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Essay

On S-2 Prime Hyperideals of Commutative Hyperrings

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Abstract

In this paper, we present the notion of S-2-prime hyperideals, which provides a unifying generalization of 2-prime and S-prime hyperideals within multiplicative hyperrings. We explore their key algebraic properties and investigate their connections with other hyperideal classes. We emphasize the unique aspects that differentiate S-2-prime hyperideals, illustrating their role in expanding the theoretical framework of hyperideal structures. We examine how these hyperideals behave under hyperring homomorphisms, extensions, and standard algebraic operations, demonstrating that many known properties of prime, 2-prime, and S-prime hyperideals extend naturally to the S-2-prime setting. We provide illustrative examples to highlight important differences and to offer practical insight into their structure. Overall, we enhance the theoretical understanding of hyperideals in multiplicative hyperrings and establish a framework for future research in this area.

Keywords: 2- prime hyperideals; S- prime hyperideals; S-2- prime hyperideals

1. Introduction

Hyperstructures constitute a broader framework than classical algebraic structures. In classical algebra, a binary operation assigns a single element to each pair of elements, whereas in the context of hyperstructures, such an operation produces a set of elements [1]. This theory was first introduced by F. Marty [2]. Algebraic hyperstructures are utilized across multiple fields, such as geometry, artificial intelligence, and cryptography, providing adaptable frameworks to represent complex and uncertain systems. A comprehensive reference on the applications of hyperstructures is the book Applications of Hyperstructures Theory by P. Corsini and V. Leoreanu [3]. Hyperrings were initially proposed by Krasner in 1983, where addition is interpreted as a hyperoperation and multiplication remains standard, thereby broadening the framework of classical ring theory [9]. Earlier, in 1982, Rota introduced the notion of multiplicative hyperrings [10]. In 1994, Vougiouklis studied hyperstructures extensively in his book "Hyperstructures and Their Representations"[18]. In 1997, Procesi and Rota introduced the concept of a prime hyperideal in a multiplicative hyperring [14]. In 2012, Dasgupta, on the other hand, has studied the prime ideals of multiplicative hyperring extensively [11]. Davvaz has covered hyperrings extensively in his book "Hyperring Theory and Applications" [1]. Algebraic hyperstructures have applications in various fields [15], [16], [17], [20]. In 2014, Ameri and Norouzi studied commutative hyperrings as a generalization of classes of hyperrings [12]. Ameri and Kordi (2017) first defined the concept of coprime hyperideals within the context of multiplicative hyperrings [13]. In 2016, Beddani introduced notion 2-prime ideals [8]. In 2017, Ay, Yesilot and Sonmez defined the prime expansion, which is associated to prime hyperideals [19]. Hamed and Malek (2020) proposed the notion of S-prime ideals, extending the concept of traditional prime ideals [7]. The concept of S-prime hyperideals, extending prime hyperideals, was introduced by Ghiasvand and Farzalipour in 2021 [4]. Subsequently, Anbarloei defined 2-prime hyperideals in 2022 as another generalization of prime ideals [5]. More recently, in 2024, Yavuz introduced S-2 prime ideals, which unify the notions of 2-prime ideals and S-prime ideals [6].

In recent years, researchers have explored various generalizations of hyperideal concepts within

multiplicative hyperring theory. Prime, 2-prime, and S -prime hyperideals, in particular, have been studied in detail to better understand the fundamental algebraic properties of multiplicative hyperrings. These concepts not only extend classical ideal theory to hyperstructures but also help to investigate the connections between different hyperideals. Although the study of general prime hyperideals remains an active area of research, a comprehensive structure that combines the features of S -prime and 2-prime hyperideals in multiplicative hyperrings has not been sufficiently addressed in the literature. Motivated by this, we introduce a broader hyperideal, S -2- prime hyperideal, and examine its fundamental properties.

In this study, we generalize the concepts of S -2-prime ideals from commutative ring theory, along with S -prime and 2-prime hyperideals in the context of multiplicative hyperrings. Throughout the paper, we let $\tilde{T}$ stand for a multiplicative hyperring and S indicate a multiplicatively closed subset of $\tilde{T}$. The paper is organized into four main sections. The first section includes a literature review. The next section provides preliminary information related to our main topic. In the third section of the study, the definition of an S -2-prime hyperideal is presented, along with an illustrative example. Building on the notions of S -prime hyperideals and 2-prime hyperideals found in earlier works, it is shown by theorem that these hyperideals are also S -2 hyperideals; however, the converses are demonstrated to be false by means of counter examples. Then, we give a new characterization of S -2-prime hyperideal in Teorem 3 Subsequently, we investigate the relationship between S -2-prime hyperideals and matrice space Teorem 4 Also, we examine this ideals properties under homomorphism see Teorem 5, 6 Moreover, In Theorem 7, we examine these ideals in direct product rings, and in Theorem 8, we give their generalization. Moreover, studying S -2-prime hyperideals provides deeper insight into the interplay between S -prime and 2-prime hyperideals. This framework clarifies their algebraic structure and opens up potential applications in areas where hyperstructure theory is relevant, such as geometry, artificial intelligence, and cryptography. In addition, our work not only defines S -2-prime hyperideals but also investigates their fundamental properties, relationships with existing hyperideals, and broader generalizations, highlighting the original contribution of this study. The final section presents the results obtained from this study.

2. Preliminaries

This section provides some preliminary information for the topic. Below are the definitions of hypergroup, multiplicative hyperring, hyperideal, prime hyperideal, good homomorphism, and multiplicatively closed subset.

Definition 1. [1] Let $\tilde{T}$ be a nonempty set, and let $\star : \tilde{T} \times \tilde{T} \rightarrow P^*(\tilde{T})$ denote a hyperoperation. The pair $(\tilde{T}, \star)$ is then called a hypergrouoid.

For any two nonempty subsets E and F of $\tilde{T}$ and $x \in \tilde{T}$, we define

$$E \star F = \bigcup_{e \in E, f \in F} e \star f, E \star x = E \star \{x\} \text{ and } x \star F = \{x\} \star F$$

Definition 2. [1] A hypergrouoid $(\tilde{T}, \star)$ is called a semihypergroup if for all e, f, t of $\tilde{T}$ we have $(e \star f) \star t = e \star (f \star t)$, which means that

$$\bigcup_{u \in e \star f} u \star t = \bigcup_{v \in f \star t} e \star v$$

Definition 3. [1] A hypergrouoid $(\tilde{T}, \star)$ is called a quasihypergroup if, for every element $e \in \tilde{T}$, the equalities $e \star \tilde{T} = \tilde{T} \star e = \tilde{T}$ hold.

Definition 4. [1] A hypergrouoid $(\tilde{T}, \star)$ that at the same time fulfills the conditions of a semihypergroup and a quasihypergroup is designated a hypergroup.

Definition 5. [1] A triple $(\dot{T}, +, \star)$ is called a multiplicative hyperring if

- (1) $(\dot{T}, +)$ is an abelian group;
- (2) $(\dot{T}, \star)$ is a semihypergroup;
- (3) For all $e, f, t \in \dot{T}$, we have $e \cdot (f + t) \subseteq e \star f + e \star t$ and $(f + t) \star e \subseteq f \star e + t \star e$;
- (4) For all $e, f \in \dot{T}$, we have $e \star (-f) = (-e) \star f = -(e \star f)$.

If we have equalities instead of inclusions in (3), then we say that the multiplicative hyperring is strongly distributive.

In this study, unless otherwise specified, we will take the multiplicative hyperring as strongly distributive.

Definition 6. [1] We say that D is a hyperideal of $(\dot{T}, +, \star)$ if $D - D \subseteq D$ and for all $x, y \in D, r \in \dot{T}, x \star r \cup r \star x \subseteq D$.

Definition 7. [11,21] Let $D \neq \dot{T}$ be a hyperideal of the multiplicative hyperring $(\dot{T}, +, \star)$. Then D is called a prime hyperideal of $\dot{T}$ if, for any $e, f \in \dot{T}$, the inclusion $e \star f \subseteq D$ implies that $e \in D$ or $f \in D$.

Notation 1. [21] Let Λ be a subset of a hyperring $\dot{T}$, and let $A_i \mid i \in J$ denote the collection of all hyperideals of $\dot{T}$ containing Λ . The hyperideal generated by Λ is defined as the set of elements belonging to every A_i and is denoted by $\langle \Lambda \rangle$. If $\Lambda = \lambda_1, \lambda_2, \dots, \lambda_n$, this hyperideal is written as $\langle \lambda_1, \lambda_2, \dots, \lambda_n \rangle$.

Proposition 1. [11] Let $(\dot{T}, +, \star)$ be a commutative multiplicative hyperring. Then for any $e, f \in \dot{T}$, $\langle e \rangle \langle f \rangle \subseteq \langle e \star f \rangle$.

Definition 8. [1] Let $(\dot{T}, +, \star)$ be a multiplicative hyperring, and let D be a hyperideal of $\dot{T}$. We define addition on cosets in the usual manner, and multiplication as follows:

$$(e + D) \diamond (f + D) = \{t + D \mid t \in e \star f\}$$

on the set $\dot{T}/D = \{e + D \mid e \in \dot{T}\}$ of all cosets of D . Then $(\dot{T}/D, +, \diamond)$ forms a multiplicative hyperring, and it retains strong distributivity if $\dot{T}$ possesses it.

Definition 9. [1] A homomorphism (good homomorphism) between two multiplicative hyperrings $(\dot{T}, +, \star)$ and $(\ddot{T}, \ddot{+}, \ddot{\star})$ is a map $f : \dot{T} \rightarrow \ddot{T}$ such that for all m, n of $\dot{T}$, we have $f(m + n) = f(m) \ddot{+} f(n)$ and $f(m \star n) \subseteq f(m) \ddot{\star} f(n)$ ($f(m \star n) = f(m) \ddot{\star} f(n)$ respectively).

Definition 10. [5] Let D be a proper hyperideal of a multiplicative hyperring $\dot{T}$. We say that D is a 2-prime if for all $e, f \in \dot{T}$, $e \star f \subseteq D$ implies $e^2 \subseteq D$ or $f^2 \subseteq D$.

Definition 11. [4] A nonempty subset S of $(\dot{T}, +, \star)$, a multiplicative hyperring with identity 1, is called multiplicatively closed when it obeys the following conditions:

- (i) $1 \in S$,
- (ii) $s_1 \star s_2 \cap S \neq \emptyset$ for all $s_1, s_2 \in S$.

Definition 12. [4] Let $\dot{T}$ be a multiplicative hyperring, $S \subseteq \dot{T}$ be a multiplicatively closed subset of $\dot{T}$ and D be a hyperideal of $\dot{T}$ such that $D \cap S = \emptyset$. We say that D is S -prime if there exists an $s \in S$ such that for all $e, f \in \dot{T}$ with $e \star f \subseteq D$, we have $s \star e \subseteq D$ or $s \star f \subseteq D$.

Notation 2. [1] Let $(\dot{T}, +, \star)$ be a hyperring with identity 1. We consider the relation Ψ given by $e \Psi f$ if and only if $e, f \subseteq \Theta$, where Θ is a finite sum of finite products of elements from $\dot{T}$, that is,

$$e\Psi f \iff \exists p_1, p_2, \dots, p_n \in \dot{T} \text{ such that } \{e, f\} \subseteq \sum_{j \in J} \prod_{i \in I_j} p_i, J \subseteq \{1, 2, \dots, n\}.$$

We denote by Ψ^* the transitive closure of Ψ . This relation Ψ^* is the smallest equivalence relation on the multiplicative hyperring $(\dot{T}, +, \star)$ for which the quotient $\dot{T}/\Psi^*$ consists of all finite sums of products of elements from $\dot{T}$. Accordingly, the definition of Ψ^* on $\dot{T}$ can be restated as follows:

$e\Psi f \iff \exists p_1, p_2, \dots, p_n \in \dot{T}$ with $p_1 = e$ and $p_{c+1} = f$ and $\theta_1, \theta_2, \dots, \theta_c \in \Theta$ such that $\{p_g, z_{g+1} \subseteq \theta_g$ for $g \in \{1, 2, \dots, c\}$.

Suppose that $\Psi^*(e)$ is the equivalence class containing $e \in \dot{T}$. Then, both the sum $\oplus$ and the product $\odot$ in $\dot{T}/\Psi^*$ are defined as follows: $\Psi^*(e) \oplus \Psi^*(f) = \Psi^*(t)$ for all $t \in \Psi^*(e) + \Psi^*(f)$ and $\Psi^*(e) \odot \Psi^*(f) = \Psi^*(d)$ for all $d \in \Psi^*(e) \star \Psi^*(f)$. Then $\dot{T}/\Psi^*$ forms a ring, called the fundamental ring of $\dot{T}$.

Proposition 2. [4] Let $\dot{T}$ be a multiplicative hyperring with scalar 1. Then $S \subseteq \dot{T}$ is a multiplicatively closed subset of $\dot{T}$ if and only if S/Ψ^* is a multiplicatively closed subset of the ring R/Ψ^* .

Notation 3. [22] Recall that the ideal generated by the μ^{th} powers of elements of a proper ideal D is denoted by $D_{[\mu]} = \langle \{d^\mu : d \in D\} \rangle$. It is clear that $D_{[\mu]} \subseteq D^\mu \subseteq D$, and equality holds when $\mu = 1$. Moreover, if $\mu!$ is a unit in $\dot{T}$, then we have $D_{[\mu]} = D^\mu$.

Notation 4. [13] Let $\dot{T}$ be a multiplicative hyperring and let $M_n(\dot{T})$ represent the set of all hypermatrices over $\dot{T}$. For any $E = (e_{ij})_{nn}$ and $F = (f_{ij})_{nn}$ in $P^*(M_n(\dot{T}))$, $E \subseteq F$ precisely when $e_{ij} \subseteq f_{ij}$ for each i, j .

3. S-2- Prime Hyperideals

In this section, we will define S-2- prime hyperideals and examine their algebraic properties with examples.

Definition 13. Let $\dot{T}$ be a multiplicative hyperring. $S \subseteq \dot{T}$ be a multiplicatively closed subset of $\dot{T}$ and D be a proper hyperideal of $\dot{T}$ such that $D \cap S = \emptyset$. We say that D is S-2 prime if there exists an $s \in S$ such that for all $e, f \in \dot{T}$ with $e \star f \subseteq D$, we have $s \star e^2 \subseteq D$ or $s \star f^2 \subseteq D$.

Example 1. Let $(\mathbb{Z}_4, +, \cdot)$ is a ring and $D = \langle \bar{2} \rangle$ be an ideal of $\mathbb{Z}_4$. We define $e \star f = ef + D$ hyperoperation for $e, f \in \mathbb{Z}_4$.

$\star$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{0}$	D	D	D	D
$\bar{1}$	D	$\{\bar{1}, \bar{3}\}$	D	$\{\bar{1}, \bar{3}\}$
$\bar{2}$	D	D	D	D
$\bar{3}$	D	$\{\bar{1}, \bar{3}\}$	D	$\{\bar{1}, \bar{3}\}$

$(\mathbb{Z}_4, +, \star)$ is a multiplicative hyperring. Now, we consider the $S = \{\bar{1}, \bar{3}\}$ multiplicative closed subset of $\mathbb{Z}_4$ and $D = \{\bar{0}, \bar{2}\}$ hyperideal of $\mathbb{Z}_4$. Its easy to see that $S \cap D = \emptyset$. For $e, f \in \mathbb{Z}_4$ and $\bar{3} \in S$ that satisfy the requirement $e \star f \subseteq D$, $\bar{3} \star e^2 = \bar{3}e^2 + D \subseteq D$ or $\bar{3} \star f^2 = \bar{3}f^2 + D \subseteq D$. Thus, D is an S-2 prime hyperideal of $\mathbb{Z}_4$.

Example 2. Let $R = (\mathbb{Z}, +, \cdot)$. We define $e \star f = \{2ef, 3ef\}$ for all $e, f \in \dot{T}$. Then $(\mathbb{Z}, +, \star)$ is a multiplicative hyperring. $S = \{7^n | n \in \mathbb{N}\}$ be a multiplicatively closed subset of $\dot{T}$ and $D = 11\mathbb{Z}$ be a hyperideal of $\dot{T}$. It easy to see that $D \cap S = \emptyset$. Put $s = 1$. Then D is an S-2 prime hyperideal of $\dot{T}$.

Theorem 1. Let $\dot{T}$ be a multiplicative hyperring, $S \subseteq \dot{T}$ a multiplicatively closed subset, and D a hyperideal of $\dot{T}$ such that $D \cap S = \emptyset$. If D is an S-prime hyperideal of $\dot{T}$, then it is also an S-2-prime hyperideal of $\dot{T}$.

Proof. Let D is a S - prime hyperideal of $\dot{T}$. Then, there exists $s \in S$ such that for all $e, f \in \dot{T}$ with $e \star f \subseteq D$ then $s \star e \subseteq D$ or $s \star f \subseteq D$. Let $s \star e \subseteq D$. Since $s \star e \subseteq D$ and D is a hyperideal of $\dot{T}$, $(s \star e) \star e \subseteq D \star e \subseteq D$. According to (2) and (3) in definition 5 $(s \star e) \star e = s \star (e \star e) = s \star e^2 \subseteq D$. Let $s \star f \subseteq D$. Then $s \star f \star f \subseteq D$. Since $f \star f = f^2$ then $s \star f^2 \subseteq D$. Thus D is an S – 2- prime hyperideal of $\dot{T}$. $\square$

The theorem does not necessarily hold in the reverse. A corresponding example is presented below.

Example 3. Let $(\mathbb{Z}_{12}, +, \cdot)$ be a ring and $D = \langle \bar{9} \rangle$ be an ideal of $(\mathbb{Z}_{12}, +, \cdot)$ and $S = \{\bar{1}, \bar{2}\}$ be a multiplicative closed subset of $(\mathbb{Z}_{12}, +, \cdot)$. We define $e \star f = ef + D$ hyperoperation on hyperring $(\mathbb{Z}_{12}, +, \star)$. For $e, f \in \mathbb{Z}_{12}$, let $e \star f \subseteq D$. We have there exists an $s \in S$, $s \star e^2 \subseteq D$ or $s \star f^2 \subseteq D$. But, when $\bar{3} \star \bar{3} \subseteq D$ there is not exists an $s \in S$ such that $s \star \bar{3} \subseteq D$.

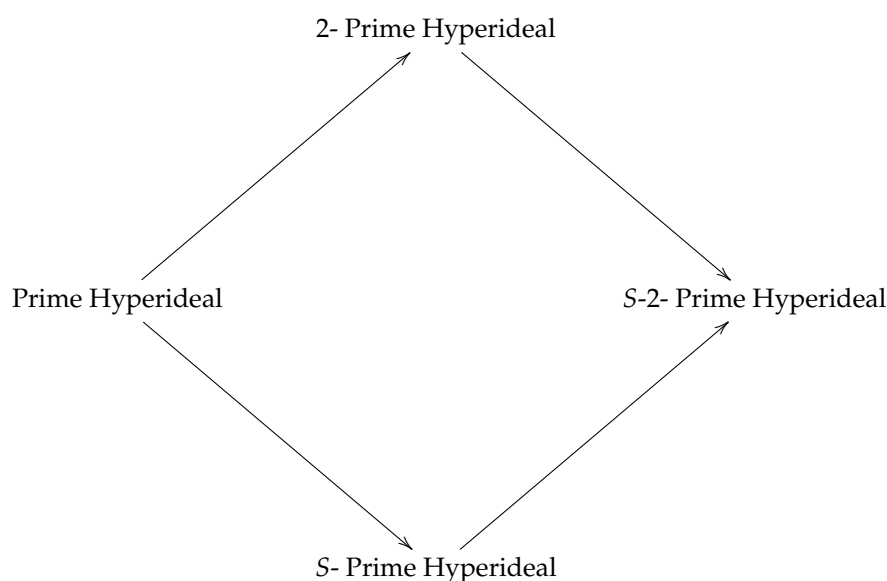
Theorem 2. Let $\dot{T}$ be a multiplicative hyperring, and let S be a multiplicatively closed subset of $\dot{T}$. Let D be a hyperideal of $\dot{T}$ such that D has empty intersection with S . If D is a 2-prime hyperideal in $\dot{T}$, then D is also an S -2-prime hyperideal of $\dot{T}$.

Proof. Let D is a 2- prime hyperideal of $\dot{T}$. Then there exists $e \star f \subseteq D$ for all $e, f \in R$ then $e^2 \subseteq D$ or $f^2 \subseteq D$. We take an S multiplicatively closed subset of $\dot{T}$ such that $S \cap D = \emptyset$. For $s \in S$, if $e^2 \subseteq D$ then $s \star e^2 \subseteq D$ or if $f^2 \subseteq D$ then $s \star f^2 \subseteq D$. $\square$

The theorem does not necessarily hold in the reverse. A corresponding example is presented below.

Example 4. Let $(\mathbb{Z}_{10}, +, \cdot)$ be a ring and $D = \langle \bar{6} \rangle$ be an ideal of $(\mathbb{Z}_{10}, +, \cdot)$ and $S = \{\bar{1}, \bar{3}, \bar{7}, \bar{9}\}$ be a multiplicatively closed subset of $(\mathbb{Z}_{10}, +, \cdot)$. We define $e \star f = ef + D$ hyperoperation on hyperring $R = (\mathbb{Z}_{10}, +, \star)$. For $\bar{2}, \bar{3} \in \dot{T}$, $\bar{2} \star \bar{3} \subseteq D$. For $s = \bar{3}$, $\bar{3} \star \bar{2}^2 \subseteq D$ or $\bar{3} \star \bar{3}^2 \subseteq D$ is true but neither of $\bar{2}^2 \subseteq D$ nor $\bar{3}^2 \subseteq D$. Then P is an S – 2- prime hyperideal but D is not a 2- prime hyperideal of $\dot{T}$.

Earlier studies have established that every prime hyperideal simultaneously qualifies as a 2-prime hyperideal [5] and an S -prime hyperideal [4]. In this study, using both of the above theorems, we have shown that both 2-prime hyperideals and S are also S -2- prime hyperideals. As can be seen from the examples above, this diagram is one-sided and its converse is not always true.



Definition 14. It is noted that the hyperideal formed by taking the μ^{th} powers of elements from a proper hyperideal D is denoted by $D_{[\mu]} = \langle d^\mu : d \in D \rangle$. It is evident that $D_{[\mu]} \subseteq D^\mu \subseteq D$, with equality occurring when $\mu = 1$. Moreover, if $\mu!$ is invertible in $\dot{T}$, then $D_{[\mu]} = D^\mu$.

Theorem 3. Let $\dot{T}$ be a multiplicative hyperring, $S \subseteq \dot{T}$ a multiplicatively closed subset, and D a hyperideal of $\dot{T}$ such that $D \cap S = \emptyset$. Then D is an S -2-prime hyperideal of $\dot{T}$ if and only if for any two hyperideals I and J of $\dot{T}$ with $IJ \subseteq D$, there exists an element $s \in S$ such that $s \star I_{[2]} \subseteq D$ or $s \star J_{[2]} \subseteq D$.

Proof. ($\Rightarrow$): Let D is a S -2 prime hyperideal. Then, there exists $s \in S$ such that for all $e, f \in \dot{T}$ with $e \star f \subseteq D$ then $s \star e^2 \subseteq D$ or $s \star f^2 \subseteq D$. Let for all $t \in S$, there exists hyperideals E, F of $\dot{T}$ with $EF \subseteq D$, $t \star E_{[2]} \not\subseteq D$ and $t \star F_{[2]} \not\subseteq D$. Since $s \in S$ there exists hyperideals, I, J of $\dot{T}$ with $IJ \subseteq D$ $t \star I_{[2]} \not\subseteq D$ and $t \star J_{[2]} \not\subseteq D$. Thus there exists $e \in I$ and $f \in J$ such that $s \star e^2 \not\subseteq D$ and $s \star f^2 \not\subseteq D$ with $e \star f \subseteq IJ \subseteq D$ that it contradicts with hypothesis.

($\Leftarrow$): Let $e \star f \subseteq D$ where $e, f \in \dot{T}$. Thus $\langle e \star f \rangle \subseteq D$. By [11], we have $\langle e \rangle \star \langle f \rangle \subseteq \langle e \star f \rangle$ and so $\langle e \rangle \star \langle f \rangle \subseteq D$. Thus there exists an $s \in S$ such that $s \star \langle e \rangle_{[2]} \subseteq D$ or $s \star \langle f \rangle_{[2]} \subseteq D$ by hypothesis. Therefore, $s \star e^2 \subseteq D$ or $s \star f^2 \subseteq D$. Hence D is a S -2 prime hyperideal of $\dot{T}$ and the proof is complete. $\square$

Corollary 1. Let $\dot{T}$ be a multiplicative hyperring, $S \subseteq \dot{T}$ a multiplicatively closed subset, and D a hyperideal of $\dot{T}$ with $D \cap S = \emptyset$. If there exists an element $s \in S$ such that for any hyperideals $L_1, L_2, \dots, L_k$ of $\dot{T}$, whenever $L_1, L_2, \dots, L_k \subseteq D$, there is some $v \in \{1, 2, \dots, k\}$ for which $s \star L_{v[2]} \subseteq D$, then D is an S -2-prime hyperideal of $\dot{T}$.

Proof. Let $IL_1, L_2, \dots, L_k$ are hyperideals of $\dot{T}$. By previous theorem let $e \star f \subseteq D$ where $e, f \in \dot{T}$. Thus $\langle e \star f \rangle \subseteq D$. Then we have $\langle e \rangle \star \langle f \rangle \subseteq \langle e \star f \rangle$ and so $\langle e \rangle \star \langle f \rangle \subseteq D$. So, let $e_1 \star e_2 \star \dots \star e_k \subseteq D$ where $e_1, e_2, \dots, e_k \in \dot{T}$. Thus $\langle e_1 \star e_2 \star \dots \star e_n \rangle \subseteq D$. Then we have $\langle e_1 \rangle \star \langle e_2 \rangle \star \dots \star \langle e_k \rangle \subseteq D$. By hypothesis there exists an $s \in S$ such that $s \star \langle e_1 \rangle_{[2]} \subseteq D$, $s \star \langle e_2 \rangle_{[2]} \subseteq D, \dots, s \star \langle e_{k-1} \rangle_{[2]} \subseteq D$ or $s \star \langle e_k \rangle_{[2]} \subseteq D$. Thus $s \star e_{1[2]} \subseteq D, s \star e_{2[2]} \subseteq D, \dots, s \star e_{k-1[2]} \subseteq D$ or $s \star e_{k[2]} \subseteq D$. Therefore, D is a S -2 hyperideal of $\dot{T}$. $\square$

Proposition 3. Let $\dot{T}$ be a multiplicative hyperring, $S \subseteq \dot{T}$ a multiplicatively closed set, and D a hyperideal of $\dot{T}$ with $D \cap S = \emptyset$. Suppose there exists $s \in S$ such that for any elements $e_1, \dots, e_k \in \dot{T}$, if $e_1 \star \dots \star e_k \subseteq D$, then at least one index $c \in \{1, \dots, k\}$ satisfies $s \star e_c^2 \subseteq D$. Under these conditions, D qualifies as an S -2-prime hyperideal of $\dot{T}$.

Proof. Let there exists an $s \in S$ such that for all $e_1, e_2, \dots, e_k$ of $\dot{T}$, if $e_1 \star e_2 \star \dots \star e_k \subseteq D$, then $s \star e_c^2 \subseteq D$ for some $c \in \{1, 2, \dots, k\}$. Take $k = 2$. Then there exists an $s \in S$ such that for e_1, e_2 of $\dot{T}$, if $e_1 \star e_2 \subseteq D$, then $s \star e_1^2 \subseteq D$ or $s \star e_2^2 \subseteq D$. Then D is an S -2- prime hyperideal of $\dot{T}$. $\square$

Proposition 4. Let $\dot{T}$ be a multiplicative hyperring, $S \subseteq \dot{T}$ a multiplicatively closed subset, and D a hyperideal of $\dot{T}$ disjoint from S . Then

- (i) Let O be a hyperideal of $\dot{T}$ with $O \cap S = \emptyset$. Whenever D is an S -2-prime hyperideal of $\dot{T}$, the product OD also constitutes an S -2-prime hyperideal of $\dot{T}$.
- (ii) Let $\dot{T} \subseteq \ddot{T}$ be an extension of $\dot{T}$. If K is an S -2-prime hyperideal of $\ddot{T}$, then its intersection with $\dot{T}$, $K \cap \dot{T}$, also forms an S -2-prime hyperideal of $\dot{T}$.

Proof. (i) Let $t \in O \cap S$. Let $e \star f \subseteq OD$ where $e, f \in \dot{T}$. Hence there exists an $s \in S$ such that $s \star e^2 \subseteq D$ or $s \star f^2 \subseteq D$. Thus $t \star s \star e^2 \subseteq OD$ or $t \star s \star f^2 \subseteq OD$. Since $(t \star s) \cap S \neq \emptyset$, so there is $u \in (t \star s) \cap S$. Thus $u \star e^2 \subseteq OD$ or $u \star f^2 \subseteq OD$. So OD , is an S -2 prime hyperideal of $\dot{T}$.

- (ii) Let $e \star f \subseteq K \cap \dot{T}$ where $e, f \in \dot{T}$. Since K is an S -2 prime hyperideal of $\dot{T}$, there exists $s \in S$ such that $s \star e^2 \subseteq K$ or $s \star f^2 \subseteq K$. Therefore $s \star e^2 \subseteq K \cap \dot{T}$ or $s \star f^2 \subseteq K \cap \dot{T}$. So, $K \cap \dot{T}$ is an S -2 prime hyperideal of $\dot{T}$.
□

Example 5. Let $R = (\mathbb{Z}_4, +, \star)$ be a hyperring, and let $D = \langle \bar{2} \rangle$ be a hyperideal of $\dot{T}$. We define a hyperoperation $\star$ on $\dot{T}$ by $e \star f = ef + D$ for all $e, f \in \dot{T}$.

$$M_2[\mathbb{Z}_4] = \left\{ \begin{pmatrix} \kappa & \lambda \\ \mu & \nu \end{pmatrix} \mid \kappa, \lambda, \mu, \nu \in \mathbb{Z}_4 \right\}$$

For $\kappa_1 = \bar{1}$, $\kappa_2 = \bar{2}$ and $\lambda_1, \lambda_2, \mu_1, \mu_2, \nu_1, \nu_2 = \bar{0}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \subseteq M_2[\mathbb{Z}_4], \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \subseteq M_2[\mathbb{Z}_4]$$

$$\begin{pmatrix} \kappa_1 \star \kappa_2 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 \star 2 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 + D & 0 \\ 0 & 0 \end{pmatrix} \subseteq M_2[D]$$

Let $S = \{\bar{1}, \bar{3}\}$. Take $s = \bar{1}$. Then

$$\begin{pmatrix} 1 \star 1^2 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \{\bar{1}, \bar{3}\} & 0 \\ 0 & 0 \end{pmatrix} \not\subseteq M_2[D]$$

but

$$\begin{pmatrix} 1 \star 2^2 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \subseteq M_2[D]$$

Here $1 \star 1^2 \not\subseteq D$ but $1 \star 2^2 \subseteq D$.

A proof of this example is given below.

Theorem 4. Let $\dot{T}$ be a multiplicative hyperring with identity 1, $S \subseteq \dot{T}$ be a multiplicatively closed subset of

$\dot{T}$ and D be a hyperideal of $\dot{T}$. If $M_n(D)$ is an $M_n(S) = \left\{ \begin{pmatrix} s & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \mid s \in S \right\}$ -2 prime hyperideal of

$M_n(R)$, then D is an S -2 prime hyperideal of $\dot{T}$.

Proof. $M_n(S)$ is a multiplicatively closed subset of $M_n(\dot{T})$. Let $e \star f \subseteq D$ where $e, f \in \dot{T}$. Then

$$\begin{pmatrix} e \star f & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \subseteq M_n(\dot{T})$$

We have

$$\begin{pmatrix} e \star f & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} = \begin{pmatrix} e & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \begin{pmatrix} f & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix}$$

Since $M_n(D)$ is an $M_n(S)$ -2 prime hyperideal, then

$$\begin{pmatrix} e & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix}^2 = \begin{pmatrix} e^2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \subseteq M_n(D)$$

so $e^2 \subseteq D$ or

$$\begin{pmatrix} f & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix}^2 = \begin{pmatrix} f^2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \subseteq M_n(D)$$

$f^2 \subseteq D$.

Since $M_n(D)$ is an $S - 2$ prime hyperideal

$$\begin{pmatrix} s & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \begin{pmatrix} e & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix}^2 = \begin{pmatrix} s & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \begin{pmatrix} e^2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \subseteq M_n(D)$$

or

$$\begin{pmatrix} s & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \begin{pmatrix} f & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix}^2 = \begin{pmatrix} s & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \begin{pmatrix} f^2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \subseteq M_n(D)$$

Hence

$$\begin{pmatrix} s \star e^2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \subseteq M_n(D)$$

or

$$\begin{pmatrix} s \star f^2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \vdots & 0 \end{pmatrix} \subseteq M_n(D)$$

Therefore $s \star e^2 \subseteq D$ or $s \star f^2 \subseteq D$. Hence D is an S -2 prime hyperideal of $\dot{T}$.

□

Let D be a hyperideal of a multiplicative hyperring $\dot{T}$, and let $e \in \dot{T}$. Define $(D : e) = \{r \in \dot{T} \mid r \star e \subseteq D\}$. Then, for every $e \in \dot{T}$, the set $(D : e)$ forms a hyperideal of $\dot{T}$.

Proposition 5. Let $\dot{T}$ be a multiplicative hyperring, $S \subseteq \dot{T}$ a multiplicatively closed subset, and D a hyperideal of $\dot{T}$ with $D \cap S = \emptyset$. If there exists $s \in S$ such that $(D : s)$ is a 2-prime hyperideal of $\dot{T}$, then D is an S -2-prime hyperideal of $\dot{T}$.

Proof. Let $(D : s)$ is a 2 prime hyperideal of $\dot{T}$. Assume that $e \star f \subseteq D$ for some $e, f \in \dot{T}$. Since D is a hyperideal, then $D \subseteq (D : s)$. Thus $e \star f \subseteq (D : s)$, so since $(D : s)$ is a 2-prime, $e^2 \subseteq (D : s)$ or $f^2 \subseteq (D : s)$. Therefore $s \star e^2 \subseteq D$ or $s \star f^2 \subseteq D$. Hence D is an S -2 prime hyperideal of $\dot{T}$.

□

Theorem 5. Let $\dot{T}$ be a multiplicative hyperring with identity 1, and let $S \subseteq \dot{T}$ be a multiplicatively closed subset. Then D is an S -2-prime hyperideal if and only if D/Ψ^* is an S/Ψ^* -2-prime ideal of $\dot{T}/\Psi^*$.

Proof. ($\Rightarrow$): Let D is an S -2 prime hyperideal of $\dot{T}$. Let $x \otimes y \in D/\Psi^*$ where $x, y \in \dot{T}/\Psi^*$. Thus there exists $e, f \in \dot{T}$ such that $x = \Psi^*(e)$ and $y = \Psi^*(f)$. Hence $x \otimes y = \Psi^*(e \star f) \in D/\Psi^*$ and so $e \star f \subseteq D$. Then there exists an $s \in S$ such that $s \star e^2 \subseteq D$ or $s \star f^2 \subseteq D$ since D is an S -2 prime hyperideal of $\dot{T}$. Therefore $\Psi^*(s)\Psi^*(e)^2 \in D/\Psi^*$ or $\Psi^*(s)\Psi^*(f)^2 \in D/\Psi^*$. Thus D/Ψ^* is an S/Ψ^* -2 prime ideal of $\dot{T}/\Psi^*$.

($\Leftarrow$): Assume that D/Ψ^* is an S/Ψ^* -2 prime ideal of $\dot{T}/\Psi^*$. Let $e \star f \subseteq D$ where $e, f \in \dot{T}$. Then $\Psi^*(e \star f) \in D/\Psi^*$. Thus $\Psi^*(e)\Psi^*(f) \in D/\Psi^*$. Thus there exists $s \in S$ such that $\Psi^*(s)\Psi^*(e)^2 \in D/\Psi^*$ or $\Psi^*(s)\Psi^*(f)^2 \in D/\Psi^*$ since D/Ψ^* is an S/Ψ^* -2 prime ideal of $\dot{T}/\Psi^*$. Thus, $\Psi^*(s \star e^2) \in D/\Psi^*$ or $\Psi^*(s \star f^2) \in D/\Psi^*$. Therefore $s \star e^2 \subseteq D$ or $s \star f^2 \subseteq D$. Hence, D is an S -2 prime hyperideal of $\dot{T}$. The proof is complete. □

The following theorem states that if D , a prime hyperideal of $\dot{T}$, is an $f(S)$ -2-prime hyperideal, then its preimage under f is an S -2-prime hyperideal of $\dot{T}$.

Theorem 6. Let $\dot{T}$ be a multiplicative hyperring and let S be a multiplicatively closed subset of $\dot{T}$. Consider a homomorphism $f : \dot{T} \rightarrow \dot{T}$ of hyperrings such that the image $f(S)$ contains no zero element. If D is an $f(S)$ -2-prime hyperideal in $\dot{T}$, then its preimage under f , denoted $f^{-1}(D)$, forms an S -2-prime hyperideal in $\dot{T}$.

Proof. $f(S)$ is a multiplicatively closed subset of $\dot{T}$. Let D be an $f(S)$ -2 prime hyperideal of $\dot{T}$. Hence there exists an $s \in S$ such that for all $\mu, \nu \in \dot{T}$ if $\mu \star \nu \subseteq D$, then $f(s) \star \mu^2 \subseteq D$ or $f(s) \star \nu^2 \subseteq D$. Let $O = f^{-1}(D)$. Hence we have $O \cap S = \emptyset$ since $D \cap f(S) = \emptyset$. Let $c \star d \subseteq O$ where $c, d \in \dot{T}$. Thus $f(e \star f) = f(c) \star f(d) \subseteq D$ which implies that $f(s) \star f(c)^2 \subseteq D$ or $f(s) \star f(d)^2 \subseteq D$. Therefore $f(s \star c^2) \subseteq D$ or $f(s \star d^2) \subseteq D$ since f is a good homomorphism. Hence $s \star c^2 \subseteq f^{-1}(D) = O$ or $s \star d^2 \subseteq f^{-1}(D) = O$. The proof is complete. □

Proposition 6. Let $\dot{T}$ be a multiplicative hyperring, with $S \subseteq \dot{T}$ a multiplicatively closed subset, and let D be a hyperideal of $\dot{T}$ such that D and S are disjoint. Suppose O is a proper hyperideal of $\dot{T}$ containing D , and let $\bar{S}$ denote the image of S in R/D , with $(O/D) \cap \bar{S} = \emptyset$. Then, O is an S -2-prime hyperideal of $\dot{T}$ if and only if the quotient O/D is an $\bar{S}$ -2-prime hyperideal of R/D .

Proof. ($\Rightarrow$): Let O be an S -2 prime hyperideal. Then there exists an $s \in S$ such that for all $e, f \in \dot{T}$ with $e \star f \subseteq O$, then $s \star e^2 \subseteq O$ or $s \star f^2 \subseteq O$. Let $(e + D) \otimes (f + D) \subseteq O/D$ where $e + D, f + D \in \dot{T}/D$. Hence $e \star f \subseteq O$ and, so $s \star e^2 \subseteq O$ or $s \star f^2 \subseteq O$. Thus $(s + D) \otimes (e + D)^2 \subseteq O/D$ or $(s + D) \otimes (f + D)^2 \subseteq O/D$. Therefore, O/D is an $\bar{S}$ -2 prime hyperideal of R/D .

($\Leftarrow$): Since $(O/D) \cap \bar{S} = \emptyset$, we can easily prove that $O \cap S = \emptyset$. Let $e \star f \subseteq O$ where $e, f \in \dot{T}$. Thus

$(e + D) \otimes (f + D) \subseteq O/D$. There exists an $\bar{s} = s + D \in \bar{S}$ such that $(s + D) \otimes (e + D)^2 \subseteq O/D$ or $(s + D) \otimes (f + D)^2 \subseteq O/D$. Hence we conclude $s \star e^2 \subseteq O$ or $s \star f^2 \subseteq O$. $\square$

We will now examine the properties of S -2- prime hyperideals in the cartesian product of multiplicative hyperrings to better understand their behavior under such constructions. In the subsequent theorem, we extend this definition to the product of n multiplicative hyperrings and explore their properties accordingly.

Theorem 7. Let $\hat{T} = \hat{T}_1 \times \hat{T}_2$ and $S = S_1 \times S_2$ where S_1 and S_2 are multiplicatively closed subsets of $\hat{T}_1$ and $\hat{T}_2$, respectively. Suppose $D = D_1 \times D_2$ is a hyperideal of $\hat{T}$. Then D is an S -2 prime hyperideal of $\hat{T}$ if and only if D_1 is an S_1 -2 prime hyperideal of $\hat{T}_1$ with $D_2 \cap S_2 \neq \emptyset$ or D_2 is an S_2 -2 prime hyperideal of $\hat{T}_2$ with $D_1 \cap S_1 \neq \emptyset$.

Proof. ($\Rightarrow$): Let $D = D_1 \times D_2$ is an S -2 prime hyperideal of $\hat{T}$. Since $(1, 0) \star (0, 1) \subseteq D$, there exists $s = (s_1, s_2) \in S$ such that $(s_1, s_2) \star (1, 0)^2 \subseteq D$ or $(s_1, s_2) \star (0, 1)^2 \subseteq D$. Thus $D_1 \cap S_1 \neq \emptyset$ or $D_2 \cap S_2 \neq \emptyset$. We may assume that $D_1 \cap S_1 \neq \emptyset$. As $D \cap S = \emptyset$, we have $D_2 \cap S_2 = \emptyset$. Let $e \star f \subseteq D_2$ for some $e, f \in \hat{T}_2$. Since $(0, e) \star (0, f) \subseteq D$ and D is an S -2 prime hyperideal of $\hat{T}$, $(s_1, s_2) \star (0, e)^2 \subseteq D_2$ or $(s_1, s_2) \star (0, f)^2 \subseteq D_2$. Hence we get $s_2 \star e^2 \subseteq D$ or $s_2 \star f^2 \subseteq D$. Thus D_2 is an S_2 -2 prime hyperideal of R_2 . In the other case, we can easily show that D_1 is an S_2 -2 prime hyperideal of R_1 .

($\Leftarrow$): Conversely, assume that $D_1 \cap S_1 \neq \emptyset$ and D_2 is an S_2 -2 prime hyperideal of R_2 . Therefore there exists an $s_1 \in D_1 \cap S_1$. Let $(e, f) \star (o, d) \subseteq D$ for some $e, o \in \hat{T}_1$ and $f, d \in \hat{T}_2$. This refers to $f \star d \subseteq D_2$ and there exists an $s_2 \in S$ such that $s_2 \star f^2 \subseteq D_2$ or $s_2 \star d^2 \subseteq D_2$. Put $s = (s_1, s_2) \in S$. Thus we get $(s_1, s_2) \star (e, f)^2 \subseteq D$ or $(s_1, s_2) \star (o, d)^2 \subseteq D$. Therefore P is an S -2 prime hyperideal of $\hat{T}$. Similarly if $D_2 \cap S_2 \neq \emptyset$ and D_1 is an S_1 -2 prime hyperideal of $\hat{T}$, then we can achieve a similar conclusion. $\square$

Theorem 8. Let $k \geq 2$, and consider the hyperring $\hat{T} = \hat{T}_1 \times \cdots \times \hat{T}_k$ with $S = S_1 \times \cdots \times S_k$, where each S_i is a multiplicatively closed subset of $\hat{T}_i$. Assume that D_i is a proper hyperideal of $\hat{T}_i$ for every $i = 1, \dots, k$. Define $D = D_1 \times \cdots \times D_k$. Then D forms an S -2-prime hyperideal of $\hat{T}$ if and only if there exists an index $r \in 1, \dots, k$ such that D_r is an S_r -2-prime hyperideal of $\hat{T}_r$, and for all other indices $i \neq r$, the intersection $D_i \cap S_i$ is nonempty.

Proof. Consider a positive integer $k \geq 2$. The statement holds for $k = 2$ by the previous theorem. Assume that the statement is valid for any integer p with $2 \leq p < k$. Let $p = k$ and $\hat{T} = \hat{T}_1 \times \cdots \times \hat{T}_{k-1}$, $\hat{S} = S_1 \times \cdots \times S_{k-1}$ and $\hat{D} = D_1 \times \cdots \times D_{k-1}$. Since $D = \hat{D} \times D_k$, $\hat{T} = \hat{T} \times \hat{T}_k$, and $S = \hat{S} \times S_k$, it follows from the assumption that either D_k is an S_k -2-prime hyperideal of $\hat{T}_k$ with $\hat{D} \cap \hat{S} \neq \emptyset$, or $\hat{D}$ is an $\hat{S}$ -2-prime hyperideal of $\hat{T}$ with $D_k \cap S_k \neq \emptyset$. If $\hat{D} \cap \hat{S} \neq \emptyset$ and D_k is an S_k -2-prime hyperideal of $\hat{T}_k$, then the argument is concluded. Then, assume that $D_k \cap S_k \neq \emptyset$ and $\hat{D}$ forms an $\hat{S}$ -2-prime hyperideal within $\hat{T}$. From induction hypothesis for $p = k - 1$, we have that D_j is an S_j -2- prime hyperideal of $\hat{T}_j$ for some $j \in \{1, 2, \dots, k - 1\}$ and $D_i \cap S_i \neq \emptyset$ for each $i \in \{1, 2, \dots, k - 1\} - \{j\}$. Hence, the necessary condition is satisfied.

For the reverse direction, assume that D_1 is an S_1 -2-prime hyperideal of $\hat{T}_1$ corresponding to some element $\bar{s} \in S_1$, and for each $i \neq 1$, there exists $s_i \in D_i \cap S_i$. Define $s = (\bar{s}, s_2, \dots, s_k)$. Then it follows that $D = D_1 \times \cdots \times D_k$ forms an S -2-prime hyperideal of $\hat{T}$. $\square$

4. Conclusions

Since hyperideals play a central role in multiplicative hyperrings, many researchers have explored their generalizations, focusing particularly on prime, 2-prime, and S -prime hyperideals to better understand the fundamental algebraic properties of these structures. These generalizations provide the motivation for our study, in which we introduce a new hyperideal, S -2- prime hyperideal, that

encompasses the features of both S -prime and 2-prime hyperideals. We examine the fundamental properties of S -2- prime hyperideal in detail and investigate its relationships with other hyperideal classes, highlighting similarities and distinctions. Our results demonstrate that several properties established for prime, 2-prime, and S -prime hyperideals also hold for S -2- prime hyperideal, while it additionally exhibits new characteristics not addressed by previous notions. Moreover, we consider the behavior of S -2- prime hyperideal under various operations, homomorphisms, and ring extensions, and suggest potential avenues for future research, such as exploring weakly S -2-prime hyperideal generalizations or δ -expansion methods to further understand its structure. Overall, our work contributes to a more comprehensive understanding of hyperideals in multiplicative hyperrings and opens new directions for subsequent studies.

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