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Article

The Weak Gravity Conjecture as an Entropy Theorem

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Abstract

We establish the Weak Gravity Conjecture as a theorem derived from quantum information principles. Using only the Quantum Null Energy Condition (QNEC) and generalized entropy, we show that any consistent semiclassical theory of gravity must contain a charged state with charge-to-mass ratio $q/m \geq 1$. The argument proceeds by contradiction: if all charged states are strictly subextremal, extremal charged horizons form a one-sided boundary of the semiclassical state space. On such horizons, the affine-parameter half-sided modular inclusion structure required for null entropy variations becomes algebraically obstructed, rendering the horizon algebra modularly terminal. Terminality forces an upper divergence of the second null derivative of the outside entropy along complete null generators, which is incompatible with QNEC when the local stress tensor remains finite. Our result reframes a central swampland conjecture as a consequence of semiclassical entropy consistency, without invoking string theory, holography, or dynamical decay assumptions.

Keywords: weak gravity conjecture; swampland conjecture; QNEC; modular theory

1. Introduction

It is widely believed that General Relativity (GR) is an effective low-energy description of an underlying ultraviolet (UV) complete theory of quantum gravity. This perspective has motivated the formulation of nontrivial constraints on infrared (IR) effective theories that can consistently admit a UV completion. Such constraints are collectively known as *swampland conjectures* [1–7]. Effective theories that fail to satisfy these criteria are said to lie in the *swampland* [8].

One of the most prominent swampland constraints is the Weak Gravity Conjecture (WGC) [2], which asserts that any consistent theory of quantum gravity must contain a charged state whose charge-to-mass ratio satisfies $q/m \geq 1$ (in appropriate units). Despite extensive evidence and broad applicability, the WGC has remained a conjecture, lacking a derivation from first principles.

From a seemingly different direction, black hole thermodynamics indicates that entropy plays a fundamental role in gravitational dynamics. Classically, black hole entropy is given by one quarter of the horizon area—the Bekenstein–Hawking entropy [9,10]. However, quantum fields can violate the classical area increase theorem [11], motivating the introduction of the *generalized entropy*,

$$S_{\text{gen}} = \frac{A}{4G} + S_{\text{out}},$$

defined as the sum of the horizon area term and the entanglement entropy of quantum fields outside the horizon. Consistency of entropy evolution under null deformations has since emerged as a powerful organizing principle in semiclassical gravity.

This insight led to the formulation of the Quantum Focussing Conjecture (QFC) [12], which posits that the quantum expansion—defined as the first null variation of S_{gen} —is non-increasing along null congruences. Under standard assumptions, a direct consequence of the QFC is the Quantum Null Energy Condition (QNEC), which provides a lower bound on the null–null component of the stress–energy tensor in terms of the second null derivative of the outside entropy. Remarkably, QNEC has since been proven to hold in broad classes of relativistic quantum field theories [13–15].

In semiclassical gravity, the generalized entropy S_{gen} must therefore admit a consistent and extendible definition on the physical state space under null deformations. In this work, we show that this requirement, together with the Quantum Null Energy Condition, enforces the Weak Gravity bound.

Our result reframes the Weak Gravity Conjecture as a consequence of local quantum–information constraints rather than a dynamical postulate. The argument relies only on generalized entropy, null deformations, and QNEC, and makes no appeal to string theory, holography, or decay heuristics.

2. Generalized Entropy

For a null cut $\Sigma(\lambda)$ generated by an affine null parameter λ , we define the generalized entropy

$$S_{\text{gen}}(\lambda) = \frac{A(\lambda)}{4G} + S_{\text{out}}(\lambda), \quad (1)$$

where S_{out} is the renormalized von Neumann entropy of quantum fields outside the cut. Throughout, we work in the standard semiclassical regime in which S_{gen} is well defined and admits second derivatives under smooth null deformations in the distributional sense [16].¹

Moreover, the null–null component of the stress tensor $\langle T_{kk} \rangle$ is finite on the generators of interest, and the Quantum Null Energy Condition (QNEC),

$$\langle T_{kk} \rangle \geq \frac{1}{2\pi} \frac{d^2 S_{\text{out}}}{d\lambda^2}, \quad (2)$$

holds.

We further assume the existence of a complete null generator along which $S_{\text{gen}}(\lambda)$ approaches a finite *terminal* value, in the sense that no modularly well-defined extension exists beyond the endpoint (formalized in Lemma 2 below). This is not an independent postulate: if the second null variation of $S_{\text{gen}}(\lambda)$ (equivalently, of $S_{\text{out}}(\lambda)$ in our setup, since the geometric area term does not produce an upper divergence along the generator considered) were *uniformly bounded above* near the endpoint in the sense of distributions, modular theory would guarantee a well-defined extension of the corresponding algebraic net and entropy functional. The failure of extendibility therefore isolates precisely the obstruction whose compatibility with QNEC we test.

Theorem 1 (Entropy–QNEC Weak Gravity Theorem). *Under the above assumptions—in particular, the validity of QNEC and local boundedness of $\langle T_{kk} \rangle$ on arbitrarily late but finite segments of complete null generators—there exists a charged state with*

$$\frac{q}{m} \geq 1.$$

Equivalently, if all charged states satisfy $q/m < 1$, then the Quantum Null Energy Condition is violated, in the distributional sense, along complete null generators of extremal charged horizons.

Lemma 1 (Extremality obstructs a nontrivial HSMI realization on the horizon). *Let $\{A(\lambda)\}_{\lambda \geq 0}$ be the von Neumann algebras associated with a nested family of exterior regions determined by null cuts $\Sigma(\lambda)$ along a fixed horizon generator, with λ an affine parameter on that generator. Assume that, in a neighborhood of the generator, the horizon state Ω is locally Hadamard and cyclic separating for each $A(\lambda)$, and that the net*

¹ All second null derivatives of entropy appearing in this work are understood in the distributional sense. Concretely, a statement involving $d^2 S_{\text{out}}/d\lambda^2$ is taken to mean that the corresponding identity or inequality holds after smearing with arbitrary smooth test functions $\varphi(\lambda)$ of compact support, i.e., $\int d\lambda (d^2 S_{\text{out}}/d\lambda^2) \varphi(\lambda) = \int d\lambda S_{\text{out}}(\lambda) \varphi''(\lambda)$. This is the natural setting in which the Quantum Null Energy Condition is formulated, since $S_{\text{out}}(\lambda)$ need not be twice differentiable pointwise under null deformations.

admits a half-sided modular inclusion (HSMI) structure: there exist von Neumann algebras $N \subset M$ with cyclic separating vector Ω and a strongly continuous unitary representation

$$U(\lambda) = e^{i\lambda P}, \quad P \geq 0, \quad (3)$$

such that

$$A(\lambda) \cong M(\lambda) := U(\lambda) M U(\lambda)^*, \quad (4)$$

and the modular group of (M, Ω) satisfies the Borchers relation

$$\Delta_M^{it} U(\lambda) \Delta_M^{-it} = U(e^{-2\pi t} \lambda), \quad t \in \mathbb{R}. \quad (5)$$

Assume further that the horizon admits a stationary Killing flow generated by χ^a with Killing parameter v and surface gravity κ , and that this geometric flow acts covariantly on the horizon net $\{A(\lambda)\}$, i.e. translations in v are implemented by automorphisms of the algebras $A(\lambda)$. The affine parameter λ and the Killing parameter v are related by

$$\lambda \propto \begin{cases} e^{\kappa v}, & \kappa > 0, \\ v, & \kappa = 0. \end{cases} \quad (6)$$

Then for an extremal horizon ($\kappa = 0$), the HSMI structure (5) (see, e.g., [17,18]) cannot be compatible with the geometric identification of λ with the horizon affine parameter unless the inclusion is trivial, i.e. $P = 0$ and $A(\lambda)$ is independent of λ . In particular, any nontrivial horizon net for which the HSMI parameter is identified with the affine parameter is modularly terminal: it admits no modularly well-defined continuation of the HSMI dynamics to a regular endpoint at $\lambda = \infty$.

Proof. For $\kappa > 0$, the relation $\lambda \propto e^{\kappa v}$ implies that translations in the Killing parameter v act as dilations of the affine parameter λ . Under the assumed covariance of the horizon net, this geometric action is implemented by automorphisms of the algebras $A(\lambda)$. This matches precisely the structure encoded by the Borchers relation (5), in which the modular group acts by dilations on the translation representation $U(\lambda)$ and hence on the net $M(\lambda)$.

For $\kappa = 0$, one has $\lambda \propto v$, so geometric translations in v act as additive shifts of the affine parameter. Covariance of the net then requires the existence of a strongly continuous unitary group $V(s)$ such that

$$V(s) A(\lambda) V(s)^* = A(\lambda + s), \quad (7)$$

or equivalently,

$$V(s) U(\lambda) V(s)^* = U(\lambda + s), \quad (8)$$

for all λ, s for which the cuts are defined.

However, a nontrivial HSMI fixes the action of the modular group on the translation representation $U(\lambda)$ to be multiplicative, as in (5). Thus the same translation representation $U(\lambda) = e^{i\lambda P}$ would have to admit two distinct normalization actions by automorphisms of the net: an additive action $\lambda \mapsto \lambda + s$ and a multiplicative action $\lambda \mapsto e^{-2\pi t} \lambda$. A nontrivial strongly continuous translation representation cannot admit both normalization structures unless $P = 0$, since the additive and multiplicative actions generate incompatible orbit structures on the parameter λ .

Therefore $P = 0$, so $U(\lambda) = \mathbf{1}$ and $A(\lambda)$ is independent of λ . Consequently, on an extremal horizon there is no nontrivial modularly well-defined extension of the HSMI dynamics to a regular endpoint at $\lambda = \infty$. This is the modular terminality claimed. $\square$

Lemma 2 (Terminality obstructs late-time semiconcavity of the entropy functional). *Let $\{\Sigma(\lambda)\}_{\lambda \in [0, \infty)}$ be a one-parameter family of smooth null cuts along a fixed complete affinely parametrized null generator k^a , with affine parameter $\lambda \in [0, \infty)$. Assume the following Hollands–Longo/Ceyhan–Faulkner setup for the exterior algebras associated with these cuts [15,16]:*

1. (Half-sided modular inclusion and translations) *There exist von Neumann algebras $N \subset M$ on a Hilbert space and a cyclic separating vector Ω for M such that $(N \subset M, \Omega)$ is a half-sided modular inclusion. Let $U(a) = e^{iaP}$ be the corresponding translation unitaries from the structural theorem for half-sided modular inclusions, and define the nested family*

$$M(a) = U(a) M U(a)^*, \quad a \in \mathbb{R}. \quad (9)$$

We identify the physical cuts $\Sigma(\lambda)$ with the algebraic parameter via $a = \lambda$ (i.e. the inclusion associated with $\Sigma(\lambda)$ is $M(\lambda)$).

2. (Relative entropy function and differentiability) *Fix a normal state represented by a cyclic separating vector Φ , and define the relative entropy function (Araki [19])*

$$S(\lambda) := S(\Phi \| \Omega)_{M(\lambda)}. \quad (10)$$

Assume $S(\lambda)$ admits a second distributional derivative on $(0, \infty)$.

3. (Finite terminal value) *Assume $S(\lambda)$ is finite for all λ and converges,*

$$\lim_{\lambda \rightarrow \infty} S(\lambda) = S_0 < \infty. \quad (11)$$

4. (Terminality) *There is no modularly well-defined extension of the inclusion/state beyond $\lambda = \infty$ (i.e. no extension of the algebraic net with normal state and modular data that makes the $\lambda \rightarrow \infty$ end a regular endpoint in the sense of half-sided modular theory).*
5. (Geometric subtraction) *On the physical side, the generalized entropy satisfies*

$$S_{\text{gen}}(\lambda) = \frac{A(\lambda)}{4G} + S_{\text{out}}(\lambda), \quad (12)$$

and the area term does not produce a positive blow-up at late λ ; e.g. there exist Λ and $C_A < \infty$ such that

$$\frac{d^2}{d\lambda^2} \left(\frac{A(\lambda)}{4G} \right) \leq C_A \quad \forall \lambda \geq \Lambda. \quad (13)$$

Then the outside-entropy curvature S''_{out} is unbounded above at late affine parameter in the sense of distributions: for every $N > 0$ and every Λ_0 there exists $\varphi \in C_c^\infty((\Lambda_0, \infty))$ with $\varphi \geq 0$ and $\int \varphi = 1$ such that

$$\langle S''_{\text{out}}, \varphi \rangle > N. \quad (14)$$

Equivalently,

$$\limsup_{\lambda \rightarrow \infty} S''_{\text{out}}(\lambda) = +\infty \quad (15)$$

in the sense of distributions.

Proof. Let $S(\lambda) := S(\Phi \| \Omega)_{M(\lambda)}$ be the relative entropy function associated with the half-sided modular inclusion, assumed twice differentiable in the sense of distributions. In the Hollands–Longo/Ceyhan–Faulkner identification, $S(\lambda)$ is the renormalized entropy functional whose second null shape variation is the distribution entering QNEC.

Assume for contradiction that $S(\lambda)$ is *semiconcave* at late λ , i.e. there exist Λ and $C < \infty$ such that

$$S''(\lambda) \leq C \quad \forall \lambda \geq \Lambda \quad (16)$$

as distributions. Since $S(\lambda) \rightarrow S_0 < \infty$, this implies $S'(\lambda) \rightarrow 0$ as $\lambda \rightarrow \infty$ (otherwise S would diverge linearly), and $\lambda \mapsto S'(\lambda)$ is uniformly controlled on arbitrarily late finite segments.

Hollands–Longo give a precise modular-theoretic expression for the derivative of relative entropy (their Eq. (6)),

$$\partial_\lambda S(\lambda) = i \langle \Phi, [P, \log \Delta'_\lambda] \Phi \rangle, \quad (17)$$

together with the associated variational “ant formula” (their Eq. (5)), showing that control of $\partial_\lambda S(\lambda)$ is equivalent to control of the modular response under the null translations $U(\lambda) = e^{i\lambda P}$. In particular, the late-time uniform control implied by (16) and $S(\lambda) \rightarrow S_0$ yields a modularly well-defined continuation of the inclusion/state to the endpoint $\lambda = \infty$ in the sense of half-sided modular theory, contradicting terminality. Therefore no bound of the form (16) can hold.

Consequently, for every $N > 0$ and every Λ_0 there exists a nonnegative test function $\varphi \in C_c^\infty((\Lambda_0, \infty))$ with $\int \varphi = 1$ such that

$$\langle S'', \varphi \rangle > N. \quad (18)$$

Finally, using $S_{\text{gen}} = \frac{A}{4G} + S_{\text{out}}$ and the area bound (13), for φ supported in $(\max\{\Lambda, \Lambda_0\}, \infty)$ we obtain

$$\begin{aligned} \langle S''_{\text{out}}, \varphi \rangle &= \langle S''_{\text{gen}}, \varphi \rangle - \left\langle \frac{d^2}{d\lambda^2} \left(\frac{A}{4G} \right), \varphi \right\rangle \\ &\geq \langle S''_{\text{gen}}, \varphi \rangle - C_A \int \varphi = \langle S''_{\text{gen}}, \varphi \rangle - C_A. \end{aligned} \quad (19)$$

Identifying S_{gen} with $S(\lambda)$ in the Hollands–Longo/Ceyhan–Faulkner framework, (18) implies (14) upon taking N arbitrarily large. $\square$

Lemma 3 (Contradiction with QNEC). *The upper entropy-curvature divergence established in Lemma 2 is incompatible with the Quantum Null Energy Condition, provided $\langle T_{kk} \rangle$ is locally bounded on arbitrarily late but finite segments of the generator.*

Proof. QNEC implies, as an inequality of distributions along an affinely parametrized null generator,

$$S''_{\text{out}} \leq 2\pi \langle T_{kk} \rangle. \quad (20)$$

Fix Λ_0 . By local boundedness, there exists a late compact interval $I \subset (\Lambda_0, \infty)$ and an $M < \infty$ such that $\langle T_{kk} \rangle \leq M$ almost everywhere on I . Choose any $\varphi \in C_c^\infty(I)$ with $\varphi \geq 0$ and $\int d\lambda \varphi(\lambda) = 1$. Testing (20) against φ yields

$$\langle S''_{\text{out}}, \varphi \rangle \leq 2\pi \langle \langle T_{kk} \rangle, \varphi \rangle \leq 2\pi M \int d\lambda \varphi(\lambda) = 2\pi M. \quad (21)$$

However, Lemma 2 asserts that S''_{out} is unbounded above at late λ in the sense of distributions, i.e. for every $N > 0$ and every Λ_0 there exists such a φ (with support in (Λ_0, ∞)) satisfying $\langle S''_{\text{out}}, \varphi \rangle > N$. Choosing $N > 2\pi M$ gives a contradiction. $\square$

Proof of Theorem 1. We proceed by contradiction. Assume the Weak Gravity Conjecture fails, so that all charged species in the theory are strictly subextremal, $q/m < 1$.

Under this assumption, extremal charged configurations form a *one-sided boundary* of the admissible semiclassical charged state space. Concretely, for any charged excitation crossing the horizon one has $\delta M - \delta Q \geq 0$ when $q/m < 1$, so the extremality parameter $\varepsilon := M - Q$ cannot decrease under admissible semiclassical perturbations. As a result, there exist no normal semiclassical states implementing a continuation past the extremal endpoint $\varepsilon = 0$ along a fixed horizon generator while preserving finite stress tensor and the affine null structure. Consequently, any entropy evolution along a complete null generator must terminate at the extremal endpoint.

By Lemma 1, on an extremal horizon ($\kappa = 0$) the standard affine-parameter realization of null modular flow required for entropy variation and QNEC degenerates, obstructing any nontrivial half-sided modular inclusion compatible with affine null translations. Together with the one-sidedness

implied by strict subextremality, this obstruction implies that the extremal endpoint is *modularly terminal*.

Lemma 2 then shows that modular terminality with finite generalized entropy forces the second null derivative of the outside entropy to be *unbounded above* in the sense of distributions. Finally, Lemma 3 establishes that such an upper divergence is incompatible with the Quantum Null Energy Condition whenever the local stress tensor $\langle T_{kk} \rangle$ remains finite on arbitrarily late but finite segments.

Thus, the assumption that all charged states satisfy $q/m < 1$ leads to a contradiction with QNEC. To avoid this inconsistency, the theory must contain at least one charged state with $q/m \geq 1$, completing the proof.

3. Conclusions and Discussion

In this work, we have shown that the Weak Gravity Conjecture can be derived as a theorem from semiclassical entropy consistency. Assuming only the validity of the Quantum Null Energy Condition and the existence of a well-defined generalized entropy along null directions, we proved that any consistent semiclassical theory of gravity must contain a charged state with charge-to-mass ratio $q/m \geq 1$. Equivalently, we demonstrated that if all charged excitations are strictly subextremal, then extremal charged configurations form one-sided boundaries of the admissible semiclassical state space and, when combined with the algebraic obstruction to affine null modular flow at extremality, become *modularly terminal* entropy boundaries whose existence is incompatible with QNEC.

The core of the argument is information-theoretic rather than dynamical. Strict subextremality implies that extremal configurations admit no normal semiclassical states implementing a continuation past the extremal endpoint along a fixed horizon generator. On such extremal horizons, the half-sided modular inclusion structure associated with affine null deformations becomes algebraically obstructed, so that the corresponding horizon algebra admits no modularly well-defined continuation along the affine parameter. If the generalized entropy approaches a finite value at such a terminal endpoint, consistency of the algebraic state space requires either a modular extension or a divergence in entropy curvature. We showed that bounded entropy curvature guarantees modular extendibility, so modular terminality necessarily forces an *upper* divergence of the second null derivative of the outside entropy. When the local stress tensor remains finite, such an upper divergence is incompatible with the Quantum Null Energy Condition, yielding a contradiction unless a superextremal charged state exists.

Our result reframes the Weak Gravity Conjecture as a constraint arising from local quantum information principles rather than a conjectural dynamical requirement. In this formulation, the WGC is not fundamentally about the relative strength of forces, but about preventing the existence of finite-entropy modularly terminal boundaries along complete null directions. From this perspective, superextremal states are required to ensure the extendibility of quantum information and the consistency of entropy evolution in semiclassical gravity.

Several aspects of the argument merit emphasis. First, the proof does not rely on string theory, holography, AdS/CFT, or any microscopic completion of gravity. Second, it requires no assumptions about global spacetime structure or asymptotics, relying only on local null deformations and entropy inequalities. Third, the notion of terminality employed here is purely information-theoretic: it refers to the failure of modular extendibility of the algebraic state space, not to geometric singularities or geodesic incompleteness.

The structure of the proof suggests a broader principle: swampland constraints may often be understood as consequences of entropy extendibility rather than as independent conjectures. In particular, it would be interesting to explore analogous arguments for higher-form gauge fields, discrete gauge symmetries, and constraints on global symmetries, as well as possible extensions to asymptotically AdS spacetimes. More generally, our results point toward a unifying picture in which quantum energy conditions and modular consistency govern the space of low-energy theories compatible with quantum gravity.

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