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Phenomenological theory of the Stern-Gerlach experiment

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Abstract. We propose a phenomenological theory of spin behavior in a magnetic field, which explains from the point of view of classical physics the two-valued result of the Stern-Gerlach experiment. The behavior of the spin and intrinsic magnetic moment of an electron wave of an atom in an external magnetic field is considered. We show that in a weak magnetic field, the intrinsic magnetic moment of an electron wave is always oriented parallel to the magnetic field strength vector, while in a strong magnetic field, depending on the initial orientation of the intrinsic magnetic moment, two orientations are realized: either parallel or antiparallel to the magnetic field strength vector. Within the framework of classical electrodynamics, the calculation of the motion of an atomic beam in an inhomogeneous magnetic field is carried out, which reproduces the results of the Stern-Gerlach experiment.

Keywords: Stern-Gerlach experiment, phenomenological theory of spin behavior in a magnetic field, classical field theory, self-consistent Maxwell-Pauli theory.

1. Introduction

According to classical electrodynamics, particles with non-zero magnetic moment are deflected, due to the magnetic field gradient, from a straight path.

In the Stern-Gerlach experiment [1], silver atoms were passed through a spatially inhomogeneous magnetic field. When the magnetic field was null, the silver atoms were deposited as a single band on the detecting glass slide (Fig. 1). When the field was made stronger, the middle of the band began to widen and eventually to split into two, so that the glass-slide image looked like a lip-print, with an opening in the middle, and closure at either end (Fig. 1).

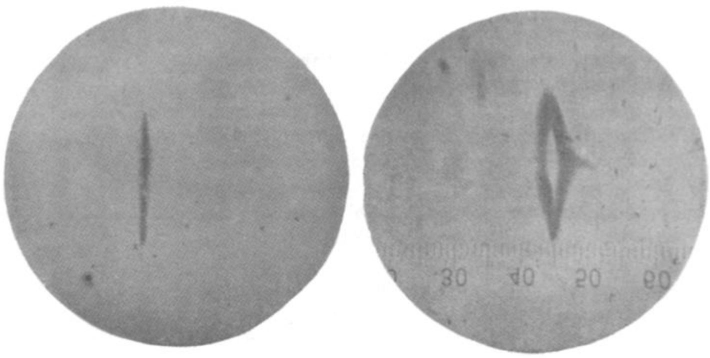

Fig.1. Atomic beam image on the detecting glass slide in Stern-Gerlach experiments in the absence of a magnetic field (left) and in the presence of an inhomogeneous magnetic field (right) (adopted from [1]).

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The Stern-Gerlach experiments led to the conclusion that atoms have their own magnetic moment (spin). Moreover, if the direction of the magnetic moment were randomly uniform, then a continuous spot would be observed on the detecting glass slide, and not the splitting the atomic beam into two parts, as in the Stern-Gerlach experiments. This indicated that the intrinsic magnetic moment is quantized and in a magnetic field can have only two directions: parallel and antiparallel to the magnetic field strength vector. It is for this reason that in the Stern-Gerlach experiments in an inhomogeneous magnetic field, the silver atoms were deflected up or down depending on their spin before they struck a detector screen. The screen revealed discrete points of accumulation, rather than a continuous distribution owing to their quantized spin.

In 1927, T.E. Phipps and J.B. Taylor reproduced the effect using hydrogen atoms in their ground state [2].

The Stern-Gerlach experiment [1] is a seminal experiment in quantum mechanics. He played a huge role in the development of modern concepts underlying quantum mechanics and quantum physics in general. In fact, it convinced physicists of the reality of spatial quantization and allowed formulating one of the basic ideas of quantum mechanics, and of all modern physics, namely, the idea of a half-integer electron spin. The Stern-Gerlach experiment is one of those keystones that form the foundation of modern quantum theory.

The Stern-Gerlach experiment is not only of historical significance. At present, it is considered as a thought tool when discussing such fundamental questions for quantum mechanics as the interpretation of quantum mechanics, quantum measurements, EPR paradox, Bell's theorem, Bell's inequality, the possibility of constructing hidden-variable theories [3-7], quantum computing [8]etc.

The Stern–Gerlach experiment is considered to be the prototype of a quantum measurement, demonstrating the observation of a single, real value (eigenvalue) of an initially unknown physical property. According to quantum measurement theory, the wave function representing the atom magnetic moment is in a superposition of those two directions entering the magnet. At the moment of the measurement, only one spin direction eigenvalue is recorded (the so called collapse of the wave function) [9].

Note that, although it is believed that the Stern-Gerlach experiment demonstrates the purely quantum properties of atoms, in its analysis, starting from the Stern's work [10], the methods of classical mechanics and classical electrodynamics are used.

Thus, the spin of an atom in a Stern–Gerlach experiment is treated as a quantum degree of freedom, while the atom moving through a magnetic field is described by the classical (Maxwellian) electrodynamics [10].

From the point of view of classical physics, the behavior of the intrinsic magnetic moment in a magnetic field seems inexplicable. According to classical electrodynamics, the most energetically favorable is the parallel orientation of the intrinsic magnetic moment relative to the vector of the external magnetic field. Indeed, the potential energy of a magnetic dipole in a magnetic field is equal to $U = -(\boldsymbol{\mu}\mathbf{H})$, where $\boldsymbol{\mu}$ is the magnetic moment, $\mathbf{H}$ is the strength of the external magnetic field, while any system tends to the state with the lowest potential energy. In the case under consideration, the potential energy minimum corresponds to $(\boldsymbol{\mu}\mathbf{H}) = |\boldsymbol{\mu}||\mathbf{H}|$. From the point of view of classical electrodynamics and classical mechanics, the spontaneous antiparallel orientation of the vectors $\boldsymbol{\mu}$ and $\mathbf{H}$, corresponding to the maximum potential energy U, is energetically unfavorable: although such an orientation is stationary, it is unstable.

The statement that the spin in a magnetic field can take only two orientations (parallel and antiparallel to the field) in quantum mechanics is considered as a postulate, i.e. as a statement that does not require (or rather, does not have) proof. In other words, this property of the spin cannot currently be theoretically derived from any first principles.

Attempts [11-25] were made to give a rational explanation for the results of the Stern-Gerlach experiments, i.e. to explain the physical mechanism (reason) why in an external magnetic field the intrinsic magnetic moment of half of the atoms in the atomic beam is oriented parallel to the magnetic field strength vector, while the other half is antiparallel.

None of the previous attempts to rationally explain the results of the Stern-Gerlach experiments was found to be satisfactory.

The self-consistent theory of Maxwell-Pauli and Maxwell-Dirac developed in [26, 27] and its analysis [28] prompted the author of this work to a new idea that can explain, at least from a mathematical point of view, the two-valued results of the Stern - Gerlach experiments.

2. Spin behavior in a magnetic field

In [26, 27], a self-consistent Maxwell-Pauli and Maxwell-Dirac theory was developed, in which the intrinsic angular momentum (spin) of an electron wave and the intrinsic magnetic moment associated with it are considered as real physical vectors that have a simple and clear classical meaning. In particular, the spin and intrinsic magnetic moment of an electron wave in a hydrogen atom are determined by the relations [26]

$$\mathbf{S} = \frac{\hbar}{2}\boldsymbol{\nu} \tag{1}$$

and

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$$\boldsymbol{\mu} = -\mu_B \mathbf{v} \tag{2}$$

where $\mathbf{v}$ is the unit vector: $|\mathbf{v}| = 1$.

From relations (1) and (2) it follows that for an electron wave in a hydrogen atom, the vector of its intrinsic angular momentum (spin) $\mathbf{S}$ and the vector of its intrinsic magnetic moment $\boldsymbol{\mu}$ are antiparallel.

In the absence of a magnetic field, the orientation of the vector $\mathbf{v}$ can be any and does not change with time.

It was shown in [28] that in an external magnetic field the unit vector $\mathbf{v}$ satisfies the equation

$$\dot{\mathbf{v}} = \gamma_e \mathbf{v} \times \mathbf{H} - 2\alpha \mathbf{v} \times \dot{\mathbf{v}} + \frac{b}{c} \mathbf{v} \times \ddot{\mathbf{v}} \tag{3}$$

where $\mathbf{H}$ is the strength of the external magnetic field; $\gamma_e = -\frac{e}{m_e c}$ is the intrinsic (spin) gyromagnetic ratio of the electron wave; $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant; b is some parameter having the dimension of length. In [28], within the framework of the self-consistent Maxwell-Pauli theory [25], it was obtained

$$b = \alpha \int \int R |\psi(\mathbf{r})|^2 |\psi(\mathbf{r}')|^2 dV' \, dV \tag{4}$$

where $R = |\mathbf{r} - \mathbf{r}'|$, $\psi(\mathbf{r})$ is the wave function describing the state of the atom.

As shown in [28], Eq. (3) holds only for the spin magnetic moment, but not for the orbital magnetic moment of an electron wave in an atom.

For $b = 0$, equation (3) has the form the Landau–Lifshitz–Gilbert equation [29, 30] with a damping parameter 2α.

Note that a phenomenological equation similar to (3) was considered in [31–35] as applied to magnetization dynamics of ferromagnets and ferrimagnets.

In this paper, we consider equations (3) as applied to an individual atom that has only a spin magnetic moment.

It is further assumed that the second and third terms on the right side of Eq. (3) are much smaller than the first one, which describes the classical Larmor precession of the spin in an external magnetic field with a frequency

$$\boldsymbol{\Omega}_\nu = \frac{e}{m_e c} \mathbf{H} \tag{5}$$

Multiply equation (3) on the left vectorially by $\mathbf{v}$. As a result of simple transformations, one obtains

$$\mathbf{v} \times \dot{\mathbf{v}} = \gamma_e \mathbf{v} \times \mathbf{v} \times \mathbf{H} + 2\alpha \dot{\mathbf{v}} + \frac{b}{c} \mathbf{v} \times \mathbf{v} \times \ddot{\mathbf{v}} \tag{6}$$

Let us substitute (3) into the right-hand side of equation (6). As a result, one obtains

$$\mathbf{v} \times \dot{\mathbf{v}} = \frac{\gamma_e}{(1+4\alpha^2)} \mathbf{v} \times \mathbf{v} \times \mathbf{H} + \frac{2\alpha\gamma_e}{(1+4\alpha^2)} \mathbf{v} \times \mathbf{H} + \frac{2\alpha b}{(1+4\alpha^2)c} \mathbf{v} \times \ddot{\mathbf{v}} + \frac{b}{(1+4\alpha^2)c} \mathbf{v} \times \mathbf{v} \times \ddot{\mathbf{v}} \tag{7}$$

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Taking into account (7), we reduce equation (3) to the form

$$\dot{\mathbf{v}} = \frac{\gamma_e}{1+4\alpha^2}\mathbf{v}\times\mathbf{H} + \frac{b}{(1+4\alpha^2)c}\mathbf{v}\times\ddot{\mathbf{v}} - \frac{2\alpha\gamma_e}{1+4\alpha^2}\mathbf{v}\times\mathbf{v}\times\mathbf{H} - \frac{2\alpha b}{(1+4\alpha^2)c}\mathbf{v}\times\mathbf{v}\times\ddot{\mathbf{v}} \tag{8}$$

To calculate the derivative $\ddot{\mathbf{v}}$, we take into account that the second and subsequent terms on the right-hand side of Eq. (8) are small compared to the first one. As a result, one can write approximately

$$\dot{\mathbf{v}} \approx \frac{\gamma_e}{1+4\alpha^2}\mathbf{v}\times\mathbf{H} \tag{9}$$

$$\ddot{\mathbf{v}} \approx \frac{\gamma_e}{1+4\alpha^2}\dot{\mathbf{v}}\times\mathbf{H} \approx -\left(\frac{\gamma_e}{1+4\alpha^2}\right)^{\mathbf{2}}\mathbf{H}\times\mathbf{v}\times\mathbf{H} = -\left(\frac{\gamma_e}{1+4\alpha^2}\right)^{\mathbf{2}}\left(\mathbf{v}\mathbf{H}^2 - \mathbf{H}(\mathbf{v}\mathbf{H})\right) \tag{10}$$

Substituting (10) into (8), one obtains

$$\dot{\mathbf{v}} = \frac{\gamma_e}{1+4\alpha^2}\left(1 + \frac{b\gamma_e}{(1+4\alpha^2)^2c}(\mathbf{v}\mathbf{H})\right)\mathbf{v}\times\mathbf{H} - \frac{2\alpha}{1+4\alpha^2}\gamma_e\left[1 + \frac{b\gamma_e}{(1+4\alpha^2)^2c}(\mathbf{v}\mathbf{H})\right]\mathbf{v}\times\mathbf{v}\times\mathbf{H} \tag{11}$$

Note that, as follows from Eq. (11), the true frequency of the Larmor precession of the spin in an external magnetic field differs from (5) and is equal to

$$\mathbf{\Omega'}_v = \frac{1}{1+4\alpha^2}\left(1 - \frac{1}{(1+4\alpha^2)^2}\frac{eHb}{m_ec^2}\cos\theta\right)\frac{e}{m_ec}\mathbf{H} \tag{12}$$

Assuming the vector $\mathbf{H}$ to be constant, we multiply equation (11) by it:

$$\mathbf{H}\dot{\mathbf{v}} = -\frac{2\alpha}{1+4\alpha^2}\gamma_e\left[1 + \frac{b\gamma_e}{(1+4\alpha^2)^2c}(\mathbf{v}\mathbf{H})\right]\left[(\mathbf{v}\mathbf{H})^2 - \mathbf{H}^2\right] \tag{13}$$

Taking into account that $(\mathbf{v}\mathbf{H}) = H\cos\theta$, where θ is the angle between the vectors $\mathbf{v}$ and $\mathbf{H}$, one obtains

$$\frac{d\theta}{dt} = -\frac{2\alpha}{1+4\alpha^2}\gamma_e\mathrm{H}\left(1 + \frac{b\gamma_e}{(1+4\alpha^2)^2c}H\cos\theta\right)\sin\theta \tag{14}$$

At

$$H > -\frac{\left(1+4\alpha^2\right)^2c}{b\gamma_e} = (1+4\alpha^2)^2\frac{m_ec^2}{be} \tag{15}$$

the right-hand side of equation (14) changes sign as the angle θ changes from zero to π. In this case, when

$$\cos\theta < -\frac{\left(1+4\alpha^2\right)^2c}{b\gamma_eH} \tag{16}$$

the right-hand side of equation (14) is positive, and the angle θ increases until it reaches the value $\theta = \pi$. That is, the magnetic moment (2) of an electron wave in an atom turns in a magnetic field until it becomes parallel to the vector $\mathbf{H}$. On the contrary, when

$$\cos\theta > -\frac{\left(1+4\alpha^2\right)^2c}{b\gamma_eH} \tag{17}$$

the right-hand side of equation (14) is negative, and the angle θ decreases until it reaches the value $\theta = 0$. That is, the magnetic moment of an electron wave in an atom turns in a magnetic field until it becomes antiparallel to the vector $\mathbf{H}$.

We introduce nondimensional time

$$\tau = -\frac{2\alpha}{1+4\alpha^2}\gamma_e H t = \frac{2\alpha}{(1+4\alpha^2)}\frac{eH}{m_e c}t \tag{18}$$

Then equation (14) can be written in the form

$$\frac{d\theta}{d\tau} = (1-\lambda\cos\theta)\sin\theta \tag{19}$$

where

$$\lambda = -\frac{b\gamma_e}{(1+4\alpha^2)^2 c}H = \frac{1}{(1+4\alpha^2)^2}\frac{eHb}{m_e c^2} \tag{20}$$

The formal solution of equation (19) has the form

$$\int_{\theta_0}^{\theta}\frac{d\theta}{(1-\lambda\cos\theta)\sin\theta} = \tau \tag{21}$$

After integration, one obtains

$$-\frac{1}{\lambda(1-\lambda^2)}\ln\frac{1-\lambda\cos\theta}{1-\lambda\cos\theta_0} + \frac{1}{2(1-\lambda)}\ln\frac{1-\cos\theta}{1-\cos\theta_0} - \frac{1}{2(1+\lambda)}\ln\frac{1+\cos\theta}{1+\cos\theta_0} = \tau \tag{22}$$

where θ_0 is the initial angle between the spin vector and the vector **H**.

Let us introduce the critical angle θ_{cr} satisfying the equation

$$\lambda\cos\theta_{cr} = 1 \tag{23}$$

Then conditions (15)-(17) can be formulated as follows.

For $0 \le \lambda \le 1$, the right-hand side of equation (19) has a constant sign (positive) at any angles θ, while for $\lambda > 1$, the right-hand side of equation (19) is positive for $\theta > \theta_{cr}$ and negative for $\theta < \theta_{cr}$.

That is, at $0 \le \lambda \le 1$, the spin vector in a magnetic field always acquires an orientation antiparallel to the vector **H**, regardless of the initial angle θ_0. When $\lambda > 1$, the angle θ increases and tends to π if $\theta > \theta_{cr}$ and vice versa decreases and tends to zero if $\theta < \theta_{cr}$.

The dependence of the critical angle θ_{cr} on the parameter λ is shown in Fig. 2. In the calculations, $\theta_0 = \theta_{cr} \pm \Delta\theta$ was used as the initial condition, where $0 < \Delta\theta \ll \theta_{cr}$.

Solutions of equation (19) are shown in Fig. 3.

We introduce the function

$$f(\tau) = \frac{1}{\Delta\theta}\frac{d\theta}{d\tau} = \frac{(1-\lambda\cos\theta)\sin\theta}{\Delta\theta} \tag{24}$$

where

$$\Delta\theta = \begin{cases} -\theta_{cr}, & \text{for } \theta < \theta_{cr} \\ \pi - \theta_{cr}, & \text{for } \theta > \theta_{cr} \end{cases} \tag{25}$$

Function $f(\tau)$ is a bell-shaped one, and

$$\int_0^\infty f(\tau)d\tau = 1 \tag{26}$$

Then the characteristic nondimensional time $\Delta\tau$ of turn of the spin vector can be defined as

$$\Delta\tau = 1/f_{\max} \tag{27}$$

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where

$$f_{\max} = \frac{(1-\lambda\cos\theta_m)\sin\theta_m}{\Delta\theta} \tag{28}$$

is the maximum value of the function $f(\theta)$; θ_m is the angle θ at which the maximum of the function $f(\theta)$ is reached. The angle θ_m is determined from the equation

$$\lambda\cos 2\theta_m = \cos\theta_m \tag{29}$$

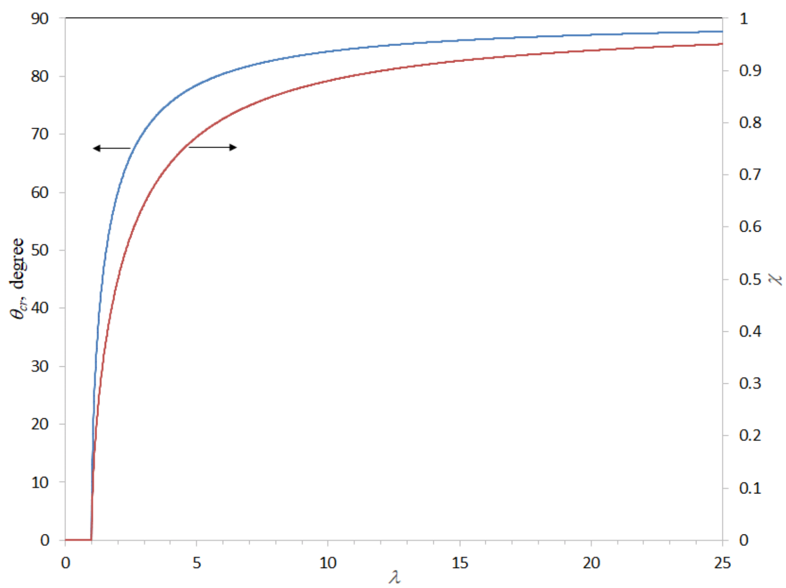

Fig. 2. Dependence of the critical angle θ_{cr} and the degree of uniformity χ of the distribution of the spin vector orientation on the parameter λ.

Dependences of $\Delta\tau$ on the parameter λ are shown in Figs. 4.

The characteristic dimensional time of the spin vector reversal

$$\Delta t = t_\nu \Delta\tau \tag{30}$$

where taking into account (18),

$$t_\nu = (1+4\alpha^2)\frac{m_e c}{2\alpha e H} = 3.9\times 10^{-6} H^{-1} \tag{31}$$

Here, the magnetic field **H** is measured in gauss, and the time is measured in seconds.

Using (5), one obtains

$$|\boldsymbol{\Omega}_\nu| t_\nu = \frac{1}{2}(1+4\alpha^2)\alpha^{-1} \gg 1 \tag{32}$$

That is, the period of the Larmor precession of the vector **ν** is much shorter than the characteristic time of the turn of this vector in an external magnetic field. From this point of view, we can say that the Larmor precession of the vector **ν** occurs quasi-stationary.

Thus, according to equation (11), the vector **ν** (vector of intrinsic angular momentum **S**) of a hydrogen atom in an external magnetic field performs a Larmor precession around the vector **H** with an angular frequency (5) and, at the same time, slowly turns, tending to take an orientation parallel or antiparallel to the vector **H** depending on its initial orientation.

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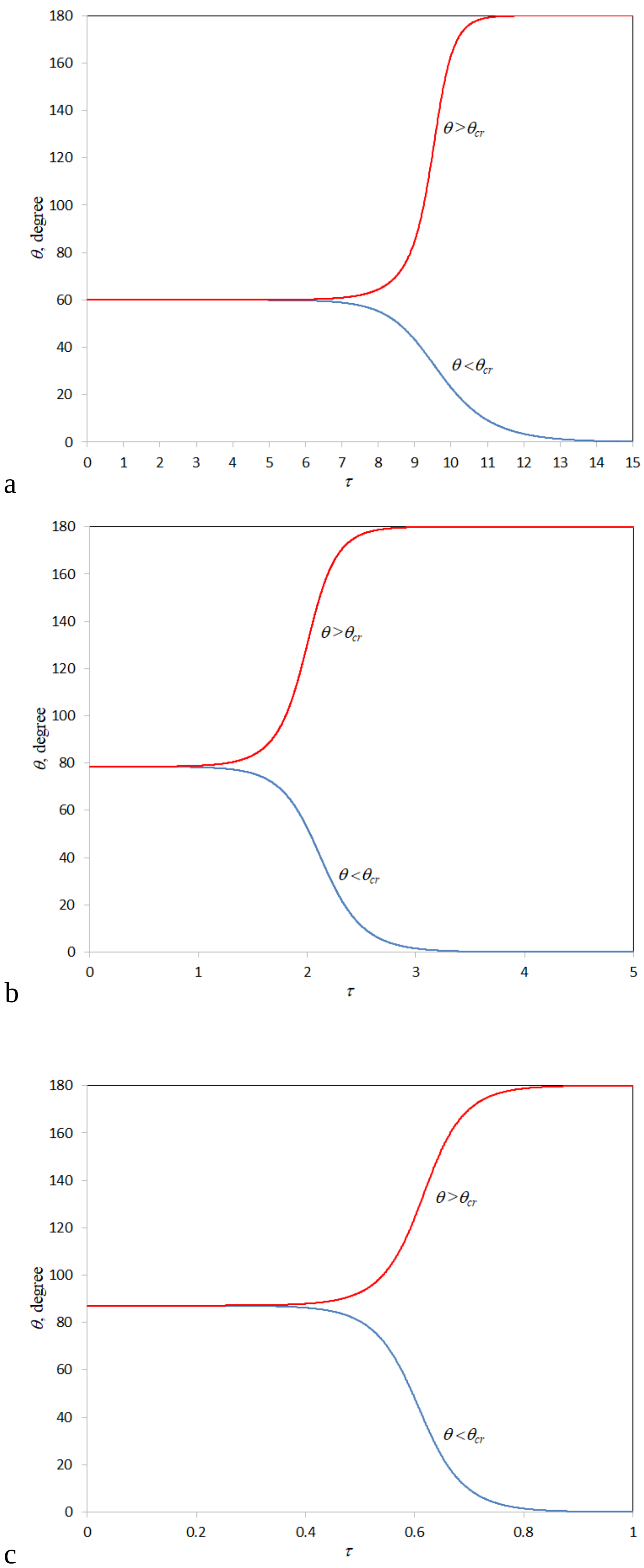


Fig.3. Dependences of the angle θ on the nondimensional time $\tau = \frac{2\alpha}{(1+4\alpha^2)}\frac{eH}{m_e c}t$ for different values of the parameter λ: a) $\lambda = 2, \theta_{cr} = 60°$; b) $\lambda = 5, \theta_{cr} = 78.46°$; c) $\lambda = 20, \theta_{cr} = 87.13°$

In the Stern-Gerlach experiment, the magnetic field had a strength of $H < 15000$ gauss. Thus, in the Stern-Gerlach experiment, $t_\nu > 5.2 \times 10^{-10}\ s$, i.e. this is a fairly fast process. At the thermal speed of the atom ~100 m/s, the atom has time to cover a distance of >5×10^{-8} m=0.05 μm during this time. That is, once in such a field, an atom will almost instantly acquire an orientation in which its intrinsic magnetic moment will be directed either parallel or antiparallel to the magnetic field, depending on its initial orientation.

Consider a statistical ensemble of atoms. Let at the initial moment of time all atoms have a random and uniformly distributed orientation along the angle θ: $\rho(\theta_0, 0) = 1/\pi$, where $\rho(\theta, \tau)d\theta$ is the probability that the angle between the vector of the intrinsic angular momentum of the atom and the vector **H** lies in the range $[\theta, \theta + d\theta]$. This distribution will change over time: $\rho(\theta, \tau)$. Taking into account that $\rho(\theta_0, 0)d\theta_0 = \rho(\theta, \tau)d\theta$, one obtains

$$\rho(\theta, t) = \rho(\theta_0, 0)\left(\frac{\partial\theta_0}{\partial\theta}\right)_t \qquad (33)$$

Differentiating (21) with respect to θ_0 at constant τ, one obtains

$$\left(\frac{\partial\theta_0}{\partial\theta}\right)_\tau = \frac{(1-\lambda\cos\theta_0)\sin\theta_0}{(1-\lambda\cos\theta)\sin\theta} \qquad (34)$$

Thus,

$$\rho(\theta, \tau) = \frac{1}{\pi}\frac{(1-\lambda\cos\theta_0)\sin\theta_0}{(1-\lambda\cos\theta)\sin\theta} \qquad (35)$$

where $\theta_0 = \theta_0(\theta, \tau)$ according to (21).

Dependences $\rho(\theta)$ for different values of the nondimensional time τ for different values of the parameter λ are shown in Fig. 5.

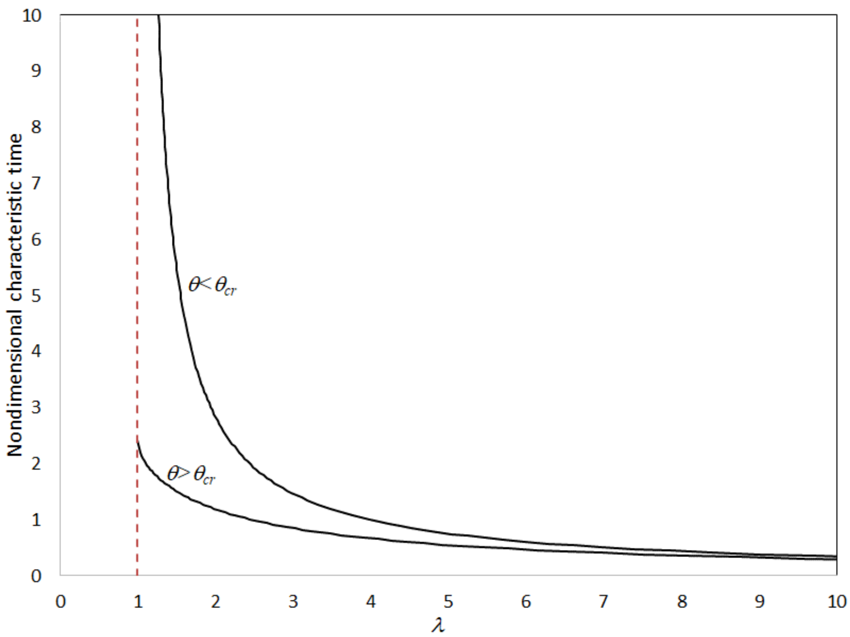
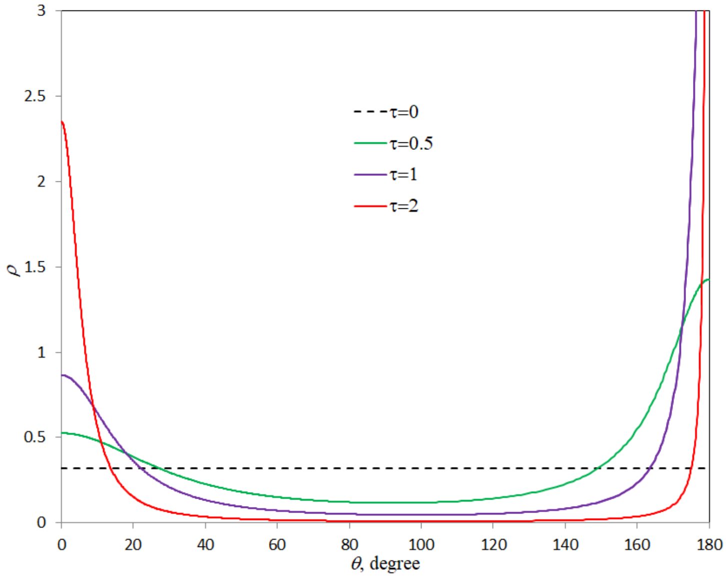
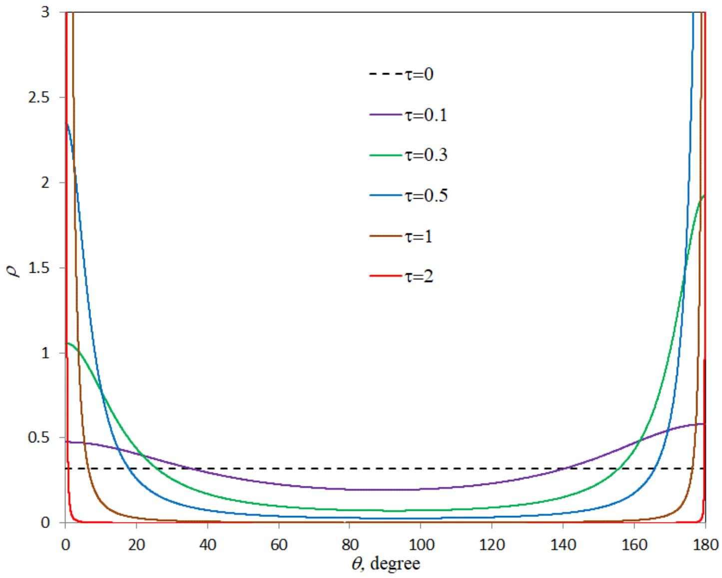


Fig. 4. Dependence of the characteristic nondimensional time $\Delta\tau$ of the spin vector turn on the parameter λ.



a



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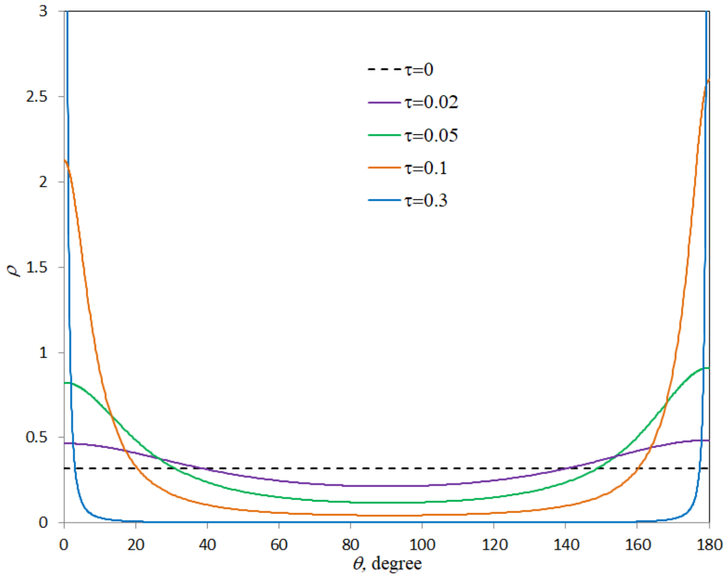

c

Fig. 5. Dependences of the distribution density $\rho(\theta)$ at different instants of nondimensional time $\tau = \frac{2\alpha}{(1+4\alpha^2)} \frac{eH}{m_e c} t$ for different values of the parameter λ: a) $\lambda = 2$; b) $\lambda = 5$; c) $\lambda = 20$.

It can be seen from Fig. 5 that the spin vector is nonuniformly distributed in directions: the fraction of atoms whose spin vector is antiparallel to the vector **H** is always greater than the fraction of atoms with a spin orientation parallel to the vector **H**, and this nonuniformity depends on the parameter λ.

The nonuniform distribution of the spin vector in directions in a magnetic field can be characterized by the parameter (degree) of uniformity

$$\chi = \frac{N_{\uparrow}}{N_{\downarrow}} \tag{36}$$

where $N_{\uparrow}$ and $N_{\downarrow}$ is the number of atoms whose spin vector has an orientation parallel and antiparallel to the vector **H**, respectively.

If initially (before entering the magnetic field) the atoms had a random and equally probable orientation of the spin vector in space, then

$$N_{\uparrow} = N\theta_{cr}/\pi, N_{\downarrow} = N(\pi - \theta_{cr})/\pi \tag{37}$$

where N is the total number of atoms.

Then the degree of uniformity of the orientation of the spin vector in the magnetic field

$$\chi = \frac{\theta_{cr}}{\pi - \theta_{cr}} \tag{38}$$

The dependence of the degree of uniformity χ on the parameter λ is shown in Fig. 2. As the parameter λ increases, the distribution of atoms on the spin directions relative to the vector **H** becomes more uniform: the number of atoms with a spin orientation parallel to the vector **H** tends to the number of atoms with an antiparallel spin orientation.

Thus, taking into account (2), we can consider that in a sufficiently strong external magnetic field the atomic beam is divided into two equal parts: one of which has a parallel orientation of intrinsic magnetic moments with respect to the vector **H**, while the other has an antiparallel orientation. Then, according to classical electrodynamics, an atom having intrinsic magnetic moment **μ** in a inhomogeneous magnetic field will be affected by the force

$$\mathbf{F} = \nabla(\boldsymbol{\mu}\mathbf{H}) = \pm\mu_B \nabla H \tag{39}$$

where $H = |\mathbf{H}|$; the signs "+" and "-" correspond to the parallel and antiparallel orientation of the vector **μ** with respect to the vector **H**.

If the field **H** is inhomogeneous (as in the Stern-Gerlach experiments), this will lead to the separation of the beam of atoms in space into two parts, one of which corresponds to the parallel while the other to the antiparallel orientation of the intrinsic magnetic moment of the electron wave of the atom with respect to the vector **H**.

Thus, the fulfillment of condition (15), in fact, means an explanation of the two-valued results of the Stern-Gerlach experiments.

If condition (15) is not satisfied, then the intrinsic magnetic moment of the atom is always oriented parallel to the field **H**. In this case, the atomic beam does not split into two parts in an inhomogeneous external magnetic field.

3. Numerical simulation of the Stern-Gerlach experiment

For illustration and completeness of presentation, we will calculate the Stern-Gerlach experiment, assuming that the intrinsic magnetic moment of an electron wave in an atom is always oriented either parallel or antiparallel to the magnetic field strength vector according to the phenomenological theory proposed in this paper.

In this case, taking into account (39), the equations of motion of an electrically neutral atom in a magnetic field have the form

$$m\frac{d\mathbf{v}_\pm}{dt} = \pm\mu_B\nabla H \tag{40}$$

$$\frac{d\mathbf{r}_\pm}{dt} = \mathbf{v}_\pm \tag{41}$$

The sketch of the Stern-Gerlach experiment is shown in Fig. 6.

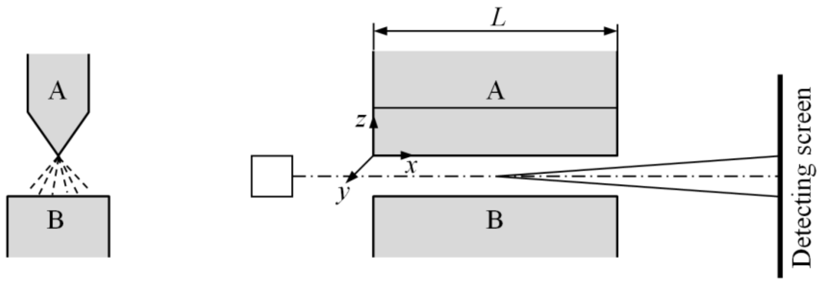

Fig. 6. Sketch of the Stern-Gerlach experiment. Dashed lines are magnetic field lines.

Assuming that the width of the magnets is much less than their length in the direction of the atomic beam propagation, it can be approximately assumed that the magnetic field is uniform in the direction of the beam propagation (i.e., along the x axis):

$$\mathbf{H} = \left(0, H_y(y,z), H_z(y,z)\right) \tag{42}$$

Considering that the purpose of this calculation is not to model a specific Stern-Gerlach experiment, but to illustrate the theory developed in the previous section, we consider a model inhomogeneous magnetic field described by the equations

$$H_y = q\frac{y}{(y^2+z^2)^{3/2}}, H_z = q\frac{z}{(y^2+z^2)^{3/2}} \tag{43}$$

Where $q > 0$ is some constant characteristic of the magnet.

Then

$$H = \frac{q}{y^2+z^2} \tag{44}$$

It is assumed here that the line $y = z = 0$ coincides with the nose of the upper magnet (Fig. 6) and the motion of the atomic beam occurs in the region $z < 0$.

We assume that at the entrance to the magnetic field (i.e., in the cross section $x = 0$) the velocities of all atoms are the same (monovelocity beam) and equal to $\mathbf{v}(0) = (V_0, 0, 0)$. In other words, in these calculations, we do not take into account the velocity distribution of atoms associated with the final temperature of the source.

For the convenience of calculations, we pass to nondimensional variables, leaving the previous notations:

$$t \to \frac{V_0 t}{L}, \mathbf{r} \to \frac{\mathrm{r}}{L}, \mathbf{v} \to \frac{\mathbf{v}}{V_0} \tag{45}$$

Then equation (41) in nondimensional variables will retain its form, while equation (40), taking into account (44), will take the form

$$\frac{dv_x}{dt} = 0 \tag{46}$$

$$\frac{dv_y}{dt} = \mp\beta \frac{y}{(y^2+z^2)^2} \tag{47}$$

$$\frac{dv_z}{dt} = \mp\beta \frac{z}{(y^2+z^2)^2} \tag{48}$$

where

$$\beta = \frac{2\mu_B q}{m V_0^2 L^2} \tag{49}$$

From equations (41) and (46), taking into account the initial conditions, one obtains

$$v_x = 1 \tag{50}$$

$$x = t \tag{51}$$

From (51) it follows that the exit from the magnet corresponds to

$$x_L = t_L = 1 \tag{52}$$

When modeling the Stern-Gerlach experiment, the motion of each individual atom was successively calculated according to equations (47), (48), (50) and (51) for a fixed value of the parameter β.

To do this, at the entrance to the magnetic field ($x = 0$) for each atom, the initial conditions were set:

- initial coordinates of the atom $y(0)$, $z(0)$, uniformly generated by the random number generator in the range: $y(0) = [-d/2, d/2], z(0) = [-D - \Delta/2, -D + \Delta/2]$ (it is assumed that atoms enter the magnet through a narrow slit of length d and width $\Delta \ll d$, parallel to the y axis and located symmetrically with respect to the z axis: the center of the slit has coordinates

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$y = 0, z = -D$; it is considered that the atomic beam is distributed uniformly over the entire area of the slit);

- initial speed of the atom $\mathbf{v}(0) = (1, 0, 0)$;

- the orientation of the intrinsic magnetic moment (spin) of the atom with respect to the external magnetic field, i.e. the sign "+" or "-" in equations (47) and (48). To do this, it is assumed that at the exit from the source of atoms (at the entrance to the magnetic field), the intrinsic magnetic moment (spin) of the atom can have an arbitrary (random, equiprobable) orientation in space. When it enters a magnetic field (i.e., in the $x = +0$ cross section), the spin of an atom instantly changes its orientation, acquiring either parallel ("-" sign in equations (47) and (48)), or antiparallel ("+" sign in equations (47) and (48)) orientation with respect to the magnetic field strength vector **H**. When an atom moves inside a magnet, the orientation of the spin with respect to the vector **H** (the sign in equations (47) and (48)) is saved.

Taking into account (37), one obtains the probabilities of parallel $p_\uparrow$ and antiparallel $p_\downarrow$ orientations of the spin vector with respect to the vector **H**:

$$p_\uparrow = \theta_{cr}/\pi, p_\downarrow = (\pi - \theta_{cr})/\pi \qquad (53)$$

Thus, the signs "+" and "-" in equations (47) and (48) are calculated using the random numbers generator and ratios (53).

After leaving the magnet ($x > 1$), the atoms move by inertia along rectilinear trajectories described by the equations

$$y = y(1) + (x - 1)v_y(1), z = z(1) + (x - 1)v_z(1) \qquad (54)$$

where $y(1)$ and $z(1)$ are the coordinates of the atom at the exit of the magnet (i.e. at $x = 1$); $v_y(1)$ and $v_z(1)$ are the velocities of the atom at the exit of the magnet.

The calculations were carried out with the following nondimensional parameters:

$$\lambda = 2;\ \beta_0 = 0.001; D = 0.2; d = 1;\ \Delta = 0.02 \qquad (55)$$

The number of particles involved in the calculations, $N = 10000$.

The results of calculations are shown in Figs. 7-9.

The results of the calculations allow drawing the following conclusions:

(i) The shape of the atomic beam image on the detecting screen, obtained in calculations for the case when the detecting screen is located directly at the exit from the magnet ($x = 1$, Fig. 8), qualitatively reproduces the results of the Stern-Gerlach experiment (Fig. 1). It can be expected that quantitative agreement can also be achieved if a real magnetic field **H**(y, z) is used in the calculations and the velocity distribution of atoms corresponding to the source temperature is taken into account.

(ii) The thickness of the lines on the detecting screen is practically independent on the parameter λ, because it is determined by the width of the slit. The intensity of the lines depends on the parameter λ: the fewer particles fall on the line, the paler it is. In the Stern-Gerlach experiments, the lower (left in Fig. 1) line corresponding to $\theta = 0$ looks paler, although this can be attributed to image quality.

(iii) As shown in Fig. 8 and 9, the picture on the detecting screen for the same experiment changes significantly when the x position of the screen changes. Thus, the picture on the detecting screen, obtained in the Stern-Gerlach experiments (Fig. 1), could be observed only when the detecting screen was placed in the immediate vicinity of the magnet ($x = 1$, Fig. 8). If the detecting screen were located at large distances from the magnet ($x > 1$), then, as can be seen from Fig. 9, the observed picture would be more complex, and it would apparently be difficult to draw a conclusion about the spatial quantization of the electron spin and the magnitude of intrinsic magnetic moment on the basis of these experiments.

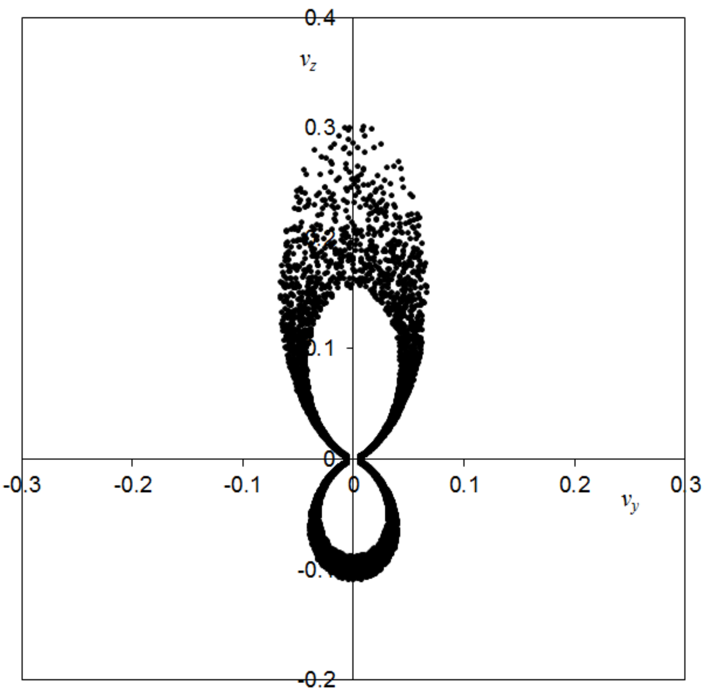

Fig. 7. Distribution of velocities $v_y(1)$ and $v_z(1)$ of atoms at the exit of the magnet ($x = 1$) for parameters (55).

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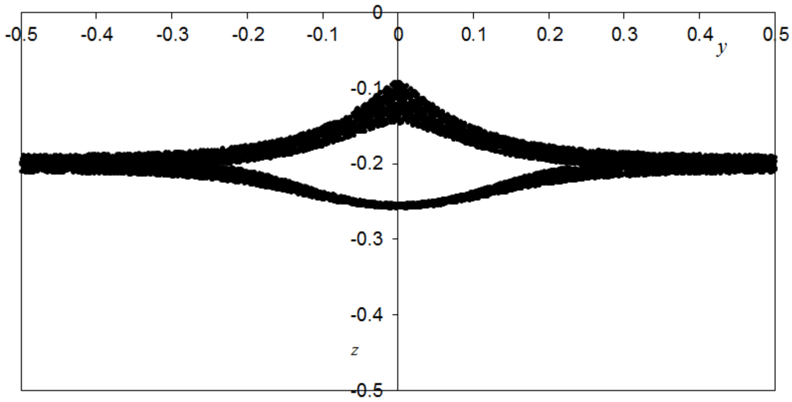

Fig. 8. Distribution of coordinates $y = y(1)$ and $z = z(1)$ of atoms at the exit of the magnet ($x = 1$) for parameters (55).

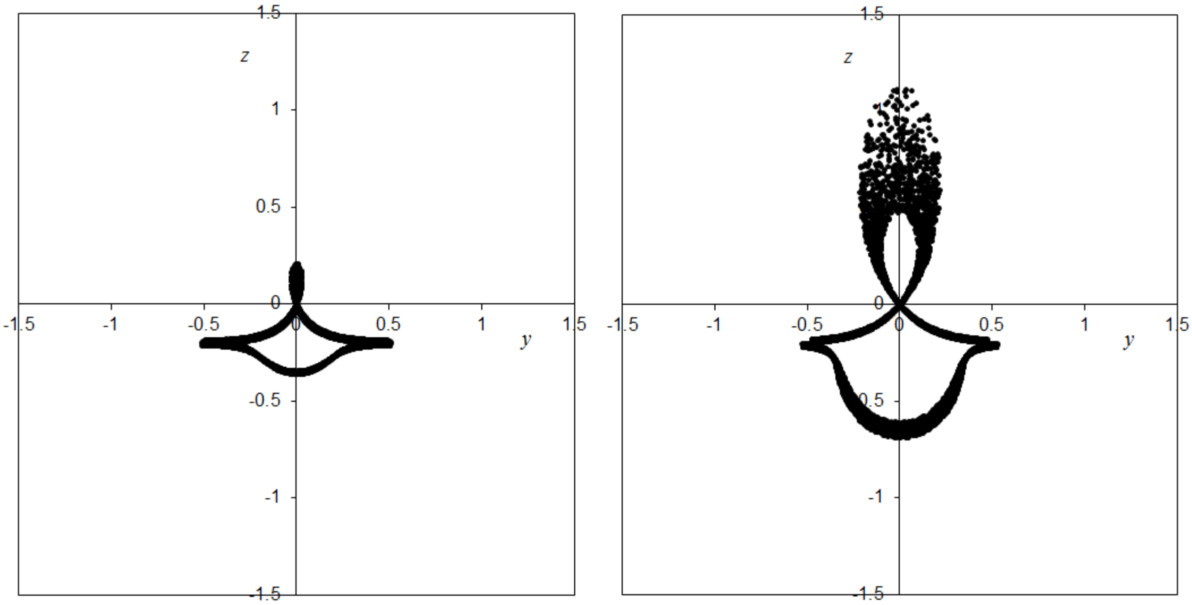

Fig. 9. Distribution of coordinates y and z of atoms in cross sections $x = 2$ (left) and $x = 5$ (right) for parameters (55).

4. Concluding remarks

Thus, we have shown that the phenomenological theory of the behavior of the intrinsic magnetic moment (spin) of an electron wave of an atom in an external magnetic field proposed in this paper allows naturally explaining the two-valued result of the Stern-Gerlach experiments and calculating the Stern-Gerlach experiment in all details based on classical mechanics and classical electrodynamics without resorting to quantization.

The theory under consideration predicts the existence of a threshold value of the magnetic field strength

$$H_{th} = (1+4\alpha^2)^2 \frac{m_e c^2}{be} \tag{56}$$

If the magnetic field strength $H < H_{th}$, the intrinsic magnetic moment of the electron wave of the atom will always be oriented parallel to the magnetic field strength vector, and in an inhomogeneous magnetic field the two-valued result characteristic of the Stern-Gerlach experiment [1, 2] will not be observed. In this case, the atoms will be deposited as a single band on the detecting screen, just as in the absence of a magnetic field, and only at $H > H_{th}$ in the Stern-Gerlach experiments will a two-valued result be observed, in which the atomic beam is split into two parts. This conclusion of the theory under consideration can be verified experimentally, although I am well aware of the difficulties that experimenters will encounter along the way, taking into account the small magnitude of the effect being studied.

As mentioned above, formally, equation (3) follows from the self-consistent Maxwell-Pauli theory [26-28].

Let us estimate the right-hand side of inequality (15) for the hydrogen atom in the ground state. Calculating the integral (4), one obtains [28]

$$b = \alpha \frac{35}{16} a_B \tag{57}$$

Substituting (57) into the right side of (15), one obtains

$$(1+4\alpha^2)^2 \frac{m_e c^2}{be} = 2 \times 10^{13}\text{Gauss} \tag{58}$$

which is a billion times greater than the value of the magnetic field strength in the Stern-Gerlach experiments. Thus, condition (15) is not satisfied in the Stern-Gerlach experiments. This means that the theory considered in this paper is still phenomenological, and cannot be derived directly from the self-consistent Maxwell-Pauli theory [26-28].

At the same time, equation (14), and hence equation (3), with a different value of the coefficient b, shows a possible mechanism leading to a two-valued result in the Stern-Gerlach experiments. Such an explanation of the Stern-Gerlach experiments is visual, plausible and seems very tempting.

It is possible that a certain modification of the self-consistent Maxwell-Dirac theory [26-28] will make it possible in the future to strictly derive equation (3) from a more general theory.

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