Reconciling the Cosmological Constant with the Energy Density of Quantum Field Theories of the Zeropoint

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Abstract

This paper results from our investigation into novel means of electromagnetic propulsion. It requires the basis of our claims to be put on a sound theoretical footing regarding the purported momentum exchange with the electromagnetic field. One of these concerns is the huge discrepancy between the energy density of the Zeropoint and its purported manifestation as the Cosmological Constant. Here we state that it is manifestly wrong to introduce the zeropoint at zero order into the stress-energy tensor, because it is something which describes zero particle count. As a fluctuation, it belongs in a higher order Taylor expansion in frequency of the stress-energy tensor. Furthermore in the 3rd order in the Einstein constant our procedure is some 9 orders of magnitude too small. We make up this difference by suggesting that vacuum energy is much higher still and that more degrees of freedom exist in physics beyond the Standard Model or that there is interaction energy between the modes.

Keywords: Zeropoint Energy, Cosmological Constant, Stress Energy Tensor, Einstein Field Equations, Standard Model, Dark Energy, Quantum Gravity.

1. Introduction

We have been seeking to put a putative electromagnetic propulsor[1], which is based on the Feynman/Heaviside Disk/static field momentum conjecture[2-6], on a sound theoretical footing[7-9] and as such, believe it viable to talk about "dumping" momentum to the "zeropoint" of the electromagnetic field. We are in the process of trying to show how the zeropoint behaves like a superfluid or supersolid with mechanical properties, such as the ability propagate waves and thermalize momentum imparted to it from the propulsor. The effect of zeropoint fluctuations is not contested, it is behind the physics of spontaneous emission, the Lamb Shift, Van der Waals forces[10] but short of new physics to explain Dark Energy and universal expansion, the zeropoint is seen as the explanation for this... save a huge difference[11] in the magnitude of this energy density compared to the cosmological constant[†]. Understanding this has then fallen within the remit of the electromagnetic propulsion project.

Particle physics has been described as ever more cunning applications of the quantised harmonic oscillator[12]; the basic Hamiltonians of quantised harmonic oscillators for boson and fermion fields with their ladder operators are listed here:-

$$H_B = \frac{1}{2}\hbar\omega\left(a^\dagger a + aa^\dagger\right) = \left(a^\dagger a + \frac{1}{2}\right)\hbar\omega \quad \text{eqn. 1}$$

$$H_F = \frac{1}{2}\hbar\omega\left(b^\dagger b - bb^\dagger\right) = \left(b^\dagger b - \frac{1}{2}\right)\hbar\omega \quad \text{eqn. 2}$$

Quantum mechanics involves differences in energy, so the zeropoint terms don't matter, even then there is normal ordering[10] to remove these terms. However in General Relativity it would seem that the absolute value of the energy density of these fields is relevant by the central equation,

$$\mathbf{R}_{\mu\nu} - \frac{1}{2}R\mathbf{g}_{\mu\nu} = \frac{8\pi G}{c^4}\mathbf{T}_{\mu\nu} \quad \text{eqn. 3}$$

And this energy density is of the order[13, 14],

$$\rho_{0,boson} = \frac{\hbar}{2}\left(\frac{1}{2\pi}\right)^3 \int_0^K k^2\sqrt{k^2+m^2}\,dk$$
$$\rho_{0,fermion} = -2\frac{\hbar}{2}\left(\frac{1}{2\pi}\right)^3 \int_0^K k^2\sqrt{k^2+m^2}\,dk \quad \text{eqns. 4}$$

Where K is a momentum cut-off. If the masses are neglected, the integrals can be estimated by integrating to the Planck Frequency:-

$$\rho_0 = \frac{\hbar}{8\pi^2 c^3}\int_0^{\omega_P} \omega^3 d\omega \quad \text{eqn. 5}$$

And this figure is huge, of the order of 10^{113} J/m^3 (later we'll argue it should be higher still). Pauli[13] argued by relativistic invariance of the ideal gas representing the zeropoint, that there would be a momentum cut-off thus,

$$\int_0^K k^2\sqrt{k^2+m^2}\,dk = \frac{K^4}{4} + \frac{m^2K^2}{4} - \frac{m^4}{4}\log\frac{2K}{m} + O\left(\frac{1}{K}\right) \quad \text{eqn. 6}$$

[†] $\rho_{QFT} \sim 10^{113}\,J/m^3, \rho_{CC} \sim 10^{-9}\,J/m^3$ respectively.

And then sought to cancel the positive zeropoint of boson fields against the negative zeropoint of fermion fields by the constraint that the number of types of fermion particles is twice that of boson particles (factor of 2 in fermion contribution eqns. 4) by these requirements‡,

$$\sum_i \left(m_0^i\right)^2 = 2\sum_j \left(m_{1/2}^j\right)^2$$
$$\sum_i \left(m_0^i\right)^4 = 2\sum_j \left(m_{1/2}^j\right)^4 \qquad \text{eqns. 7}$$
$$\sum_i \left(m_0^i\right)^4 \log m_0^i = 2\sum_j \left(m_{1/2}^j\right)^4 \log m_{1/2}^j$$

The zero mass of the photon of the electromagnetic field would dominate the LHS; such a cancellation is impossible and the zeropoint is of the order given by eqn. 5.

The homogenous, isotropic zeropoint, with no preferred frame, is represented in the stress-energy tensor as an ideal fluid with $\rho_0 = -p$,

$$\mathbf{T}_{\mu\nu} = \begin{pmatrix} \rho_0 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \qquad \text{eqn. 8}$$

The trace of the tensor is,

$$\rho_0 - 3p = -2p \qquad \text{eqn. 9}$$

The units of the LHS of the Einstein Field Equations (EFEs, eqn. 3) are m^{-2} and a contraction on both indices gives:

$$R = -\frac{8\pi G}{c^4} T \qquad \text{eqn. 10}$$

The scalar curvature, R, is basically saying that a sphere of dust around a gravitating source would decrease in volume; specifically if the positive mass-energy in the $T_{00} > \sum T_{ii}$ terms on the RHS, it would cause the surface area and volume to decrease (eqn. 9). Conversely, in a region where the negative pressure of the zeropoint dominates positive mass-energy, space would expand and that is exactly what we observe in Hubble expansion of the Universe.

2. A Glaring Error in current approaches and then some Numerological Speculation

The huge disparity (at least 10^{120}) between the Cosmological Constant and the vacuum energy density of quantum field theories (QFT) has been described as the most embarrassing in physics, if not *all* of science. The cancellation program discussed in the introduction seems doomed to fail, so we must embrace the huge vacuum energy and somehow reconcile it with the absolute energy requirements of General Relativity.

However, if the stress-energy tensor of a particle moving along a trajectory $\mathbf{x}_{traj}(t)$ is given by§,

$$\mathbf{T}_{\mu\nu}(\mathbf{x},t) = \frac{E}{c^2} v_\mu v_\nu \delta\left(\mathbf{x} - \mathbf{x}_{traj}(t)\right)$$
$$\text{with } v_\mu = \left(1, \frac{d\mathbf{x}_{traj}(t)}{dt}\right) \qquad \text{eqn. 11}$$

then it is a glaring error by the research community to include expressions for vacuum energy densities in the zeroth order in the stress-energy tensor (density) – it makes no sense representing something that has zero particle count, unlike eqn. 11 where $E = n\hbar\omega$ is implicit for quanta of particles. It is only correct to consider the zeropoint as a *fluctuation* - it has variance but no average. We should write the stress-energy tensor as a Taylor series somehow in particle number and frequency. As the electromagnetic zeropoint will dominate, which was discussed towards the end of the introduction, eqn. 5 shall be used.

A few considerations make the process of writing the Taylor series even easier: space is approximately flat away from gravitating sources so we don't need the covariant derivative, we don't need to take the derivative with-respect-to the wave-4-vector either, as eqn. 5 has no wave-vector terms and so we can differentiate solely by frequency. Thus LHS of eqn. 3 is represented by $\mathbf{E}_{\mu\nu}(\omega)$, $\kappa = \frac{8\pi G}{c^4}$ and some mystery term U (to be discussed) so,

$$\mathbf{E}_{\mu\nu}(\omega)U^0 = \mathbf{E}_{\mu\nu}(\omega)\big|_0 U^0 + U^{-1}\frac{d\mathbf{E}_{\mu\nu}(\omega)}{d\omega}\Delta\omega + U^{-2}\frac{1}{2}\frac{d^2\mathbf{E}_{\mu\nu}(\omega)}{d\omega^2}(\Delta\omega)^2 + \ldots$$
$$\Rightarrow$$
$$\mathbf{E}_{\mu\nu}(\omega)U^0 = \kappa U^0\mathbf{T}_{\mu\nu}(\omega) + \kappa^2 U^{-1}\frac{d\mathbf{T}_{\mu\nu}(\omega)}{d\omega}\Delta\omega + \frac{1}{2}U^{-2}\kappa^3\frac{d^2\mathbf{T}_{\mu\nu}(\omega)}{d\omega^2}(\Delta\omega)^2 + \ldots$$

eqn. 12

Our motivation is the purely numerical observation that the magnitude of $\left|\kappa^3\rho_{QFT}\right|_{magn}$ is in the ballpark of ρ_{cc}.

‡ Visser[14] shows the above as a direct consequence of Lorentz invariance, the finiteness of the zeropoint arises directly from this without a momentum cut-off.

§ Note this is not a tensor density as per EFE. We are interested in the form and appeal by analogy here.

The zeroth order term (in $E_{\mu\nu}(\omega)$) on the RHS in regards to our cosmological constant problem is properly zero. The first order term would sum over positive and negative frequencies such that $\Delta\omega = 0$ and the first order term is zero too (it could also be quashed by U). This follows for every even power of $\Delta\omega$ too.

The 2nd order term is the variance in the fluctuation of the zeropoint and $(\Delta\omega)^2 > 0$, which by the Energy-time Uncertainty Principle is easily calculated to be of the order of the Planck frequency, so nothing changes from eqn. 5. However, what is U?

1. A device to mop up dimensional slackness (U^{-1} units N or J/m).
2. A device that raised to power of zero has no effect on traditional EFE and brings in particle number in the zeroth order term.
3. A device to knock out the 1st power term in κ (though this can be quashed with our argument about $\pm\Delta\omega$ too).
4. A device that makes the fluctuation of the zeropoint relevant in 3rd power of κ (2nd order in **T**) and ignores particle number count on all terms but the zeroth order (as already mentioned).
5. A device to fix $U^{-3}\kappa^3\rho_{QFT} = \rho_{cc}$ (alternatively make ρ_{QFT} even bigger).
6. A scalar, a scaling of the identity matrix, a tensor, an operator?

Appendix 1 interprets what U could be but we will run with the idea for now and the notion that the zeropoint might be some 10^9 times bigger – it certainly shouldn't be present in the EFE stress-energy tensor (as it is wanted at present) to the zeroth order, so what harm is there in suggesting it has a different value?

3. Is the Zeropoint much bigger?

We have seen that between the zeropoint energies of the bosonic and fermionic fields that the bosonic dominates (eqn. 5 and eqn. 6) and could be the source of dark energy responsible for universal expansion. Section 2 concluded that, if our approach is correct, it should be some 10^9 orders of magnitude higher still, maybe a factor of 1000 per spatial dimension. Where could this enter?
New massive and hence short-ranged fields would be out of the question, as already discussed, for reasons of mass (eqn. 6). We might then look to physics beyond the Standard Model[12]; perhaps a 5th long ranged force exists? Then, Grand Unified Theories (GUTs) suggest running coupling constants merge down from around 100 Planck lengths (figure 1), so perhaps down from there is the realm of String Theory. The contribution of the missing 10^9 might be found there.

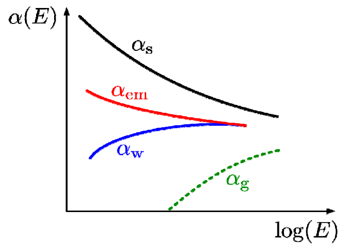

Figure 1 – Running coupling constants merging in GUTs

For instance, String Theory[12] says there are 11 dimensions: 1 time and 10 spatial dimensions: 3 space, 7 compactified. This is approximately 1 spatial dimension + 2 compactified per spatial dimension. We have 10^9 to make up, so the deficit might be made up in those degrees of freedom. Most of the zeropoint energy arises at small length scales.

We have an alternative suggestion: although eqn. 5 is calculated assuming free fields, there might be interaction between the zeropoint modes such that the zeropoint becomes:

$$H_{zpe} = \frac{1}{2}\left(\hbar\omega_1 + \hbar\int I(\omega_1, d\omega)\,d\omega\right) \quad \text{eqn. 13}$$

The suggestion is that the mode ω_1 in question is somehow convolved with the rest of the fluctuations from the other modes with an interaction term. Let us look into how this interaction term might arise.

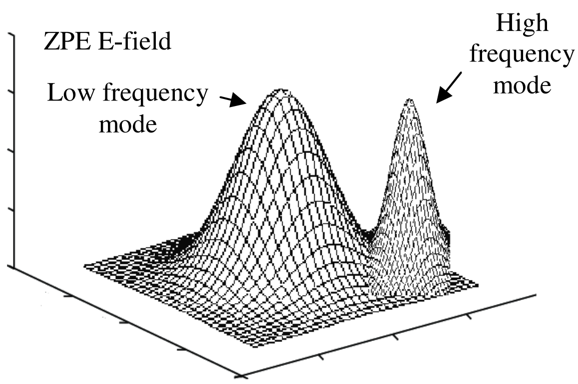

Figure 2 – Modes forming "virtual sources" – low and high frequency modes overlapping in space

The electromagnetic field is modelled as a sum of wave modes[10, 15] in three dimensions (eqns. 15). When quantised by the Uncertainty Principle it has a variance at zero photon count but no average.

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Now the energy (Hamiltonian) of the electromagnetic field is given by[2, 16, 17],

$$H = \frac{1}{2}\varepsilon_0 \int_V \left(E^2 + c^2 B^2\right) dV \qquad \text{eqn. 14}$$

The electric field operator is found by the usual procedure for quantising the electromagnetic field[10, 15],

$$\mathbf{E}(\mathbf{r},t) = i\sqrt{\frac{\hbar\omega}{2V\varepsilon_0}}\sum_{\mathbf{k}\lambda}\left(a_{\mathbf{k}\lambda}(t)e^{i\mathbf{k}\cdot\mathbf{r}} - a^{\dagger}_{\mathbf{k}\lambda}(t)e^{-i\mathbf{k}\cdot\mathbf{r}}\right)$$

$$\mathbf{B}(\mathbf{r},t) = i\sqrt{\frac{\hbar}{2\omega V\varepsilon_0}}\sum_{\mathbf{k}\lambda}\mathbf{k}\times\left(a_{\mathbf{k}\lambda}(t)e^{i\mathbf{k}\cdot\mathbf{r}} - a^{\dagger}_{\mathbf{k}\lambda}(t)e^{-i\mathbf{k}\cdot\mathbf{r}}\right) \qquad \text{eqns. 15}$$

ω is associated with each mode **k** via $\omega = ck$

And note is made of the magnitude of the zeropoint electrical field,

$$\mathrm{E}_{zpe} = \sqrt{\frac{\hbar\omega}{2V\varepsilon_0}} \qquad \text{eqn. 16}$$

It is an easy matter to relate this and eqns. 15 via eqn. 14 to eqn. 5 by the prescription[18]:

$$\sum_{k\lambda} \to \sum_{\lambda}\left(\frac{L}{2\pi}\right)^3 \int d^3k$$

So summing over the zeropoint modes,

$$\frac{1}{V}\sum_{k\lambda}\frac{1}{2}\hbar\omega_k = \frac{2}{8\pi^3}\int d^3k \frac{1}{2}\hbar\omega_k = \frac{4\pi}{8\pi^3}\int dk k^2 \frac{1}{2}\hbar\omega_k$$
$$= \frac{\hbar}{8\pi^2 c^3}\int \omega^3 d\omega \ (\text{ eqn. 5})$$

Although eqns. 15 suggest the B and E-fields at a point and time in space, the Uncertainty Relations suggest that nearby points will be correlated. Figure 2 shows this concept with the field falling off as Gaussians[8] of a low frequency mode with an high frequency mode “invading” its space. A strange way of thinking about this, even though there are no sources, is that the field and its fall-off constitute the effect of a “virtual source”; then similar to calculating the mutual electrostatic energy of a charge density in an electric field,

$$E_{\text{int}} = \frac{1}{2}\int_V \rho\phi dV \qquad \text{eqn. 17}$$

The mutual electrostatic energy of the modes (k_1, k_2) is calculated (there is no magnetic work as a function of distance),

$$E_{\text{int}} = \frac{\varepsilon_0}{2}\int_V E_{k_1}E_{k_2} dV \qquad \text{eqn. 18}$$

Furthermore, to obtain the total interaction energy for a given k_1, we must integrate over all of k_2 and for any position k_2 (higher frequency mode, figure 2) may be. This type of convolution calculation is more profitably carried out in the Fourier domain of momentum (k-space) and frequency (ω-space) by Parseval-Wiener-Khinchin theorem[19],

$$\frac{\varepsilon_0}{2}\int_V E_{k_1}E_{k_2}dV =$$
$$\int_{1/k_{Planck}}^{1/k_1}\left[\int_{k_1}^{k_{Planck}} E_{k_1}(r_0,t_0)\frac{1}{\sqrt{2\pi}\sigma_{k_1}}e^{-\frac{(k-k_1)^2}{2\sigma_{k_1}^2}} E_{k_2}(r_0+r_2,t_0)\frac{1}{\sqrt{2\pi}\sigma_{k_2}}e^{-\frac{(k-k_2)^2}{2\sigma_{k_2}^2}} e^{-ikr_2}dk\right]dr\cdot$$
$$\int_{1/\omega_{Planck}}^{1/\omega_1}\left[\int_{\omega_1}^{\omega_{Planck}} E_{\omega_1}(r_0,t_0)\frac{1}{\sqrt{2\pi}\sigma_{\omega_1}}e^{-\frac{(\omega-\omega_1)^2}{2\sigma_{\omega_1}^2}} E_{\omega_2}(r_0,t_0+t_2)\frac{1}{\sqrt{2\pi}\sigma_{\omega_2}}e^{-\frac{(\omega-\omega_2)^2}{2\sigma_{\omega_2}^2}} e^{ikt_2}d\omega\right]dt$$

eqn. 19

The square bracketed terms above represent the Gaussian spread in k and ω-space of the electric field adjacent to the first and second modes $\mathbf{E}_{k_1}(\mathbf{r},t)$ and $\mathbf{E}_{k_2}(\mathbf{r},t)$ from the Gaussian spread in r and t, which ultimately come from the Uncertainty Relations. The second mode that is interacting with the first is displaced in time and space and this is shown by the Fourier shift theorem[19] by the factors e^{-ikr_2} and e^{ikt_2}. Note that the latter factor is positive in the exponent, as causally only future time event will affect our measurement window. The integrals over *dk* and *dω* then is the Parseval-Wiener-Khinchin theorem and the energy calculation. The outer integrals over *dr* and *dt* allow us to move the second mode in space-time (figure 2) and sum for all possible arrangements. The inner square bracket terms form a 2-degree Chi-squared type distribution and we shall return in a later paper with a more detailed calculation, save to say that the size of these terms is of the order 1 or so.

The outer integrals can be approximated and rendered dimension-less by expressing distance in units of reciprocal wave-vector and time as reciprocal frequency units. Varying k_2 and ω_2 near to or greater than k_1 and ω_1, the exponential terms are nearly constant and the outer integrals reduce to ω_1/k_1 whose magnitude is *c*, that is some $3\text{x}10^8$. Thus, following the procedure used to obtain eqn. 5 we re-write it,

$$\rho_0 = \frac{\hbar}{8\pi^2 c^3}\int_0^{\omega_P}\left(|c|\, factor\right)\omega^3 d\omega \qquad \text{eqn. 20}$$

($|c|$ indicates just the magnitude and “factor” is a small constant from the inner brackets of eqn. 19). We believe that this supplies the missing 10^9 to complete the 120 orders of magnitude problem as expected from the Taylor expansion method $U^{-3}\kappa^3\rho_{QFT} = \rho_{cc}$ in this paper and the measured size of the Cosmological Constant**.

** See the footnote on the first page of this paper.

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Conclusion

The well-known Pauli theory on zeropoint says that the mass-less electromagnetic photon must have the dominant zeropoint over massive fermion fields and so we concentrated on the former to establish a relation between it and Dark Energy by a model and modification to the stress-energy tensor: it simply isn't correct to attempt to put the zero photon count of the electromagnetic zeropoint into the tensor at zeroth order.

In looking at the theoretical underpinnings of a novel propellant-less electromagnetic propulsion engine, the author has been forced to look at the reality of the zeropoint. What emerges is that the zeropoint field appears to be a superfluid or supersolid, with the interaction between the modes forming this "material" giving the system more degrees of freedom, such that the zeropoint energy is some 10^9 higher than currently calculated.

Fortuitously a direct relation between the zeropoint of quantum field theories and the observed vacuum constant (or Dark Energy) of Astronomy and Cosmology has been obtained by a polynomial expansion of the stress energy tensor in the Einstein constant to 3rd order and a Taylor expansion of the said tensor too to 2nd order. Coupled with the interaction between the modes, the ghastly chasm of at least 10^{120} orders of magnitude has been made up.

The "numerology" of the model may seem arbitrary but in its favour it is *physical* and not merely mathematical trickery: it reasonably asserts that there is interaction between the modes of the zeropoint and the model drops zeropoint energy from the stress-energy tensor in the zeroth order (and concerns over why it doesn't severely gravitate or expand the universe) and introduces it as a fluctuation in the second order $(\Delta\omega)^2$ which is correct and looks promising, as the zeropoint has a variance but no average.

We shall follow this paper with another looking at the zeropoint as a liquid-like medium to which momenergy can be dissipated – as per our requirements to give a theoretical underpinning to our putative propulsion device.

Appendix 1 – Dimensional Analysis of "U" and its interpretation

Dimensional analysis of U, the factor introduced into eqn. 12 means it has units of Newtons^{-1} or meters/Joules. This is very similar to the spring constant (an inverse spring constant in case of U),

$$dF_x = K_x dx$$

If we extend this to two dimensions:

$$dF_x \times dF_y = \left(K_x \otimes K_y\right) dx \times dy$$

The tensor $\left(K_x \otimes K_y\right)$ formed provides notions of a flexural rigidity tensor that describes the deforming of area $dA = dx \times dy$. Thus if U^{-2} had units $[N]^2/[M]^2$ in the 3rd term in eqn. 12, it would mean it is some kind of flexural rigidity but taken together with the 3rd order in κ (the Einstein-Newton constant) it becomes (maybe) just a modified Einstein-Newton constant,

$$\mathbf{E}_{\mu\nu} = \frac{1}{2} U^{-2} \kappa^3 \frac{d^2 \mathbf{T}_{\mu\nu}(\omega)}{d\omega^2} (\Delta\omega)^2$$

$$[m]^{-2} = [N]^2 \cdot [N]^{-3} \cdot [N][m]^{-2}$$

(eqn. 11)

compare with

$$\frac{1}{dA} = K^2 \frac{1}{dF_x \times dF_y}$$

$$[m]^{-2} = [N/m]^2 \cdot [N]^{-2}$$

The first option with U as a flexural rigidity constant suggests that eqn. 12 has something to do with deforming an infinitesimal area element of spacetime against an intrinsic pressure and that this is somehow related to zeropoint fluctuations. This may have something to do with the "elasticity of spacetime" too, as Sakharov put it[20]; both effects seem related to the same source - ρ_{QFT} .

Yet, mundanely, we intended to relate ρ_{QFT} to ρ_{cc} thus, $U^{-3}\kappa^3 \rho_{QFT} = \rho_{cc}$. Also ρ_{QFT} was *increased* (non-arbitrarily or trivially but given extra degrees of freedom) to make this true. U, can pick up this increase or just maintain the units and do nothing. So,

$$G = \frac{c^4}{8\pi} U \cdot \sqrt[3]{\frac{\rho_{cc,measured}}{\rho_{QFT}\left(\hbar, \pi, \omega_{Planck}\right)}}$$

The hypothesis in this paper was built to relate $\rho_{cc,measured}$ directly to $\rho_{QFT}\left(\hbar, \pi, \omega_{Planck}\right)$. One wonders if G is a combined fundamental constant of nature and mathematics.

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