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Negative Viscosity Materials Exploiting the Effect of Negative Mass

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Abstract: We introduce the media (gaseous and liquid) demonstrating the negative viscosity. Consider the vibrated plate (ω is the frequency of vibrations), which is vertically pulled through the gas, built of the core-shell "meta-molecules": the mass of the shell is M, the mass of the core is m, the core is connected to the shell with the ideal spring k. Y is the vertical axis. Vibrating of the vertical plate supplies to the core-shell meta-molecules an excess vertical momentum. If $\omega < \omega_0$ ($\omega_0 = \sqrt{\frac{k}{m}}$), this excess momentum coincides with the positive direction of axis Y; if $\omega > \omega_0$, the excess moment is oriented against axis Y; thus, the effect of negative viscosity becomes possible. No violation of energy conservation is observed; the energy is supplied to the system by the external source vibrating the plate. The effect of the negative viscosity is also possible in liquids. Frequency dependence of the viscosity is addressed. Asymptotic expressions are derived for the frequency-dependent viscosity.

Keywords: metamaterials; negative viscosity; core-shell systems; frequency; Newtonian liquid; drag force

1. Introduction

Development of optical and acoustic metamaterials opened new horizons in the materials science [1–3]. Metamaterials supplied to engineers media with the properties, which do not exist in the natural materials and alloys. Metamaterials are usually defined as artificial, periodic structures suited for control/altering mechanic [2] or electromagnetic [1,4] properties of materials to obtain prescribed features that are not observed naturally. The prefix "μετά" (a Greek word meaning 'beyond, after') indicates that the properties of the material are outside of the range of properties observed in nature [2–14]. The structural units of the metamaterials ('meta-molecules") are usually macroscopic [2–14]. These macroscopic meta-molecules can be tailored and dynamically controlled in shape, properties, chirality and size [2–13]. The lattice constant and inter-cell interaction can be artificially modified, and useful "defects" can be designed and introduced precisely at desired locations.

Numerous amazing, paradoxical and useful effects such the negative refraction of light [4–9] and negative effective mass became possible with the meta-materials [15–25]. The effect of the negative effective mass (also called "negative inertia" [25]") may be achieved under exploiting the plasma oscillations of the electron gas in metals [21,22]. The negative effective mass materials were already successfully demonstrated experimentally by embedding soft silicon rubber coated heavy spheres in thermosetting polymer, acting as the local mechanical resonators [24]. Development of metamaterials enabled unique engineering applications, including seismic devices, optical and acoustic camouflage, acoustic and thermal cloaking/absorbers [8–10,26–29]. Metamaterials enable

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achievement of controlled and exotic mechanical properties through the rational design of their microstructures. It was demonstrated recently that when compared with conventional schemes, metamaterials provide an unprecedented potential for governing the mass and energy transport processes [3,12]. We report in this communication that the paradoxical control of the momentum transport resulting in the negative viscosity, emerging from the effect of the negative mass, is possible. A "negative-viscosity" effect in a magnetic fluid was already reported [30,31]. In a Poiseuille flow, a constant magnetic field balances vorticity and impedes the rotation of individual magnetic particles embedded into the liquid [30,31]. Conversely, an alternating magnetic field favors the rotation of the particle [30,31]. The magnetic energy is transformed into the angular momentum of the magnetic particles, which is converted into a hydrodynamic motion of the fluid [30,31]. The entire effect results in a decrease of the total viscosity [30]. Negative viscosity of ferrofluids under alternating magnetic field was reported by other groups [32,33]. Negative effective viscosity also emerges in a large-scale flow maintained by a small-scale periodic force field [34]. If the field is essentially anisotropic, the system of small-scale eddies generated in the liquid is unstable to long-wave disturbances, i.e., the effective viscosity of the corresponding large-scale flow is negative [34]. If the applied periodic field is isotropic, the long-wave instability disappears; thus, the effective viscosity is positive, as reported by Sivashinsky and Yakhot [34]. We introduce the additional possible mechanism of negative viscosity, exploiting the effect of negative mass.

2. Results

Consider ideal gas built of core-shell particles/units, shown in Figure 1. The mass of the shell is M, the mass of the core is m. Particles participate in the random thermal motion (we will restrict the randomness of this motion, below). The core particle m is connected to the spherical shell M with the ideal Hookean, massless spring k. The aforementioned core-shell system (considered in our approach as "meta-molecules") gives rise to the effect of the negative mass. When the core-shell unit is exerted to the external harmonic/sinusoidal force $\vec{F}(t) = \vec{F}_0 e^{j\omega t}$ it may be superseded with an equivalent effective mass denoted m_{eff}, supplied with Eq. 1 (for the details of derivation of Eq. 1 see [15–18]:

$$m_{eff} = M + \frac{m\omega_0^2}{\omega_0^2 - \omega^2}, \tag{1}$$

where $\omega_0 = \sqrt{\frac{k}{m}}$. It is easily recognized from Eq. 1 that, when the frequency of the external harmonic force ω approaches ω_0 from above the effective mass m_{eff}, defined by Eq. (1) will be negative. The effect was also called the "negative inertia" [25]. Of course, actually, neither "negative mas" nor "negative inertia" does not exist, and this should be clearly understood. What does the effect of negative mass mean? It means that the under certain frequencies the entire core-shell system, seen as integral unit moves in the direction opposite to the direction of the external force (in other words, the core-shell unit moves against the applied harmonic force) [15–18].

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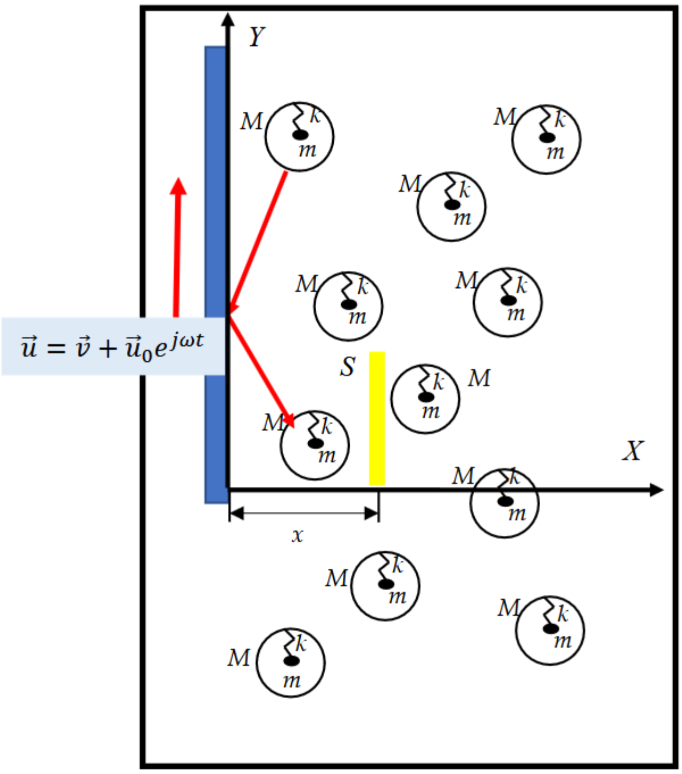

Figure 1. Plate is vertically pulled through the ideal gas of the core-shell system with the velocity $\vec{u} = \vec{v} + \vec{u}_0 e^{j\omega t}$; $|\vec{v} = const| \ll |\vec{u}_0 = const|$. Axis X is normal to the plate. Transverse transfer of momentum through the cross-section S (shown with a yellow rectangle) is considered.

Consider the vertical plate pulled vertically upward (along axis Y) through the gas of core-shell particles (i.e. meta-molecules) with the velocity $\vec{u} = \vec{v} + \vec{u}_0 e^{j\omega t}$; $|\vec{v} = const| \ll |\vec{u}_0 = const|$, as shown in **Figure 1.** For a sake of simplicity we assume the vibration of the core mass m occures along axis Y; i.e. the core-shell units are oriented vertically (the springs remain oriented vertically). Vibrating vertical plate supplies to the core-shell units an excess vertical momentum. If $\omega < \omega_0$ this excess momentum coincides with the positive direction of axis Y; however, if $\omega > \omega_0$, the excess moment is oriented against axis Y ($|\vec{v}| \ll |\vec{u}_0|$ is adopted); thus, the effect of negative viscosity becomes possible. Consider the case when $\omega > \omega_0$ is true. The vibrated plate supplies to the core-shell meta-molecules the downward momentum; thus, the gas of core-shell units supplies to the plate the excess upward moment. This effect may be described as the negative viscosity of the gas built of the core-shell elements.

Address the ideal gas of the core-shell particles possesing the number concentration n. Particles partcipate in the random thermal motion with a 3D averaged velocity c (the motion of the particles is not completely random: the vertical orientation of the springs remains untouched). Consider the transfer of excess vertical moment Δp supplied by the vertically pulled plate ($\vec{u} = \vec{v} + \vec{u}_0 e^{j\omega t}$) to the gas, built from the core-shell particles, through the vertical area S, located at the distance $\boldsymbol{x}$ from the plate (see **Figure 1**) over the time interval Δt. This transfer equals

$$\Delta p = m_{eff} v_1 \Delta N_+ - m_{eff} v_2 \Delta N_- = \frac{1}{6} m_{eff} n S c \Delta t (v_1 - v_2) = \frac{2\lambda}{6} m_{eff} n S c \Delta t \frac{v_1 - v_2}{2\lambda}, \quad (2)$$

where m_{eff} is supplied by Eq. 1, v_1 and v_2 are the velocities of excess, oriented, vertical motion of the core-shell-units before and after passing area S, ΔN_+ and ΔN_- are the number of core-shell units, passing the area along and against the positive direction of axis x correspondingly, λ is a mean free path of core shell-units. The Second Newton Law yields for the viscous stress Eq. (3):

$$\tau_{visc} = \frac{\Delta p}{S \Delta t} = -\frac{1}{3} m_{eff} \lambda n c \frac{\partial v}{\partial x} = -\eta_{eff} \frac{\partial v}{\partial x}, \quad (3)$$

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where $\eta_{eff} = \frac{1}{3} m_{eff} \lambda nc$ is the effective viscosity of the core-shell units built gas. Considering Eq. 1 and $c = \sqrt{\frac{8k_BT}{\pi(m+M)}}$ (T is the temperature of the core-shell units built gas) yields for the effective viscosity:

$$(\omega) = \frac{1}{3}\left(M + \frac{m\omega_0^2}{\omega_0^2-\omega^2}\right)\lambda n \sqrt{\frac{8\pi k_BT}{\pi(m+M)}} \quad (4)$$

The effective viscosity η_{eff} is negative when the frequency of the external harmonic force vibrating the plate ω approaches ω_0 from above. It should be emphasized that the effective viscosity supplied by Eq. (4) is frequency dependent. Such a behavior is typical for non-Newtonian liquids.[35,36]

Now consider one more possibility of the negative effective viscosity emerging in the situation illustrated with Figure 2. Consider the core-shell system which is oscillated vertically within the ideal gas with the velocity with the velocity $\vec{u} = \vec{u}_0 e^{j\omega t}$; $|\vec{u}_0| = const$; the mass of the shell is $\widetilde{M}$, the mass of the core is $\widetilde{m}$; the core mass $\widetilde{m}$ is connected to the shell with two ideal, massless, Hookean springs $\frac{k}{2}$. The core-shell is embedded into the ideal gas of the particles with the mass of μ and concentration n. The particles move with the averaged velocity $c = \sqrt{\frac{8k_BT}{\mu m}}$. Obviously the transverse transport of momentum occurs with the classical viscosity, given by $\eta = \frac{1}{3}\mu\lambda nc$, where λ is a mean free path of the particles μ. So this case looks trivial, and does not give rise to the phenomenon of the negative viscosity. However, the actual situation is more complicated. Consider the drag force F_{drag}; we address the situation when the Reynolds number is much smaller than unity ($Re = \frac{\rho u_0 a}{\eta} \ll 1$, where ρ is the density of the gas and a is the characteristic dimension of the core-shell system). Thus, the drag force may be expressed as [35]:

$$|F_{drag}| = \varrho a \eta \dot{y}, \quad (5)$$

where ϱ is the dimensionless coefficient; axis Y is vertical (see Figure 2). The second Newton Law yields:

$$m_{eff}\ddot{y} = -\varrho a \eta \dot{y}, \quad (6)$$

where $m_{eff} = \widetilde{M} + \frac{\widetilde{m}\omega_0^2}{\omega_0^2-\omega^2}$; $\omega_0 = \sqrt{\frac{k}{\widetilde{m}}}$. Considering $\dot{y}(t) = |u_0|e^{j\omega t}$ yields:

$$\left(\widetilde{M} + \frac{\widetilde{m}\omega_0^2}{\omega_0^2-\omega^2}\right) j\omega = -\varrho a \eta \quad (7)$$

Eq. 7 enables introducing of the effective complex viscosity with Eq 8:

$$\eta_{eff} = \frac{1}{\varrho a}\left(\widetilde{M} + \frac{\widetilde{m}\omega_0^2}{\omega_0^2-\omega^2}\right) j\omega \quad (8)$$

Thus, we obtain:

$$\mathrm{Im}\eta_{eff} = \frac{1}{\varrho a}\left(\widetilde{M} + \frac{\widetilde{m}\omega_0^2}{\omega_0^2-\omega^2}\right)\omega \quad (9)$$

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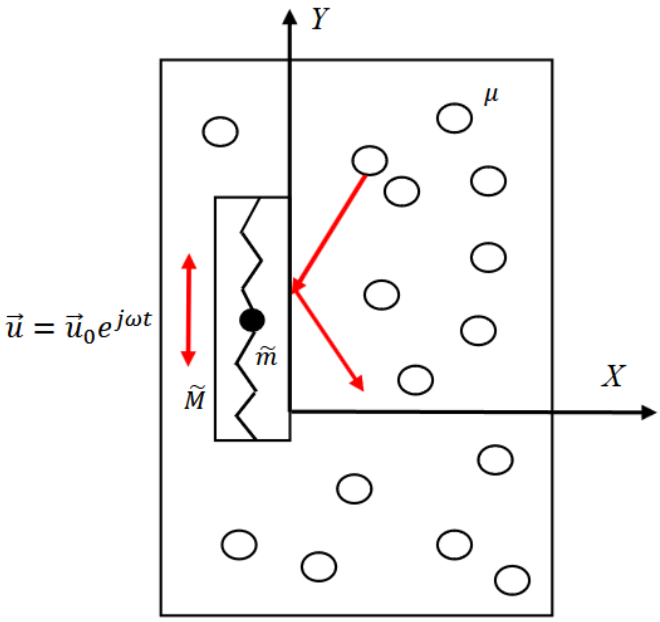

Figure 2. Core-shell system is oscillated vertically through the ideal gas of the particles with mass μ.

It is instructive to build the graph $\mathrm{Im}\eta_{eff}(\omega)$; such a graph is schematically shown in Figure 3. It is easy to demonstrate that the function $\mathrm{Im}\eta_{eff}(\omega)$ has no extremal points. It is also easy to see that $\mathrm{Im}\eta_{eff}(\omega) < 0$ takes place when ω approaches ω_0 from above (see Figure 3).

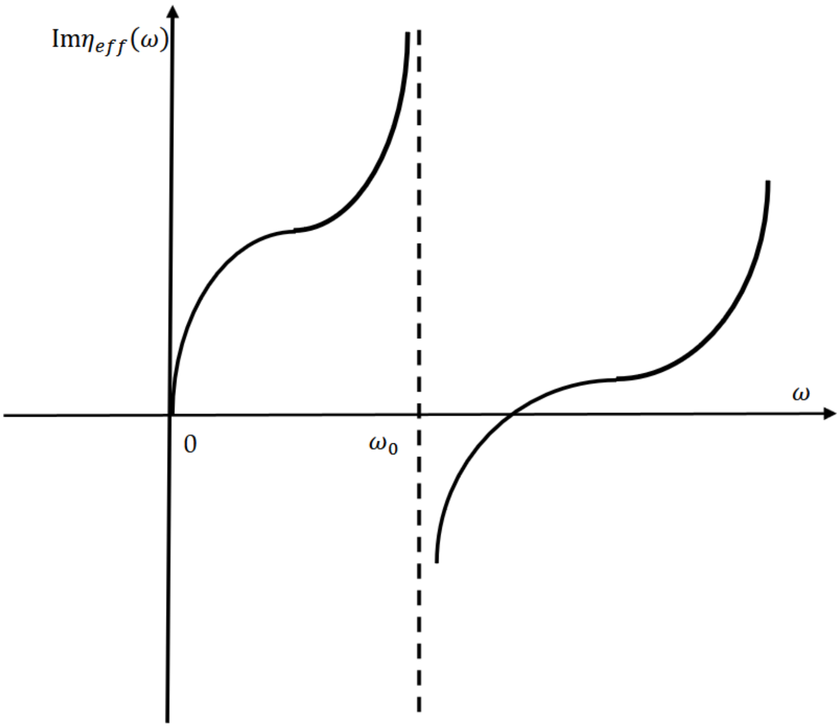

Figure 3. The dependence $\mathrm{Im}\eta_{eff}(\omega)$ established for the system shown in **Figure 2** is depicted (see Eq. (9)).

The same approach may be applied for the core-shell system (the mass of the shell is $\tilde{M}$, the mass of the core is $\tilde{m}$, the core is connected to shell with the ideal spring k) embedded into the Newtonian liquid η, depicted in Figure 4. The core-shell system is oscillated horizontally with the velocity $\vec{u}(t) = \vec{u}_0 e^{j\omega t}$; $|\vec{u}_0| = const$. In this case the second Newton Law yields (we assume that the Reynolds number is much smaller than unity):

$$m_{eff}\ddot{x} = -6\pi\eta R\dot{x}, \qquad (10)$$

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where $F_{drag} = -6\pi\eta R\dot{x}$ is the Stokes drag force, $m_{eff} = \widetilde{M} + \frac{\widetilde{m}\omega_0^2}{\omega_0^2-\omega^2}$; $\omega_0 = \sqrt{\frac{k}{\widetilde{m}}}$. Considering $\dot{x}(t) = |u_0|e^{j\omega t}$ gives rise to:

$$\left(\widetilde{M} + \frac{\widetilde{m}\omega_0^2}{\omega_0^2-\omega^2}\right) j\omega = -6\pi\eta R \quad (11)$$

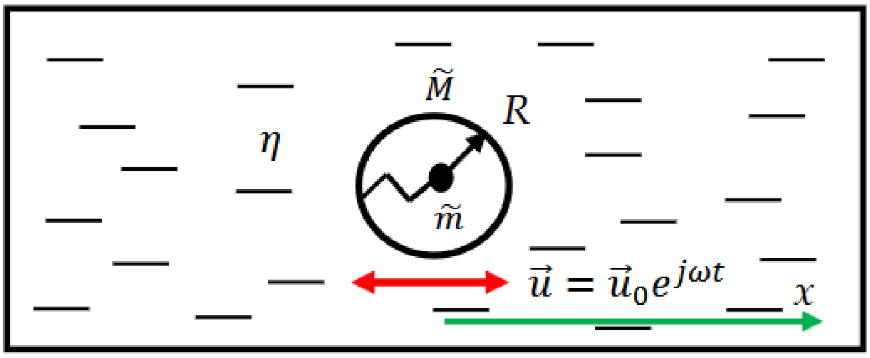

Figure 4. Core-shell system embedded into the liquid with viscosity η. The mass of the shell is $\widetilde{M}$, the mass of the core is $\widetilde{m}$, the core is connected to shell with the ideal spring k. The core-shell system is oscillated horizontally within the liquid η with the velocity with the velocity $\vec{u} = \vec{u}_0 e^{j\omega t}$; $|\vec{u}_0| = const$.

Thus, we obtain for the effective viscosity:

$$\eta_{eff} = \frac{1}{6\pi R}\left(\widetilde{M} + \frac{\widetilde{m}\omega_0^2}{\omega_0^2-\omega^2}\right) j\omega \quad (12)$$

The imaginary part of the effective viscosity is supplied with Eq. 13:

$$\mathrm{Im}\eta_{eff}(\omega) = \frac{1}{6\pi R}\left(\widetilde{M} + \frac{\widetilde{m}\omega_0^2}{\omega_0^2-\omega^2}\right)\omega \quad (13)$$

Plot $\mathrm{Im}\eta_{eff}(\omega)$ coincides qualitatively with that, shown in Figure 4. It should be emphasized, that the expressions (8) and (12) supplying the effective viscosity of the media have nothing to do with the true viscosities of the gas or liquid. Indeed, the viscosity of the media does not depend on the mass of the body, which moves through these media. However, these expressions are useful for understanding the frequency dependence of the drag force, illustrated with Figure 4. It is instructive to supply the asymptotic expressions for $\mathrm{Im}\eta_{eff}(\omega)$. In the limit of high frequencies we calculate:

$$\lim_{\omega\to\infty} \mathrm{Im}\eta_{eff}(\omega) = \frac{\widetilde{M}\omega}{6\pi R} \quad (14)$$

In the limit of low frequencies, we obtain, in turn:

$$\lim_{\omega\to 0} \mathrm{Im}\eta_{eff}(\omega) = \frac{(\widetilde{M}+\widetilde{m})\omega}{6\pi R}$$

3. Discussion

Contra-intuitive, paradoxical negative viscosity effect was already discussed in the literature. This effect occurs in in a Poiseuille flow of a ferrofluid, when a constant magnetic field balances vorticity and impedes the rotation of individual magnetic particles embedded into the liquid [30,31]. Negative viscosity emerges also in ferrofluids under alternating magnetic field [32,33]. It is also possible in a large-scale flow maintained by a small-scale periodic force field [34]. We introduce the novel mechanism of negative viscosity, which occurs in the media built of the core-shell systems, introduced in [15,16,18,19]. Consider the vibrated plate, which is pulled through the gas, built of the core-shell systems, seen as meta-molecules The effective viscosity η_{eff} is negative when the frequency of the external harmonic force vibrating the plate ω approaches ω_0 from above. It should be emphasized that the negative viscosity does not violate the conservation of energy; indeed, the plate is vibrated by the external source of energy.

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4. Conclusions

We introduce the metamaterials demonstrating the negative viscosity. The negative mass emerges when the "quasi-molecules" (i.e. "meta-molecules") are represented by the core-shell systems, in which the core mass m is connected to the shell M with the ideal massless spring k. We addressed the following model system: consider the vibrated plate, which is vertically pulled through the gas, built of the "meta-molecules"/core-shell systems. Y is the vertical axis. Vibrating of the vertical plate supplies to the core-shell "meta-molecules" an excess vertical momentum. If $\omega < \omega_0$ ($\omega_0 = \sqrt{\frac{k}{m}}$), this excess vertical component of the momentum coincides with the positive direction of axis Y, if $\omega > \omega_0$, the excess moment is oriented against axis Y. Thus, the effect of negative viscosity becomes possible. It should be stressed, that no violation of energy conservation is observed; the energy is supplied to the system by the external source vibrating the plate. The effect of the negative viscosity is also possible in liquids. We considered the core-shell system embedded in the Newtonian liquid in which the Stokes drag force acts on the vibrated core-shell unit. The entire system demonstrates the effect of the negative viscosity. Frequency dependence of the viscosity is addressed. Low- and high-frequency asymptotic expressions for the frequency dependent viscosity are derived.

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Conflicts of Interest The authors declare no conflicts of interest.

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