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Newtonian Gravity of Antimatter Particles and Gravitational Foundation Underlying Field Quantization through Quantum Spacetime Geometrization

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Newtonian Gravity of Antimatter Particles and Gravitational Foundation Underlying Field Quantization through Quantum Spacetime Geometrization

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Abstract: By incorporating quantum mechanics into gravitational theory through the so-called spacetime geometrization procedure that consists in applying the principle of least action alongside the covariance of quantum mechanical motion equations, we present a model that describes the gravitational behavior of antimatter whose existence is fundamentally rooted in quantum mechanics. This approach is based on the fact that the equivalence of gravitational and inertial mass in General Relativity can be replaced by the condition of covariance of classical equations of motion in curved spacetime. The findings suggest that to assign negative values to antimatter particles with non-zero rest mass, we must adopt new concepts for antimatter kinematics in order to preserve the energy conservation. The model indicates that while Newtonian gravity of point-like matter on macroscopic scale may be repulsive on antimatter particles, the observed kinematics appear attractive. Consequently, because we cannot directly measure the gravitational force on antimatter, it is fundamentally difficult to experimentally confirm whether gravity between matter and antimatter is inherently attractive or repulsive. The work also shows that the gravitational effect of non-punctual quantum bodies includes an additional term that is inversely proportional to their mass. The divergence of gravitational energy for infinitesimal masses may provide an explanation for the origin of field quantization in elementary particles, defining fundamental vacuum states with discrete, non-zero mass values.

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Introduction

General Relativity, a form of spacetime geometrization, is derived by utilizing two fundamental conditions: the equivalence of inertial and gravitational masses, and the principle of least action [1]. On the other hand, it is also true that the equivalence of inertial and gravitational masses corresponds to imposing the covariance of the classical equations of motion in curved spacetime. In this sense the general Relativity can be conceptualized as classical spacetime geometrization. If instead of the covariance of the classical motion equation, we assume the covariance of quantum mechanical motion equations with the minimum action condition we obtain a spacetime whose geometry (gravity) is defined by the quantum mechanical physics [2,3]. Doing so, as shown in reference [2,3] we are able to define a quantum gravity equation (QGE) that in the macroscopic limit, for the *k-th* eigenstate, reads

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{(k)\mu\nu} - \Lambda_Q g_{\mu\nu}\right) \qquad (1)$$

where, given the quantum mechanical mass density in spacetime $|\psi|^2$, generated by the wavefunction ψ, of a boson field [2]

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$$\psi_{;\mu}^{;\mu} = \left(g^{\mu\nu}\partial_\nu\psi\right)_{;\mu} = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}\left(g^{\mu\nu}\partial_\nu\psi\right) = -\frac{m^2c^2}{\hbar^2}\psi\,, \tag{2}$$

the energy tensor density $T_{(k)\mu\nu}$ for the eigenstate ψ_k, the stable states at macroscopic scale [4], reads

$$T_{(k)\mu\nu} = -|\psi_k|^2\, m^2c^4\left(\frac{\hbar}{2i}\partial_t\, ln[\frac{\psi_k}{\psi_k{}^*}]\right)^{-1}\left(\left(\frac{\hbar}{2mc}\right)^2\partial_\mu\, ln[\frac{\psi_k}{\psi_k{}^*}]\partial_\nu\, ln[\frac{\psi_k}{\psi_k{}^*}] - \left(1 - \frac{V_{qu(k)}}{mc^2}\right)g_{\mu\nu}\right) \tag{3}$$

where $g_{\nu\mu}$ is the metric tensor, where $g = | g_{\nu\mu} |^{-1}$ and where

$$V_{qu(k)} = -\frac{\hbar^2}{m}\frac{1}{|\psi_{(k)}|\sqrt{-g}}\partial_\mu\sqrt{-g}\left(g^{\mu\nu}\partial_\nu|\psi_{(k)}|\right) \tag{4}$$

is the quantum potential of the generalized Madelung-like hydrodynamic representation, [5–7] of the Klein-Gordon equation [2] that reads

$$g_{\mu\nu}\partial^\nu S\,\partial^\mu S - \hbar^2\frac{1}{|\psi|\sqrt{-g}}\partial_\mu\sqrt{-g}\left(g^{\mu\nu}\partial_\nu|\psi|\right) - m^2c^2 = 0 \tag{5}$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial q^\mu}\sqrt{-g}\left(g^{\mu\nu}|\psi|^2\frac{\partial S}{\partial q^\nu}\right) = 0\,, \tag{6}$$

where

$$S_{(k)} = \frac{\psi_k}{\psi_k{}^*} \tag{7}$$

and where

$$-\partial_\mu S_{(k)} = p_\mu \tag{8}$$

is the 4-impulse,

The off-diagonal terms in the energy density tensor (3) reflect its quantum nature, as effectively explained by the non-commutative field theory [8].

Given the physical description of antimatter provided by quantum mechanics, the gravity of quantum mechanical fields (1-3) naturally incorporates it into gravitational theory. Consequently, performing the (non-relativistic) weak gravity limit of the QGE (1) yields the Newtonian forces for both matter and antimatter.

To derive the classical Newtonian gravity between matter and antimatter, it is necessary to disregard the quantum contributions from the term Λ_Q (by setting $\Lambda_Q = 0$ [2]) and to impose the conditions for the establishing of low energy classical non-relativistic limit [7], which read

$$\hbar \to 0\,, \tag{9}$$

$$V_{qu} \to 0\,, \tag{10}$$

$$\gamma \cong 1\,. \tag{11}$$

leading to

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$$T_{\mu\nu} = T_{\mu\nu(k)} \cong \pm mc^2 |\psi_{(k)\pm}|^2 \left[-g_{\mu\nu} + \left(\frac{\hbar}{2mc}\right)^2 \partial_\mu ln[\frac{\psi_{(k)\pm}}{\psi_{(k)}{}^*{}_\pm}]\partial_\nu ln[\frac{\psi_{(k)\pm}}{\psi_{(k)}{}^*{}_\pm}] \right]$$
$$\cong \pm mc^2 |\psi_{(k)\pm}|^2 \left[-g_{\mu\nu} + \left(\frac{1}{mc}\right)^2 p_{\mu\pm} p_{\nu\pm} \right] \quad (12)$$
$$= \pm mc^2 |\psi_{(k)\pm}|^2 \left[u_{\mu\pm} u_{\nu\pm} - g_{\mu\nu} \right]$$

where $u_\mu = \frac{p_\mu}{mc}$ is the velocity field and plus and minus subscripts refer to the eigenstates of particle and antiparticle, respectively.

Therefore, from (12) the weak gravity limit of QGE reads

$$R_{\nu\mu} - \frac{1}{2} g_{\nu\mu} R_\alpha{}^\alpha = \pm \frac{8\pi G}{c^2} m |\psi_{(k)\pm}|^2 \left(u_{\mu\pm} u_{\nu\pm} \right) \quad (13)$$

leading to the trace identity

$$R_\alpha{}^\alpha - \frac{1}{2} \delta_\alpha{}^\alpha R_\alpha{}^\alpha = -R_\alpha{}^\alpha = \pm \frac{8\pi G}{c^2} m |\psi_\pm|^2 u_\alpha u^\alpha . \quad (14)$$

Thus, in the particles reference system where

$$u_{\mu_+} = u_{\mu_-} = u_\mu = (1,0,0,0), \quad (15)$$

it follows that

$$R_0{}^0 = \pm \frac{8\pi G}{c^2} m |\psi_\pm|^2 \quad (16)$$

3.1. The Antimatter Newtonian Field

Given the Newtonian gravitational potential ϕ, as a function of the component g_{00} of the metric tensor [9]

$$\frac{2\phi}{c^2} = g_{00} - 1, \quad (17)$$

whose trace, at zero order, reads

$$g_{\alpha\alpha} \cong -2, \quad (18)$$

Equation (16) for two free, quantum decoupled, classical particles [9] such as

$$|\psi|^2 = |\psi_+|^2 + |\psi_-|^2 \quad (19)$$

leads to the identity

$$R_0{}^0 = \frac{4\pi G}{c^2} m\left(|\psi_+|^2 - |\psi_-|^2\right) = R_{00} = \frac{\partial \Gamma^\alpha{}_{00}}{\partial q^\alpha} \approx -\frac{1}{2} \partial_\alpha \left(g_{\gamma\gamma} \partial_\alpha g_{00} \right) = \frac{1}{c^2} \partial_\alpha \partial_\alpha \phi \quad (20)$$

and therefore, to

$$\partial_\alpha \partial_\alpha \phi_\pm = \pm 4\pi G m |\psi_\pm|^2 . \quad (21)$$

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We have introduced the macroscopic condition by assuming that the typical inter-particles distance is much bigger than both the physical length of their quantum structure and the range of interaction of the quantum potential. In this limit, we can assume the particle and the antiparticle are point-like and located in R_+ and R_-, respectively, with spatial densities

$$|\psi_+|^2 = \delta^3(r - R_+) \tag{22}$$

$$|\psi_-|^2 = \delta^3(r - R_-), \tag{23}$$

with the normalized condition

$$\iiint \delta^3(r - R_\pm)dV = 1, \tag{24}$$

so that (21) reads

$$\partial_\alpha \partial_\alpha \phi_\pm = \pm 4\pi G m \delta^3(r - R_\pm) \ , \tag{25}$$

leading to

$$\begin{aligned} \iiint \partial_\alpha \partial_\alpha \phi_\pm V = \oiint \partial_\alpha \phi_\pm \cdot dS^\alpha = \int_0^{4\pi} \frac{\partial \phi_\pm}{\partial r} \cdot (r - R_\pm)^2 \, d\Omega \\ = \frac{\partial \phi_\pm}{\partial r} 4\pi (r - R_\pm)^2 = \pm 4\pi G m \iiint \delta^3(r - R_\pm)dV = \pm 4\pi G m \end{aligned} , \tag{26}$$

and by integration to

$$\phi_\pm = -(\pm) G \frac{m}{|r - R_\pm|} \ . \tag{27}$$

3.2. Newtonian Force Between Point-Like Matter and Antimatter Particles

Given the negative energy values (1.14) of antimatter, also the energy tensor density (12) becomes negative leading to the repulsive antimatter Newtonian potential as given by (27)

$$\phi_- = G \frac{m_-}{r - R_-} . \tag{28}$$

Nonetheless, in order to determine the characteristics of the antimatter gravity, its kinematics must also be taken under consideration.

Given that the force applied to antimatter particle with mass m_- (with negative valued energy), for energy conservation, generates an acceleration in the opposite direction, so that

$$F = -m_- \ddot{r} = -\partial_R U_-, \tag{29}$$

it follows that the antimatter Newtonian force (3. 22) on an antimatter particle test m_- results repulsive

$$F = -m_- \partial_r \phi_- = G \frac{m_- \, m_-}{(r_- - R_-)^2}, \tag{30}$$

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but the acceleration of the antiparticles

$$\ddot{r}_- = -G\frac{m_-}{(r_- - R_-)^2} = \partial_r \phi_- \tag{31}$$

results attractive (even if the force is repulsive).

Therefore, to determine the force between matter and antimatter, we must recognize that, in the matter-antimatter interaction, the force on the antiparticle does not comply with the third law of dynamics, which requires that it be equal in magnitude and opposite in direction to the force on the particle. Consequently, if we calculate the variation of the total energy with respect to the displacement of the particle and antiparticle, this variation is zero.

However, if we consider the apparent "kinematic" forces acting on the antimatter, expressed as $F_{kin} = -F = m_-\ddot{r} = \partial_R U_-$ where U_- represents the gravitational field energy of the antiparticle, the validity of the third law is restored. Thus, the kinematic force derived from the variation in the spatial position of the matter and antimatter particles is given by:

$$\begin{aligned} F_r &= -\partial_R U_+ + \partial_R U_- = -\partial_R \sum_{i,j} \frac{1}{2}\int m_i \phi_{j\neq i} dV = -\frac{1}{2}\partial_R\left(\int\int m_-\phi_+ dV - m_+\phi_- dV\right) \\ &= -\frac{1}{2}\partial_R\left(m_-\int \delta(r-R_-)\phi_+ dV - m_+\int \delta(r-R_+)\phi_- dV\right) \\ &= \frac{G}{2}\partial_R\left(m_+m_-\int \delta(r-R_-)\frac{1}{|r-R_+|}dV + m_-m_+\int \delta(r-R_+)\frac{1}{|r-R_-|}dV\right) \\ &= G\frac{m_-m_{R+}}{2}\partial_R\left(\frac{1}{|R_+-R_-|} + \frac{1}{|R_--R_+|}\right) = -G\frac{m_-m_+}{R^2} \end{aligned} \tag{32}$$

where $R = |R_+ - R_-|$. Equation (32) demonstrates that the kinematic forces between matter and antimatter are attractive. Consequently, the antiparticles experience a repulsive force.

3.3. Newtonian Gravity at Short Distance Between Two Quantum Bodies

The results (29-32) are valid as far as the wave function localization produces a mass distribution that is satisfactory well described by the Dirac's delta respect the scale of the problem so that the results are appropriate for sufficiently large distance between particles (typical of the classical macroscopic approach). On very short distance, when the physical length of the problem is of order of the quantum body mass distribution, the gravitational interaction is influenced by the quantum mass density distribution $|\psi|^2$.

Here we consider the case of quantum bodies (sufficiently large) to be described by continuous fields. Furthermore, the gravity interaction is derived by assuming the particle densities are much lighter than the Planck mass in a cube of planch length side, so that (at zero order of approximation) the spacetime can be assumed Minkowskian. Then, the gravitational force is derived by the first order curvature produced by such mass distributions.

Before deriving the gravity of a quantum mass distribution localized around a center of attractive force it must be observed that, in the classical acceptation, the gravitational force is that one experienced by a test particle that does not alter the gravitational potential. In the quantum case this is not possible since the presence of the test particle alters the gravity field of the reference particle due to their non-linear coupling in the QGE. Thence, the gravitational potential, as conceptualized in the Newtonian limit independent by the mass of the particle on which it acts, disregards this non-linearity of the QGE and derives by the gravitational fields generated by the mass density distribution of each single particle, disregarding the mutual, very small, perturbation.

Therefore, we are in the position to derive the gravitational potential of a particles as deriving by the curvature that its mass distributions $|\psi|^2$ alone produce through the gravity equation

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$$R_{\nu\mu}-\frac{1}{2}g_{\nu\mu}R_{\alpha}^{\ \alpha}=\frac{8\pi G}{c^4}\frac{mc^2|\psi|^2}{\gamma}\left(\begin{array}{l}\left(\sqrt{1-\frac{V_{qu}}{mc^2}}-1\right)g_{\mu\nu}-\Lambda_Q g_{\mu\nu}\\+\sqrt{1-\frac{V_{qu}}{mc^2}}^{-1}\left(\frac{\hbar}{2mc}\right)^2\partial_\mu ln[\frac{\psi}{\psi*}]\partial^\lambda ln[\frac{\psi}{\psi*}]g_{\lambda\nu}\end{array}\right).$$

$$=\frac{8\pi G}{c^4}\frac{mc^2|\psi|^2}{\gamma}\left(\begin{array}{l}\left(\sqrt{1-\frac{V_{qu}}{mc^2}}-1\right)g_{\mu\nu}-\Lambda_Q g_{\mu\nu}\\+\sqrt{1-\frac{V_{qu}}{mc^2}}^{-1}\left(\frac{\hbar}{2mc}\right)^2 p_\mu p^\lambda g_{\lambda\nu}\end{array}\right) \quad (33)$$

Here, for sake of completeness, we also consider the contribution that can come from the quantum pressure $-\Lambda_Q g_{\mu\nu}$ where the cosmological-like term Λ_Q, whose expression is given in ref. [2], reduces to a small constant] in quasi-Minkowskian spacetime approximation [10]. Therefore, Equation (33) for the classical case $\frac{V_{qu}}{mc^2}\ll 1$ leads to

$$R_{\nu\mu}-\frac{1}{2}g_{\nu\mu}R_{\alpha}^{\ \alpha}=\frac{8\pi G}{c^4}\frac{mc^2|\psi|^2}{\gamma}\left(\begin{array}{l}-\left(\frac{V_{qu}}{2mc^2}+\Lambda_Q\right)g_{\mu\nu}\\+\left(1+\frac{V_{qu}}{2mc^2}\right)u_\mu u_\nu\end{array}\right). \quad (34)$$

Furthermore, by using the identity

$$-\left(\frac{V_{qu}}{2mc^2}+\Lambda_Q\right)\left(\delta_\alpha^\alpha-u_\alpha u^\alpha\right)=-3\left(\frac{V_{qu}}{2mc^2}+\Lambda_Q\right) \quad (35)$$

it follows that

$$-R_{\alpha}^{\ \alpha}=\frac{8\pi G}{c^4}mc^2|\psi|^2\left(1-3\left(\frac{V_{qu}}{2mc^2}+\Lambda_Q\right)\right) \quad (36)$$

leading to the gravitational potential

$$\partial_r\phi=\frac{Gm}{(r-R)^2}\int_0^V|\psi|^2\left(1-3\left(\frac{V_{qu}}{2mc^2}+\Lambda_Q\right)\right)d^3V \quad (37)$$

where, by utilizing (1.16), the classical and quantum parts read, respectively,

$$\partial_r\phi_{Class}=\frac{Gm}{\left(r-R_j\right)^2}\left(\int_0^{(r-R)}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}|\psi|^2_{(r-R,\vartheta,\varphi)}(r-R)^2 d(r-R)cos\,\vartheta d\vartheta d\varphi\right) \quad (38)$$

$$\partial_r\phi_Q=\frac{3}{2}\frac{\hbar^2}{mc^2}\frac{G}{\left(r-R_j\right)^2}\left(\int_0^{(r-R)}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}|\psi|_{(r-R,\vartheta,\varphi)}(r-R)^2\left(\partial_\mu\partial^\mu|\psi|+2mc^2|\psi|\Lambda_Q\right)d(r-R)cos\,\vartheta d\vartheta d\varphi\right). \quad (39)$$

It is worth mentioning that the quantum contribution (39) becomes larger smaller the particle mass leading to the asymptotical expression

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$$lim_{m\to 0}\partial_r\phi_Q \approx \frac{3}{2}\frac{\hbar^2}{mc^2}\frac{G}{\left(r-R_j\right)^2}\left(\int_0^{(r-R)}\int_{-\pi/2}^{\pi/2}\int_0^{2\pi}|\psi|_{(r-R,\vartheta,\varphi)}\left(r-R\right)^2\partial_\mu\partial^\mu|\psi|\,d\left(r-R\right)\cos\vartheta d\vartheta d\varphi\right) \quad (40)$$

From (40) it is important to highlight that, to prevent gravitational energies from diverging, the mass of particles cannot decrease continuously to zero but must be quantized with minimum values. This implies the existence of elementary particles and the necessity of quantizing their fields, pointing to a gravitational constraint that drives field quantization. Furthermore, this hypothesis supports the notion that only fields require quantization, while gravity itself, as defined by the left side of gravitational equation (1), becomes indirectly a quantum field through the equivalence to the quantized fields on the right side of (1) [11].

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