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Article

LOCAL VERSUS GLOBAL TIME IN EARLY RELATIVITY THEORY

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Abstract: In his groundbreaking 1905 paper on special relativity, Einstein distinguished between local and global time in inertial systems, introducing his famous definition of distant simultaneity to give physical content to the notion of global time. Over the following decade, Einstein attempted to generalize this analysis of relativistic time to include accelerated frames of reference, which, according to the principle of equivalence, should also account for time in the presence of gravity. Characteristically, Einstein's early methodology focused on simple, intuitively accessible physical situations, often exhibiting a high degree of symmetry. In this paper, we show that while Einstein's heuristic approach led to significant insights into the nature of relativistic time, it also posed challenges for him in making the final conceptual leap to global time as a conventionally chosen fourth coordinate. We also address some misunderstandings concerning Einstein's early work on time and the gravitational redshift.

Keywords: Time and gravity; Coordinate time and proper time; Gravitational redshift

1. Introduction

In his 1905 paper introducing the special theory of relativity Einstein begins his explanation of the new kinematics with an extensive discussion of the concept of time [6] (§1). As he emphasizes, a mathematical description of motion lacks physical meaning unless we clearly understand what we mean by "time" in concrete cases. An obvious way of addressing this is to link the concept of time to data provided by clocks.

However, if we look at one clock at position A and another identical clock at position B, both stationary in an inertial frame, these only provide us with the *local times* at A and B, respectively. But to conduct physics, we surely should be able to describe the temporal evolution of processes extending from A to B. This requires more than local considerations: we should be able to compare locally defined quantities. This requires a common, *global time* and a notion of distant simultaneity. At this point, Einstein introduces his famous simultaneity criterion: clocks at A and B are synchronous if, according to their indications, the time taken by light to travel from A to B equals the time needed to travel back from B to A.

Two key points stand out in Einstein's approach. First, his discussion focuses on a simple and easily imaginable scenario, using familiar concepts. Second, there is an implicit appeal to symmetry: it is assumed that a stationary system (inertial system) exists with homogeneous, isotropic, and time-invariant spatial properties. The notion of global time introduced in this manner reflects the global symmetries of spacetime and derives its physical significance from them.

After 1905, Einstein began working on generalizing relativity theory to include accelerated systems and, through the principle of equivalence, the presence of a gravitational field [7,8]. In these papers, he used the same strategy as before: arguing from thought experiments in simple physical scenarios involving global symmetries to reach general conclusions. In 1907, the principle of equivalence makes its first appearance, as the hypothesis that the effects of gravity on any physical process can be understood as the results of describing such processes from a reference frame accelerated relative to an inertial system. Accordingly, it becomes crucial to generalize the 1905 kinematical considerations to accelerated frames of reference. As it turns out, local and global notions of time can again be introduced in such accelerated frames, by a procedure that is similar to the one used in the 1905 paper. However, there is one important difference: in an accelerated frame, clocks indicating local time do not agree with clocks showing global time. In other words, local and global clocks tick at different rates.

In his 1911 paper, Einstein revisits this issue and rederives his 1907 results by a much simpler argument. Using the difference between local and global time, he makes a new, empirically testable prediction: starlight will be deflected by heavy celestial bodies, a result for which the 1911 paper is still famous.

Clearly, in these papers Einstein did not introduce global time as an arbitrary fourth coordinate. On the contrary, he often referred to his global time as *the real time* of a reference system, so that local timekeeping devices lagging behind clocks indicating global time could be said to *truly* go slow. This perspective even appears in his final 1916 review paper [9] on the general theory of relativity.

Modern treatments of general relativity emphasize that there are no *a priori* preferred frames or *a priori* given global spatiotemporal symmetries [1]; general relativity is a “background-independent” theory. This background independence means that Einstein’s physically significant global clock time cannot be generally expected to exist. From a modern standpoint, it is sufficient to have a consistent set of global coordinates for a global spacetime description, with coordinates that do not require a direct physical interpretation. However, as Einstein tells us in his Autobiographical Notes [15] (pp. 66–67), he struggled to accept the idea of coordinates not corresponding to directly measurable distances and time intervals. The present historical account of Einstein’s early work on relativity aims to make his reluctance on this point more understandable. Additionally, this paper seeks to shed light on the details of Einstein’s treatment of local versus global time, addressing several misunderstandings in the literature. Finally, considering the development of Einstein’s thoughts on time in relativity theory may further clarify the relativistic conception of time¹.

2. From 1905 to 1907

In his 1905 paper [11] (p. 38) Einstein notes that the description of the motion of a material point requires the specification of the values of its coordinates as a function of time, and that such a description can only have physical significance if “we are quite clear as to what we understand by ‘time’.”

To achieve this clarity Einstein considers clocks constructed in such a way that they indicate the familiar time of classical mechanics. That means that a material particle moving very close to a clock, so that its position can be coordinated without problems to that clock’s indications, should obey Newton’s equations of motion (in first approximation). As pointed out in the Introduction, we have thus gained access to a local time, defined at the position of each clock. In order to compare times indicated by different clocks, located at different positions, we need a notion of distant simultaneity. This notion is supplied by Einstein’s famous definition [6] (p. 894) [11] (p. 40): “Let a ray of light start at the ‘A time’ t_A from A towards B, let it at the ‘B time’ t_B be reflected at B in the direction of A, and arrive again at A at the ‘A time’ t'_A . By definition, the two clocks synchronize if $t_B - t_A = t'_A - t_B$.”

With this simultaneity definition in place, a global time can be defined, relative to any inertial system. Indeed, we could place clocks throughout the inertial system in question, and synchronize them all with one chosen standard clock. Statements about what time it is then acquire a global meaning. For example, “it is twelve o’clock” will now refer to the infinite set of events where all synchronized local clocks indicate the same local time, twelve o’clock.

The global symmetries of special relativistic inertial reference frames (due to the flatness of special relativistic spacetime) guarantee that synchronized clocks remain in synchrony over time. Therefore, a set of synchronized local clocks, stationary in an inertial system, is also a set of global clocks in that system: local and global clocks tick at the same rate. When Einstein, with the equivalence principle in mind, began investigating the behavior of clocks in accelerated frames, he found out that this simple connection between local and global time is lost.

¹ The present paper improves on an earlier preprint concerning the same subject [2].

In 1907, Einstein wrote a review paper titled "On the Relativity Principle and the Conclusions Drawn From It" [7]. In the final part of this review, he posed the question of whether the relativity principle could be generalized to include accelerated frames of reference. To explore this, Einstein compared two reference frames: an inertial system Σ_2 with a homogeneous gravitational field causing a free-fall acceleration γ along the negative X -axis, and another frame Σ_1 uniformly accelerating in empty space with an acceleration γ along the positive X -axis. Einstein noted that for mechanical processes, the behaviors observed in Σ_1 do not differ from those in Σ_2 : all bodies experience equal acceleration in a gravitational field, so the motions in Σ_2 are identical to those in Σ_1 .

Based on this observation, Einstein proposed that *all* physical processes occur in exactly the same way in Σ_1 and Σ_2 . Thus, an inertial system with a gravitational field can be considered equivalent to an accelerated frame of reference without a gravitational field. This marked the first appearance of the celebrated principle of equivalence in the literature. It is important to stress that this principle is a hypothesis extending beyond the mechanical evidence of the equality of gravitational and inertial mass. It is applied to all physical phenomena, including electromagnetic phenomena like the propagation of light.

The equivalence principle allows predictions about what will happen in a gravitational field by considering the corresponding accelerated frame, where there is no gravitational field. This problem can be addressed using the special theory of relativity. In the 1907 paper, Einstein therefore examined the spatiotemporal relations between an inertial system S and a system Σ accelerating along the X -axis of S with constant acceleration γ . Both systems are equipped with measuring rods and clocks of identical construction. At S -time $t = 0$ the frames S and Σ coincide, with vanishing instantaneous mutual velocity. At that moment, every clock in Σ is synchronized to show the same time as the clock in S with which it momentarily coincides. The time shown by the local clocks in Σ after this initial synchronization with S defines the "local time" σ of Σ . As Einstein notes, in terms of this local time local descriptions of physical processes will be the same throughout system Σ . This follows from special relativity: acceleration does not systematically affect lengths and times, so local descriptions match those from local inertial systems that are instantaneously comoving with each clock. For example, the standard value c for the speed of light will always be found when measured locally.

However, Einstein warned that we should not regard σ as "the time" (as he calls it) of system Σ because the accelerating local clocks of Σ will fall out of sync with each other (according to the 1905 criterion). As seen from S , all clocks in Σ undergo exactly the same accelerated motion, meaning that at any instant of S -time all Σ -clocks will have ticked the same amount of time. Therefore, they will remain synchronized from S 's perspective. However, this synchronization will not hold when viewed from an inertial system S' that is instantaneously at rest relative to Σ at any S -time $t > 0$: at such an instant, Σ , and therefore also S' , will have a non-zero velocity relative to S . Consequently, the simultaneity relations in S and S' must differ, as dictated by special relativity.

In addition to the local times σ Einstein therefore introduces a "global time" in Σ , which is essentially the time of the instantaneously comoving inertial system S' . Specifically, the global time τ of an event in Σ is the time indicated by a clock at the origin of Σ at the moment that is simultaneous with the event, according to the simultaneity of the instantaneously comoving inertial system S' . Because simultaneity differs between S and S' , the local times σ and the just-defined global time τ in Σ are not the same.

The quantitative relation between σ and τ can easily be determined. Two events that take place at positions x_1 and x_2 and times t_1 and t_2 of S , respectively, are simultaneous with respect to S' if $t_1 - x_1 v / c^2 = t_2 - x_2 v / c^2$, where v is the speed of S' with respect to S . If the time difference with $t = 0$ is small, and if the velocity v is small as well, we have in first approximation $x_2 - x_1 = x'_2 - x'_1 = \xi_2 - \xi_1$, with ξ the coordinate in system Σ along the common X -axis. Moreover, we have in this approximation $t_1 = \sigma_1$, $t_2 = \sigma_2$, and $v = \gamma\tau$.

It follows that $\sigma_2 - \sigma_1 \approx (\xi_2 - \xi_1)(\gamma\tau)/c^2$. When we take the origin of the coordinate system as the place of the first event, so that $\sigma_1 = \tau$ and $\xi_1 = 0$, we obtain:

$$\sigma = \tau(1 + \frac{\gamma\xi}{c^2}). \quad (1)$$

According to the principle of equivalence, the same equation holds in a system in which there is a homogeneous gravitational field. In that case we can replace $\gamma\xi$ with the gravitational potential Φ , so that we obtain

$$\sigma = \tau(1 + \frac{\Phi}{c^2}). \quad (2)$$

Summing up, there are two timekeeping systems in Σ : a local one and a global one. Both the local time σ and the global time τ can be indicated by a set of clocks. The local time is shown by clocks that are initially synchronized with the clocks in S and then move along with Σ without further adjustments. The global time of Σ is shown by clocks synchronized in an instantaneously comoving frame S' , with the clock at the origin of the comoving frame set to show the same time as the σ -clock at the origin of Σ .

When we are interested in local processes and measurements, it is natural to use the σ -clocks, which run freely without synchronization interventions. However, when describing extended processes, such as the propagation of signals, we need a notion of simultaneity to coordinate events at different positions. In this case, the τ -clocks are very useful because equal τ values imply simultaneity in the standard special relativistic sense.

From Eq. (2), it follows that at global instant τ , a local clock at a position with gravitational potential Φ indicates a time that differs from the time of a local clock at the origin (where $\Phi = 0$) by a factor of $(1 + \Phi/c^2)$. This difference is experimentally significant: a stationary observer in Σ will see or measure that the two clocks tick at different frequencies, differing by exactly this factor. This is because the time $\Delta\tau$ needed for the light to travel to the observer is independent of τ [7] (p. 458)—there is no time dependence in the factor by which τ differs from σ , according to formulas 1 and 2. Consequently, a light signal that has a certain duration during its emission, expressed in τ , will have the same duration, again expressed in τ , when received by the observer. However, the number of pulses in the signal is determined by measuring σ . Thus, signals of the same duration in terms of τ , consisting of light with the same natural frequency as determined via σ , may have different frequencies when received by an observer, because the relation between τ and σ varies with position. This remarkable result leads to the prediction that clocks at a position with a higher gravitational potential will be observed to run faster. As Einstein interpreted it, they *really* run faster than clocks lower in a gravitational field. This statement makes sense if the global time system τ , which allows comparing two local frequencies from a third position, is considered the *real time* of reference frame Σ .

An example of the influence of gravity on the rate of clocks is provided by the behavior of atoms and molecules emitting spectral lines. From the aforementioned argument, it can be concluded that spectral light coming from the surface of the Sun will arrive on Earth with a frequency slightly shifted to the red end of the spectrum.

Another prediction is that light will be bent in a gravitational field [7] (p. 461). The derivation of this effect in the 1907 paper is rather complex. Additionally, the influence of gravity on clocks can be demonstrated in a simpler way than explained above. For these reasons, along with the opportunity for new experimental tests, Einstein returned to this topic in 1911.

3. Gravity, Time and Light in 1911

The principle of equivalence again takes a central place in the 1911 paper [8]. As Einstein shows, a simple thought experiment using the equivalence principle suffices to derive how clocks behave in a gravitational field.

Let there be two bodies A and B in a system of reference K in which there is a homogeneous gravitational field. Both systems, assumed to be very small, are located on the z -axis of K ; the acceleration due to gravity is $-\gamma$ (directed downward in the z -direction). System B is located higher in the field, at a distance h from A , so that the gravitational potential at B is greater than at A , $\Phi(B) - \Phi(A) = \gamma h$. System B emits electromagnetic radiation in the direction of A , and we are interested in how gravity influences this signal traveling from B to A .

The principle of equivalence tells us that we can replace system K with a system K' that possesses a constant acceleration γ in the positive z -direction and in which there is no gravitational field. In order to have a situation that is equivalent to the original one we have to assume that A and B are located at fixed positions on the z' -axis, with constant mutual distance h . Finally, let K_0 be an inertial system that at the moment of the emission of the radiation is instantaneously at rest with respect to K' .

When we describe the process of the emission, propagation and reception of the radiation from system K_0 , B has no velocity relative to K_0 when the radiation is emitted and the radiation takes a time h/c to arrive at A (in first approximation). A possesses the approximate speed $(h\gamma)/c = v$ when the radiation arrives at its position. Now, Einstein notes, if the radiation had the frequency ν_2 when it was emitted by B , as measured by a standard clock positioned at B and comoving with B , the radiation received in A will have a different frequency ν_1 as measured by a clock of the same construction comoving with A . Indeed, when the signal arrives at A , A (and its clock) will possess a speed v relative to K_0 , so that there will be a change in measured frequency on account of the Doppler effect. The relation between ν_1 and ν_2 is given by the Doppler formula

$$\nu_1 = \nu_2 \left(1 + \frac{\gamma h}{c^2}\right). \quad (3)$$

According to the equivalence principle this same result should be valid in system K , where a gravitational field is present. This means that light emitted at a higher value of the gravitational potential will arrive at positions with a lower potential with a higher frequency (as measured by a local clock). Rewriting Eq. (3), we find

$$\nu_1 = \nu_2 \left(1 + \frac{\Phi}{c^2}\right), \quad (4)$$

where Φ is the gravitational potential at B and the value of the potential at A has been set to 0. This is the same gravitational redshift formula as derived in the 1907 paper, Eq. (2).

The 1911 derivation of the redshift formula may seem of a different nature from that in the 1907 paper: in 1907 Einstein emphasized the necessity of a global time, whereas the 1911 derivation appears to involve only local times, measured by clocks of the same kind located at different positions in the gravitational field. However, that impression is deceptive. First, the application of the Doppler formula presupposes that we can compare the frequencies at A and B . In fact, a global time is provided by inertial system K_0 with its standard simultaneity. As explained in section 2, however, the clock that is stationary at A will not agree with this K_0 time when the radiation arrives, because during the transmission process it has obtained a velocity relative to K_0 . Conversely, if we want to describe the process from the viewpoint of a stationary observer in K , it is natural to introduce a global time via the standard synchronization procedure in K . This is essentially the same procedure as used in section 2. Compared to this global time in K , local clocks that are stationary in K will be out of step, and formula (2) applies.

Einstein discusses the situation as follows [8, pp. 905–906; pp. 105–106 in the English translation]:

On superficial consideration equation (4) seems to assert an absurdity. If there is constant transmission of light from B to A , how can any other number of periods per second arrive at A than is emitted from B ? But the answer is simple. We cannot regard ν_2 or respectively ν_1 simply as frequencies (as the number of periods per second) since we have not yet determined a time in system K . What ν_2 denotes is the number of periods per second with reference to the time-unit of the clock U at B , while ν_1 denotes the number of periods per second

with reference to the identical clock at A . Nothing compels us to assume that the clocks U in different gravitation potentials must be regarded as going at the same rate. On the contrary, we must certainly define the time in K in such a way that the number of wave crests and troughs between B and A is independent of the absolute value of time: for the process under observation is by nature a stationary one. ... Therefore the two clocks at A and B do not both give the "time" correctly. If we measure time at A with the clock U , then we must measure time at B with a clock which goes $1 + \Phi/c^2$ times more slowly than the clock U when compared with U at one at the same place. For when measured by such a clock, the frequency of the light-ray which is considered above is at its emission from B given by $\nu_2(1 + \Phi/c^2)$, and is therefore, by (4), equal to the frequency ν_1 of the same light-ray on its arrival at A .

Einstein here explicitly introduces a global time that differs from the indications of local clocks. This global time corresponds to the time τ defined in the 1907 article, while the clocks at A and B correspond to the local times σ . As in the 1905 and 1907 articles, both the local and global times are assumed to be directly measured by sets of clocks. In the 1907 article this material implementation of global time was realized by standard clocks in instantaneously comoving inertial systems, whereas in the 1911 paper the clocks indicating global time are introduced directly, via the rule that they be constructed such that they tick $1 + \Phi/c^2$ times more slowly than local clocks at the same location.

In the Collected Papers of Einstein, Volume 3, the editors comment that Einstein's train of thought in the 1911 paper is quite different from the one in 1907 [12] (p. 497). In particular, they claim that the "corrected" clocks just mentioned in the quoted passage of the 1911 paper do not figure in the 1907 article. This is incorrect: the corrected local clocks indicate global time, and as we have seen in section 2 this concept of global time played a major role in the 1907 paper. The editors also refer to Pais' biography of Einstein [14] (pp. 198–199) for a more extensive and deeper analysis. However, in the indicated passage Pais states that local clocks cannot be assumed to run at the same rates, but doesn't mention Einstein's correction procedure for calculating the "real", i.e. global time from the indications of local clocks.

The 1911 paper ends with a calculation of the bending of light in a gravitational field, and is most famous for this prediction. This calculation is based on the observation that the velocity of light, *measured in global time*, will not be constant but will vary with the gravitational potential according to

$$c = c_0(1 + \frac{\Phi}{c^2}), \quad (5)$$

where c_0 is the value at the origin (where $\Phi = 0$). Huygens's principle, applied to this situation with a variable speed of light, implies that light rays will be deflected in the direction in which the gravitational potential decreases. For a ray grazing the Sun, Einstein finds a deflection of $0.83''$, and comments [11] (p. 108) that "it would be a most desirable thing if astronomers would take up the question here raised."²

4. The Final Theory of General Relativity, 1916

In the first part of his 1916 overview of the just-finished general theory of relativity [9], Einstein paid a great deal of attention to the conceptual foundations of his new theory. To make clear that non-Euclidean spatial relations must be expected in the presence of gravitational fields, and that time will have unusual properties too, Einstein discusses the example of a frame K' that rotates (and

² The value found by Einstein in 1911 reflects the influence of gravity on time, but does not take into account that gravity also deforms the spatial geometry. The full general theory of relativity predicts a value that is twice the value predicted by the 1911 considerations.

therefore accelerates) with respect to an inertial frame K [pp. 115–116][11]. Concerning time in the rotating frame he writes, after having discussed the impossibility of retaining Euclidean geometry³:

Neither can we introduce a time in K' that meets the physical requirements if this time is to be indicated by clocks of identical construction at rest relatively to K' . To see this, let us imagine two such identical clocks, placed one at the origin of the coordinates and the other at the circumference of the circle and both considered from the “stationary” frame K . By a familiar result of the special theory of relativity, the clock at the circumference—judged from K —goes more slowly than the other, because the former is in motion and the other at rest. An observer at the common origin of coordinates, capable of seeing the clock at the circumference by means of light, would therefore see it lagging behind the clock beside him. As he will not make up his mind to let the velocity of light along the path in question depend explicitly on the time, he will interpret his observations as showing that the clock at the circumference “really” goes more slowly than the clock at the origin. So he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be.

There is a striking similarity here to Einstein’s reasoning in the 1907 and 1911 articles. The idea is again that local clocks moving along with an accelerated system, and therefore also clocks stationary in a system in which there is a gravitational field, will indicate *bonafide* local times—but local times at different positions will not combine into one physically acceptable global time. In the considered scenarios—uniform linear acceleration and uniform rotation—we should, according to Einstein, require from a physically reasonable global time that in its terms physical laws, in particular the law governing the propagation of light, will not depend explicitly on time. In the case at hand, namely a system rotating relatively to an inertial system, the observer at the origin will receive light pulses at his local times $n(1 + \epsilon) + r/c$, with $n = 1, 2, 3, \dots$; r the distance between the origin and the distant clock, c the standard value of the velocity of light, and with ϵ representing the time dilation due to the distant clock’s speed with respect to K . If our observer insists that the distant clock in his system K' is completely similar to his own clock and therefore does not lag behind, so emits light pulses at times n , he is forced to accept that the speed of light steadily decreases since the light pulses arrive later and later than expected: $c(n) = (rc)/(r + n\epsilon c)$, with $c(n)$ the velocity with which the n -th pulse travels. This is an unpalatable conclusion, Einstein tells us, and the only way out is to accept that the distant clock objectively runs slow.

It is important to note that if the speed of light is independent of global time, electromagnetic signals “transport global time intervals”. Indeed, if a source and receiver are both stationary with respect to a reference system K' , the global time interval between two emission events will equal the global time between the corresponding reception events. Therefore, an observer is able to directly “see” the global time intervals between distant emission events, and can therefore check whether distant processes go slower or faster (with respect to global time) than similar processes at his own position. In this way he will also be able to verify that distant clocks “really” tick slower or faster than his own clock.

In the concrete examples discussed by Einstein, both the local and the global time are associated with sets of clocks. However, in the formal part of the 1916 paper things become more abstract and general. In particular, the restriction to homogeneous gravitational fields (or special symmetric cases like fields corresponding to uniform rotation) is explicitly dropped. Consequently, in general no physically privileged global frames exist that can be naturally associated with global clocks. Nevertheless, there are clear traces of Einstein’s earlier treatment even when he is dealing with the final version of general relativity. The discussion of the gravitational redshift at the end of the 1916 review paper is a case in point.

³ Translation following [11], with minor adjustments.

The last section of the 1916 review, entitled “Behaviour of Rods and Clocks in the Static Gravitational Field. Bending of Light-rays. Motion of the Perihelion of a Planetary orbit”, is devoted to a number of concrete problems [11, pp. 160–164]. The influence of gravity on clocks and the gravitational redshift are now dealt with very quickly. For a unit clock that is at rest in a static gravitational field we have for one clock period $ds = 1$ and $dx_1 = dx_2 = dx_3 = 0$. Therefore, $g_{44}dx_4^2 = 1$, so that $dx_4 = 1/\sqrt{g_{44}}$. If there is a point mass with mass M at the origin of coordinates, the general relativistic field equations tell us that in first approximation $g_{44} = 1 - \kappa M/4\pi r$, with κ the gravitational coupling constant appearing in these equations ($\kappa = 8\pi G/c^2$, with G Newton’s constant) and r the radial spatial distance from the point mass. Therefore,

$$dx_4 \approx 1 + \frac{\kappa M}{8\pi r}. \quad (6)$$

Einstein concludes [11] (p. 162):

Thus the clock goes more slowly if set up in the neighbourhood of ponderable masses. From this it follows that the spectral lines of light reaching us from the surface of large stars must appear displaced towards the red end of the spectrum.⁴

Einstein’s reasoning here is the same as in his earlier discussions of the gravitational redshift, as we shall further explain in a moment. However, without this historical context misunderstandings can easily arise.

5. Assessment of the Early Redshift Derivations

Einstein’s 1916 derivation of the gravitational redshift soon became standard—it can still be found in present-day textbooks. An important role in making it widely known and popular was played by the work of Eddington. Eddington was the first to make the general theory of relativity known in the English-speaking world, and his seminal publications (first of all [4,5]) were widely read. Of the two just-mentioned publications especially the less technical *Space, Time and Gravitation* was very influential. In this book Eddington discusses the comparison of frequencies emitted by atoms of the same kind but located at different positions, e.g. on the Sun and on Earth, respectively. Eddington explains the situation as follows [5] (pp. 128–129) (italics in the original):⁵

Consider an atom momentarily at rest at some point in the solar system... If ds corresponds to one vibration ... we have $ds^2 = g_{44}dt^2$. The *time* of vibration dt is thus $1/\sqrt{g_{44}}$ times the *interval* of vibration ds .

Accordingly, if we have two similar atoms at rest at different points in the system, the interval of vibration will be the same for both; but the time of vibration will be proportional to the inverse square-root of g_{44} , which differs for the two atoms. Since $g_{44} = 1 - \frac{2M}{r}$, $\frac{1}{\sqrt{g_{44}}} = 1 + \frac{M}{r}$, very approximately.

Take an atom at the surface of the Sun, and a similar atom in a terrestrial laboratory. For the first, $1 + M/r = 1.00000212$, and for the second $1 + M/r$ is practically 1. The time of vibration of the solar atom is thus longer in the ratio 1.00000212, and it might be possible to test this by spectroscopic examination.

There is one important point to consider. The spectroscopic examination must take place in the terrestrial laboratory; and we have to test the period of the solar atom by the period of the waves emanating from it when they reach the Earth. Will they carry the period to us

⁴ Einstein subsequently shows that a light-ray grazing the Sun will be deflected by $1.7''$, twice the magnitude of the 1911 prediction, and that the orbits of the planets undergo a slow rotation, which in the case of Mercury will be $43''$ per century.

⁵ For the sake of consistency of notation we use g_{44} where Eddington wrote γ . For the comparison with Einstein’s formulas it should be noted that Eddington uses units in which $\kappa/8\pi = 1$.

unchanged? Clearly they must. The first and second pulse have to travel the same distance r , and they travel with the same velocity dr/dt ; for the velocity of light in the mesh-system used is $1 - 2M/r$, and though this velocity depends on r , it does not depend on t . Hence the difference dt at one end of the waves is the same as that at the other end.

Eddington's account closely reflects Einstein's derivations from 1907, 1911, and 1916. First, atoms at different locations act as *local* clocks—in modern terms, they measure *proper time*, justifying the use of $ds = 1$ as the interval for a unit period at all locations. However, in addition to this local time, there is also a global time, referred to as "*the time*," as Eddington and Einstein both wrote. This global time t must be used for time comparisons between different places. The comparison is straightforward: the interval ds_S of one vibration on the Sun corresponds to a lapse dt_S of global time at that position. This time interval dt_S is transmitted unchanged to the laboratory on Earth since the speed of light, expressed in global time, does not depend on t . Because dt_S is slightly larger than 1, the period of the atom on the Sun as received on Earth takes a bit longer than the period of a similar atom in the terrestrial laboratory. Therefore, there is a slight redshift in the radiation coming from the Sun.

In an influential article published in 1980, John Earman and Clark Glymour [3] criticized the early derivations of the gravitational redshift by Einstein, Eddington, and others who followed their approach. They described Einstein's 1907 derivation as cumbersome and obscure, lacking clarity regarding the meanings of "time" and "local time" [3] (p.178), without offering a detailed discussion of the 1907 article. However, they did discuss the 1911 paper and described the thought experiment in which radiation is emitted from B to A in a homogeneous gravitational field. As detailed in section 3 and mentioned by Earman and Glymour, Einstein concluded that clocks at different positions run at different rates (measured in global time), implying that the velocity of light is position-dependent (as measured in global time). Earman and Glymour do not delve into Einstein's reasoning on this point but comment [3] (pp.181–182):

All of the heuristic derivations of the red shift can be faulted on various technical grounds. But to raise such objections is to miss the purpose of heuristic arguments, which is not to provide logically seamless proofs but rather to give a feel for the underlying physical mechanisms. It is precisely here that most of the heuristic red shift derivations fail—they are not good heuristics. For they are set in Newtonian or special relativistic space-time; but the red shift strongly suggests that gravitation cannot be adequately treated in a flat space-time. Einstein's resort to the notions of a variable speed of light and variable clock rates in a gravitational field can be seen as an acknowledgment, albeit unconscious, of this point; but as we will now see, these notions served to obscure the role of curvature of space-time as the light ray moves from source to receiver.

It is true that the 1907 and 1911 papers only use the principle of equivalence in Newtonian or Minkowski spacetime and are therefore flawed from the point of view of the finished general theory of relativity. There is no principled discussion of inhomogeneous gravitational fields in the 1907 and 1911 papers, although Einstein does conjecture in several places that his results for homogeneous fields will also apply to inhomogeneous ones. But it appears odd to dismiss Einstein's arguments as bad heuristics on these grounds. Good heuristics usually retain major parts of old conceptual frameworks and add some new idea or perspective to suggest results that should be rigorously derived within a new theory. Einstein's heuristics did exactly that. Moreover, the specific objection that Einstein's treatment does not analyze the transmission of light from source to receiver and that the early derivations of Einstein, Eddington and their followers can therefore only be misleading [3] (p. 176) is incorrect—this should already be clear from the above Eddington quotation. But let us look at the objection in more detail.

After quoting Einstein's 1916 derivation of Eq. (6) and his conclusion ("The clock goes more slowly if set up in the neighbourhood of ponderable masses. From this it follows that the spectral lines of light reaching us from the surface of large stars must appear displaced towards the red end of the spectrum."), Earman and Glymour comment [3] (pp. 182–183):

To the modern eye, Einstein's derivation is no derivation at all, for the formula (6) expresses only a co-ordinate effect [...] Einstein provided no deduction from the theory to explain what happens to a light ray or photon as it passes through the gravitational field on its way from the Sun to the Earth. Unfortunately, Einstein's 'derivation' was dressed up by the expositors of the general theory, and it quickly became codified in the literature as the official derivation.

Earman and Glymour go on to explain Eddington's role in spreading Einstein's error and criticize the redshift derivation Eddington gave in his *Report on the Relativity Theory of Gravitation* [4]. They claim that Eddington's derivation confuses coordinate time with physical time and fails to engage the crucial question of how the radiation emitted at the Sun is received on Earth. However, the criticized derivation in Eddington's Report is virtually identical to the derivation he presents in [5], which we have reproduced above. As should be clear from that passage, Eddington did pay attention to the propagation of the light signal and remarked expressly on the fact that the speed of light does not depend on t —this justifies the essential point in his calculation, namely that the time interval dt is transferred unchanged from Sun to Earth.

It is surprising, then, to find that Earman and Glymour present their own (correct) derivation of the redshift formula by arguing at great length that in a static gravitational field coordinates can be chosen in such a way that the coordinate time interval is transmitted without change. Via a rather roundabout use of this premise they finally arrive at a formula that is equivalent to Einstein's [3] (pp. 184–185).

Apparently, the background of the confusion is that Earman and Glymour have looked at Einstein's and Eddington's formulas with all too modern eyes. In modern expositions of general relativity coordinates are considered to be conventional markers of events; in particular, the time coordinate x_4 is not required to have a physical interpretation in terms of clocks. Formulas like (1) then indeed may strike one as expressing nothing but a mathematical statement about an uninterpreted coordinate. But Einstein did not approach the subject from that perspective. It is true that Einstein in his definitive work on general relativity took a decisive step towards the modern view, through his insistence that arbitrary coordinate systems may be used [9] (p. 776). But the ideas of his earlier heuristic work, in which he deemed it essential to give "time" a clear and concrete physical meaning, still lingered on. From this older point of view the physical properties of global time in static gravitational fields are obvious, whereas they seem in need of justification if one looks at the mathematical formulas from a modern standpoint.

6. Conclusion: Coordinates and Time

In 1921, five years after his review article on the general theory of relativity, Einstein delivered the Stafford Little Lectures at Princeton University. These lectures, published in 1922 as "The Meaning of Relativity" [10], covered both the special and general theories of relativity. Revisiting the behavior of rods and clocks in a gravitational field [10] (pp. 90–92), Einstein observed that only in local inertial systems can coordinates be chosen to match "naturally measured lengths and times." In the symmetric case of a static field generated by a central mass, with coordinates adapted to the global symmetry, a unit measuring rod will not always match a unit coordinate interval. As Einstein explained, its "coordinate length" will be shortened. He further noted that this coordinate length, and its variation based on location and orientation, depends on the chosen coordinate system. So, this is a coordinate effect in the modern sense.

Regarding time, Einstein remarked that the interval between two beats of a unit clock ($ds = 1$) corresponds to a longer "time" ($dx_4 > 1$) "in the unit used in our system of coordinates". He then continued [10] (p. 92):

The rate of a clock is accordingly slower the greater is the mass of the ponderable mass in its neighbourhood. We therefore conclude that spectral lines which are produced on the Sun's surface will be displaced towards the red, compared to the corresponding lines produced on the Earth, by about $2 \cdot 10^{-6}$ of their wave-lengths.

This argument mirrors the one in his 1916 paper, which, as we have seen, is challenging to understand if we think of dx_4 as a completely arbitrary coordinate, and of the slowing down as a mere coordinate effect.

Evidently, by 1921, Einstein was well aware of the arbitrary nature of coordinates, as evidenced by his discussion on measuring rods. Nevertheless, he maintained his earlier strategy of making the concept of time as physically concrete as possible. This approach had significantly aided him in developing the special theory of relativity and taking the initial steps towards general relativity. Furthermore, in specific cases with global symmetries (such as a static gravitational fields) this strategy of introducing a global time remains helpful even in the final form of general relativity. Recognizing this methodological motif in Einstein's early work clarifies much of his contributions and highlights the difficulties he faced in relativity in its most general form, where a global time realized by physical clocks is no longer a helpful concept.

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References

1. Dieks, D., Another Look at General Covariance and the Equivalence of Reference Systems, *Studies in History and Philosophy of Modern Physics*, **37**, 2006, 174–191.
2. Dieks, D., Time Coordinates and Clocks: Einstein's Struggle, arXiv:1801.09297 [physics.hist-ph], 2018.
3. Earman, J. and Glymour, C., The Gravitational Red Shift as a Test of General Relativity: History and Analysis, *Studies in History and Philosophy of Science*, **11**, 1980, 175–214.
4. Eddington, A.S., *Report on the Relativity Theory of Gravitation*. London: Fleetwood Press, 1918; 2nd edition, 1920.
5. Eddington, A.S., *Space, Time and Gravitation*. Cambridge: Cambridge University Press, 1923.
6. Einstein, A., Zur Elektrodynamik bewegter Körper, *Annalen der Physik* **17**, 1905, 891–921. Reprinted in [16], pp. 275–310. English translation in [11], pp. 35–65.
7. Einstein, A., Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen, *Jahrbuch der Radioaktivität und Elektronik* **4**, 1907, 411–462. Reprinted in [16], pp. 433–488. English translation in: Schwartz, H.M., Einstein's comprehensive 1907 essay on relativity, *American Journal of Physics*, **45**, 1977, 512–517, 811–817, 899–902.
8. Einstein, A., Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes, *Annalen der Physik* **35**, 1911, 898–908. English translation in [11], pp. 97–108. Reprinted in [12], pp. 485–497.
9. Einstein, A., Die Grundlage der allgemeinen Relativitätstheorie, *Annalen der Physik* **49**, 1916, 769–882. Reprinted in [13], pp. 283–339. English translation in [11], pp. 100–164.
10. Einstein, A., *The Meaning of Relativity*. Princeton: Princeton University Press, 1922.
11. Einstein, A., Lorentz, H.A., Weyl, H., and Minkowski, H., *The Principle of Relativity*. New York: Dover Publications, 1952.
12. Klein, M.J., et al. (eds), *The Collected Papers of Albert Einstein*, Vol. 3. Princeton: Princeton University Press, 1993.
13. Kox, A.J., et al. (eds.), *The Collected Papers of Albert Einstein*, Vol. 6. Princeton: Princeton University Press, 1996.
14. Pais, A., *Subtle Is the Lord. The Science and Life of Albert Einstein*. Oxford, Oxford University Press, 1982.
15. Schilp, P.A. (ed.), *Albert Einstein, Philosopher-Scientist*. La Salle: Open Court, 1949.
16. Stachel, J. (ed.), *The Collected Papers of Albert Einstein*, Vol. 2. Princeton: Princeton University Press, 1989.

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