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A Critical Analysis of the Quantum Nonlocality Problem: On the Polemic Assessment of What Bell Did

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Abstract: Despite their Nobel prize-winning empirical confirmation, the interpretation of the Bell inequality remains controversial. By carefully analyzing Bell's work on nonlocality, we show how the correct appreciation of Bell's rationale calls for reformulating a widespread argument on quantum nonlocality, yielding a more consistent formulation of the problem. We delve into a formal proof of quantum mechanics' violation of local causality strikingly overlooked in favor of less appropriate arguments. For completeness, we also mention a few alternatives that may rationally justify considering quantum mechanics as a local theory.

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1. Introduction

Bell, of course, like Einstein, believed that quantum mechanics implies *"Spooky action at a distance."* But contrary to a widespread opinion, we claim that the correct argument supporting quantum nonlocality should not be based on the Bell inequality.

The main goal of this work is to bring up a rather trivial point that is, however, almost universally overlooked, namely, the difference between the proof of quantum nonlocality and quantum completeness.

We show that an objective reading of Bell's works on quantum foundations reveals he appropriately recognized the difference. Otherwise, we obtain a controversial approach to quantum nonlocality that justifies its rejection by the quantum localists adopting a radical opposite stance.

A characteristic exposition of these antagonist positions is given through an exchange between Tim Maudlin, with his article "What Bell did" which suggested the title of this paper, and Reinhard Werner's comments, exemplifying the innumerable articles claiming quantum nonlocality based on the Bell inequality, on the one hand, and its rejection for the same reason on the other Maudlin (2014a,b); Werner (2014a,b).

Despite our agnostic attitude towards quantum nonlocality, we argue that Bell's and Einstein's beliefs in the nonlocal character of quantum mechanics cannot be so easily dismissed on naive prejudices about realism or "classicality" as many localists interpret.

We stress that the correct argument sustaining quantum nonlocality should be exclusively based on quantum mechanics' objective predictions irrespective of eventual interpretations regarding the nature of the quantum state. It involves neither the Bell inequality, hidden variables, nor the metaphysical existence of ghostly entities like the elements of physical reality. Differently, the argument is not merely unconvincing and obscure but somewhat inconsistent and easy to debunk. Ironically, Einstein, who is usually erroneously identified with a superfluous notion of reality, dismissed the EPR reasoning and personally never based his arguments on the celebrated *elements of physical reality* Howard (1985).

We contend that Bell would have probably agreed with us since there is no solid evidence to support that he interpreted his inequality as a direct proof of quantum nonlocality. On the contrary, there is unambiguous proof where, on a few occasions, he explicitly argued for quantum nonlocality without his inequality, then introduced the latter only to prove the impossibility of a rational local completion. In any case, if we want to interpret he used both strategies, our point is that the latter is the correct one.

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Sections 2 and 3 show that, in his most lucid writings, Bell distinctly formulated his inequality only after explicitly establishing the nonlocal character of quantum mechanics, either by previously assuming Bell (1964) or directly proving it Bell et al. (1985); Bell (2001). So, the controversial approach of using a "classical inequality" to infer properties of quantum theory is not ascribable to John Bell.

Section 4 analyzes Bell's actual quantum nonlocality argument which only appeared in 1975 more than ten years after the publication of his celebrated 1964 paper. We highlight an approach that puts on a more formal basis the reasons why quantum mechanics is not locally causal.

Since we are dealing with quantum mechanics interpretation, we also have logically consistent counterarguments for quantum locality. If only for completeness, we briefly review some well-known counterarguments in section 5. Of course, none of such "correct" counterarguments rely on precepts such as "...the Bell inequality is based on classical physics..." or "...the Bell inequality assumes realism..."; not because they are necessarily incorrect but because they are irrelevant. Finally, we present our conclusions in sections 6 and 7.

2. The 1964 Bell Theorem

A usual and widespread interpretation asserts that Bell formulated his inequality to prove that quantum mechanics is not a local theory, presenting the Bell theorem as a quantum nonlocality theorem. But an attentive reading of Bell's 1964 argument reveals two crucial facts that prove otherwise Bell (1964):

1. Bell already considered quantum mechanics as nonlocal from the beginning, i.e., before formulating his inequality. Indeed, in the third line of the introduction, he wrote: *"These additional variables were to restore to the theory causality and locality."* That is, the inclusion of hidden variables into the theory was supposed to modify it and recover locality instead of proving its nonlocality.
2. Bell starts the conclusion section by saying: *"In a theory in which parameters are added to quantum mechanics...."*; so, clearly, he was not inferring properties of quantum mechanics, but only of a modified theory in which parameters are added.

So, Bell's stunning conclusion is not that quantum mechanics is nonlocal, which he took for granted, but that we cannot fix its nonlocality. Bell resumed the argument where EPR has left it: if local, quantum mechanics must be incomplete. Since the orthodox interpretation asserts quantum mechanics is complete, then, according to EPR, it must be nonlocal.

The previous inference was implicit in his expression in a) asserting that additional variables were necessary to restore locality. Then, following an EPR-like reasoning, Bell derived a deterministic hidden variable model and proved, through his inequality, the untenability of a local completion. Therefore, the impossibility of an acceptable local completion only proves that we cannot modify orthodox quantum mechanics to make it local.

Thus, accepting the EPR reasoning, as Bell did, the inequality is unnecessary to prove quantum nonlocality, so claiming that Bell's inequality proves it (which Bell did not) would only be circular reasoning.

On the other hand, Bell's theorem is a mere mathematical theorem that should be free of any polemic if we strictly follow Bell's rationale, namely, that a local completion is untenable. Richard Feynman put that clearly when he said that Bell's theorem,

> It is not an important theorem. It is simply a statement of something we know is true – a mathematical proof of it. Whitaker (2016)

However, we disagree with Feynman on the unimportance of the Bell theorem since it was a significant advance in the Bohr-Einstein debate that, by 1964, remained stagnant for almost thirty years.

3. Bell's Theorem after 1964

Bell's arguments evolved over the years. In later works, he abandoned the polemic EPR reasoning, dissipating the fog around it. However, a persistent view exists that dispenses with Bell's later arguments. That view advocates a controversial reading of Bell's 1964 reasoning advertising the Bell theorem as a quantum nonlocality theorem. We call this view "radical non-localist".

According to the radical non-localist stance, the EPR argument is unassailable. They even consider it an "analytic concept", i.e., we cannot coherently deny it Maudlin (2014b). However, as we observed in the previous section, be it analytic or not, when we accept the EPR reasoning, we do not need the inequality to prove quantum nonlocality.

Except on the occasions where he left the issue ambiguous, Bell explicitly separated his arguments of quantum nonlocality from his inequality, which he used to prove the impossibility of a local completion. Above, ambiguous means he neither claimed his inequality proved quantum nonlocality nor said otherwise. To prove our previous assertion, next, we chronologically review some of Bell's papers discussing nonlocality after 1964.

3.1. Introduction to the Hidden Variable Problem

In 1971 he wrote the paper "Introduction to the hidden-variable question" Bell (2004). Here Bell did not mention quantum nonlocality but investigated the de Broglie-Bohm hidden variables theory and highlights its explicit nonlocal character as "the difficulty".

Bell concluded after formulating his inequality and proving that quantum mechanics violates it:

> Thus the quantum-mechanical result cannot be reproduced by a hidden-variable theory which is local in the way described.

Literally, "the quantum-mechanical result cannot be reproduced by a hidden-variable theory" is different from concluding quantum mechanics is not local. Besides, the adjective "local" refers to the hidden-variable theory.

3.2. The Theory of Local Beables

This article appeared in 1975. [1] Here Bell abandoned the EPR reasoning and introduced the concept of local causality. He argued that quantum mechanics violates this form of locality in section 3 without mentioning any inequality. He starts that section by asserting:

> Ordinary quantum mechanics, even the relativistic quantum field theory, is not locally causal in the sense of (2).

(2) above refers to local causality. Then he develops his quantum nonlocality argument. It is similar to the one given by Einstein in 1927.[2] In the same section, immediately after establishing the nonlocal character of quantum mechanics, Bell explored the problem of adding hidden variables. Then in section 4, "Locality inequality", he derived a stochastic Bell-CHSH inequality. Finally, in section 5, he established the impossibility of a local completion by proving that quantum mechanics violates his inequality, concluding:

> So quantum mechanics is not embeddable in a locally causal theory as formulated above.

That is different from concluding, "So quantum mechanics is not a local theory". Otherwise, why would he bother to prove quantum mechanics violates local causality two sections before without using any inequality or hidden variables? Because he was well aware of the logical loophole of concluding quantum nonlocality from his inequality. As Stapp once clearly explained Stapp (2012):

> Thus whatever is proved is not a feature of quantum mechanics, but is a property of a theory that tries to combine quantum theory with quasi-classical features that go beyond what

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[1] Bell's work is reproduced in Bell et al. (1985).

[2] Einstein's argument is reproduced by Laudisa 2019 and also by Harrigan and Spekkens 2010.

> is entailed by quantum theory itself. One cannot logically prove properties of a system by establishing, instead, properties of a system modified by adding properties alien to the original system.

Above, "properties alien to the original system" rigorously mean variables that do not legitimately pertain to quantum mechanics. Although some have observed that the hidden variables can include the quantum state Gisin (2012); Laudisa (2018); Norsen (2011), the problem persists with the other "additional variables". As we observe in the Appendix 8.3, the Bell inequality cannot be formulated without additional variables foreign to quantum mechanics, notwithstanding that one of those variables may include the quantum state.

3.3. Bertlmann's Socks

In 1981 Bell wrote his celebrated paper "Bertlmann's socks and the nature of reality" Bell (1981). On this occasion, Bell did not explicitly prove quantum nonlocality before formulating the inequality. He based his arguments on EPR. But he also uses common causes to explain local correlations.

This is one of the papers where he left ambiguous whether his inequality violation should be interpreted as proof of quantum nonlocality. Can we assume that Bell changed his mind about the meaning of his inequality? We do not think so because, in his last paper (cf. 3.4), he returned to his previous formulations, i.e., either accepting (1964) or proving (1975) quantum nonlocality without introducing hidden variables or mentioning any inequality.

In Bell (1981), Bell chose intuition and ease of interpretation over logical rigor. In Bell's own words, this paper was one of those that:

> ...are nontechnical introductions to the subject. They are meant to be intelligible to nonphysicists. Bell and Aspect (2004)

That is why he spent great effort explaining the difference between quantum and classical entanglement through naive analogies, such as those of Mr. Bertlmann's socks.

3.4. La Nouvelle Cuisine

This is Bell's last paper which appeared in 1990 Bell (2001). Here again, Bell's view of his inequality and quantum nonlocality is crystal clear. This time Bell mentions EPR in the two sections that concern us here. In section 8, when proving that "Ordinary quantum mechanics is not locally causal" without mentioning any inequality and without actually using an EPR argument. Then, in section 10, when explicitly introducing hidden variables as local common causes, for proving, through his inequality, that :

> Quantum mechanics cannot be embedded in a locally causal theory

Again, the order in which he presents his argument, first establishing quantum nonlocality without using any inequality and then proving the impossibility of a local completion through his inequality, is unambiguous and uncontroversial.

4. Bell's Proof of Quantum Nonlocality

In this section, we closely analyze Bell's arguments on the nonlocal character of quantum mechanics. In his 1975 paper, "The theory of local beables" Bell et al. (1985), Bell gave an explicit argument for quantum nonlocality for the first time. This paper has four outstanding characteristics that were missing in 1964:

- A formal definition of locality that is directly applicable to quantum mechanics. He called it *local causality* (LC).
- An argument showing quantum mechanics violates LC and hence is nonlocal. The argument, of course, does not involve his inequality.

- A justification for assuming *statistical independence* (SI) in his hidden variable model. In 1964, SI was an *ad hoc* implicit assumption.
- An absence of any reference to the EPR paper.

Next, we briefly address each of these characteristics.

4.1. Local Causality

Bell's definition of LC is a formalization of the idea that, according to relativity theory, interactions can happen only at a finite speed. It means that causes cannot have an instantaneous effect on distant events. *He formulated LC so that it can be applied to not deterministic theories like quantum mechanics. It is a locality argument that avoids a purportedly classical EPR-like reasoning. A concept directly applicable to orthodox quantum mechanics without distorting its nature.*

For the particular case that concerns us, i.e., the singlet state correlations in a Bell-type experiment, LC takes the following form. Let $P(A,B \mid a,b)$ be the probability of a joint measurement giving the results $A,B \in \{-1,+1\}$ conditional on the respective measurements directions a,b. The laws of probabilities require

$$P(A,B \mid a,b) = P(A \mid B,a,b)P(B \mid a,b) \tag{1}$$

So far, it is just about probabilities. Let us now add some physics and assume that both observers, Alice and Bob, choose their measurement directions at the last moment so that both measurements are spacelike separated events. Then LC requires that neither the results A,B nor the measurement settings a,b made on one side can affect the state of affairs on the other side. However, we cannot exclude the existence of correlations. In the r.h.s of (1), we can have that

$$P(A \mid B,a,b) \neq P(A \mid a) \tag{2}$$

$$P(B \mid a,b) \neq P(B \mid b) \tag{3}$$

notwithstanding that events A and a are spacelike separated from B and b. However, relativistic causality requires the correlations implied by (2) and (3) to be explained by local common causes λ. They are local because they are supposed to lie at the intersection of the backward light cones of the measurement events. Once the common causes λ are specified, the inclusion of spacelike separated parameters in the l.h.s of (2) and (3) become redundant

$$P(A \mid B,a,b,\lambda) = P(A \mid a,\lambda) \tag{4}$$

$$P(B \mid a,b,\lambda) = P(B \mid b,\lambda) \tag{5}$$

Including λ in (1)

$$P(A,B \mid a,b,\lambda) = P(A \mid B,a,b,\lambda)P(B \mid a,b,\lambda) \tag{6}$$

Replacing (4) and (5) in (6)

$$P(A,B \mid a,b,\lambda) = P(A \mid a,\lambda)P(B \mid b,\lambda) \tag{7}$$

The last equation is also known as the screening-off condition. It is the formal expression of the intuitive idea behind relativistic locality and is Bell's definition of LC for the case at hand.

The common cause λ is usually called "hidden variables"; however, it is somewhat misleading to believe the λ variables are necessarily unknown parameters. The only condition they need to comply with is lying at the intersection of the backward light cones of the measuring events to constitute a local explanation of the correlations. It is also utterly misleading to think they are EPR elements of physical reality; on the contrary, their role is to eliminate any EPR-like argument. Furthermore, local causality is independent of the stochastic properties of the common causes. More concretely, they are independent of the statistical independence hypothesis.

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Although Bell did not mention Reichenbach, his λ variables are according to Reichenbach's common cause principle Hitchcock and Rédei (2020). The last point is relevant because, as we shall see later, one possibility to block the argument in favor of quantum nonlocality is to reject Reichenbach's principle of common causes Cavalcanti and Lal (2014).

4.2. Quantum Nonlocality

After defining local causality, Bell gave an argument explaining why, when considered complete, quantum mechanics violates it. Bell's argument is similar to the one given by Einstein in 1927.[3]

We can recast Bell's and Einstein's arguments in more formal terms through the mathematical formulation of local causality. The crucial point is that (7) avoids the polemic around an EPR-like classical argument. If quantum mechanics is complete and local, the locally causal explanation of its correlations must lie within the quantum state. In our case

$$\mid \psi\rangle = \frac{1}{\sqrt{2}}(\mid +\rangle\otimes\mid -\rangle - \mid -\rangle\otimes\mid +\rangle) \tag{8}$$

Thus, if locally causal, ordinary quantum mechanics must satisfy (7) when

$$\lambda =\mid \psi\rangle \tag{9}$$

However, choosing $a = b$, $A = 1$, and $B = -1$, an elementary quantum mechanical calculation gives

$$\begin{array}{rcl} P(1,-1 \mid a,a,\mid \psi\rangle) & = & P(1 \mid a,\mid \psi\rangle)P(-1 \mid a,\mid \psi\rangle) \\ \frac{1}{2} & \neq & \frac{1}{2}\frac{1}{2} = \frac{1}{4} \end{array} \tag{10}$$

Given that (10) is not widely known as a non-classical argument for quantum nonlocality, and some find (9) puzzling or inappropriate, the appendix contains a detailed explanation.

Since in (10) $1/2 \neq 1/4$, ordinary quantum mechanics lacks a locally causal explanation of its correlations, i.e., *the quantum state alone cannot screen-off events on one side from spacelike separated events on the other far away side. Hence, it conspicuously fails the LC locality criterion.*

Note the difference between (10) and the EPR argument. The first one relies exclusively on quantum mechanical objective predictions. It is independent of the wave function interpretation and wave function collapse. In particular, it is independent of the ontic or epistemic nature of the quantum state, depending only on the quantum formalism irrespective of any interpretation. It is an argument in line with the Copenhagen approach, an operational definition that does not rely on metaphysical assumptions.

Formally, that is the counterargument against claims asserting the singlet correlations find a local common cause explanation in their preparation with the same generating event Griffiths (2020). There is no doubt they find an explanation in their preparation. Unfortunately, that explanation is not locally causal because all we know from its preparation is its quantum state, and as (10) proves, it does not contain a common cause explanation. Nor does the magic of superposition justify those correlations, at least in a locally causal way Boughn (2017).

4.3. Statistical Independence

As we mentioned above, in Bell's 1964 paper, he implicitly assumed the hidden variables distribution function[4] $P(\lambda)$ was not conditional on the experimental settings a and b.

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[3] Einstein's argument is reproduced by Laudisa in Laudisa (2019) and also by Harrigan and Spekkens in Harrigan and Spekkens (2010).

[4] Note that the distribution function of the λ common causes is irrelevant for the definition of local causality. $P(\lambda)$ is necessary only to derive the Bell inequality.

$$P(\lambda \mid a, b) = P(\lambda) \tag{11}$$

We can justify (11) by requiring the experimental settings to be independent of the same common factors λ affecting the results

$$P(a, b \mid \lambda) = P(a, b) \tag{12}$$

According to Bayes theorem we have

$$P(a, b \mid \lambda)P(\lambda) = P(\lambda \mid a, b)P(a, b) \tag{13}$$

Then from (12) and (13) we get (11). The ansatz (12) seems to be a reasonable assumption justifying (11).

Thus, (12) and (11) are equivalent and are known as *statistical independence*, *measurement independence*, *freedom*, or *no-conspiracy*. We shall come back to SI in sect. 5.3.

4.4. The EPR Paper

Although Bell conceived his 1964 paper as a continuation of the EPR argument, one of the virtues of his 1975 formulation is not referencing the EPR paper. Besides presumably being a classical-like argument, the EPR reasoning contains an unnecessary construction that has been the source of much superfluous metaphysical speculation, namely, the *elements of physical reality*.

The reality criterion has a highly metaphysical burden because it assumes the existence of physical magnitudes from the mere possibility of predicting their values, notwithstanding that we do not indeed measure them. They are unnecessary because they are employed neither to prove quantum nonlocality (10) nor to derive the Bell inequality Lambare and Franco (2021).

Bohr attacked the reality criterion Bohr (1935). Einstein did not write the EPR paper, and he did not like how it came out. In a letter to Schrödinger he wrote Howard (1985):

> But still it has not come out as well as I really wanted; on the contrary, the main point was, so to speak, buried by erudition.

Einstein based his argument for incompleteness in his separation principle and avoided any reference to the reality criterion. Thus, it is worth noticing that Einstein and Bell distanced themselves from the original EPR elements of physical reality criterion. Even in 1964, when Bell referenced the EPR article, he never mentioned the elements of physical reality.

5. Quantum Locality

We briefly mention three counterarguments that may justify considering quantum mechanics as a local theory. Only the third one contemplates the use of the Bell theorem and is related to the Bell inequality. We mention them only for completeness as possible logically admissible counterarguments.

These arguments are, of course, well-known and not new. There are also others we do not mention, such as the many-worlds interpretation and QBism, that also claim to preserve quantum locality.

5.1. Rejecting Local Causality

Jarrett Jarrett (1984) helped clarify the nature of local causality by decomposing it into the conjunction of two different conditions, which Shimony respectively called parameter independence (PI) and outcome independence (OI) [5]

$$LC \equiv PI \wedge OI \tag{14}$$

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[5] Jarrett used the terms "locality" and "completeness", implying that PI alone is locality. Shimony terminology is better because it is more neutral.

Shimony also proposed the more picturesque expressions controllable and uncontrollable nonlocality, respectively. We refer the non-specialist to Ref. Shimony (1993) for a detailed explanation of these concepts.

Jarret proved that a theory complying with PI is no-signaling. He also showed quantum mechanics respects PI, hence is no-signaling. However, quantum mechanics violates OI, thus violating LC.

We can effectively block the argument in favor of quantum nonlocality by adopting parameter independence as the appropriate concept for locality and rejecting outcome independence as a necessary condition.

In summary, by accepting no-signaling (parameter independence) as a sufficient criterion for locality, we reject the more stringent condition of local causality, recovering quantum mechanics locality. Of course, those who claim quantum mechanics is not local will not accept the definition Norsen (2009).

However, more rational discussions are possible by explicitly acknowledging the different criteria. It is fair to note that even some who can be considered radical nonlocalists accept that quantum nonlocality is not right out “action at a distance” Gisin (2023).

5.2. Rejecting Causation

Causation in physics was criticized by Bertrand Russell in 1912 Russell (1913) and was proposed to solve the quantum nonlocality problem by Van Fraassen 1982. There is no action at a distance simply because there is no need for a causal explanation.

According to Van Fraassen *“In some cases, the methodological tactic of developing a causal theory will achieve this aim of empirical adequacy, in other cases it will not, and that is just the way the world is.”* He claimed that a mythical picture of causal processes got grip on our imagination.

Van Fraassen 1982 was also puzzled by the *“incredible metaphysical extravaganzas to which this subject has led”* that we suppose in part coincide, or are consequences of, what we refer to as incorrect arguments here.

5.3. Completing Quantum Mechanics

This approach is different from the former two because it implies going beyond orthodox quantum mechanics. If we are willing to accept local causality as the correct locality concept and recover a causal explanation, we must consider quantum mechanics as an emergent theory.

We can complete quantum mechanics with local hidden variables if we reject statistical independence. The 1975 version of the Bell theorem is

$$LC \wedge SI \rightarrow Bell \quad inequaqlity \tag{15}$$

Thus, we can retain local causality in a hidden variables theory by rejecting statistical independence. Indeed, well-known local hidden variables models exist reproducing the singlet correlations violating statistical independence Degorre et al. (2005); Feldmann (1995).

Whether statistical independence is a necessary physical condition is a contentious issue. According to some physicists, its rejection is a rational position Hall (2016); Hossenfelder and Palmer (2020); ’t Hooft (2021). Others, including John Bell Bell (1981), sustain its rejection as inadmissible since it purportedly compromises the experimental freedom implying unreasonable conspiracies.

6. Conclusions

The incorrect mixing of two distinct issues, the proofs of quantum nonlocality on the one hand and quantum completion on the other, has diverted the debate from the correct arguments and hindered it from advancing to more rational alternatives; giving a wrong perspective of the real interpretational difficulties.

We have argued for two key questions: a) the correct quantum nonlocality proof is not based on the Bell inequality which should be sensibly interpreted as a no-local-hidden variables theorem, and b)

John Bell's quantum nonlocality argument was also not based on his inequality, which he used only to prove the impossibility of a local completion.

The first issue (a) is a consistency puzzle. It is formally sustained on Bell's concept of local causality that orthodox operational quantum mechanics certainly violates without the need for introducing any elements extraneous to quantum theory.

The second one (b) has a historical character. Although it is more subject to interpretation, we claim it follows from an unbiased and rigorous reading of Bell's writings.

We conclude there are valid reasons to support both views, locality, and nonlocality. However, the correct arguments supporting or rejecting either of the two opposite positions are not as definite or trivial as the usual specious explanations uphold, exacerbating the different attitudes.

7. Final Remarks

Although we may find it comforting to base the claim of quantum nonlocality on the Bell inequality to dismiss it as the consequence of different worldviews, classical vs. quantum Griffiths (2021); Khrennikov (2019); Werner (2014a), accepting the correct argument is not insurmountable and may hint at deeper insights.

Distant simultaneity and nonlocality are closely related concepts. Both lack direct and objective physical determination. Admitting a certain degree of convention is necessary to maintain a coherent level of discourse. That is why the locality problem will remain controversial. However, it is essential to recognize its contentious nature for the correct motives instead of incorrect or dubious reasonings.

Relativity taught us to reject absolute simultaneity because of its lack of objective determination, likewise, quantum mechanics may be teaching us to reject action at a distance for the same reason, concretized through its nonsignaling property. Quantum mechanics may require a revision of our notion of causality, just as relativity prompted us to revise our concept of simultaneity.

The other possibility is that quantum mechanics is emergent and, because of Bell's theorem, that would require the acceptance of superdeterminism.[6] These options are still valid open questions, and pretending they are closed or inexistent is not the best scientific attitude.

8. Appendix: Common Causes and the Quantum State

The correct significance and use of the hidden variables are largely misinterpreted, so the following sections are intended to clarify their use and meaning.

8.1. Common Causes Meaning

Some researchers find it perplexing that the quantum state $|\psi\rangle$ can be considered a common cause in the definition of local causality. That is owed to the incorrect metaphysical meaning usually attached to the λ variables as preexisting EPR elements of physical reality Nisticò (2014) or a necessarily classical concept.

Sustaining that λ is by necessity an element foreign to quantum mechanics amounts to forbidding the application of the local causality concept to quantum mechanics. It is particularly convenient for summarily dismissing its vexing nonlocal character decreeing it local by construction Werner (2014a). But the physical meaning of λ is not limited to classical or metaphysical concepts other than representing local common causes. Bell also explained that the hidden variables may include the quantum state Bell (1981):

> It is notable that in this argument nothing is said about the locality, or even localizability, of the variable λ. These variables could well include, for example, quantum mechanical state vectors, which have no particular localization in ordinary space-time.

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[6] By superdeterminism, we mean violating the mathematical condition (11) without implying any particular interpretation regarding its admissibility.

Unfortunately, researchers often grossly overlook Bell's explanation of the meaning of the λ variables. They generally identify λ with metaphysical entities such as EPR preexisting values or believe they must necessarily be unknown parameters. We suspect such misunderstandings arise from the concrete example he used in his 1964 paper Bell (1964) where λ was a spin three-vector or from naive analogies he employed in his less technical expositions such as those in "Bertlmann's socks and the nature of reality" Bell (1981).

8.2. Formal Proof of Quantum Nonlocality

The particular case $\lambda = \mid \psi\rangle$ is necessary to formalize Bell's (and Einstein's) qualitative arguments of quantum nonlocality according to the rigorous definition of local causality. This step is necessary to test whether quantum mechanics gives a locally causal explanation of its correlations. *Thus, there is no valid argument against submitting quantum mechanics, within its own rules, to the local causality test.*

To see whether ordinary quantum mechanics complies with the local causality criterion, all we have to do is set $\lambda = \mid \psi\rangle$ in (7) with $\mid \psi\rangle$ given by (8), where $\mid +\rangle$ and $\mid -\rangle$ denote the spin eigenstates in the z-direction. We assume that motion takes place in y direction with setting angles a and b lying the $x - z$ plane measured with respect to the z axis. If $\mid a, +\rangle$ and $\mid a, -\rangle$ are the spin eigenstates in the a direction

$$\mid a,+\rangle = +\cos\frac{a}{2}\mid +\rangle + \sin\frac{a}{2}\mid -\rangle \tag{16}$$

$$\mid a,-\rangle = -\sin\frac{a}{2}\mid +\rangle + \cos\frac{a}{2}\mid -\rangle \tag{17}$$

Analogously for the particle measured at the other laboratory, we have

$$\mid b,+\rangle = +\cos\frac{b}{2}\mid +\rangle + \sin\frac{b}{2}\mid -\rangle \tag{18}$$

$$\mid b,-\rangle = -\sin\frac{b}{2}\mid +\rangle + \cos\frac{b}{2}\mid -\rangle \tag{19}$$

The joint probability according the quantum formalism is

$$P(A,B\mid a,b,\mid\psi\rangle) = \langle\psi\mid(\mid a,A\rangle\otimes\mid b,B\rangle\langle a,A\mid\otimes\langle b,B\mid)\mid\psi\rangle \tag{20}$$

Letting $A = +1$, $B = -1$ according to (8), (16) and (19)

$$P(+1,-1\mid a,b,\mid\psi\rangle) = \langle\psi\mid(\mid a,+\rangle\otimes\mid b,-\rangle\langle a,+\mid\otimes\langle b,-\mid)\mid\psi\rangle \tag{21}$$

$$= \frac{1}{\sqrt{2}}(\langle +\mid a,+\rangle\langle -\mid b,-\rangle - \langle -\mid a,+\rangle\langle +\mid b,-\rangle)\frac{1}{\sqrt{2}}()^{*}$$

$$= \frac{1}{2}(\cos\frac{a}{2}\cos\frac{b}{2} + \sin\frac{a}{2}\sin\frac{b}{2})^2 \tag{22}$$

$$= \frac{1}{2}\cos^2(\frac{a-b}{2}) \tag{23}$$

Where $()^{*}$ represents the complex conjugate of the first factor in parenthesis. If we further assume $a = b$, (23) gives

$$P(+1,-1\mid a,a,\mid\psi\rangle) = \frac{1}{2} \tag{24}$$

When we perform a measurement only in Alice's laboratory, the quantum formalism prescribes

$$P(+1,a,|\psi\rangle) = \langle\psi\mid(\mid a,+\rangle\langle a,+\mid\otimes I)\mid\psi\rangle \tag{25}$$

$$= \langle\psi\mid[(\mid a,+\rangle\langle a,+\mid\otimes I)\mid\psi\rangle] \tag{26}$$

$$= \langle\psi\mid\left[\frac{1}{\sqrt{2}}(\mid a,+\rangle\langle a,+\mid+\rangle\otimes\mid-\rangle-\mid a,+\rangle\langle a,+\mid-\rangle\otimes\mid+\rangle)\right]$$

$$= \langle\psi\mid\left[\frac{1}{\sqrt{2}}(\cos\frac{a}{2}\mid a,+\rangle\otimes\mid-\rangle-\sin\frac{a}{2}\mid a,+\rangle\otimes\mid+\rangle)\right] \tag{27}$$

$$= \frac{1}{2}\left[\cos\frac{a}{2}\langle+\mid a,+\rangle+\sin\frac{a}{2}\langle-\mid a,+\rangle\right] \tag{28}$$

$$= \frac{1}{2}\left[\cos^2\frac{a}{2}+\sin^2\frac{a}{2}\right] \tag{29}$$

$$= \frac{1}{2} \tag{30}$$

Where $I = \mid+\rangle\langle+\mid+\mid-\rangle\langle-\mid$ is the identity operator in the one particle two-dimensional Hilbert-space. In a similar way, performing a measurement only on Bob's laboratory we find

$$P(-1,b,|\psi\rangle) = \langle\psi\mid(I\otimes\mid b,-\rangle\langle b,-\mid)\mid\psi\rangle=\frac{1}{2} \tag{31}$$

From (24), (30), and (31), we obtain (10) formally proving that ordinary quantum mechanics lacks a local common cause explanation for its correlations.

8.3. The unconvincing argument

The violation of local causality by quantum mechanics through the singlet state is well known and was observed by other authors Laudisa (2018); Norsen (2011). However, most non-localists give it only a subsidiary importance. They prefer to turn to the Bell inequality as their main argument. On the other hand, localists conveniently overlook it.

A typical example of this approach is given, for instance, by Norsen 2011 who ultimately presents the CHSH inequality as a quantum nonlocality proof when, after taking for granted statistical independence, he declares:

> ...the empirically violated Clauser-Horne-Shimony-Holt inequality can be derived from Bell's concept of local causality alone, without the need for further assumptions involving determinism, hidden variables, "realism," or anything of that sort.

In our opinion, that move is unwarranted and justifies the opposite stance held by localists. Indeed, the CHSH inequality cannot be formulated without hidden variables or common causes not present in quantum mechanics precisely because quantum mechanics violates (7), as proved by (10). Certainly, proving Bell-type inequalities require writing joint probabilities as

$$P(A,B\mid a,b)=\int P(A\mid a,\lambda)P(B\mid b,\lambda)P(\lambda)d\lambda \tag{32}$$

which is impossible without going beyond quantum mechanics. Admittedly, it is a trivial logical loophole. However, an endemic loophole that is frequently exploited by localists to debunk even the most lucid quantum nonlocality presentations like the one by Brunner et al. Brunner et al. (2014) where again (7) is correctly explained but finally (32) is highlighted as the "locality constraint", declaring

> This is the content of Bell's theorem, establishing the nonlocal character of quantum theory and of any model reproducing its predictions.

Localists usually respond that since the inequality based on (32) is not about quantum mechanics, it signals the nonlocality "of any model reproducing its predictions", except quantum mechanics.

References

Bell, J.S., A. Shimony, M.A. Horne, and J.F. Clauser. 1985. An Exchange on Local Beables. *Dialectica 39*(2), 85–110.

Bell, J. S. 1964. On the Einstein-Podolsky-Rosen paradox. *Physics 1*, 195–200. https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195.

Bell, J. S. 1981. Bertlmann's socks and the nature of reality. *Journal of Physque 42*, 41–61. https://doi.org/10.1051/jphyscol:1981202.

Bell, J. S. 2001. *La nouvelle cuisine*, pp. 216–234. Farrer Road, Singapore 912805: WORLD SCIENTIFIC. https://doi.org/10.1142/4757.

Bell, J. S. 2004. *Speakable and Unspeakable in Quantum Mechanics*, Chapter Introduction to the hidden variable question, pp. 36–37. Cambridge: Cambridge University Press.

Bell, J. S. and Alain Aspect. 2004. *Preface to the first edition* (2 ed.)., pp. 11–13. Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9780511815676.001.

Bohr, Niels. 1935. Can quantum-mechanical description of physical reality be considered complete? *Phys.Rev. 48*, 696–702.

Boughn, S. 2017. Making sense of Bell's theorem and quantum nonlocality. *Found. of Phys. 47*, 640–657. https://doi.org/10.1007/s10701-017-0083-6.

Brunner, Nicolas, Daniel Cavalcanti, Stefano Pironio, Valerio Scarani, and Stephanie Wehner. 2014, Apr. Bell nonlocality. *Rev. Mod. Phys. 86*, 419–478. https://doi.org/10.1103/RevModPhys.86.419.

Cavalcanti, Eric G and Raymond Lal. 2014, oct. On modifications of Reichenbach's principle of common cause in light of Bell's theorem. *Journal of Physics A: Mathematical and Theoretical 47*(42), 424018. https://doi.org/10.1088/1751-8113/47/42/424018.

Degorre, Julien, Sophie Laplante, and Jérémie Roland. 2005, Dec. Simulating quantum correlations as a distributed sampling problem. *Phys. Rev. A 72*, 062314. https://doi.org/10.1103/PhysRevA.72.062314.

Feldmann, Michel. 1995. New loophole for the Einstein-Podolsky-Rosen paradox. *Foundations of Physics Letters 8*(1), 41–53. https://doi.org/10.1007/BF02187530.

Gisin, Nicolas. 2012, 01. Non-realism: Deep Thought or a Soft Option? *Foundations of Physics 42*, 80–85. https://doi.org/10.1007/s10701-010-9508-1.

Gisin, Nicolas. 2023, February. Quantum non-locality: from denigration to the Nobel prize, via quantum cryptography. *Europhysics News 54*(1), 20–23. https://doi.org/10.1051/epn/2023103.

Griffiths, Robert. 2020. Nonlocality claims are inconsistent with Hilbert-space quantum mechanics. *Physical Review A 101*, 022117. https://doi.org/10.1103/PhysRevA.101.022117.

Griffiths, Robert B. 2021, Dec. Reply to "Comment on 'Nonlocality claims are inconsistent with Hilbert-space quantum mechanics' ". *Phys. Rev. A 104*, 066202. https://doi.org/10.1103/PhysRevA.104.066202.

Hall, Michael J. W. 2016. *The Significance of Measurement Independence for Bell Inequalities and Locality*, pp. 189–204. Cham: Springer International Publishing. https://doi.org/10.1007/978-3-319-31299-6_11.

Harrigan, Nicholas and Robert W. Spekkens. 2010. Einstein, Incompleteness, and the Epistemic View of Quantum States. *Foundations of Physics 40*, 125–157. https://doi.org/10.1007/s10701-009-9347-0.

Hitchcock, Christopher and Miklós Rédei. 2020. Reichenbach's Common Cause Principle. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Spring 2020 ed.). : Metaphysics Research Lab, Stanford University.

Hossenfelder, Sabine and Tim Palmer. 2020. Rethinking superdeterminism. *Frontiers in Physics 8*, 139. https://doi.org/10.3389/fphy.2020.00139.

Howard, Don. 1985. Einstein on locality and separability. *Studies in History and Philosophy of Science Part A 16*(3), 171–201. https://doi.org/https://doi.org/10.1016/0039-3681(85)90001-9.

Jarrett, Jon P. 1984. On the Physical Significance of the Locality Conditions in the Bell Arguments. *Noûs 18*(4), 569–589. https://doi.org/10.2307/2214878.

Khrennikov, Andrei. 2019. Get Rid of Nonlocality from Quantum Physics. *Entropy 21*(8), 806. https://doi.org/10.3390/e21080806.

Lambare, J. P. and R. Franco. 2021. A Note on Bell's Theorem Logical Consistency. *Found Phys 51*(84). https://doi.org/10.1007/s10701-021-00488-z.

Laudisa, F. 2018. Stop making sense of Bell's theorem and nonlocality? *European Journal for Philosophy of Science 8*, 293–306. https://doi.org/10.1007/s13194-017-0187-z.

Laudisa, F. 2019. Counterfactual Reasoning, Realism and Quantum Mechanics: Much Ado About Nothing? *Erkenn 84*, 1103–1118. https://doi.org/10.1007/s10670-018-9997-4.

Maudlin, Tim. 2014a. Reply to comment on What Bell did. *J. Phys. A 47*, 424012. https://doi.org/10.1088/1751-8113/47/42/424012.

Maudlin, Tim. 2014b. What Bell did. *J. Phys. A 47*, 424010. https://doi.org/10.1088/1751-8113/47/42/424010.

Nisticò, Giuseppe. 2014. Non Locality Proofs in Quantum Mechanics Analyzed by Ordinary Mathematical Logic. *Int J Theor Phys 53*, 3475–3487. https://doi.org/10.1007/s10773-013-1822-x.

Norsen, Travis. 2009. Local Causality and Completeness: Bell vs. Jarrett. *Found. Phys. 39*(3), 273–294. https://doi.org/10.1007/s10701-009-9281-1.

Norsen, Travis. 2011. John S. Bell's concept of local causality. *American Journal of Physics 79*(12), 1261–1275. https://doi.org/10.1119/1.3630940.

Russell, Bertrand. 1913. On the notion of cause. *Proceedings of the Aristotelian Society 13*, 1–26. https://doi.org/10.1093/aristotelian/13.1.1.

Shimony, Abner. 1993. *Controllable and uncontrollable non-locality*, Volume 2, pp. 130–139. Cambrige: Cambridge University Press. https://doi.org/10.1017/CBO9781139172196.010.

Stapp, Henry P. 2012. Quantum Locality? *Foundations of Physics 42*, 647–655. https://doi.org/10.1007/s10701-012-9632-1.

't Hooft, Gerard. 2021, May. Fast Vacuum Fluctuations and the Emergence of Quantum Mechanics. *Foundations of Physics 51*. https://doi.org/10.1007/s10701-021-00464-7.

Van Fraassen, B. C. 1982. The Charybdis of realism: epistemological implications of Bell's inequality. *Synthese 52*, 25–38. https://doi.org/10.1007/BF00485253.

Werner, R. F. 2014a. Comment on "What Bell did". *J. Phys. A 47*, 424011. https://doi.org/10.1088/1751-8113/47/42/424011.

Werner, R. F. 2014b. What Maudlin replied to. https://doi.org/10.48550/ARXIV.1411.2120.

Whitaker, Andrew. 2016. RICHARD FEYNMAN AND BELL'S THEOREM. *American Journal of Physics 84*(7), 493–494. https://doi.org/10.1119/1.4948268.