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Article

Entropic Cosmology with New Entropy

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Abstract

We study entropic cosmology with new nonadditive entropy of the apparent horizon $S_K = S_{BH}/(1 + \sigma S_{BH})^2$ where S_{BH} is the Bekenstein–Hawking entropy. Making use of the thermodynamics-gravity correspondence a new model of dark energy is analyzed. We obtain the generalised Friedmann equations for Friedmann–Lemaître–Robertson–Walker spatially flat universe with the barotropic matter fluid possessing the equation of state (EoS) $w = p/\rho$. The dark energy pressure p_D , the energy density ρ_D , and the deceleration parameter q of the universe are computed. At some model parameters w and σ we obtain the normalized density parameter of the matter $\Omega_m \approx 0.315$ and the deceleration parameter $q_0 \approx -0.535$ for the current epoch that are in agreement with the Planck data. We show that our model has the phantom divide for the EoS of dark energy. The model under consideration, with the help of the thermodynamics-gravity correspondence, describes the universe inflation and the late time of the universe acceleration. It is proven that the suggested entropic cosmology being equivalent to cosmology within the teleparallel gravity with the function $F(T)$ which we computed.

Keywords: entropic cosmology; entropy; dark energy; barotropic matter; deceleration parameter; teleparallel gravity

1. Introduction

The current universe acceleration can be explained by introducing the cosmological constant Λ in the Einstein equation which plays the role of the dark energy. Such standard cosmology leads to large scale homogeneity and isotropy. The latest observational data show the deviation from such scenario with the constant cosmological constant with better describing the acceleration of universe by dynamical cosmological constant (the dark energy). There are discrepancies with the Λ CDM model called cosmic tensions [1–4]. At the same time the thermodynamics of the apparent horizon can describe the acceleration of the universe [5–16] due to a correspondence between gravity and thermodynamics. It was shown [17,18] that the entropy is expressed through the horizon area and the temperature is connected with the surface gravity [19–22]. The Friedmann equations can be found by the virtue of the first law of apparent horizon thermodynamics because the apparent horizon (or the Hubble horizon) of the FLRW space-time represents a thermodynamic system [5,7,22–24]. Nonadditive entropies [25–32], due to the long-range nature of gravity, were considered and they lead to the generalized Friedmann equations. Some holographic dark energy models were studied in [33–38]. Here, we propose new apparent horizon non-extensive entropy $S_K = S_{BH}/(1 + \sigma S_{BH})^2$ where S_{BH} being the Bekenstein–Hawking entropy. We use the equation of state (EoS) for barotropic perfect fluid, $p = w\rho$, where p is the matter pressure and ρ is the energy density of the matter. The modified Friedmann equations, dark energy density and pressure were obtained. We show that the early and late time universe acceleration take place. The normalized density parameter of the matter $\Omega_m \approx 0.315$ and the deceleration parameter $q_0 \approx -0.535$ for the current epoch are found for some model parameters which are in agreement with the Planck data. It is shown that our model describes the phantom divide for the EoS of dark energy. The entropic cosmology considered is equivalent to cosmology based on the teleparallel gravity with the function $F(T)$ evaluated.

We use units with $\hbar = c = k_B = 1$.

2. The Entropy

Let us introduce new entropy

$$S_K = - \sum_{i=1}^W \frac{p_i \ln p_i}{(1 - \sigma \ln p_i)^2}, \quad (1)$$

with W being a number of states and each state has a probability p_i with the distribution $\{p_i\}$ and σ is a free dimensionless parameter. The summation in Equation (1) is given by all possible microstates of the system. After putting $\sigma = 0$ in Equation (1), it is converted into the Gibbs entropy

$$S_G = - \sum_{i=1}^W p_i \ln(p_i). \quad (2)$$

We suppose that each microstate is populated with equal probability, and therefore $1/p_i = W$ ($i = 1, 2, \dots, W$) and Equation (2) is converted into the Boltzmann entropy $S_B = \ln(W)$. Taking into account that $1/p_i = W$, one obtains from Equation (1) the equation

$$S_K = \frac{\ln(W)}{(1 + \sigma \ln(W))^2}. \quad (3)$$

It is worth noting that the Bekenstein–Hawking entropy reads $S_{BH} = \ln(W)$ and from Equation (3) we obtain

$$S_K = \frac{S_{BH}}{(1 + \sigma S_{BH})^2}. \quad (4)$$

At $\sigma = 0$ the entropy (4) becomes the Bekenstein–Hawking entropy $S_{BH} = A/(4G)$, where $A = 4\pi R_h^2$ is the horizon area. For two probabilistically independent systems A and B , one has $p_{ij}^{B+C} = p_i^B p_j^C$ and we obtain nonadditive entropy $S_K(B+C) \neq S_K(B) + S_K(C)$. It should be noted that the Bekenstein–Hawking entropy S_{BH} as well as other entropies [25–28] has a singularity at $H = 0$, but the entropy S_K does not possess a singularity, $S_K = 0$ at $H = 0$. For small σ the entropy becomes

$$S_K = S_{BH} - 2\sigma S_{BH}^2 + 3\sigma^2 S_{BH}^3 + O(\sigma^3). \quad (5)$$

Thus, corrections to the entropy S_{BH} decrease the S_K entropy and the same effect takes place in quantum gravity [39]. We suppose that entropy (4) includes the quantum gravity corrections to S_{BH} .

3. The Thermodynamics of the Apparent Horizon

The FLRW spatially flat universe possesses the metric

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega_2^2), \quad (6)$$

where $a(t)$ is a scale factor and $d\Omega_2^2$ represents the 2-dimensional unit sphere line element. The apparent horizon radius ($R_h = a(t)r$) is given by

$$R_h = \frac{1}{H}, \quad (7)$$

with the Hubble parameter of the universe being $H = \dot{a}(t)/a(t)$ and $\dot{a}(t) = \partial a / \partial t$. It is worth noting that the apparent horizon corresponds to the Hubble horizon. The first law of apparent horizon thermodynamics is

$$dE = -T_h dS_h + W dV_h, \quad (8)$$

where W is the work density, $E = \rho V_h = (4\pi/3)\rho R_h^3$ and ρ is the energy density of a matter. The work density is given by [22–24]

$$W = -\frac{1}{2} \text{Tr}(T^{\mu\nu}) = \frac{1}{2}(\rho - p), \quad (9)$$

the p is the pressure of the matter and the apparent horizon temperature is

$$T_h = \frac{H}{2\pi} \left| 1 + \frac{\dot{H}}{2H^2} \right|. \quad (10)$$

From first law of apparent horizon thermodynamics (8), and using Equations (9) and (10) we obtain

$$\frac{H}{2\pi} \left| 1 + \frac{\dot{H}}{2H^2} \right| dS_h = -\frac{4\pi}{3H^3} d\rho + \frac{2\pi(\rho + p)}{H^4} dH. \quad (11)$$

Making use of the continuity equation

$$\dot{\rho} = -3H(\rho + p), \quad (12)$$

and taking into account Equation (11), one finds

$$\frac{H}{2\pi} \left| 1 + \frac{\dot{H}}{2H^2} \right| \dot{S}_h = -\frac{4\pi\dot{\rho}}{3H^3} \left(1 + \frac{\dot{H}}{2H^2} \right). \quad (13)$$

4. The Friedmann Equations

From Equations (12) and (13) and using the inequality $1 + \dot{H}/(2H^2) > 0$, we obtain

$$\frac{H}{2\pi} \dot{S}_h = \frac{4\pi(\rho + p)}{H^2}. \quad (14)$$

We utilize the entropy

$$S_h = S_K = \frac{S_{BH}}{(1 + \gamma S_{BH})^2}, \quad (15)$$

where $S_{BH} = \pi R_h^2/G = \pi/(GH^2)$. Then from Equations (14) and (15) we obtain the generalized Friedmann equation

$$\frac{\dot{H}H^4(H^2 - b)}{(H^2 + b)^3} = -4\pi G(\rho + p), \quad (16)$$

where $b = \sigma\pi/G$. After integrating Equation (16) and with the help of Equation (12) one finds the second generalized Friedmann equation

$$H^2 - \frac{b^2(4b + 5H^2)}{(H^2 + b)^2} - 4b \ln\left(\frac{H^2 + b}{b}\right) = \frac{8\pi G}{3}\rho, \quad (17)$$

with the integration constant $C = 4b \ln(b)$. When $\sigma = 0$ ($b = 0$) Equation (17) is converted into the Friedmann equation of general relativity. Equation (17) can be represented as

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda_{eff}}{3}, \quad (18)$$

where

$$\Lambda_{eff} = \frac{3b^2(4b + 5H^2)}{(H^2 + b)^2} + 12b \ln\left(\frac{H^2 + b}{b}\right), \quad (19)$$

and Λ_{eff} can be treated as the dynamical cosmological constant. We plotted the Λ_{eff} versus H at $b = 1, 2, 3$ in Figure 1, where the Planckian units with $G = c = \hbar = k_B = 1$ are used.

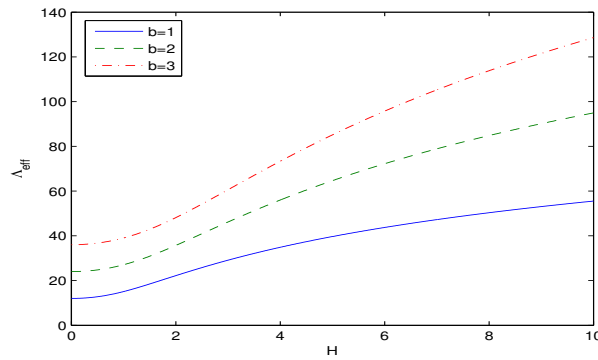


Figure 1. The function Λ_{eff} versus H at $b = \pi\sigma/G = 1, 2, 3$. According to Figure 1 Λ_{eff} increases as b increases. As $H \rightarrow 0$ ($R_h \rightarrow \infty$) one has $\Lambda_{eff} \rightarrow 12b$.

Figure 1 shows that when b increases, at fixed H , the Λ_{eff} also increases and we have $\lim_{H \rightarrow 0} \Lambda_{eff} = 12b$. As a result, at small H (the large apparent horizon radius R_h), the Λ_{eff} is constant. Making use of Equation (18) we obtain the density of dark energy

$$\rho_D = \frac{3b}{8\pi G} \left[\frac{b(4b + 5H^2)}{(H^2 + b)^2} + 4 \ln \left(\frac{H^2 + b}{b} \right) \right]. \quad (20)$$

The normalized density parameters is given by $\Omega_m = \rho/(3M_P^2 H^2)$ and $\Omega_D = \rho_D/(3M_P^2 H^2)$ with $M_P = 1/\sqrt{8\pi G}$ being the reduced Planck mass. Then from Equations (17) and (20) we have $\Omega_m + \Omega_D = 1$. By virtue of Equation (17) one obtains the normalized density for the matter

$$\Omega_m = 1 - \frac{b}{H^2} \left[\frac{b(4b + 5H^2)}{(H^2 + b)^2} + 4 \ln \left(\frac{H^2 + b}{b} \right) \right]. \quad (21)$$

With the dimensionless variable $x = H^2/b$, Equation (21) is converted into

$$\Omega_m = 1 - \frac{4 + 5x}{x(1 + x)^2} - \frac{4}{x} \ln(1 + x). \quad (22)$$

We plotted Ω_m versus x in Figure 2.

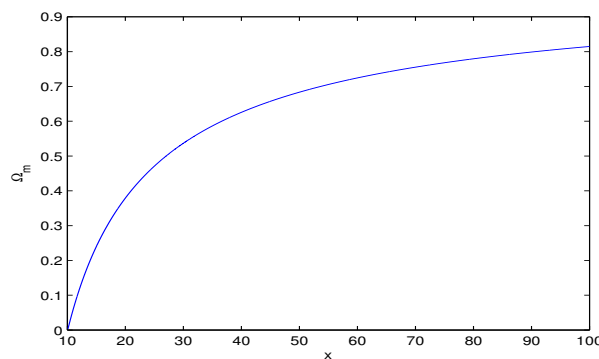


Figure 2. The Ω_m versus x is shown in figure. The Ω_m increases if x increases and when $x \rightarrow \infty$ ($R_h \rightarrow 0$) one has $\Omega_m \rightarrow 1$.

According to Figure 2 as $x \rightarrow \infty$ ($H \rightarrow \infty$, $R_h \rightarrow 0$) we have $\Omega_m \rightarrow 1$. For the current epoch, in accordance with the Planck data, $\Omega_{m0} \approx 0.315$ [40]. At $\Omega_{m0} = 0.315$ the solution to Equation (22) is $x \approx 17.4$ and we find the entropy parameter σ :

$$\sigma = \frac{bG}{\pi} = \frac{GH_0^2}{17.4\pi} \approx 0.018 GH_0^2. \quad (23)$$

Assuming that there is not mutual interaction between various cosmos components, and using Equation (12) for dark energy, we obtain

$$p_D = -\frac{\dot{\rho}_D}{3H} - \rho_D. \quad (24)$$

From Equations (20) and (24) it follows that the pressure corresponding to the dark energy is given by

$$p_D = -\frac{b(b^2 + 3bH^2 + H^4)\dot{H}}{4\pi G(b + H^2)^3} - \frac{3b}{8\pi G} \left[\frac{b(4b + 5H^2)}{(H^2 + b)^2} + 4 \ln\left(\frac{H^2 + b}{b}\right) \right]. \quad (25)$$

By virtue of Equations (16), (17) and (25) we find

$$p_D = \frac{3b(b^2 + 3bH^2 + 4H^4)(1+w)}{8\pi GH^4(H^2 - b)} \left[H^2 - \frac{b^2(4b + 5H^2)}{(H^2 + b)^2} - 4b \ln\left(\frac{H^2 + b}{b}\right) \right] - \frac{3b}{8\pi G} \left[\frac{b(4b + 5H^2)}{(H^2 + b)^2} + 4 \ln\left(\frac{H^2 + b}{b}\right) \right]. \quad (26)$$

Making use of Equations (20) and (26), EoS $w_D = p_D/\rho_D$ for dark energy is

$$w_D = \frac{(b^2 + 3bH^2 + 4H^4)(1+w)}{H^4(H^2 - b)} \left[\frac{H^2(H^2 + b)^2}{b(4b + 5H^2) + 4((H^2 + b)^2 \ln(H^2/b + 1))} - 1 \right] - 1. \quad (27)$$

With the variable $x = H^2/b$, Equation (27) becomes

$$w_D = \frac{(1 + 3x + 4x^2)(1+w)}{x^2(x-1)} \left[\frac{x(x+1)^2}{4 + 5x + 4(x+1)^2 \ln(x+1)} - 1 \right] - 1. \quad (28)$$

The w_D versus x is plotted in Figure 3.

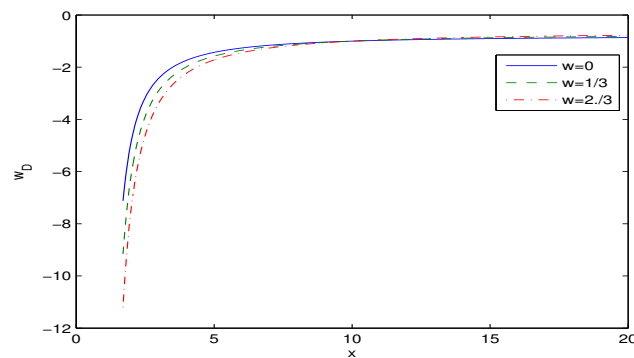


Figure 3. The w_D versus $x = H^2/b$ is depicted at $w = 0, 1/3, 2/3$. When x is large, the EoS parameter w_D possesses the limit $\lim_{x \rightarrow \infty} w_D = -1$.

From Equation (28) we have $\lim_{x \rightarrow \infty} w_D = -1$. As a result, the dark energy EoS $w_D = -1$ for the large Hubble parameter H (the small R_h) that corresponds for the inflation epoch. According to Figure 3 the phantom phase with $w_D < -1$ takes place. Making use of Equation (28) the equation for the phantom divide $w_D = -1$ is given by

$$x(1+x)^2 - 4 - 5x - 4(x+1)^2 \ln(x+1) = 0. \quad (29)$$

The approximate solution to Equation (29) being $x = H^2/b \approx 10.06$ (see also Figure 3) and the model realizes the scenario with the crossing of the phantom divide for the dark energy EoS. Making use of

the redshift $z = a_0/a(t) - 1$ with a_0 being a scale factor at the current time, utilizing the continuity equation (12) and EoS $p = w\rho$, we obtain the density energy of the matter as follows:

$$\rho = \rho_0(1+z)^{3(1+w)}, \quad (30)$$

where ρ_0 is the energy density of the matter at the present time. By virtue of Equations (17) and (30) one finds the equation

$$H^2 - \frac{b^2(4b+5H^2)}{(H^2+b)^2} - 4b \ln\left(\frac{H^2+b}{b}\right) = \frac{8\pi G\rho_0}{3}(1+z)^{3(1+w)}. \quad (31)$$

With the aid of Equation (31) we obtain the redshift

$$z = \left[\frac{3}{8\pi\rho_0 G} \left(H^2 - \frac{b^2(4b+5H^2)}{(H^2+b)^2} - 4b \ln\left(\frac{H^2+b}{b}\right) \right) \right]^{1/(3(1+w))} - 1. \quad (32)$$

By using dimensionless parameters $\bar{H} = H/\sqrt{G\rho_0}$, $\bar{b} = b/(G\rho_0)$, Equation (32) is converted into

$$z = \left[\frac{3}{8\pi} \left(\bar{H}^2 - \frac{\bar{b}^2(4\bar{b}+5\bar{H}^2)}{(\bar{H}^2+\bar{b})^2} - 4\bar{b} \ln\left(\frac{\bar{H}^2+\bar{b}}{\bar{b}}\right) \right) \right]^{1/(3(1+w))} - 1. \quad (33)$$

In Figure 4 we depicted the reduced Hubble parameter $\bar{H}$ versus redshift z .

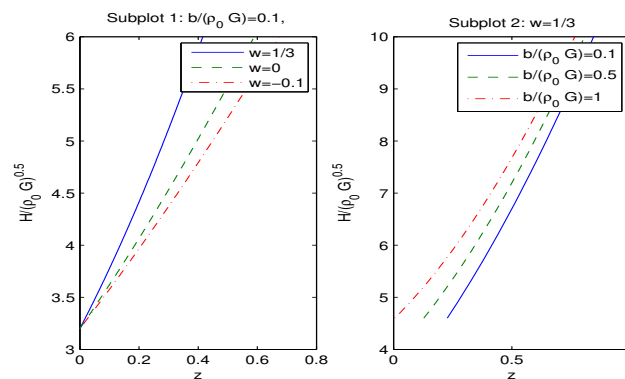


Figure 4. **Left panel:** The $\bar{H}$ versus z with $\bar{b} = 0.1$, $w = 1/3, 0, -0.1$. Figure shows that $\bar{H}$ increases when z increases. When EoS parameter for the matter w increases, at fixed $\bar{H}$, the redshift z decreases. **Right panel:** According to figure, when parameter $\bar{b}$ increases (at fixed z), the reduced Hubble parameter $\bar{H}$ also increases.

Figure 4 shows that when redshift z increases the reduced Hubble parameter $\bar{H}$ also increases. In accordance with the left panel of Figure 4, when parameter w increases (at fixed $\bar{H}$) the redshift z decreases. Right panel of Figure 4 shows that if parameter $\bar{b}$ increases (at fixed z) the reduced Hubble parameter $\bar{H}$ also increases.

Let us consider the deceleration parameter

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2}. \quad (34)$$

When $q < 0$ the acceleration phase of the universe takes place and when $q > 0$ the deceleration phase occurs. From Equations (16), (17) and (34) we obtain

$$q = \frac{4\pi G\rho_0(1+w)(H^2+b)^3}{H^6(H^2-b)}(1+z)^{3(1+w)} - 1. \quad (35)$$

With the help of Equations (31) and (35) one finds

$$q = \frac{3(1+w)(H^2+b)^3}{2H^6(H^2-b)} \left(H^2 - \frac{b^2(4b+5H^2)}{(H^2+b)^2} - 4b \ln\left(\frac{H^2+b}{b}\right) \right) - 1. \quad (36)$$

By using dimensionless variable $x = H^2/b$ Equation (36) takes the form

$$q = \frac{3(1+w)(1+x)^3}{2x^2(x-1)} \left(1 - \frac{4+5x}{x(1+x)^2} - \frac{4}{x} \ln(1+x) \right) - 1. \quad (37)$$

Taking into consideration the value $x = H_0^2/b \approx 17.4$ ($\sigma \approx H_0^2 G / (17.4\pi)$) that reproduces the normalized density of the matter field at the current time $\Omega_{m0} \approx 0.315$ and the deceleration parameter $q_0 \approx -0.535$ [40], one finds the solution to Equation (37) for the EoS parameter of the matter $w \approx -0.2157$.

The deceleration parameter q versus the x is depicted in Figure 5.

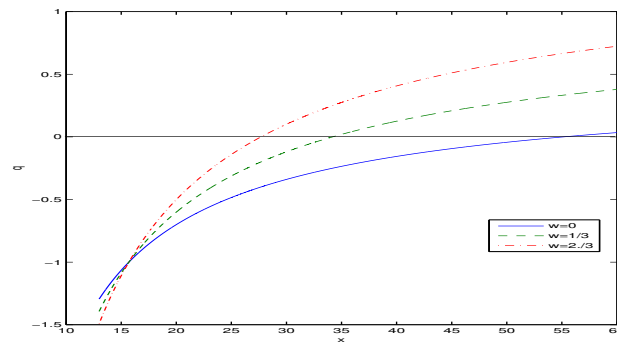


Figure 5. The deceleration parameter q versus x with $w = 0, 1/3, 2/3$. Figure shows that the deceleration parameter q increases when the parameter x increases. If the EoS parameter of the matter w increases (at fixed x) the deceleration parameter q also increases. We observe two phases, universe acceleration at $q < 0$ and deceleration at $q > 0$.

According to Figure 5 two phases takes place, the universe acceleration and deceleration. By virtue of Equation (36), the asymptotic is

$$\lim_{H \rightarrow \infty} q = \frac{3w+1}{2}. \quad (38)$$

According to Equation (38) when $w > -1/3$ ($q > 0$) (at small R_h) the universe decelerates. The universe acceleration at small R_h occurs at $w < -1/3$. From Equation (37) at $q = 0$, the equation for the transition phase is given by

$$w = \frac{2x^3(x-1)}{3(1+x)(x^3+2x^2-4x-4-4(1+x)^2 \ln(1+x))} - 1. \quad (39)$$

The EoS parameter for the matter w versus x is depicted in Figure 6.

According to Figure 6 when $1 > x > 0$ we have $w > -1$. Figure 6 shows that when $x = H^2/b$ increases from $x = 1$ the EoS parameter w decreases from $w = -1$ (de Sitter space) and becomes the phantom space.

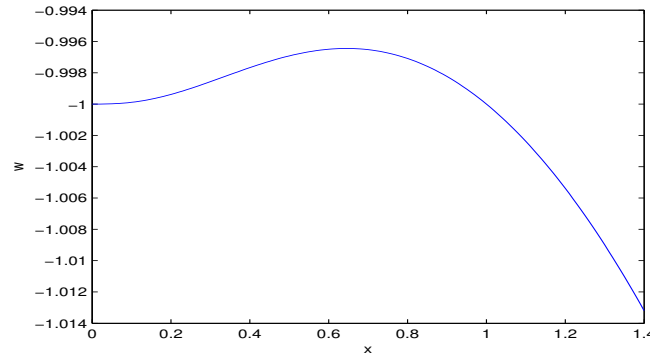


Figure 6. According to figure at $1 > x > 0$ we have $w > -1$. When x increases from $x = 1$, the EoS parameter w decreases from $w = -1$ (the de Sitter space) and w corresponds to the phantom space.

4.1. Inflation

At the phase of inflation the matter density of energy and pressure are zero, $\rho = p = 0$. The dark energy density results the universe to accelerate. From Equation (16) we obtain $\dot{H} = 0$, and during inflation the Hubble parameter is constant. The scale factor becomes $a(t) = a_0 \exp(Ht)$ corresponding to the de Sitter space-time. At the de Sitter stage the eternal inflation occurs. The Hubble parameter at the inflation phase being $H \approx 10^{-3} M_{Pl}$ ($M_{Pl} = 1/\sqrt{8\pi G}$) and $GH^2 \approx 4 \cdot 10^{-8}$. Making use of Equation (17), at $\rho = 0$ with the variable $x = H^2/b$ we find the equation

$$x - \frac{4 + 5x}{(x + 1)^2} - 4 \ln(x + 1) = 0, \quad (40)$$

corresponding to Equation (29) for the phantom divide. The solution to Equation (40) is $x = 10.0559$ with the relation

$$H^2 = 10.0559b. \quad (41)$$

From the Hubble parameter at the inflation phase $H \approx 10^{-3} M_{Pl}$, we obtain parameter b from Equation (41) and the entropy parameter $\sigma = bG/\pi \approx 1.27 \times 10^{-9}$.

5. F(T)-Gravity from Generalized Entropy

The scalar torsion T in teleparallel gravity is a field analogous to the curvature R in the Einstein theory [43,44]. The teleparallel theory of gravity explores the Weitzenböck connection and in the $F(T)$ theory the field equations are the second-order. The torsion field T is defined as [45,46],

$$T = S_\rho^{\mu\nu} T_{\mu\nu}^\rho. \quad (42)$$

The tensors $S_\rho^{\mu\nu}$ and $T_{\mu\nu}^\rho$ are given by

$$\begin{aligned} S_\rho^{\mu\nu} &= \frac{1}{2} \left(K_{\rho}^{\mu\nu} + \delta_\rho^\mu T_{\alpha}^{\alpha\nu} - \delta_\rho^\nu T_{\alpha}^{\alpha\mu} \right), \\ K_{\rho}^{\mu\nu} &= -\frac{1}{2} \left(T_{\rho}^{\mu\nu} - T_{\rho}^{\nu\mu} - T_{\rho}^{\mu\nu} \right), \\ T_{\mu\nu}^\rho &= e_i^\rho \left(\partial_\mu e_\nu^i - \partial_\nu e_\mu^i \right), \end{aligned} \quad (43)$$

with e_ν^i ($i = 0, 1, 2, 3$) being a vierbein field. The metric tensor is $g_{\mu\nu} = \eta_{ij} e_\mu^i e_\nu^j$, where η_{ij} is the flat metric of the tangent spacetime. For FLRW metric (5) $e_\mu^i = \text{diag}(1, a, a, a)$ and the torsion scalar is $T = -6H^2$. The variation of the action with respect to e_μ^i (the Lagrangian is $F(T)$) gives the equation [47]

$$\frac{1}{6} [F(T) - 2TF'(T)]|_{T=-6H^2} = \left(\frac{8\pi G}{3} \right) \rho. \quad (44)$$

From equations (17) and (44) we obtain

$$F'(T) - \frac{F(T)}{2T} = \frac{1}{2} + \frac{18b^2(24b - 5T)}{T(6b - T)^2} + \frac{12b}{T} \ln\left(1 - \frac{T}{6b}\right). \quad (45)$$

Integrating Equation (45) one finds

$$F(T) = T - \frac{3b(48b - 7T)}{T - 6b} - 24b \ln\left(1 - \frac{T}{6b}\right) + (8\sqrt{6} - 7\sqrt{1.5})\sqrt{-bT} \arctan\left(\sqrt{-\frac{T}{6b}}\right), \quad (46)$$

with the integration constant $C = 0$. Because $T = -6H^2 < 0$, we have used the relation $i \tanh^{-1}(ix) = -\arctan(x)$. It should be noted that $F(T)$ in Equation (46) is the real function. The teleparallel gravity models with some functions $F(T)$ were investigated in [48,49] The entropic cosmology with entropy (15) can be considered as a cosmology within the teleparallel gravity with the function (46).

6. Conclusions

We have proposed a new entropy $S_K = S_{BH}/(1 + \gamma S_{BH})^2$ with S_{BH} being the Bekenstein–Hawking entropy and S_K becomes zero when the apparent horizon radius R_h vanishes. The entropic cosmology with our entropy leads to inflation and current acceleration of the universe. The matter barotropic perfect fluid and the spatial flat FLRW universe are assumed. Making use of the first law of apparent horizon thermodynamics we obtained the modified Friedmann equations which have a term corresponding to the density of dark energy. Also, such term can be considered as a dynamical cosmological constant. We assume that there is no interaction between various components of dark energy (the dark energy density and pressure ρ_D and p_D obey ordinary conservation law). The p_D and the dark energy EoS $w_D = p_D/\rho_D$ have been computed with $\lim_{H \rightarrow \infty} w_D = -1$. This shows that at the small apparent horizon radius R_h the inflation of the universe occurs. In our model the universe may have two phases, acceleration and deceleration, because of the dark energy. The deceleration parameter was computed which shows that the acceleration at the current epoch takes place. At the entropy parameter $\sigma \approx 1.27 \times 10^{-9}$ and $w = -0.2157$ the deceleration parameter possesses the value $q_0 \approx -0.535$ and the normalized density parameter is $\Omega_{m0} \approx 0.315$ that are in accordance with the Planck data at the current epoch [40]. In addition, our model has the phantom divide for the EoS of dark energy. We have showed that entropic cosmology with our entropy can be considered as the cosmology within the teleparallel gravity with the function $F(T)$ obtained.

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