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Interaction Between Matter and Radiation in Special Theory of Relativity

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Abstract: While deriving the necessary condition for working of a laser, Einstein coefficients are introduced for various interactions between matter and radiation. In this article, this derivation is discussed within the frame of special theory of relativity. Whether a study of spectral analysis can confirm validity of relativistic temperature transformations or not is also discussed.

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1. Introduction

The three interactions between matter and radiation viz., spontaneous emission, stimulated absorption and stimulated emission are discussed while deriving the necessary condition for working of a laser, which is population inversion. In the first step of the derivation, for N number of atoms , rate of emission and rate of absorption are expressed using Einstein coefficients for the interactions mentioned above. [1] Let the collection of atoms be in frame S' moving along X-axis at a relativistic speed v in frame S. In the following sections, relation between Einstein coefficients, temperature, frequency of radiation measured by the observers in inertial frames are discussed.

2. Einstein Coefficients in Inertial Frames

In frame S', among N atoms, let n_1 number of atoms be in energy level 1 of energy E_1' and n_2 be in energy level 2 of energy E_2'. The energy of the photon emitted when an atom goes from level 2 to level 1 is $h\nu'$. In S' frame, if Einstein coefficients are $A_{21}', B_{12}', B_{21}'$ and spectral density is $\rho'(\nu')$, then

$$\text{rate of emission} = n_2(A_{21}' + B_{21}'\rho'(\nu')) \tag{1}$$

$$\text{rate of absorption} = n_1 B_{12}'\rho'(\nu') \tag{2}$$

The comparison of the spectral density at thermal equilibrium of N atoms to that of a black-body at temperature T' yields

$$e^{h\nu'/kT'} - 1 = \frac{A_{21}'}{B_{21}'\rho'(\nu')} \tag{3}$$

In frame S, the energy of level 1 is E_1 and of level 2 is E_2 and the energy of photon emitted in the transition from level 2 to level 1 is $h\nu$. For Einstein coefficients A_{21}, B_{12}, B_{21} and spectral density $\rho(\nu)$, a similar treatment will yield

$$e^{h\nu/kT} - 1 = \frac{A_{21}}{B_{21}\rho(\nu)} , \tag{4}$$

where T is temperature of collection of N atoms measured in frame S.

Since A_{21}' is probability per atom per unit time that an atom will undergo spontaneous emission, it can be written as $A_{21}' = p'/t'$, where p' is probability per atom of spontaneous emission in S'. In frame S, the Einstein coefficient can be similarly expressed as $A_{21} = p/t$, where p is probability per atom of spontaneous emission in S. Since time dilation states that $t = t'\gamma$, where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, Einstein coefficient for spontaneous emission in frame S' can be written as

$$A'_{21} = \frac{p'\gamma}{t} \,. \tag{5}$$

In principle, probability per atom of transition of spontaneous emission should not be dependent on observer. Hence, for $p = p'$, the Einstein coefficients in inertial frames are

$$A'_{21} = A_{21}\gamma \,. \tag{6}$$

Similar treatment yields stimulated emission transition rate per atom as

$$B'_{21}\rho'(\nu') = B_{21}\rho(\nu)\gamma \,, \tag{7}$$

and stimulated absorption transition rate per atom as

$$B'_{12}\rho'(\nu') = B_{12}\rho(\nu)\gamma \,. \tag{8}$$

Thus, the transition rate per atom is larger in frame S' for spontaneous emission, stimulated emission and stimulated absorption.

3. Relativistic Temperature Transformations

The ratio of transition rate per atom of spontaneous emission and stimulated emission in frame S' and S are equal. Thus, equations (3), (4), (6), and (7) yield

$$\frac{\nu'}{\nu} = \frac{T'}{T} \,. \tag{9}$$

The Plank-Einstein temperature transformation is $T = T'\gamma$. [2–4] This yields

$$\nu = \nu'\gamma \,, \tag{10}$$

which implies that the frequency measured in S frame is larger than frequency measured in frameS'.

According to relativistic Doppler effect,

$$\nu = \nu'\gamma\left(1 + \frac{v}{c}\cos\theta\right) ., \tag{11}$$

where $v\cos\theta$ is component of velocity of a source along the line of sight of observer. For an approaching source, from eq. (10) and eq. (11) with $\theta = 0^0$, it can be concluded that observed frequency due to Doppler effect is larger than observed frequency resulting from relativistic temperature. For a source, that is moving at an angle of 90^0, the frequency due to Doppler effect and relativistic temperature are equal. For a receding source source ($\theta = 180^0$), the observed frequency due to Doppler effect is smaller than observed frequency resulting from relativistic temperature. Thus, for observed frequency, the Doppler effect is dominant over the relativistic temperature effect.

As Ott relativistic transformation of temperature is $T' = T\gamma$ [5], eq. (9) yields

$$\nu = \frac{\nu'}{\gamma} \,. \tag{12}$$

As frequency predicted by Ott transformation in eq. (12) is smaller than frequency predicted by Einstein-Plank (10), the Doppler effect will be always dominate the relativistic temperature effect for Ott transformation. As Landsberg transformation of temperature states $T = T'$ [6], the question of

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relativistic temperature effect does not arise. For partition function based transformation $T' = T\gamma^{1/3}$ [7],

$$\nu = \frac{\nu'}{\gamma^{1/3}} , \tag{13}$$

which is smaller than frequency predicted by Ott transformation.

Thus, the experimental confirmation of temperature transformation cannot be obtained by studying the shift in frequency.

4. Conclusion

As a result of time dilation, the probability of transition per atom per unit time that it will undergo spontaneous emission, stimulated emission or stimulated absorption in S' frame is larger than that of measured in frame S. The derivation of population inversion in inertial frames S' and S leads a relation between frequency of radiation and temperature in these frames. Various relativistic temperature transformations yield observable frequency of radiation in frame S either lower or equal to the one predicted by Doppler effect. Hence it can be concluded that spectral study is not able to confirm relativistic temperature transformation.

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