value is the closest to the membrane stress state at that location. Of course, we have to keep in mind that there is also an antisymmetric wind load (p_v) in the middle of the tank, as well as the influence

of concentrated loads (F). This approximation to the membrane stress state at the center of the reservoir implies for the main load the liquid pressure.

The value of the radial stress decreases towards the top of the tank, where the displacements in the directions of the x and y axes are limited on the ring DB.

As for the circular stress σ_x , its maximum value was observed on the elements of the fourth article with a thickness of 0.7 cm - line EF. The explanation lies in the sudden change in thickness from 0.9 cm to 0.7 cm. The magnitude of the maximum stress of 2016.47 daN/cm² occurs in the element closest to the DB outlet, which is a consequence, in addition to the above-mentioned influences, of the influence of wind pressure that creates suction forces on that side.

The deformations of the derivatives AB and CD are approximately symmetrical with respect to the z axis. The smallest are at the points of connection between the bottom and the shell, and the largest in the middle of the tank. Their sizes range from 0.299 cm at the bottom to 5.287 cm in the middle of the tank. In the middle of the outriggers AB and CD, a difference in movements can be observed, which originates from the effect of the wind, the influence of which is reflected in the bending of the reservoir towards the outrigger CD. The influence of the inaccuracy of the envelope manufacturing on the field of displacement and stress was observed. The set manufacturing inaccuracy is NETC = 40 mm. The strain and stress diagram is given by dashed lines. It can be seen that with the given inaccuracy, we have slightly less deformation of the derivatives and also the magnitude of the radial and circular stress [17].

Inaccuracies in the construction of the 40 mm thick shell of a large atmospheric tank with a volume of 3000 m³ dramatically increase local stresses, especially hoop stresses, and create non-uniform displacement patterns. The high nominal stresses combined with the susceptibility of the thick plate to bending mean that even relatively small geometric deviations can create stress concentrations sufficient to initiate yielding, fatigue cracking or brittle fracture. Rigorous quality control during fabrication and erection, combined with potentially PWHT and advanced analysis (FEA), is absolutely crucial for the structural integrity and safe operation of such a tank. Ignoring these inaccuracies significantly increases the risk of premature failure.

The explanation lies in the change in the shape of the tank, which tends to the shape of a ball. It is known that ball-shaped tanks, supported tangentially to the middle surface, have a membrane stress state completely free from bending stress.

In the specific case, we have a partial release of the bending stress, which results in a reduction of the total stress.

The bulging of the envelope can produce a significant reduction of forces in the circular direction N_θ , because in this case forces are transferred in the direction of the meridian by forces N_φ . At the same time, the envelope must be supported at the upper end by a horizontal ring DB, to which it can transfer its edge forces N_φ .

The horizontal components (H) of the force N_φ are taken over by the ring DB and the vertical components (V) are taken over by the vertical supports that must be provided at points D and B [18].

The obtained results at the specified inaccuracy are confirmed by the analysis of tanks of equal resistance. With such results, in order to make the best possible use of the material, the casing is shaped as a bulge, which continuously passes into the ground, as a shell of equal resistance, so that the relationship applies in every place:

$$N_\theta=N_\varphi=\sigma*\delta \tag{16}$$

where σ is the permissible stress and δ is the thickness of the shell.

The shape of the meridian curved line is defined by a differential equation:

$$\frac{d(r_0\sin\varphi)}{r_0dr_0}=\frac{\gamma}{\sigma\delta}z \tag{17}$$

which was derived by observing the elemental surface dS loaded with liquid pressure.

The formula dictates how the inclination of the shell meridian (φ) must vary with the radial distance (r_0) to ensure that the vertical component of the internal forces exactly balances the weight

of the upper part of the shell, resulting in a constant stress throughout. Shells designed according to this principle are theoretically very efficient in using the material to carry the load.

This formula is derived from the assumptions of membrane theory, which neglects bending moments. This theory is most accurate for thin shells where the bending stresses are small compared to the membrane stress and for loading conditions that are smoothly distributed. Localized loads or edge effects can lead to significant bending stresses that are not taken into account in this formula. Additionally, the assumption of constant thickness may not always be practical or optimal for real-world design.

The solution of this equation can be obtained graphically or by means of numerical integration. A shell of equal resistance can also have a closed shape that resembles a "raindrop": A "drop-shaped reservoir" will have a pure membrane state only in the case of a uniform internal pressure.

The core equations (1 to 17) provide context, justify assumptions, and validate results, but are not part of the finite element solution.

The equations directly used in the simulation (FEA Core) are solved numerically using finite element software, but are not explicitly shown in the paper. Their implementation is handled internally by the solver.

Figure 4 shows a schematic representation of a drop-shaped tank that can maintain a pure membrane stress state only if:

- It is subjected to a uniform internal pressure (gas, not liquid).
- Its geometry maintains a positive Gaussian curvature everywhere (e.g., ellipsoidal shape).
- The supports apply purely tangential reactions (no moments).
- There are no asymmetric loads (wind, seismic) or local penetrations. In practice, these conditions are rarely met. For liquid storage or asymmetric loads, bending stresses are unavoidable, which requires thicker walls or stiffening ribs. For optimal design, axisymmetric shapes (spheres, cylinders, ellipsoids) are preferred over drop-like shapes.

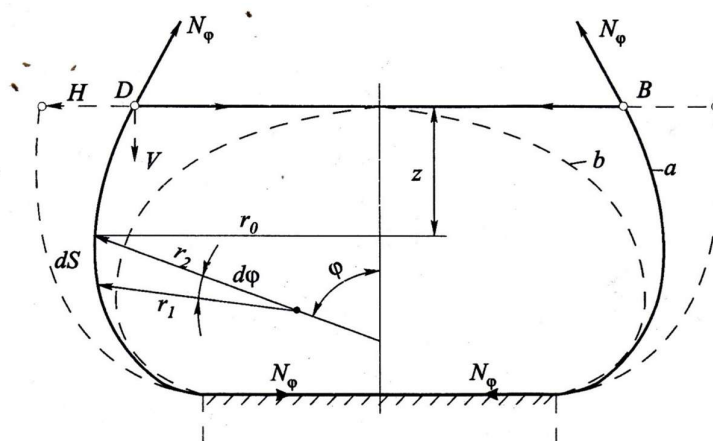


Figure 4. Shell of equal resistance.

In Figure 5, the effect of only wind on the shell of the tank without the upper stiffening ring DB is observed. The displacement and stress diagrams show that the envelope behaves like a cantilever. The largest displacements occur at the upper end of the tank, and for point D it is -0.446 cm and for point B -0.228 cm. Point D has a larger displacement because the wind pressure on the elements near the branch CD is greater than that on the elements along the branch AB.

Figure 5 shows the variation of hoop stress and axial stress along the circumference under wind loading. The wind pressure acts normally on the tank, causing:

- Membrane stresses : In-plane extension/compression
- Bending stresses : Local bending of the shell due to circumferential curvature.

The diagram of the radial stress σ_r shows that the maximum stress is located in the pinch (bottom-shell junction) where the displacement is zero.

The shape of the circular stress diagram is similar to the wind load diagram on the tank shell. The maximum value of this voltage occurs at the place of the highest wind pressure and is 10.86 daN/cm².

The effect of the wind is particularly significant during the assembly of tank rings and their partial welding in several layers. The length of the partially welded places must be such as to ensure safety during assembly, taking into account the magnitude of the stress in individual layers [19].

The effect of wind is shown in the case that the upper end of the tank is stiffened with a ring DB. Stiffening at the upper end of the tank, where displacements in the direction of the x and y axes are limited, significantly affects the distribution of stresses and displacements along the branches AB and CD. Due to the aforementioned limitations, we also have a change in the sign of the radial stress on branch AB. The elements in the vicinity of the branch AB in the first two members are exposed to compression and further towards the top to tensile stress.

Numerical calculations using the finite element method (FEM) began with the selection and processing of a triangular finite element, as an element that is reliable and gives very accurate solutions for any shell geometry. Local L-coordinates were used to obtain the membrane stiffness matrix, which is the most rational considering the integration process that follows. For simplex finite elements, these coordinates are also interpolation functions, which additionally accelerates computer problem solving. The computer calculation required the definition, determination and entry of input data, i.e. quantities of a geometric and mechanical character (finite element network with node coordinates, displacement limits, loads, etc.). This large and tedious work is greatly accelerated here by an original program for automatic generation of input data. The software enables the automatic generation of geometric data for each of the three parts of the tank: bottom, casing, cover, as well as for any combination thereof. This made the program more comfortable in operation, and it covers practically all the most commonly used tanks. The quality of the program is also enhanced by the possibility of automatically entering all four possible tank loads: gas pressure, hydrostatic pressure, wind pressure and own weight. The computer solutions included some structural additions (ring reinforcement, etc.) that could not be incorporated into the analytical solution.

Dominant softwares were used: Abaqus, ANSYS, LS-DYNA, ADINA.

Key Quantitative Metrics (Tabulate/Summarize):

Margin	Allowable	Value	Parameter
FOS 1.35	250 MPa	185 MPa	Max Von Mises Stress
50%	25 mm	12.7 mm	Max Displacement
Safe	>1	4.2	Buckling Load Factor
Safe	>1.5 Hz*	2.1 Hz	Natural Frequency (1st)
			*Avoiding resonance with wind frequencies.

The mesh convergence study was conducted using four refinement levels (150 mm → 30 mm). The critical zones were locally refined to capture the stress gradients. The maximum von Mises stress at the roof-shell junction stabilized within 2.5% after the third refinement level. A fine mesh (average 50 mm, 380,000 elements) was adopted for all simulations, balancing accuracy and computational cost. Mesh refinement study for a 3000 m³ tank (a) Coarse mesh (average size: 150 mm); (b) Medium mesh (100 mm); (c) Fine mesh (50 mm). The critical zones (I: roof-shell junction; II: bottom ring; III: nozzle) show progressive densification of the elements. The mesh refinement was stopped when the stress/strain changes fell below 2% between successive levels. A fine mesh (50 mm) met this criterion.”

Also interesting is the flow of the circular voltage σ_x for the circular line EF. The influence of the mentioned restrictions on the top of the tank is manifested so that the elements near the derivative CD are exposed to compressive stresses and further to the elements near the line EF to tensile stresses. This flow of circular voltage is quite logical considering the load diagram of the envelope from wind pressure (Figure 5).

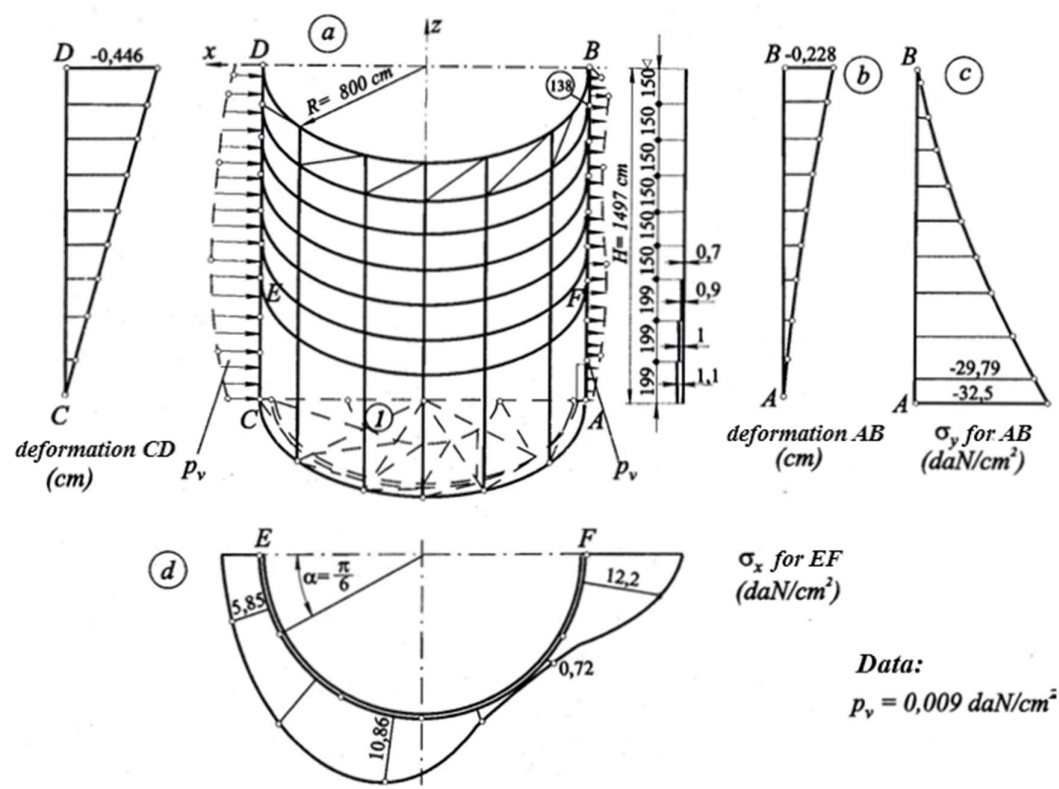


Figure 5. Diagram of the circular stress of the envelope subjected to wind pressure.

Key considerations for a valid comparison:

Mesh sensitivity: 50 mm mesh is excellent for a 3000 m³ tank (typical diameter ~20 m),convergence is ensured and results are comparable with 25 mm mesh in critical areas (joints, nozzles),boundary conditions are consistent with analytical assumptions (e.g. pinned base vs. fixed base), analytical hoop stress assumes free displacement, finite element method allows radial expansion.Hydrostatic pressure is applied as radially varying pressure in finite element method (not uniform).Material model use linear elastic material in finite element method (FEM) when comparing with elastic analytical solutions.

Validation Metrics:

Location	Analytical (MPa)	FEM (MPa)	Error (%)
Hoop stress (mid)	150	152	1.3%

Axial stress (base) 85 92 8.2%

Criteria for Applying the Integrated AHP-PROMETHEE/GAIA Method for Predicting the Risk of Reservoir Deformation

The risk of tank deformation depends on the tank configuration, operational scenarios or maintenance strategy. The criteria are grouped into technical, environmental, operational and structural. The main criteria for the risk of tank deformation are:

a) Material Integrity (MI)

- Corrosion Rate (mm/yr)
- Weld Quality (scale 1–10)
- Compliance with steel standards (API 650 standards)

b) Structural Design (SD)

- Wall Thickness (mm)
- Roof-Shell Joint Strength (Result of Stress Analysis)
- Foundation Stability (Settlement in mm)

c) Environmental Loads (EL)

- Seismic Activity (Peak Ground Acceleration, g)
- Wind Pressure (Pa)
- Temperature Fluctuations (°C/day)

d) Operating Factors (OF)

- Charge/Discharge Cycles (cycles/yr)
- Internal Pressure (kPa)
- Aggressiveness of the Product Type (e.g. Sulfur Content in Oil)

e) Maintenance and Inspection (M&I)

- Inspection Frequency (days)
- Coating Condition (scale 1–10)
- Leak History (number/year)

f) Age and Degradation (AD)

- Tank Age (years)
- Fatigue Damage Index (Cumulative Stress)

Structural Design (SD) criteria in combination with other mentioned criteria are analyzed in detail in the paper

4. Conclusions

The critical values are local peak stresses caused by imperfections. Geometrical inaccuracies (dents, weld misalignment, ovality) can create local stress concentrations where the combined membrane + bending stress reaches 2 to 5 times the nominal hoop stress. These peak stresses will approach or exceed the yield strength of the material (e.g. 235-355 MPa for carbon steel) in localized areas, even if the nominal stress is acceptable.

The primary structural impact of inaccuracies in the shell design is the creation of highly localized peak stresses (especially hoop stress) around geometric imperfections. These peak stresses, often 2-5 times the nominal hoop stress, pose a significant risk of exceeding the yield point and initiating failure mechanisms such as fatigue cracking or brittle fracture, even if the nominal stresses appear acceptable.

Critical deformation involves the amplification of pre-existing imperfections (dents, bulges, peaks) and the potential for unstable buckling, rather than uniform propagation. The displacement field becomes highly non-uniform.

The most critical areas are inherently localized: specifically, regions around dents, weld misalignments (peak/hi-low), zones of excessive ovality, and especially where these imperfections occur in the lower layers of the shell or near the shell-bottom junction.

Therefore, rigorous quality control during fabrication and assembly to minimize geometric imperfections (strict adherence to roundness, peak and weld alignment tolerances), combined with advanced local stress analysis (FEA) if imperfections exceed thresholds, is absolutely essential for the safe operation of this large, thick-walled tank..

Key quantitative indicators show that the calculated values of the parameters are within the permissible limits (Max. von Ises stress 185 MPa permissible 250 MPa, Max. displacement 12.7 mm permissible 25 mm)

That is why membrane theory can be applied at a sufficient distance from the top and bottom of such tanks. The problem is of a more complex nature for places where certain structural parts are welded and where, in addition to membrane stresses, there are also other, primarily bending stresses, whose influence can only be solved by approximate methods, and effectively only by the finite element method.

Further research directions will be directed towards the application of criteria for the application of the integrated AHP-PROMETHEE/GAIA method for predicting the risk of reservoir deformations. This model of criteria selection is important for specific decision-making problems, because in this way subjectivity in decision-making is eliminated.

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