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Multi-Stage Probabilistic Transmission Expansion Planning Under Generation Uncertainty and N-1 Security Using the Pack-Based Grey Wolf Optimizer

Edimar José de Oliveira [1,*], Lucas Santiago Nepomuceno [1], Arthur Neves de Paula [2], Raphael Paulo Braga Poubel [3] and Leonardo Willer de Oliveira [1]

[1] Electrical Engineering Department, Federal University of Juiz de Fora, Juiz de Fora, Minas Gerais 36036-900, Brazi
[2] Institute of Technological Sciences, Federal University of Itajubá, Itabira, Minas Gerais 35903-087, Brazil
[3] Department of Electrical Engineering, Federal Center for Technological Education of Minas Gerais (CEFET-MG), Belo Horizonte 30421-169, Brazil
* Correspondence: edimar.oliveira@ufjf.br

Abstract

Multi-Stage Transmission Network Expansion Planning (MS-TNEP) is critical for adapting power grids to long-term renewable integration. However, simultaneously incorporating N-1 security, active power losses, and spatial generation uncertainties imposes prohibitive computational complexity. This paper proposes a probabilistic MS-TNEP model evaluated over a 20-year horizon. To overcome intractability, a hybrid decomposition framework is employed, delegating discrete combinatorial investment decisions to an upper-level metaheuristic while resolving operational feasibility, power losses via fictitious nodal demand, and N-1 contingencies through lower-level linear programming. Furthermore, a novel Pack-Based Grey Wolf Optimizer (PBGWO) is introduced to enhance convergence in this constrained domain. The approach is validated on the modified Garver and the 46-bus Southern-Brazilian systems under multiple wind and conventional generation scenarios. Comparative analysis against the Genetic Algorithm, standard GWO, and Whale Optimization Algorithm reveals that PBGWO consistently identifies the optimal expansion schedules.

Keywords: transmission network expansion planning; multi-stage planning; probabilistic optimization; N-1 security criterion; metaheuristics; grey wolf optimizer; wind power integration

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1. Introduction

Transmission Network Expansion Planning (TNEP) is the process that determines where, how much and when new equipment should be inserted into the power grid. Its central objective is to efficiently interconnect generation centers to load centers at the lowest investment cost [1]. The definition of this cost and the expansion strategies depend directly on the structure of the power sector. In centralized environments, TNEP seeks to meet demand growth in coordination with generation, minimizing global operation and investment costs. In decentralized markets, the focus is on providing an infrastructure that supports competitive transactions and guaranties the secure power dispatch of multiple agents [2]. Regardless of the market environment, investment decisions vary according to the adopted time frame. Static planning designs the network for a single future scenario, consolidating all interventions in a single step. Multi-stage planning overcomes this simplification by establishing the optimal timing for each project on a subdivided horizon [1].

The modeling of the network defines the mathematical basis of TNEP. Even in its DC formulation, the planning is already configured as a non-convex Mixed-Integer Non-Linear Programming (MINLP) problem. This intrinsic characteristic arises from the product between discrete investment variables and continuous variables associated with the power flow. Adopting the AC model substantially amplifies this complexity due to the non-linear nature of its equations, particularly when evaluating

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the impacts of active power dissipation in lossy network representations. Similarly, the transition to the multi-stage formulation multiplies the problem dimensionality by coupling decisions across multiple stages. To enable the resolution of large-scale systems, the literature frequently employs mathematical linearization and reformulation techniques on AC and DC models. Alternatively, decomposition techniques are used to decouple investment decisions from operation decisions, guaranteeing numerical tractability in network evaluation [3].

In addition to network modeling, the reliability of planning directly impacts the size of mathematical problems. Operational security is commonly ensured by the deterministic N-1 criterion. This technical constraint guaranties uninterrupted demand supply even after isolated failure of any system equipment. Incorporating this premise requires the formulation of multiple post-contingency scenarios. As a result, the number of operational constraints multiplies, and the complexity of the model grows exponentially [4].

The combinatorial explosion driven by security guidelines and temporal multi-stages makes the choice of the solution method a critical step. Exact mathematical optimization approaches often encounter practical convergence limits when evaluating large-scale systems. Given this limitation, the literature is extensively based on metaheuristic techniques [5]. These algorithms perform a robust and efficient exploration of the non-convex search space. They mitigate the risk of premature convergence to local optima and provide high-quality solutions at a tractable computational cost.

1.1. Literature Review

The presented literature review is structured based on studies focused on solving the multi-stage TNEP. Articles are classified according to six main criteria. The first criterion is the market environment, which defines the adopted market perspective. In a centralized environment, planning and operational decisions are made by the independent operator. The main objective of this model is to reduce total investment and operation costs systemically, as well as to maintain secure grid operation. In contrast, the deregulated or competitive environment is characterized by an unbundled power sector. In this scenario, generators and transmission investors seek to maximize their individual profits in a competitive market, and the planner must deal with the uncertainties of these agents actions to ensure social welfare. The second criterion evaluates the network model and the horizon, referring to the power flow model used (such as DC, linearized AC, or full AC) and the way the planning time horizon is discretized.

The third criterion addresses how the N-1 security criterion is considered, being divided into three categories. The first category is not considered when planning does not provide security against equipment failures. The second is to check, when a list of circuits undergoing contingency is used, applying the failure to each of these circuits to then evaluate if the system can operate securely. The third is explicit constraint, when the contingency model is treated through a linear formulation and solved directly using commercial solvers. The fourth criterion classifies the work by the treatment of uncertainties, being deterministic when uncertainty is not considered in the optimization process, probabilistic, stochastic, or robust.

The fifth criterion evaluates the topological evolution of the system. The evolution is classified as static when the geographical topology of loads and generators remains unchanged over the planning horizon, with only the increment of their values from the initial year over the years. On the other hand, the evolution is classified as multi-stage when new generators and loads are added, distinguishing the final topology from the initial geometric topology.

Finally, the sixth criterion categorizes the solution method. This is classified as a decomposition when the multi-stage TNEP problem is divided into an investment subproblem and an operation subproblem. This strategy is widely used in the literature, where metaheuristic techniques are traditionally responsible for defining investment plans in the investment subproblem, while classical optimization methods are applied to solve the operation subproblem. The Mixed-Integer Linear Programming (MILP) category covers works where linearizations and reformulations of the original MINLP formulation are used, converting them into linear formulations tractable by classical optimization commercial

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solvers. In addition, there are methods based purely on the global use of metaheuristics or hybrid methods.

Table 1 summarizes the classification of the reviewed works according to established methodological criteria.

Table 1. Comparison of the literature on Multi-Stage Transmission Network Expansion Planning.

Reference	Market Environment	Network Model	N-1 Criterion	Uncertainty Treatment	Horizon / Division	Topological Evolution	Solution Method
[6]	Deregulated	DC	Explicit Constraint	Deterministic	20 years / 4 st.	Static	Decomposition (GA)
[7]	Deregulated	DC	Checking/Penal.	Stochastic	15 years / 3 st.	Multi-Stage	Decomposition (NSGA II)
[8]	Centralized	DC	Not considered	Deterministic	3 stages	Static	Decomposition (EGA)
[9]	Centralized	Linearized AC	Explicit Constraint	Deterministic	15 years / 3 st.	Static	MILP
[10]	Centralized	DC	Not considered	Stochastic	25 years / 25 st.	Static	MILP
[11]	Centralized	DC	Not considered	Deterministic	10 years / 10 st.	Static	Decomposition (HS)
[12]	Centralized	DC	Checking/Penal.	Deterministic	10 years / 10 st.	Static	Decomposition (TSHA)
[13]	Centralized	DC	Checking/Penal.	Fuzzy-Robust	8 to 12 years / 4 st.	Multi-Stage	Decomposition (PSO)
[14]	Centralized	DC	Not considered	Deterministic	5-year blocks	Static	Decomposition (ASSO)
[15]	Centralized	Full AC	Checking/Penal.	Deterministic	3 years / 3 st.	Static	Decomposition (MABC)
[16]	Centralized	DC	Checking/Penal.	Deterministic	25 years / 5 st.	Multi-Stage	Decomposition (LSHADE)
[17]	Centralized	AC and DC	Not considered	Deterministic	10 years / 10 st.	Static	Decomposition (EPSO)
[18]	Centralized	AC	Not considered	Deterministic	21 years / 3 st.	Static	Decomposition (IBBA)
[19]	Centralized	DC	Not considered	Deterministic	3 stages	Static	Decomposition (FDBCOA)
[20]	Deregulated	DC	Checking/Penal.	Stochastic	9 years / 3 st.	Multi-Stage	Decomposition (Tri-Level)
Proposal	**Centralized**	**DC**	**Checking/Penal.**	**Probabilistic**	**20 year/ 3 st.**	**Multi-Stage**	**Decomposition (PBGWO)**

Within the scope of the centralized environment with DC modeling, the works focus on proposing formulations or algorithmic strategies to bypass the high combinatorial complexity of the planning.

The work [8] addresses centralized DC planning with the objective of minimizing investment costs. The planning horizon comprises 3 continuous stages under a static topological evolution. The solution method is based on decomposition, using the Enhanced Genetic Algorithm (EGA) to propose investments and linear programming for operation. The strategy applies a constructive heuristic to generate a strictly feasible initial population and a local search phase to remove redundant lines.

The study [10] addresses the centralized DC co-planning of line expansion and energy storage systems. The planning horizon covers 25 years with annual steps under static topological evolution. The uncertainty treatment is stochastic, formulating multiple probabilistic scenarios to handle wind generation variation and load growth. The solution method uses a purely Mixed-Integer Linear Programming (MILP) mathematical formulation, modeling the temporal degradation of the storage capacity.

Research [11] addresses centralized DC planning focused on minimizing investment costs and energy deficits. The planning horizon has 10 annual stages under static topological evolution. The N-1 criterion is not considered and a small tolerance for load shedding is accepted. The solution method adopts decomposition that combines spatial and temporal dimensions. The strategy applies the Harmony Search (HS) metaheuristic to find a set of optimal static routes and uses the exact Branch and Bound algorithm to define the installation chronology.

The approach [12] addresses centralized DC planning to minimize investment and operating costs. The planning horizon evaluates 10 annual stages under static topological evolution. The N-1 criterion is evaluated through checking and penalizing against a predetermined list of contingencies. The solution method operates by decomposition, where the Lagrange multipliers of the optimal power flow guided by the Tree Searching Heuristic Algorithm (TSHA) define the creation of candidate routes, while the Genetic Algorithm (GA) acts sequentially to refine security.

The work [13] addresses centralized DC planning aimed at minimizing investment under chance constraints. The planning horizon covers 8 to 12 years divided into 4 stages, under multi-Stage topological evolution with the emergence of new loads. The uncertainty treatment is fuzzy-robust. Fuzzy theory parameterizes the impreciseness of the load forecast, while interval optimization handles renewable volatility. The decomposition solution method uses Particle Swarm Optimization (PSO) for investments and the Improved Branch and Bound (IBB) algorithm to solve the operation problem with robust optimization characteristics.

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The study [14] addresses the centralized DC planning. The planning horizon operates in 5-year multistage blocks under static topological evolution. The solution method is based on decomposition using the Adaptive Salp Swarm Optimization (ASSO) metaheuristic for the investment subproblem and linear programming for the operation.

The work [16] addresses the centralized DC planning order to minimize investment and load shedding. The planning horizon covers the period from 2016 to 2040, divided into 5 stages under multi-Stage topological evolution with the addition of distributed generators. The uncertainty treatment is deterministic, but it uses the Adaptive Neuro-Fuzzy Inference System (ANFIS) network to predefine the single long-term load forecast scenario. The N-1 criterion is evaluated by checking and penalizing load shedding. The decomposition solution method applies the LSHADE-SPACMA evolutionary algorithm to structure grid investments.

The model [19] addresses centralized DC planning focused purely on reducing investment costs. The planning horizon evaluates 3 continuous stages under static topological evolution. The solution method is based on decomposition by applying the Enhanced Coati Optimization Algorithm (FDBCOA). The central strategy integrates the Fitness Distance Balance (FDB) method coupled with opposition-based learning (OBL) to explore and balance the search space.

Research [20] addresses coordinated centralized DC planning between the transmission network and the Energy Hubs. The planning horizon has 9 years divided into 3 stages, under multi-Stage topological evolution resulting from user response to prices. The uncertainty treatment is stochastic. Demand and price uncertainties use normal PDFs, while solar radiation and wind use Beta and Weibull PDFs. The N-1 criterion is evaluated by checking and penalizing, modeled via Bernoulli PDF to simulate line and equipment failure rates. The solution method uses tri-level decomposition solved iteratively by the diagonalization and scenario reduction algorithm.

Advancing to models that incorporate the physical fidelity of AC modeling (active and reactive powers, voltage limits, and transmission losses within a lossy network representation), the literature deals with the high computational effort through linearizations or hybridizations.

The work [9] addresses centralized AC planning with the objective of co-optimizing transmission lines and reactive power sources (VAr). The planning horizon comprises 15 years divided into 3 stages under static topological evolution. The uncertainty treatment is deterministic. The N-1 criterion acts as an explicit constraint in the equations. The solution method is based on a pure MILP formulation. The AC flow is linearized by the branch flow model, allowing the exact solver to evaluate investments in lines and compensators simultaneously.

The article [15] addresses centralized AC planning focused on minimizing new line costs and maintaining the voltage stability index. The planning horizon covers 3 years in 3 annual stages under static topological evolution. The uncertainty treatment is deterministic. The N-1 criterion is tested through a contingency check and voltage limits. The decomposition solution method uses the Modified Artificial Bee Colony (MABC) metaheuristic. The strategy applies a four-phase filtering that evaluates the topology in DC flow and relaxed AC flow before submitting the plans to the exact AC flow calculation.

The study [18] addresses tecno-economic centralized AC planning seeking to minimize investments, electrical losses, and reactive compensation. The planning horizon extends over 21 years in 3 stages of 7 years, under static topological evolution. The uncertainty treatment is deterministic because the demand evolution comes from a single scenario generated by the ANFIS forecasting network. The solution method applies decomposition using the Improved Binary Bat Algorithm (IBBA), which incorporates V-shaped transfer functions and an adaptive search space for binary variables.

The approach [17] addresses centralized planning in AC and DC networks to minimize investment with penalties for load shedding. The planning horizon operates in 10 annual stages under static topological evolution. The uncertainty treatment is deterministic. The solution method presents a hybrid model. The strategy actively explores the boundary between feasible and infeasible solution regions, where the MBH heuristic iteratively moves the exact relaxed DC flow solution toward the AC feasible solution derived from the EPSO metaheuristic.

Finally, in the context of planning in a deregulated environment, methodologies abandon the exclusive perspective of minimizing physical losses to incorporate competitive market behavior and risk allocation in DC modeling.

The work [6] addresses the competitive market through a two-level problem focused on maximizing the profit of the transmission company (upper level) and maximizing social welfare (lower level). The planning horizon has 20 years divided into 4 stages under static topological evolution. The uncertainty treatment is deterministic. The N-1 criterion operates as an explicit security constraint. The decomposition solution method associates a Genetic Algorithm (GA) with niche technology to define investments, while economic operation and security are validated by the classical Primal-Dual Interior-Point Method.

Article [7] addresses the multiobjective deregulated DC planning aimed at minimizing the social cost, the adjustment cost, and the maximum risk of regret (robustness). The planning horizon has 15 years in 3 stages, under multi-Stage topological evolution generated by the uncoordinated installation of combined cycle power plants. The uncertainty treatment is stochastic. Investment decisions in thermoelectric generation by private entities are treated through a formulation based on internal risk scenarios. The N-1 criterion acts by checking for contingencies in the dispatch. The decomposition solution method uses the NSGA II evolutionary algorithm combined with fuzzy logic-based on decision-making processes to select the best compromise solution.

1.2. Contributions

Although Multi-Stage Transmission Network Expansion Planning (MS-TNEP) has been widely explored in the literature, there is a lack of approaches that simultaneously integrate the N-1 security criterion with long-term uncertainties tied to the spatial trajectories of both conventional and renewable generation expansion. This limitation is typically justified by the explosion in computational complexity imposed on the model. In this context, this work aims to fill this methodological gap. The specific contributions of this article are as follows:

1. The formulation of a probabilistic multi-Stage TNEP model that simultaneously integrates the iterative calculation of active power losses in the DC network via the fictitious nodal demand method, rigorous N-1 contingency security constraints, and multiple spatial scenarios for the growth of both wind and conventional power generation capacity.
2. The application of a hybrid decomposition technique to make the computational effort of the proposed model viable. This method delegates the combinatorial search for investment routes to a metaheuristic at the upper level, while the operational feasibility, power losses, and N-1 criterion checking under multiple scenarios are solved via linear programming at the lower level.
3. The proposal of a novel variant of the Grey Wolf Optimizer, named Pack-Based Grey Wolf Optimizer (PBGWO), aimed at improving the performance, robustness, and convergence capacity of the traditional GWO when solving the highly complex MS-TNEP problem.
4. The adaptation of four originally continuous metaheuristics to the discrete domain to handle the integer variables associated with transmission line investments. These algorithms include the *Grey Wolf Optimizer* (GWO) [21], and the *Whale Optimization Algorithm* (WOA) [22].

2. Mathematical Formulation

The mathematical formulation of the problem is presented in Equations (1)–(13). This formulation represents a mixed-integer nonlinear programming (MINLP) problem considering the expansion scenarios of the generation capacity throughout the multi-Stage planning horizon.

$$\min \quad \left[\sum_{t\in\mathcal{T}} \frac{1}{(1+r)^t} \sum_{(i,j)\in\Omega_C} IC_{ij} \cdot v_{ijt} + \sum_{t\in\mathcal{T}} \sum_{c\in\mathcal{K}} \sigma_c \sum_{i\in\mathcal{N}} \left(C_i^G \cdot g_{itc} + C_i^{LS} \cdot r_{itc} \right) \right] \tag{1}$$

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subject to:

$$\sum_{t\in\mathcal{T}} v_{ijt} \leq 1 \quad \forall (i,j) \in \Omega_C \tag{2}$$

$$y_{ijt} = \sum_{\tau=1}^{t} v_{ij\tau} \quad \forall (i,j) \in \Omega_C, \forall t \in \mathcal{T} \tag{3}$$

$$\sum_{j\in\Omega_i} f_{ijtc} + g_{itc} + r_{itc} + (W_{it} - w_{itc}^{cut}) = d_{it} + \frac{1}{2}\sum_{j\in\Omega_i} p_{ijtc}^{loss} \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}, \forall c \in \mathcal{K} \tag{4}$$

$$f_{ijtc} = u_{ijc} \cdot B_{ij}(\theta_{itc} - \theta_{jtc}) \quad \forall (i,j) \in \Omega_E, \forall t \in \mathcal{T}, \forall c \in \mathcal{K} \tag{5}$$

$$f_{ijtc} = y_{ijt} \cdot u_{ijc} \cdot B_{ij}(\theta_{itc} - \theta_{jtc}) \quad \forall (i,j) \in \Omega_C, \forall t \in \mathcal{T}, \forall c \in \mathcal{K} \tag{6}$$

$$p_{ijtc}^{loss} = R_{ij}(f_{ijtc})^2 \quad \forall (i,j) \in \Omega_E \cup \Omega_C, \forall t \in \mathcal{T}, \forall c \in \mathcal{K} \tag{7}$$

$$-u_{ijc} \cdot F_{ij}^{\max} \leq f_{ijtc} \leq u_{ijc} \cdot F_{ij}^{\max} \quad \forall (i,j) \in \Omega_E, \forall t \in \mathcal{T}, \forall c \in \mathcal{K} \tag{8}$$

$$-y_{ijt} \cdot u_{ijc} F_{ij}^{\max} \leq f_{ijtc} \leq y_{ijt} \cdot u_{ijc} \cdot F_{ij}^{\max} \quad \forall (i,j) \in \Omega_C, \forall t \in \mathcal{T}, \forall c \in \mathcal{K} \tag{9}$$

$$0 \leq g_{itc} \leq G_i^{\max} \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}, \forall c \in \mathcal{K} \tag{10}$$

$$0 \leq r_{itc} \leq d_{it} \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}, \forall c \in \mathcal{K} \tag{11}$$

$$0 \leq w_{itc}^{cut} \leq W_{it} \quad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}, \forall c \in \mathcal{K} \tag{12}$$

$$\theta_{ref,tc} = 0 \quad \forall t \in \mathcal{T}, \forall c \in \mathcal{K} \tag{13}$$

The objective function defined in (1) minimizes the total expected planning cost, composed of the present value of investments in new lines and the expected operating cost. This includes conventional generation and penalties for load shedding and contingency scenarios.

The constraint (2) establishes the investment limit in candidate lines. The constraint (3) defines the operating state of the candidate line, which couplings the investment decisions temporally.

The constraint (4) represents the system active power nodal balance, incorporating transmission losses and net renewable injection. Constraints (5) and (6) represent the Second Kirchhoff Law for the DC power flow in existing and candidate lines, respectively. The constraint (7) calculates the active ohmic losses in the transmission lines.

Constraints (6) and (7) introduce bilinear products between the binary state variable y_{ijt} and the continuous variable θ_{itc}, and strictly quadratic equalities, respectively. This formulation characterizes the full problem as a highly non-convex MINLP (Mixed-Integer Non-Linear Programming) problem intractable by conventional exact methods for large-scale systems, underpinning the need for decomposition methods and metaheuristics.

The constraints (8) and (9) impose the thermal flow limits of existing and candidate lines, respectively. In these equations, the N-1 security criterion is intrinsically modeled by the binary availability parameter u_{ijc}, isolating the failed equipment. Constraints (10)-(12) determine the maximum operational limits for conventional generation dispatch, load shedding, and wind power curtailment. Finally, the constraint (13) establishes the voltage angle in the reference bus of the system.

3. Solution Methodology

To solve the MS-TNEP problem under uncertainties in the capacity growth of both wind and conventional generation, this work adopts a strategy based on model decomposition. The original problem is partitioned into an investment subproblem guided by metaheuristics and a probabilistic operation subproblem solved by linear programming techniques. This decomposition isolates the combinatorial complexity of construction decisions and bypasses the intrinsic non-linearity of the full model.

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3.1. *Investment Subproblem*

The investment subproblem acts as the master level of optimization. The metaheuristic proposes matrix expansion plans, which uniquely define the line construction schedule. The algorithm aims to navigate the search space by minimizing the solution fitness function, which consolidates the present value of the infrastructure construction cost and the expected operation and penalty cost evaluated in subproblems.

3.1.1. Solution Encoding

The structure of each individual (candidate solution) is defined using the matrix format formulated in (14). Each column of the matrix indicates a planning horizon stage (t) and each row represents an expansion candidate corridor (k). The integer variable $v_{k,t}$ defines the number of lines added to the candidate corridor k at stage t. N_T is the total number of horizon stages and N_C is the total number of candidate corridors.

$$Ind = \begin{bmatrix} v_{1,1} & v_{1,2} & \cdots & v_{1,N_T} \\ v_{2,1} & v_{2,2} & \cdots & v_{2,N_T} \\ \vdots & \vdots & \ddots & \vdots \\ v_{N_C,1} & v_{N_C,2} & \cdots & v_{N_C,N_T} \end{bmatrix} \tag{14}$$

3.1.2. Adaptation for the Discrete Domain

The GWO and WOA metaheuristics were initially proposed to operate inherently in continuous search spaces. Applying these algorithms in the discrete domain of MS-TNEP requires mapping and repairing the decision variables. For this purpose, a strategy is proposed to adapt them to the MS-TNEP. In this strategy, the population of these metaheuristics evolves mathematically in the continuous domain with respect to the structures of their mathematical operators. Immediately before evaluating the fitness objective function, the continuous matrix of each individual is subjected to the procedure consisting of the following two steps.

1. **Discretization:** Continuous matrix values are rounded to the nearest integer, applying a lower bound of zero to prevent negative investment decisions.
2. **Physical Constraint Repair:** The method verifies the feasibility of expansion per corridor. If the sum of lines built in a corridor over the time horizon exceeds the maximum limit of parallel circuits allowed for that branch, the algorithm iteratively subtracts one unit from a randomly drawn stage among those with active constructions. The process is repeated until the maximum corridor limit is respected.

This repaired individual matrix constitutes the feasible expansion plan evaluated by the operation subproblem. To ensure the fluidity and coherence of the metaheuristic's spatial search, the continuous-valued matrix of the individual is carried over across generations, while the repaired discrete matrix is used exclusively for fitness evaluation and leader selection. Although the GA possesses a structure that naturally accommodates discrete search spaces, the physical constraint repair step is still applied to it.

3.2. *Operation Subproblem, Generation Expansion Scenarios, and N-1 Security*

The operation subproblem receives the investment matrix from the master level and updates the network topology for each stage t. Under this topology, a multiscenario DC Optimal Power Flow (OPF) problem is formulated. The set of evaluated scenarios simultaneously incorporates spatial trajectories for the growth of both wind and conventional power generation capacity, alongside the N-1 security criterion.

Specifically, this methodology considers multiple probabilistic spatial scenarios of generation expansion over a 20-year horizon, discretized into multiple time stages. These prospective generation expansion plans can be obtained *a priori* through dedicated generation expansion planning studies.

The final planning solution must be feasible for all scenarios of load demand, generation availability, and N-1 contingencies.

To integrate electrical transmission losses into the linear model, the fictitious nodal injections method is adopted through an iterative DC-OPF executed for each scenario c.

1. **Initialization:** The DC-OPF is solved in the first iteration, disregarding the active network losses.
2. **Loss Calculation:** With the nodal voltage angles (θ) obtained in the current iteration, the active loss in each circuit is calculated by $P_{losses,ij} = g_{ij}(\theta_i - \theta_j)^2$, where g_{ij} is the line conductance.
3. **Nodal Balance Update:** The active loss of each circuit is divided equally and inserted as an additional fictitious load on the sending bus i and the receiving bus j.
4. **Convergence:** The DC-OPF is executed again with the updated demand vector. The iterative process ends when the variation in total system losses between consecutive iterations is less than a predefined tolerance.

The mathematical feasibility of the nodal balance against congestion and severe scenarios is guaranteed by inserting slack variables into the model. Load shedding is represented by fictitious generators located in load buses, while renewable generation curtailment is modeled by fictitious demands at buses with installed renewable capacity.

The N-1 contingency is evaluated by zeroing the transmission capacity limit and the susceptance of the respective failed line in the analyzed scenario. If flow reconfiguration requires the dispatch of slack variables, its occurrence and volume of penalties are recorded.

The fitness is calculated by the sum of the investment cost of the proposed grid with the total expected operating cost, which includes conventional dispatch and the load shedding volume weighted by the probability of occurrence of all scenarios $c \in \mathcal{K}$.

3.3. Pack-Based Grey Wolf Optimizer (PBGWO)

The standard Grey Wolf Optimizer establishes a single social hierarchy in which the entire population follows a set of global leaders, as illustrated in Figure 1. This centralized mathematical approach can lead to premature convergence when evaluating highly non-convex and multimodal search spaces such as the multi-Stage TNEP. To overcome this limitation and enhance the global search capability, this work proposes a modification named Pack-Based Grey Wolf Optimizer.

The PBGWO introduces a parallel exploration mechanism by dividing the population into multiple independent packs. This structural change allows the algorithm to explore different regions of the discrete search space simultaneously. The methodology incorporates three main operational stages into the standard evolutionary cycle.

1. **Multi-Stage Population Clustering:** At the beginning of each generation, the continuous population is partitioned into distinct packs using the K-means clustering algorithm. The number of packs is defined proportionally to the population size. If spatial diversity is insufficient for the K-means convergence, the algorithm automatically applies a uniform random distribution as a safety fallback to maintain the structures of the packet.
2. **Local Leadership and Movement:** The fitness of all individuals is evaluated to identify the Alpha, Beta, and Delta leaders exclusively within each specific pack. The position update of each wolf is then calculated based only on the leaders of its respective pack rather than the global leaders. This creates multiple parallel search fronts. If a pack temporarily lacks a local leader, the global best solution guides its members to ensure continuous evolution.
3. **Mutation Operator:** To preserve genetic diversity and prevent stagnation within isolated packs, a mutation mechanism is applied. A predefined percentage of the population is randomly selected at each generation. For these individuals, a fraction of their matrix decision variables undergoes random perturbation while strictly respecting the maximum upper bounds of the candidate corridors.

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After the movement and mutation stages, the new continuous positions generated by the PBGWO undergo the same formatting procedure detailed in Section 3.1.2. The continuous values are discretized, and the physical constraints of the corridors are fixed. This ensures that all expansion plans evaluated remain physically feasible before being submitted to the operational subproblem. The complete flowchart of the proposed PBGWO is presented in Figure 2.

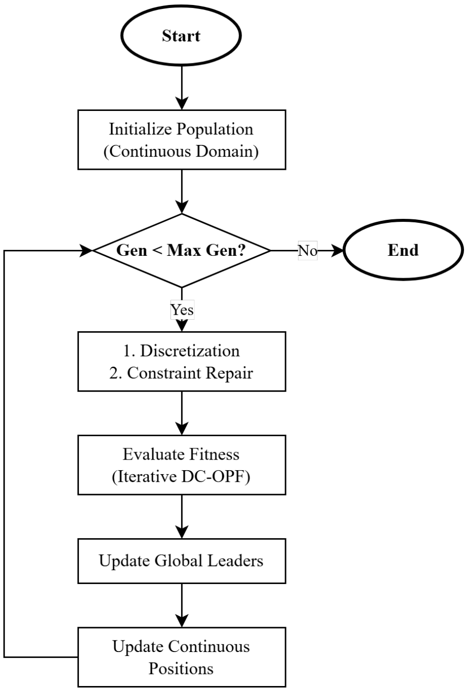

Figure 1. Flowchart of the standard continuous metaheuristics adapted for the discrete MS-TNEP.

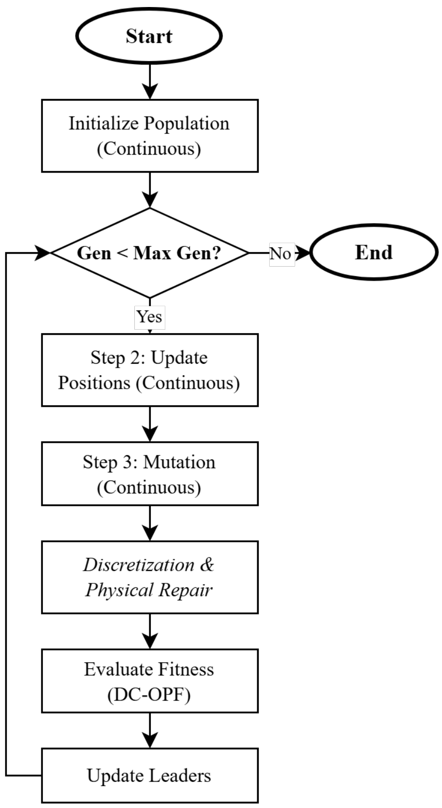

Figure 2. Flowchart of the proposed Pack-Based Grey Wolf Optimizer (PBGWO).

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4. Case Studies and Results

In this section, the performance of the optimization methodology is evaluated in two test systems. These are the 6-bus Garver system and a 46-bus Southern-Brazilian equivalent system. To demonstrate the effectiveness and robustness of the model, the simulations in each system are divided into four progressive case studies. For each test system, two case studies (Case I and Case II) are conducted to evaluate the system under normal operation and under N-1 security criterion, respectively.

All computational simulations were conducted on a computer with an Intel Core *i7* processor (2.7 GHz), and the algorithms were implemented in the MATLAB® software. Due to the difference in topological dimensionality and combinatorial complexity of the problems, the search parameters were adjusted according to the system. The algorithmic parameters were configured with a population of 100 individuals for both cases, adopting a stopping criterion of 200 generations for the Garver system and 500 generations for the Southern-Brazilian system. To attest to robustness and allow statistical analysis, 10 independent simulations were performed for each metaheuristic.

Regarding the Genetic Algorithm (GA) operating as a methodological reference in this work, the population evolution is based on the traditional operators of selection, crossover, mutation, and elitism. Specifically, the algorithm was parameterized with an 80% crossover rate, a fixed 1% mutation rate in genes (matrix decision variables), and 10% elitism. The selective pressure operated via the tournament method.

4.1. Garver System

The original Garver system, proposed in [23], consists of 6 buses, 6 existing circuits in the base topology, and 15 candidate expansion corridors, allowing a maximum installation of five parallel circuits per branch. The Garver system is adapted in this article to evaluate multi-Stage planning under uncertainty of expansion of the generation capacity. The time horizon is extended to 20 years, discretized into 10-year intervals. The model incorporates the expansion of wind farms mapped into four equiprobable prospective scenarios ($\sigma_c = 0.25$). Additionally, the demand is scaled over time, starting from 760 MW in year 0 and reaching 1,520 MW in year 20. Table 2 details the expansion schedule in cumulative megawatts for each probabilistic scenario.

Table 2. Garver System Generation Capacity Additions per Scenario in MW.

Scenario	Bus	Technology	Year 10 Addition	Year 20 Addition
1	2	Wind	100	100
	3	Conventional	0	250
	5	Wind	100	100
2	3	Conventional	0	250
	4	Wind	100	300
3	1	Conventional	50	0
	2	Wind	50	50
	3	Conventional	50	250
	4	Wind	50	50
	5	Wind	50	50
4	1	Conventional	100	100
	3	Conventional	100	350

Tables 3 and 4 detail the step-wise construction schedules representing the best investment plans obtained by each metaheuristic for Case I and Case II, respectively. In Case I, both PBGWO and GWO successfully converged to the identical optimal route, achieving an investment cost of US$ 211.63 million. Similarly, in Case II, both algorithms identified the same best-found plan, corresponding to US$ 234.88 million. The impact of the N-1 contingency analysis is clearly noticeable, driving a proportional increase in the expansion requirements to ensure systemic reliability under failure states.

Table 3. Garver System Best Construction Schedule per Metaheuristic for Case I.

Algorithm	Inv. (NPV / Base) MM US$	Detailed Expansion Plan [Corridor (Circuits)]
PBGWO & GWO	**211.63** / 290.00	**Year 0:** L2-6 (1), L3-5 (2), L4-6 (2). **Year 10:** L2-6 (3). **Year 20:** L3-5 (2), L4-6 (1).
GA	**247.23** / 353.00	**Year 0:** L2-3 (1), L2-6 (1), L3-5 (1), L4-6 (2). **Year 10:** L2-6 (3), L3-5 (1), L4-6 (1). **Year 20:** L3-5 (1), L4-5 (1).
WOA	**298.92** / 380.00	**Year 0:** L2-3 (3), L3-5 (1), L4-6 (3). **Year 10:** L2-6 (4), L3-5 (3), L4-6 (1).

Table 4. Garver System Best Construction Schedule per Metaheuristic for Case II Security N-1.

Algorithm	Inv. (NPV / Base) MM US$	Detailed Expansion Plan [Corridor (Circuits)]
PBGWO & GWO	**234.88** / 310.00	**Year 0:** L2-6 (2), L3-5 (1), L4-6 (2). **Year 10:** L2-6 (2), L3-5 (2), L4-6 (1). **Year 20:** L2-3 (1), L3-5 (1).
GA	**267.23** / 373.00	**Year 0:** L1-5 (1), L2-3 (1), L2-6 (1), L3-5 (1), L4-6 (2). **Year 10:** L2-6 (3), L3-5 (1), L4-6 (1). **Year 20:** L3-5 (1), L4-5 (1).
WOA	**365.78** / 443.00	**Year 0:** L2-6 (3), L4-5 (1), L4-6 (3). **Year 10:** L2-3 (1), L3-4 (1), L3-5 (3), L5-6 (1).

Following the assessment of the optimal routings, Table 5 presents the comprehensive statistical performance of the metaheuristics in solving the Garver system under both operational conditions.

Table 5. Comparative Statistical Performance of Metaheuristics for the Garver System.

Algorithm	Case I Multiscenario (N-0)			Case II Multiscenario (N-1)		
	Best (MM US$)	Average (MM US$)	Std. Dev.	Best (MM US$)	Average (MM US$)	Std. Dev.
PBGWO	**211.63**	**216.47**	**6.68**	**234.88**	**237.25**	**6.58**
GWO	**211.63**	243.15	36.41	**234.88**	256.38	27.13
GA	247.23	356.05	122.11	267.23	366.49	87.50
WOA	294.18	480.97	158.17	365.78	705.78	289.02

4.2. Southern Brazilian System

The second test system represents an equivalent of the National Interconnected System (SIN) of the southern region of Brazil. This large-scale topology comprises 46 buses, 62 circuits in the base network, and 79 candidate corridors, allowing the installation of up to two parallel circuits per branch. The inherent complexity and high dimensionality of this network make it a suitable benchmark to evaluate the effectiveness and computational scalability of the proposed methodologies.

Similar to the previous system, the southern equivalent was adapted to assess multi-Stage planning under uncertainty over generation capacity expansion. The time horizon spans 20 years, discretized into 10-year intervals. The model incorporates a massive expansion of wind farms mapped into four equiprobable prospective scenarios ($\sigma_c = 0.25$). Additionally, the total demand continuously increases over the planning horizon, starting from a base load of 2,059.47 MW at the initial stage (year 0), scaling to 4,118.94 MW in year 10, and culminating at 6,864.90 MW in year 20. The detailed step-wise increase in generation capacity for both conventional and wind sources across the different scenarios is presented through dedicated tables in the results section.

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Case II (N-1 Security) introduces the most complex formulation in this study, forcing the algorithm to propose an expansion schedule capable of withstanding severe failures in the critical set of lines {19-21, 32-43, 19-32, 46-19, and 46-16}, following a methodology similar to [12]. Table 6 details the cumulative schedule of generation capacity expansion in megawatts for each formulated probabilistic scenario.

Table 6. Southern-Brazilian System Generation Capacity Additions per Scenario in MW.

Scenario	Bus	Technology	Year 10 Addition	Year 20 Addition
1	12	Wind	200	200
	24	Wind	200	200
	43	Wind	200	600
2	2	Wind	100	100
	12	Wind	200	200
	20	Wind	100	100
	24	Wind	200	200
	33	Wind	100	100
	42	Wind	200	200
	43	Wind	300	300
3	12	Wind	200	200
	24	Wind	200	200
	41	Wind	400	400
	43	Wind	500	700
	45	Wind	200	200
4	6	Wind	300	300
	11	Wind	200	200
	12	Wind	200	200
	24	Wind	200	200
	25	Wind	200	200
	43	Wind	300	300

Tables 7 and 8 detail the step-wise construction schedules proposed by the metaheuristics for Case I and Case II, respectively. Given the vast search space of the 46-bus topology, each algorithm converged to unique spatial configurations, highlighting the multimodal nature of the multi-Stage transmission expansion planning problem.

Table 7. Southern-Brazilian System Best Construction Schedule per Metaheuristic for Case I.

Algorithm	Inv. (NPV / Base) MM US$	Detailed Expansion Plan [Corridor (Circuits)]
PBGWO	**19.66** / 80.71	**Year 10:** L5-6 (1), L20-21 (1), L46-6 (1). **Year 20:** L5-6 (1), L5-8 (1), L13-20 (1), L14-22 (1), L18-20 (1), L22-26 (1), L42-43 (1).
GWO	**22.43** / 86.31	**Year 10:** L5-6 (2), L20-21 (1), L46-6 (1). **Year 20:** L5-8 (1), L14-26 (1), L18-20 (1), L36-37 (1), L42-43 (1).
GA	**23.03** / 96.12	**Year 10:** L5-6 (1), L18-20 (1), L46-6 (1). **Year 20:** L2-3 (1), L14-26 (1), L20-21 (1), L42-43 (1), L46-3 (1).
WOA	**73.95** / 204.82	**Year 0:** L20-21 (1), L42-44 (1), L46-3 (1). **Year 10:** L2-3 (1), L4-9 (1), L14-15 (1), L26-29 (1), L41-43 (1). **Year 20:** L2-3 (1), L5-8 (1), L14-22 (1), L14-26 (1), L18-20 (1), L28-43 (1), L35-38 (1), L40-45 (1), L42-43 (1).

Table 8. Southern-Brazilian System Best Construction Schedule per Metaheuristic for Case II Security N-1.

Algorithm	Inv. (NPV / Base) MM US$	Detailed Expansion Plan [Corridor (Circuits)]
GA	**61.73** / 241.01	**Year 0:** L24-34 (1). **Year 10:** L5-6 (1), L19-21 (1), L20-21 (1), L40-42 (1), L46-6 (1). **Year 20:** L5-6 (1), L5-8 (1), L13-18 (1), L18-19 (1), L18-20 (1), L28-31 (1), L31-41 (1), L32-43 (1), L37-39 (1), L40-41 (1), L42-43 (1), L46-6 (1).
PBGWO	**61.75** / 239.86	**Year 0:** L24-34 (1). **Year 10:** L5-6 (1), L19-21 (1), L20-21 (1), L31-32 (1), L46-6 (1). **Year 20:** L2-3 (1), L13-18 (1), L13-20 (1), L18-19 (1), L18-20 (1), L28-41 (1), L32-43 (1), L40-41 (1), L42-43 (1), L46-3 (1).
GWO	**76.08** / 217.28	**Year 0:** L18-20 (1), L24-34 (1). **Year 10:** L5-6 (1), L19-21 (1), L20-21 (1), L32-43 (1), L46-6 (1). **Year 20:** L2-3 (1), L13-18 (1), L31-41 (1), L40-41 (1), L42-43 (1), L46-3 (1).
WOA	**158.93** / 662.73	**Year 0:** L13-20 (1), L14-15 (1), L19-21 (1). **Year 10:** L5-6 (1), L17-19 (1), L18-19 (1), L20-21 (1), L26-29 (1), L28-41 (1), L29-30 (1), L31-32 (1), L41-43 (1), L46-6 (1). **Year 20:** L2-3 (1), L4-9 (1), L4-11 (1), L5-9 (1), L8-13 (1), L13-18 (1), L14-22 (1), L14-26 (1), L16-17 (1), L16-28 (1), L18-20 (1), L19-25 (1), L21-25 (1), L24-25 (1), L24-34 (1), L27-29 (1), L27-36 (1), L28-30 (1), L28-31 (1), L28-43 (1), L31-32 (1), L31-41 (1), L36-37 (1), L37-42 (1), L40-41 (1), L40-42 (1), L41-43 (1), L42-43 (1), L42-44 (1), L44-45 (1), L46-3 (1), L46-11 (1).

Following the presentation of the optimal routing, Table 9 summarizes the statistical performance of the metaheuristics across both evaluated cases. In Case I (normal operation), the proposed PBGWO demonstrated superior optimization capabilities, finding the best global plan with a Net Present Value (NPV) of US$ 19.66 million. It significantly outperformed GWO (US$ 22.43 million) and GA (US$ 23.03 million). Furthermore, PBGWO exhibited the highest consistency, yielding the lowest standard deviation (2.08) among all algorithms.

In Case II, the enforcement of the strict N-1 security criterion drastically constrained the operative flexibility, forcing a substantial increase in transmission investments to accommodate the massive wind generation scenarios under contingent states. In this highly complex scenario, the GA and PBGWO achieved virtually tied minimum cost plans (US$ 61.73 million and US$ 61.75 million, respectively). However, an analysis of the average performance reveals the robustness of the proposed approach: PBGWO maintained strong statistical consistency, presenting an average cost of US$ 67.98 million and a low standard deviation of 6.92. In contrast, GA exhibited high variance (standard deviation of 19.06) and a significantly worse average cost (US$ 94.23 million). This indicates that while the GA occasionally found a marginal minimum due to search variance, the PBGWO reliably converges to high-quality solutions. The WOA algorithm struggled significantly with the high dimensionality of the problem, presenting the worst overall performance.

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Table 9. Comparative Statistical Performance of Metaheuristics for the Southern-Brazilian System.

Algorithm	Case I Multiscenario (N-0)			Case II Multiscenario (N-1)		
	Best (MM US$)	Average (MM US$)	Std. Dev.	Best (MM US$)	Average (MM US$)	Std. Dev.
PBGWO	**19.66**	**22.76**	**2.08**	61.75	**67.98**	**6.92**
GWO	22.43	26.40	3.16	65.24	87.70	15.90
GA	23.03	28.99	5.80	**61.73**	94.23	19.06
WOA	67.20	134.35	52.87	158.93	232.53	48.86

5. Conclusions

This paper presented a framework for Probabilistic Multi-Stage Transmission Network Expansion Planning (MS-TNEP). The proposed model coordinates long-term transmission investments over a 20-year horizon, incorporating uncertainties about expansion of generation capacity, active power losses, and N-1 security constraints. To manage the computational complexity of this mixed-integer non-linear formulation, a hybrid decomposition strategy was employed, separating the combinatorial investment decisions from the linear operational feasibility checks.

To solve the upper-level investment problem, a Pack-Based Grey Wolf Optimizer (PBGWO) was introduced. Its performance was compared with standard metaheuristics (GWO, GA, and WOA) using modified Garver and 46-bus Southern-Brazilian equivalent systems. Given that metaheuristics cannot ensure convergence to the global optimum, the evaluation focused on the quality of the best-found solutions and the statistical consistency of the algorithms across multiple independent runs.

The computational results indicate that the PBGWO is a competitive approach for high-dimensional MS-TNEP problems. Under normal operating conditions (Case I), the PBGWO identified the lowest-cost expansion schedules and exhibited the lowest standard deviation among the algorithms tested. Under the strict N-1 security criterion (Case II), the complexity of the search space increased significantly, restricting operational flexibility. In this scenario, while GA and PBGWO achieved virtually tied minimum cost plans, the PBGWO demonstrated greater statistical reliability. It maintained a lower average execution cost and a reduced standard deviation, indicating a consistent convergence toward high-quality, feasible solutions.

In summary, the integration of the hybrid decomposition method with the PBGWO provides a scalable approach for multi-Stage grid planning, accommodating generation uncertainties and security constraints. Future research will focus on integrating Energy Storage Systems (ESS) to defer transmission investments and employing full AC power flow models to assess reactive power support and voltage stability within the multi-Stage planning horizon.

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